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3D gravity inversion with Euler deconvolution as a priori information

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Abstract. It is difficult to obtain high-resolution images by 3D gravity inversion, because the problem is extremely underdetermined – there are too many model parameters. In order to reduce the number of model parameters we propose a 3D gravity inversion scheme utilising Euler deconvolution as a priori information. The essential point of this scheme is the reduction of the nonuniqueness of solutions by restricting the inversion space with the help of Euler deconvolution. We carry out a systematic exploration of the growing body process, but only in the restricted space within a certain radius of the Euler solutions. We have tested our method with synthetic gravity data, and also applied it to a real dataset, to delineate underground cavities in a limestone area. We found that we obtained a more reasonable subsurface density image by means of this combination between the Euler solution and the inversion process.

Key words: Euler deconvolution, gravity inversion, nonuniqueness.

Introduction

The microgravity survey, which offers direct information about density distribution, is one of the most powerful methods for detecting subsurface cavities. Microgravity surveys have been successfully applied to cavity detection since Colley (1963), as shown by Butler (1984), Bishop et al. (1997), and Styles and Thomas (2001).

However, the inverse problem, namely the determination of subsurface anomalous density distribution, has an inherent nonuniqueness because there are usually insufficient measurement points compared with the number of unknown source parameters required, as well as the effect of errors in the observed data. Therefore, many authors have adopted a priori information such as density variation as a function of depth (Oldenburg, 1974), a constant density contrast (Pedersen, 1979), or the indirect assumption of compactness of the anomalous mass (Last and Kubik, 1983).

For high-resolution inversion of microgravity data, the subsurface should be decomposed into small-sized cells. Dividing the subsurface into small cells inevitably brings about nonuniqueness because of the limited number of observed gravity data.

In this paper, in order to overcome the increased number of model parameters, we adopt Euler solutions as a priori information to constrain the gravity inversion. The exploration of model parameter space, using prescribed density contrasts, is conducted within the areas where Euler solutions exist. This reduces the nonuniqueness of the solutions by searching only a limited area.

Gravity inversion with Euler deconvolution

Euler deconvolution is one of the most widely used automatic interpretation methods for gravity or magnetic data because of its ability to obtain the location of sources without any prior conditions. Euler's equation in potential field (T) was defined by equation (1) (Thompson, 1982).

$$(x-x_0)\frac{\partial T}{\partial x}+(y-y_0)\frac{\partial T}{\partial y}+(z-z_0)\frac{\partial T}{\partial z}=-NT \quad (1)$$

By solving Euler's equation, the source location (x_0, y_0, z_0) is obtained for a prescribed structural index (N). However, Euler deconvolution tends to produce too many spurious solutions, depending on the choice of structural index or window size, so it is difficult to interpret gravity data using only the Euler solution method. In this work, we have utilised Euler solutions not as direct interpretations, but as a priori information in the inversion process.

Camacho et al. (2000) proposed a growing body inversion method that the anomalous volumes are searched by means of an expansion approach. The growing body inversion method determines the volume of bodies that have a pre-determined density contrast. This algorithm is based on the strong assumption that subsurface can be divided into prisms of known sizes and positions, and the density of those prisms is estimated within the pre-established density contrast. Here, the anomalous prisms are chosen to enlarge initial estimates of the source body, until a satisfactory agreement with the observed field is reached.

We modified the growing body inversion method by using Euler deconvolution solutions as a priori information. The systematic exploration by the growing body inversion routine is carried out in that part of model space within a certain radius around the Euler solutions. Therefore, the number of model parameters to be searched is dramatically reduced.

Summing up the proposed algorithm in brief is as follows:

1. Prepare surface gravity data;
2. Perform Eulerdeconvolution on the input data;
3. Select the inversion space, with the help of the Euler solutions;
4. Conduct growing body inversion (Camacho et al., 2000).

