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ESTIMATION OF RIDE QUALITY OF A PASSENGER CAR WITH NONLINEAR SUSPENSION

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ABSTRACT–The nonlinear characteristics of a suspension is directly related to the ride quality of a passenger car. In this study, the nonlinear characteristics of a spring and a damper of a passenger car is analyzed by dynamic experiments using the MTS single-axial testing machine. Also, a mathematical nonlinear dynamic model for the suspension is devised to estimate the ride quality using the K factor. And the effect on the variation of the parameters of the suspension is examined. The results showed that the dynamic viscosity of the oil in a damper was the parameter that most influeced the ride quality of a passenger car for the ride quality of a passenger car.

KEY WORDS : Ride quality, Nonlinear suspension system, Billings' method, Monte carlo method, Damper, K factor, Single-axial testing machine

1. INTRODUCTION

A passenger car has bounce, pitching, rolling, and yawing motion by road excitation and handling conditions. Among these motions, bounce motion which depends mainly on suspension characteristics is the most influential factor of ride quality. Thus, an analysis of suspension characteristics is important in car design.

For the analysis of car suspension, Clark (Clark, 1962) suggested a two dimensional four degree-of-freedom model and Kohr (Kohr, 1961) suggested a three dimensional seven degree-of-freedom model. Recently, for the analysis of the relative effect between parts, ADAMS, a multi-body dynamic analysis program, has been used (Rao and Giannopoulos, 1981). However, these studies showed that the spring and damper of suspension systems usually have nonlinear characteristics. Therefore, linear models are inadequate for accurate dynamic analysis of car motion (Reybrouch, 1994).

A spring can have nonlinearity due to the contact between pitches. When a damper is compressed or tensioned, damping force shows hysteresis, which is inherently nonlinear due to the friction between the cylinder and piston of a damper.

Despite that, the ride quality of a car with the consideration of the nonliinearities of suspension systems has not been accounted in design stage. Instead, repeated experiments and tuning work by test drivers have been done in car makers. However, trial and error can be minimized if the nonlinear characteristics of a suspension system are considered.

In this study, the nonlinear dynamic characteristics of a spring and a damper of a passenger car were analyzed by experiment using the MTS single-axial testing machine. A mathematical model for the suspension system was devised based on the experimental data to analyze the ride quality of a car and the effects of the suspension parameters.

2. SUSPENSION EXPERIMENT

The nonlinear characteristics of a spring and a damper of a passenger car was analyzed by dynamic experiments using the MTS single-axial testing machine (MTS model 248.03). The experimental set-up is shown in Figure 1. The spring and damper are excited by a hydraulic actuator and their forces are measured by a load cell. Also, the temperature sensor was attached to check the temperature change of the damper during the experiment.

2.1. Spring

The spring used in the experiment was a conical spring, which can induce stiffness nonlinearity. Also, the contact between pitches due to a large deflection can result in nonlinear characteristics.

Figure 2 shows the experimental set-up for the measurement of spring stiffness. The maximum input spring displacement was was 0.125 m, since the maximum input

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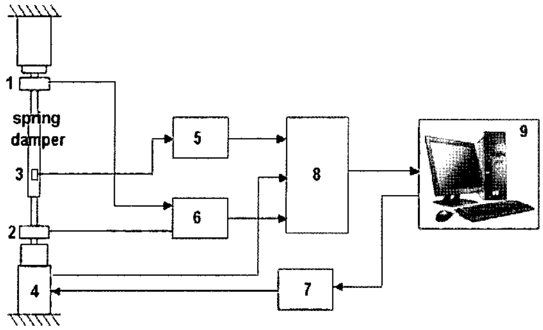


1, 2 : Load cell, 3 : Temperature sensor, 4 : Actuator,
5 : Temperature sensor amp., 6 : Load cell amp.,
7 : Controller, 8 : A/D board, 9 : Data acquisition

Figure 1. Experimental set-up.

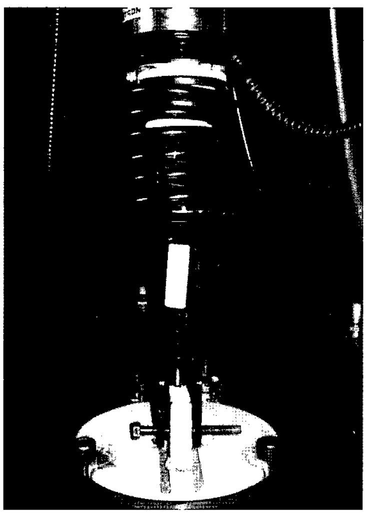


Figure 2. Experimental set-up for spring.

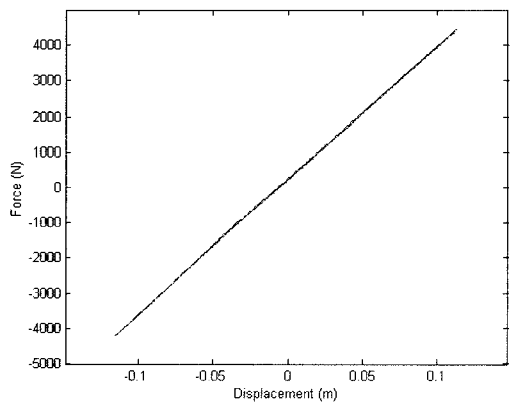


Figure 3. Force-displacement curve of the spring.

force of the car suspension from roads is generally under 4,000 N (Choi *et al.*, 2003). Figure 3 shows the spring force-displacement curve, which is almost linear. The pitches were not in contact since the input displacement was not so large in order to reflect the common running situation. The experimental result shows that the linear assumption of the spring is adequate for common situations.

