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기하학적 NP-hard 문제에 대한 근사 접근법

(An Approximation Scheme For A Geometrical NP-Hard Problem)

김 준 모*

(Joonmo Kim)

요 약

센서네트워크 중에는 센서노드들이 넓은 지역에 걸쳐 정해진 위치에 산재되어야 하는 경우가 있다. 이런 경우 센서노드들을 interconnect하기 위한 최소개수의 연결노드들을 추가하는 문제가 대두되며, 이는 The Minimum number of Steiner Points라는 추상화된 문제로 귀결된다. 이 문제는 NP-hard 문제이므로, 본 논문에서는 문제가 내포하는 기하학적인 성질을 이용하여 연결노드의 최소개수에 근접하는 방안을 제시한다. 센서네트워크에서 노드의 개수를 줄임으로써 네트워크 내부에서 오가는 메시지의 교환량이 대폭 감소하게 된다.

Abstract

In some wireless sensor networks, the sensor nodes are required to be located sparsely at designated positions over a wide area, introducing the problem of adding minimum number of relay nodes to interconnect the sensor nodes. The problem finds its abstract form in literature: the Minimum number of Steiner Points. Since it is known to be NP-hard, this paper proposes an approximation scheme to estimate the minimum number of relay nodes through the properties of the abstract form. Reducing the number of nodes in a sensor network, the amount of data exchange over the net will be far decreased.

Keywords : Sensor networks, interconnection, deployment, optimizations.

I. Introduction

Sensor networks are wireless networks in an ad hoc fashion. There are many sensor nodes in a sensor network, and a sensor node is autonomous device equipped with sensing, processing, and communication capabilities. Sensor nodes are spread over an area to gather information there and send it to a data sink, the user of the information. There are many different purposes of using sensor networks, and accordingly many ways of placing the sensor nodes over the area[1][2]. In a military battle field, the sensor nodes would be dropped from an air plane and then they will form a self-configured network. However, deployment of redundant nodes may cause much more message exchanges than enough, resulting in as much resource consumptions including powers. Another kind of placing sensor nodes is the one for the temperature surveillance over a wide wild area for ecological purposes[3]. In this case, one may need to figure out the locations of some sensor nodes based on the geological information. This paper deals with the case that the sensor nodes are likely to be located sparsely over a wide area, and some sensors may have reasons to be located at specific locations. So, in the general case of measuring the temperatures over an ecological area, the sensor nodes are assumed to be located sparsely over a wide area and each of them can be put at any point.

* 정회원, 단국대학교 전자컴퓨터공학부
(Electronics & Computer Engineering, Dankook University)

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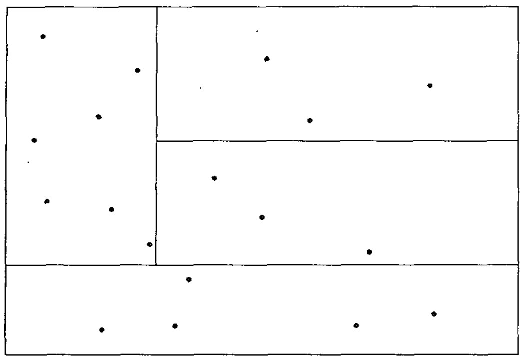
To make such a sparse placement of sensor nodes over a wide area, we should introduce relay nodes so that the sensor nodes should be interconnected when some sensor nodes are located beyond the transmission radius r of their near-by neighbors. Interconnection here means to have close enough distance for each node to more than one of its neighbors to be able to communicate with each other in the wireless network. We may regard that the relay nodes have the same hardware to the sensor nodes, but their function of sensing may be turned off. Assuming that the locations of the sensor nodes are predetermined by given conditions, the interconnection turns out to be the problem of getting the locations of the relay nodes among the sparsely scattered sensor nodes so that the minimum number of relay nodes should be deployed over the area. This interconnection problem finds its abstract form in literature, which is named as the Minimum number of Steiner Points (denoted STP-MSP). Since STP-MSP is *NP*-hard[4], challenges have been made to get the best approximation ratio to the optimal solutions. This paper proposes an approximation scheme that shows: one may build a polynomial-time algorithm that gives the approximation ratio 2 to the optimal number of relay nodes. The algorithm is not presented explicitly. Instead, the scheme involves enough descriptions on how it can be implemented. With the adaptations to the practical conditions that may take place in the real-world sensor networks, the output from the proposed scheme will make the best ever layout toward the optimal deployment.