In order to create a high-resolution image, gravity field inversion, especially in three dimensions, must have an excessively large number of model parameters. The problem usually becomes extremely underdetermined, and thus inevitably increases the nonuniqueness of the solution. Therefore, in order to reduce nonuniqueness, we add a priori information to the inversion constraints by selecting model parameters near where Euler deconvolution solutions have been found.

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Model parameters are totally independent in gravity inversion in which the density distribution of discrete subsurface cells is determined. Therefore, elimination of model parameters by using Euler solutions is in effect reducing nonuniqueness.

Model study

Figure 1 shows a synthetic gravity anomaly generated by five prisms whose density contrast is $-1.5\,\mathrm{t\,m^{-3}}$. The depth to top of all prisms is 10 m, and their thickness is 10 m. Our inversion scheme, that is, three-dimensional growing body inversion within the space selected by Euler deconvolution, was applied to the gridded data. The subsurface was divided into six layers of rectangular prismatic cells. First, we applied three-dimensional Euler deconvolution to the model data. Most of the Euler solutions occurred in the vicinity of prisms. The inversion scheme constructed model spaces within a certain distance of the Euler solutions, and then searched systematically in those spaces. Figure 2 shows the inversion results. The left panel shows the result of inversion without constraint on model space, and the right is the result with the Euler solution constraint. The solutions should appear in the layers with depth to top of 8, 12, and 18 m, because the synthetic prisms were located between 10 and 20 m in depth. The inversion without Euler solution information did not detect any of the five prismatic sources in the top 8 m layer. In the 18 m layer, it detected sources but could not resolve each prism. On the other hand, the result using Euler solutions as a priori information to growing body inversion detected and resolved the five prismatic sources separately in each of the 8, 12, and 18 m layers.

Application to a microgravity survey to map cavities in the Muan area

Our new scheme was also applied to real field microgravity data, acquired to delineate cavities in limestone at Muan, in the south-west part of Korea. The survey site, a Deokbo rice field, is located 3 km north of downtown Muan. The area is being used as a rice field, and has subsidence problems because of excessive pumping for agricultural irrigation. Ground subsidence occurred in parts of Muan county in the late 1990s. Buildings leaned over, and the ground surface sank into holes, which aroused concerns among the local population. The subsidence led to restrictions being placed on land utilisation.

The geology is briefly described as follows. Pre-Cambrian basements, schists, and granitic gneiss, are overlain by age-unknown metasediments. These are intruded by Jurassic granites, and are covered by Cretaceous volcanic sediments and intrusions. The metasedimentary sequences are mainly composed of basal quartzite and schists intercalated with thinly bedded, lens-shaped limestone and quartzite beds. Gwangju Fault, a NNE-SSW-trending strike-slip fault with minor normal faults, runs through the area. According to drilling results (Figure 5), a complicated network of cavities and cracks have developed as a result of limestone dissolution by underground water flows.

The gravity data were collected by a Scintrex CG-3 AutoGrav gravity meter at ~800 stations, with a spacing of 5 m along 17 profiles on paddy paths at about a 30 m interval, which provided a semi-gridded dataset. Measurements were made very carefully because of the soft ground surface at the stations. The surface at the stations was hardened before measurements were

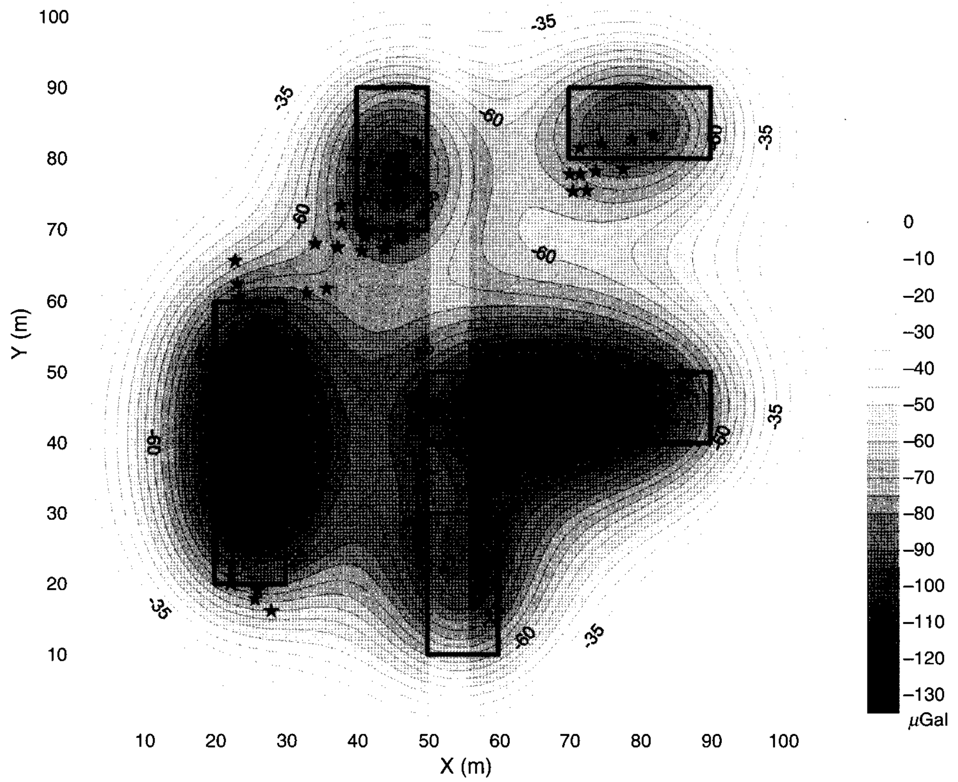

Fig. 1. Gravity anomaly due to the synthetic model. The solid line represents the boundary of prismatic source bodies. Their top is at 10 m depth and their thickness is a uniform 10 m, and the density contrast is $-1.5\,\mathrm{t\,m^{-3}}$. The star symbols indicate Euler deconvolution solutions.