II. Definitions

The problem definition of *Minimum number of Steiner Points* from [6] is: Given n terminals (mathematical points) on the Euclidean plane R^2 and a positive constant r ($\in \Re$), find a Steiner Tree interconnecting all the *terminals* with the minimum number of Steiner points such that the Euclidean length of each edge $\leq r$[5]. One may see that *STP-MSP* is analogous to the interconnection of sensor networks: the *terminals* are the sensor nodes, the *Steiner points* are the relay nodes, and r is the transmission radius. For the approximation, we are to build a scheme for the loose version of the problem. The condition of finding the minimum number of Steiner points is taken off from *STP-MSP* and so the *loose version* is: Given n *terminals* on the Euclidean plane R^2 and a positive constant r, find a *Steiner tree* that interconnects all *terminals* with *Steiner points* such that the Euclidean length of each edge $\leq r$. Then, one may show to build a scheme for the *loose version* so that the algorithm from the scheme should produce feasible solutions such that the number of the *Steiner points* of every feasible solution $\leq$ twice the optimal number of *Steiner points*. For a problem instance of the *loose version*, i.e. a set of *terminals* on R^2, one may find a set of *Steiner points* as a solution, and draw circles so that each *Steiner point* should be the center of each circle let a Steiner-Cover be the resulting set of circles. There exist many *Steiner-Covers* for a problem instance. When the *Steiner points* of a *Steiner-Cover* are optimal, the *Steiner-Cover* is called a Steiner-Cover$_{opt}$. All circles in this paper have the constant radius r. Let the bounding-box of a set of *terminals* on R^2 be the smallest rectangle enclosing the set. A rectangle is an axis-aligned one that is a partition of the *bounding-box*. The size of the *rectangle* is the length of its longer edge. A line-separator of a *rectangle* is a straight line segment which is parallel to the shorter edge of the *rectangle*: the *line-separator* partitions the *rectangle* into two of at least 1/3 of the area each.

Definition 1. (Tiling: *dividing the given area*) A *tiling* of a *rectangle* R is a binary tree (a hierarchy) of sub-*rectangles* of R. The *rectangle* R is at the root. If the size of $R = 1$ (unit distance), the hierarchy contains nothing else. Otherwise, the root contains a *line-separator* for R, and has two sub-trees that are *tilings* of the two *rectangles* into which the *line-separator* divides R.

Note that *rectangles* at depth d in the *tiling* form



· Sensor node

그림 1. 주어진 영역 분할
Fig. 1 Dividing the given area.

a partition of the root *rectangle*. The set of all *rectangles* at depth *d*+1 is a refinement of the partition obtained by putting a *line-separator* through each depth *d* *rectangle* of *size* > 1.

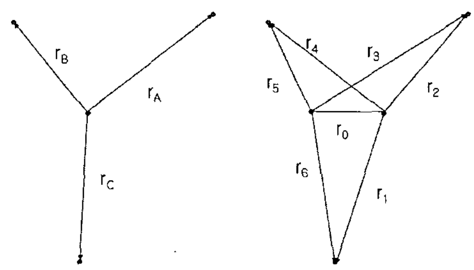
Definition2. (*portals*) A *portal* in a *tiling* is any point that lies on the perimeters of *rectangles* in the *tiling*.

A set of *portals* *P* is called *m-regular* for the *tiling* when there are *m* (an integer) equidistant *portals* on the *line-separator*. Likewise, *p* (an integer) points that lie equidistantly on the perimeter of a *circle* are named *indexed points*.

Definition3.(*m-light Steiner-Cover*) Let $m \in Z^{+}$,, *S* be a *tiling* of the *bounding-box,* and *P* be an *m-regular* set of portals on this tiling. Then, a *Steiner-Cover* in which each circle crosses with at least one *portal* in *P* at an *indexed point* is *m-light* with respect to *S*.

Ⅲ. Shorter radius

Given a set of *terminals* on R^2, there are infinitely many candidate solutions of the problem. To form a feasible scheme we are to set up a frame (partitions with *portal*s) over the set of *terminals* so that there may exist only polynomially many feasible solutions over the frame. Note that a solution is a *Steiner-*



· Sensor node

그림 2. 두 개의 센서노드에 의한 반지름 축소
Fig. 2. Shorter Radius by two Sensor nodes.