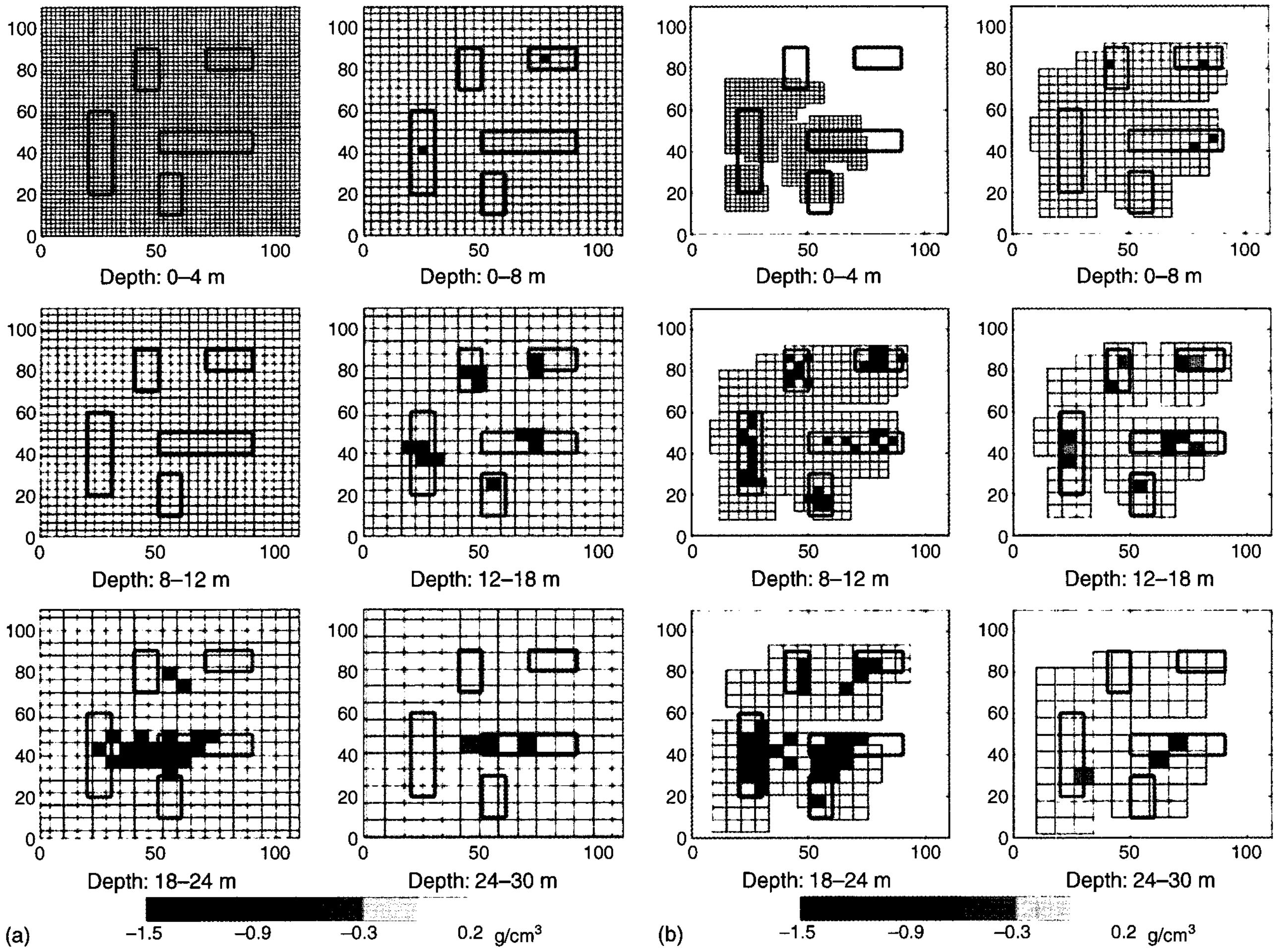

Fig. 2. Density distributions from inversions. (a) The result of growing body inversion without a priori information. (b) The inversion result when using Euler solutions as a priori information.

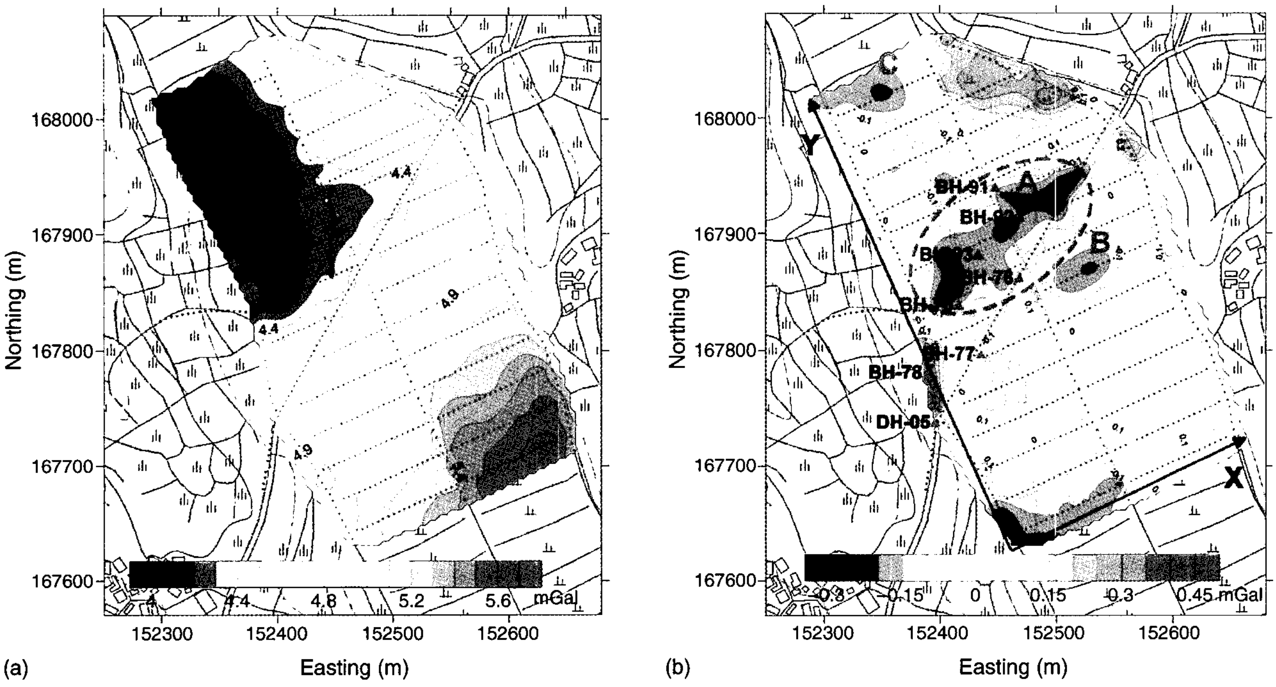

Fig. 3. (a) Bouguer gravity anomaly map and (b) residual gravity anomaly map. Red triangles indicate drill holes. Logs of drill holes are shown in Figure 5.