Cover for the given *terminals*. One may show that there is a *subset* of the polynomially many feasible solutions such that each solution in it is a *Steiner-Cover* that has twice as many circles as a *Steiner-Cover*$_{opt}$ does let *m-light Steiner-Cover*$_{2*opt}$ be a solution in the *subset*. A polynomial time Dynamic Programming (*DP*) can be designed to identify all the polynomially many feasible solutions over the frame, and the one with the minimum number of circles will be chosen as the *desired solution*. Since all the *m-light Steiner-Cover*$_{2*opt}$'s must also be checked by the *DP*, # (circles in the *desired solution*) ≤ # (circles in *m-light Steiner-Cover*$_{2*opt}$), resulting in the ratio 2 to the optimal solutions. The *proof* is grounded on the following properties. There exist *optimal solution trees* of the *STP-MSP* problem with the properties from [6]: (1) No two edges cross each other. (2) Two edges meeting at a vertex form an angle of at least 60°. (3) If two edges form an angle 60° they have the same length. Left side of Figure 2 shows the radius, the distance to nearby sensor nodes, of the central sensor node, where each of r_a, r_b and r_c are ≤ *r*. The right side of Figure 2 shows that two replaced sensor nodes linked by r_0 may interconnect the same terminals with the property that each r_1, r_2, r_3, r_4, r_5 and r_6 is ≤ *r*. Following *lemma* shows this shortening of radii.

Lemma. (*shortening radii*)
For a Sensor node in a Senor Network, let the

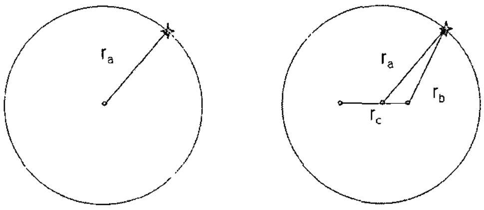


그림 3. 반지름 축소
Fig. 3 Shortening the radius

transmission radius r_a be given. Replacing the central sensor node by two ones within the distance $r_c = r/2$, where the line of r_c passes through the center. Then the triangular inequality implies: There exists r_b such that $r_b = r_a - x$, where $0 \le x \le r/2$ □

Theorem (*Structure Theorem*) For a set of *terminals* on R^2, there are infinitely many *Steiner-Cover*$_{2*opt}$'s that cover a *Steiner-Cover*$_{opt}$. The set also has an associated *tiling* of the *bounding-box* such that some *Steiner-Cover*$_{2*opt}$'s are *m-light* on this tiling. (See Figure 4)

Proof: By *Lemma*, each circle c_{opt} from an assumed *Steiner-Cover*$_{opt}$ can be covered by two other new sensor nodes: for each circle c_{opt}, one may interconnect the sensor nodes inside. From each c_{opt} covered by two other new sensor nodes we may build up a *Steiner-Cover*$_{kin}$ that covers *Steiner-Cover*$_{opt}$. Let *pair*$_{kin}$ be c_{opt} covered by two other new sensor nodes. By definition, there are infinitely many *Steiner-Cover*$_{kin}$ and all of them are actually *Steiner-Cover*$_{2*opt}$. One may build up a *tiling* over the *bounding-box* on R^2 so that a circle formed by new sensor nodes can be passed through by a *line-separator* at least once. The *tiling* can be

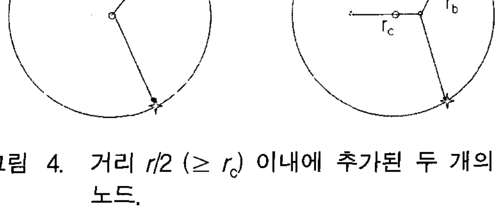


그림 4. 거리 $r/2$ ($\ge r_c$) 이내에 추가된 두 개의 노드.
Fig. 4. Two new nodes in distance $r/2$ of r_c

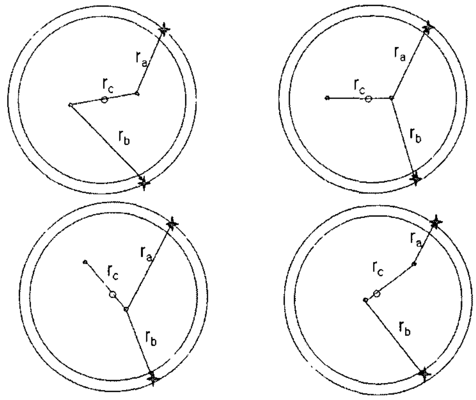


그림 5. 중앙 센서노드 주변 무한개의 pair$_{kin}$
Fig. 5. Infinitely many pair$_{kin}$ around the central sensor node.