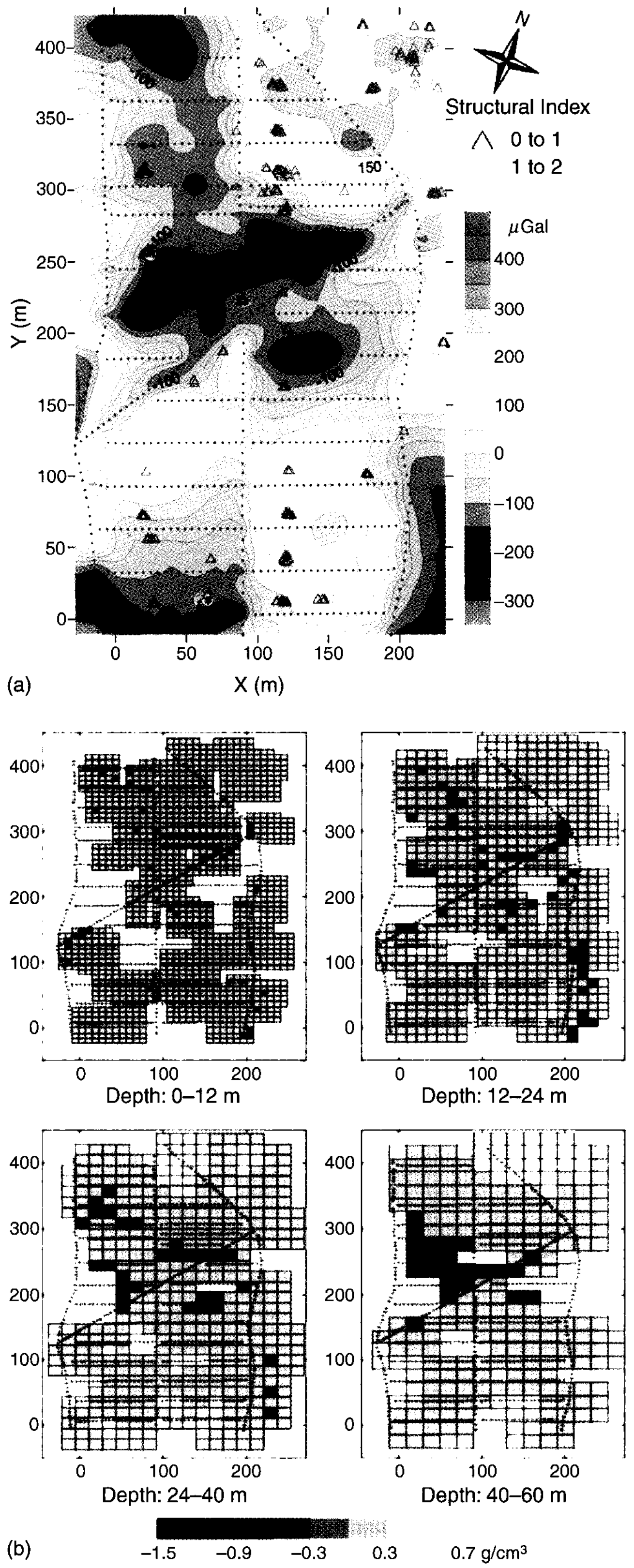

Fig. 4. (a) The residual gravity anomaly map. The dots are gravity stations, and Euler solutions are overlain. (b) Density distribution sections obtained by inversion using the growing body process with Euler solutions as a priori information.

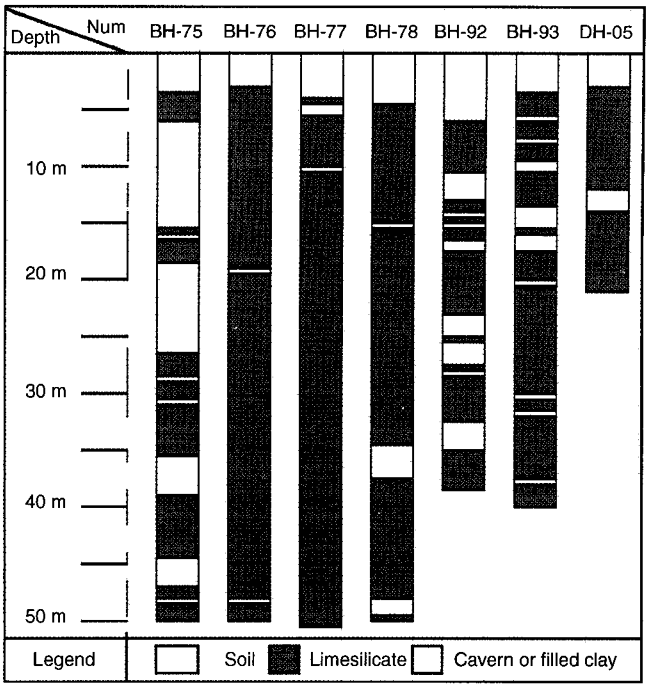

Fig. 5. Logs of drill holes. The position of the boreholes is shown in Figure 3b. Boreholes BH-75, BH-92, and BH-93, in the negative gravity anomaly zone, show caverns, either open or filled with clay.

made, to prevent the gravimeter from tilting. The positions of the stations were determined by a Leica Total Station 1100 together with DGPS using a Trimble 5700, which supported an accuracy of 3 mm. The gravimeter looped back to the base station every two hours for drift correction. The data were tied to the reference station, and earth tide, free-air, and Bouguer corrections were applied. The terrain effect was calculated more precisely by a triangular element method (Rim et al., 2005) using a digital elevation model with a 30 m grid, and after subtracting the terrain effect the Bouguer gravity anomaly map as shown in Figure 3 was obtained.

The map shows a regional variation from 3.9 mGal in the north-west to 5.8 mGal in the south-east. This trend has superimposed on it a higher frequency variation, which contains the information concerning the presence of cavities. The regional trend was separated by 3rd-order polynomial fitting to produce the residual Bouguer gravity map (Figure 4a). The map now reveals complex sets of residual anomalies with amplitudes from −0.35 to 0.5 mGal. The negative anomaly zones indicate mass-depletion regions which are associated with cavities.

Euler deconvolution solutions, classified by structural index, were also overlain on the residual anomaly map. The inversion scheme in this study, that is, three-dimensional growing body inversion within spaces around Euler solutions, was applied to the data. The subsurface was divided into four layers of rectangular prismatic cells. The result, Figure 4b, shows that cavities are well developed between 10 and 40 m in depth. In particular, the result also indicates the area where subsidence will probably occur.

As shown in Figure 5, drill holes BH-75, BH-92, and BH-93 encountered caverns or clay-filled zones in the negative gravity anomaly zone, so the data in these drilling logs also confirm these interpretations. Further, low resistivity regions generally coincide with the negative gravity regions. The resistivity distribution of the layer at the depth of 10 m, determined from a dipole-dipole survey, is also illustrated in Figure 6. Resistivities as low as ~50 Ω m seem to be related to cavities.

Conclusions

A three-dimensional gravity inversion scheme using Euler deconvolution as a priori information was devised. Euler

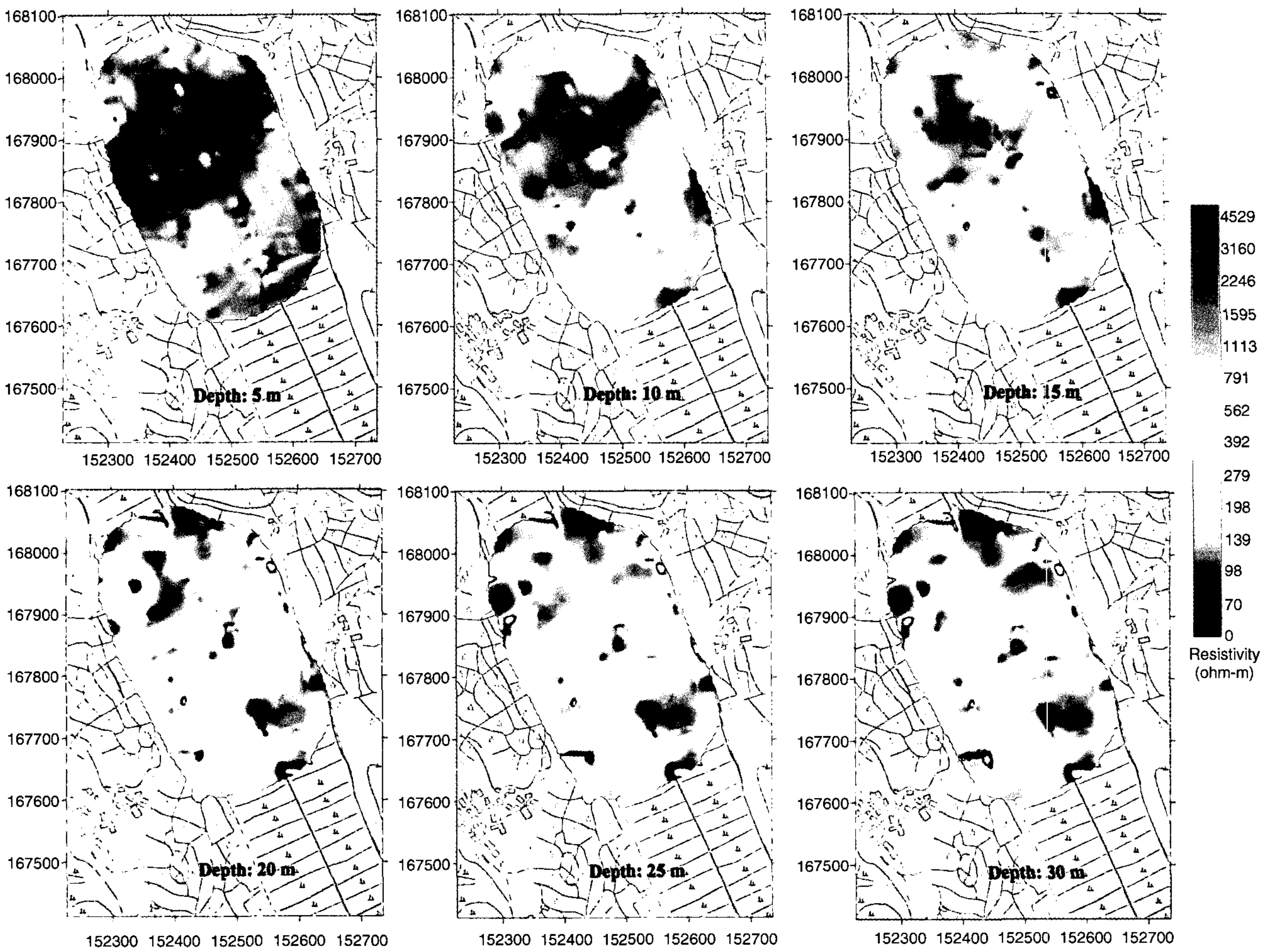

Fig. 6. Resistivity distribution in layer slices. The low-resistivity zones, which relate to caverns, open or filled with clay are coincident with negative gravity anomaly zones.

deconvolution tends to produce spurious solutions, and inversion processes sometimes lead to undesirable results because of insufficient a priori information. This study, by adding Euler solutions to gravity inversion, generates more plausible subsurface density distributions. The scheme increased the reliability of the solution by reducing the extent of the inversion model parameter space. A synthetic model study showed that the inversion algorithm of this study delineate the location of source bodies with high resolution. The scheme was applied to a real field dataset, and the model results showed the development of cavities. Three-dimensional density images of the target area showed the development and distribution of cavities. Resistivity survey data and drill logs supported these interpretations.