acquired whenever more than one bottom-most *rectangles* can be put inside a circle of radius r. Now, one may set the value w as the width between the inside circle and the outer one, and w is wider than the inter-*portal* distance d so that there should lie more than one *portals* along the part of the *line-separator*. Since there are infinitely many *pair*$_{kin}$, some of them should turn out to be *m-light pair*$_{kin}$. Therefore, there exist some *m-light Steiner-Cover*$_{2*opt}$'s on the tiling. □

A DP finds *minimum m-light Steiner-Cover* over the *tiling* in a polynomial time, where # (the circles in the *minimum m-light Steiner-Cover*) = # (the circles in *Steiner-Cover*$_{2*opt}$). The size of the bottom most rectangle of the *tiling* should be chosen to be small so that brute-force search can be made to identify all the *m-light Steiner-Covers* that cover the *terminals* in the rectangle. According to the *tiling*, two neighboring bottom most rectangles meet along a line, the *line-separator*, forming an upper-level rectangle. We may *union* two *minimum m-light Steiner-Covers* from each of the two bottom-most rectangles, forming 2^m upper-level *m-light Steiner-Cover*. For the *union*, firstly enumerate all the 2^m *combinations of portals* (*chosen portals*) out of m portals along the *line-separator*. For each case of the combinations (chosen portals), join two *minimum m-light Steiner-Covers* from each of the two bottom level rectangles respectively, where the *Steiner-Covers* should pass through the *chosen* *portals*. Each case of the combinations may have many cases of the joins. The one with the minimum number of circles among the joins is kept as the *m-light Steiner-Cover* for the case of the combinations. When joining, if duplicating circles come up from the two rectangles for an upper-level portal, one of them should be deleted. This process goes on for all the *line-separators* at the bottom-most level. Again up along the *tiling*, the upper-level rectangles also have neighboring rectangles at the same level, and the same process repeats. The process goes up until the top level, where the final *minimum m-light Steiner-Cover* is acquired. As mentioned, DP scans all the *m-light Steiner-Cover* and finds the minimum one in a polynomial time, which is described as follows. The time complexity, analogously by [7], is bounded by the number of *rectangles* in the *tiling* and other combinational factors that can be chosen to be bounded. Now, we need to show the number of entries of the lookup table for this DP is polynomial and the run time for each of the entries is poly time. An entry is mixed by the triple: (a) A rectangle, (b) A set of k_1 ($\leq 4m$) portals along the perimeter of the rectangle, and (c) The choices of k_1 circle positions, i.e., the permutation of size k_1 out of the p indexed-points, {0, 1, 2, , p-1}.

For (a), the number of distinct rectangles is at most $\binom{n}{4}$. For (b), each rectangle has 4 sides which are the parts of the line-separators of some upper level rectangles. The m portals on the line separator are evenly distanced, so they are completely determined once we know the line-separators. But the number of choices of a line-separator is at most the number of pairs of points, which is $\binom{n}{2}$. This accounts for the factor $O\left((n^2)^4\right)$. Furthermore, once we have identified the set of $\leq$ 4m portals on the four sides, the number of ways choosing a set of k1 portals is $\binom{4m}{k_1}$. So the choices in (b) is $n^8 \times \sum_{k=1}^{c}\binom{4m}{k}$, where $c = 4\cdot\frac{8L}{r}$, the maximum number of crossings. For (c), for each portal chosen above, there are p choices of a circle shapes. Hence we can upper bound the size of the lookup table by $n^4 \times n^8 \times \sum_{k=1}^{c}\binom{4m}{k}p^k = O(n^{12} \times (2p)^{4m}) = n^{O(1)}$, where m should be chosen as $m = O(\log\ n)$ to form a polynomial expression, c is a constant as $c = 4\cdot\frac{8L}{r}$, p and r are mentioned before, and L is the size of the bounding-box.

Ⅳ. Conclusion

The high-degree polynomial time of DP can be accommodated because the computation for a given instance of the Sensor Network is a batch process, not a real time one, before the deployment of the sensor nodes. If needed, to reduce the computation time, one may divide the computation into parallel ones or design a randomized algorithm, as can be referenced from the related research of [7]. The computation by the proposed scheme may produce a network layout, over which at worst twice of the minimum number of relay nodes can be deployed, far decreasing the number of message exchanges between the nodes.

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저 자 소 개

김 준 모(정회원)
1989년 서울대학교 컴퓨터공학과 학사
2001년 University of Minnesota 전산학 박사
2002년~2004년 한국정보보호진흥원 연구원
2004년~현재 단국대학교 전자컴퓨터공학부 조교수
<주관심분야 : Approximations for NP-hard problems>