Acknowledgments

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오일러 디컨벌루션을 사전정보로 이용한 3 차원 중력 역산

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요 약 : 고해상도를 가지는 지하 밀도 영상을 얻기 위한 3차원 중력 역산은 모델 변수들이 급격하게 많아지는 문제가 발생한다. 이 논문에서는 모델 변수들의 수를 줄이기 위해서 오일러 디컨벌루션의 해를 사전정보로 활용하는 3 차원 중력역산을 제안하였다. 이 논문에서 고안한 역산 알고리즘의 핵심은 오일러 디컨벌루션의 해가 얻어진 주위로 역산 공간을 제한하여 역산 해의 비유일성을 줄인 점이다. 먼저 중력 자료에 대한 3 차원 오일러 디컨벌루션의 해를 구하고, 오일러 디컨벌루션의 해가 나타나는 주위에서만 3차원 확장 탐색 역산을 수행하여 지하 밀도 영상을 구하였다. 이 3차원 중력 역산 방법은 합성 모델에 적용하여 그 성능을 검증하였고, 석회암 지대에 존재하는 공동의 분포를 밝히기 위한 고정밀 중력탐사 자료 역산에도 적용하였다. 결과적으로, 오일러 디컨벌루션의 해를 사전정보로 이용한 역산을 이용하여 분해능이 향상된 고해상도의 지하 밀도 영상을 구할 수 있었다.

주요어 : 중력 역산, 비유일성, 오일러 디컨벌루션

オイラー・デコンボリューションを既知情報として用いた重力データの3次元インバージョン

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要 旨 : 重力データの３次元インバージョンで高精度イメージを得るのは困難である。なぜなら，モデルのパラメーター数が多過ぎ，その確定が困難ないわゆる劣決定問題であるためである。そこで本論文では，モデルパラメーター数を減らすため，オイラー・デコンボリューションを既知情報として組み込んだ，重力データの次元インバージョン法を提唱する。この方法の根本概念はオイラー・デコンボリューションによってインバージョン空間を限定し，解の非一意性を緩和しようというものである。

著者らは"growing body"過程の検討を系統的に行ったが，その際インバージョン解を，オイラー・デコンボリューションの解から一定の半径以内の空間に限定した。

また，重力のモデルデータでこの方法を試し，石灰岩地帯からの実際のデータセットに適用して地下空洞の捕捉を試みた。それによりオイラー同次方程式の解とインバージョンの組み合わせによって，より妥当な地下の密度分布が得られることが確認された。

キーワード : 重力インバージョン，解の非一意性，オイラー・デコンボリューション

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