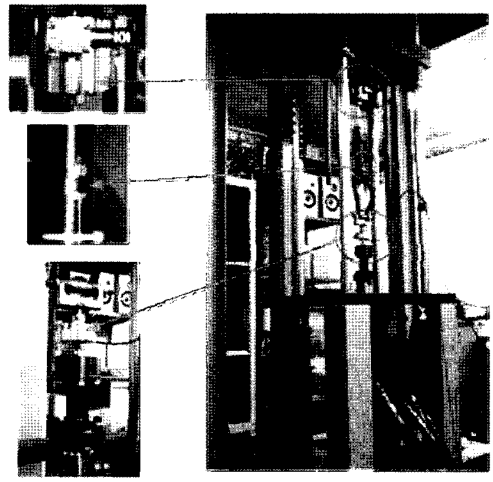


Figure 4. Experimental set-up and sensor locations for damper.

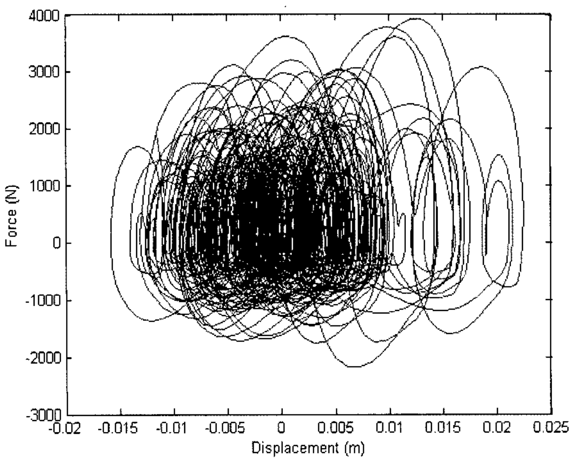
2.2. Damper

Experimental set-up and sensor locations for the measurement of the damping force are shown in Figure 4. The lower part of the damper was fixed on the actuator and the upper part of the damper was fixed just beneath the load cell to measure the damping force. Also, a temperature sensor was installed to control the temperature below 10°C to minimize the effect of temperature on the viscosity of the oil inside the damper, which can change easily with temperature.

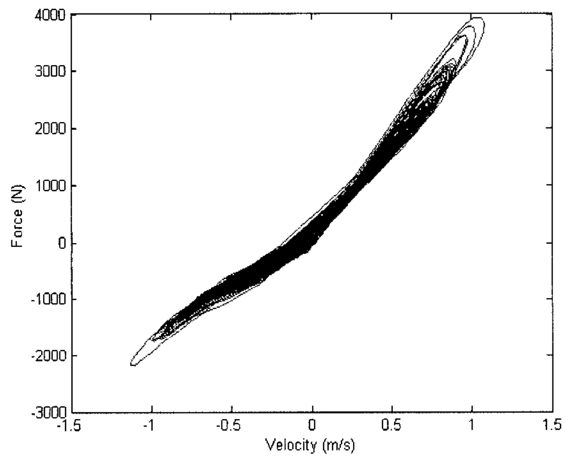
A white noise from 0 Hz to 50 Hz was used for the excitation of the damper and the damping force was measured for the excitation. Figure 5 shows the damper characteristics. Since the excitation was random, the force-displacement curve shows irregular motion. However, the force-velocity curve shows typical damper characteristics. As shown in Figure 5, the damping force has nonlinear characteristics with hysteresis.

3. NONLINEAR SYSTEM IDENTIFICATION

To develop mathematical model of a spring and damper, a nonlinear system identification technique is used, which needs input/output experimental data (Billings, 1999). In this technique, an initial model set is composed of all the possible terms of polynomials. Of course, a model that includes all the possible terms is impractical because of the large number of terms. In most cases, a complex model is not a better model. After an initial model is set



(a) Force-displacement curve



(b) Force-velocity curve

Figure 5. Nonlinear characteristics of the damper.

up, important terms should be included in the model set. The importance of each term is determined based on its contribution. After the ERR (error reduction ratio) of each term is calcualted, the terms with ERR larger than the criterion of ERR is included in the model set. The ERR (Chen *et al.*, 1989) is defined by Equation (1), where n is the number of candidate terms. The error reduction ratio is the ratio of each term to the entire response.

$$[ERR]_i = \frac{(X_i)^2}{y^2}, \ (0 \leq i \leq n) \tag{1}$$

After determining the model set, if the ERR is smaller than the target error, the optimal model set is completed or otherwise, the canceled terms are included in the model set. Figure 6 illustrates the model selection procedure. Parameter estimation of each term can be carried out by using the least square method. This step is a kind of validation procedure in which the model response

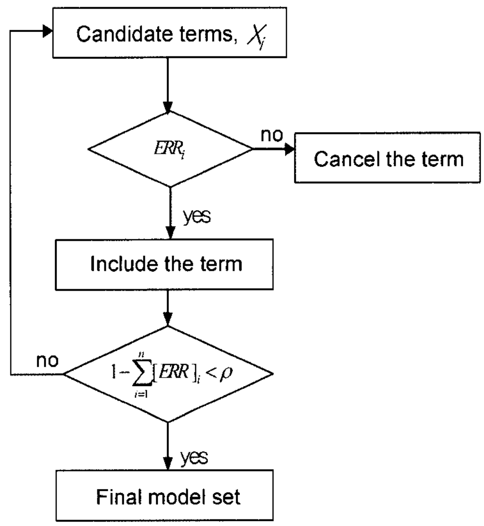


Figure 6. Model estimation process.

needs to be compared with the response of the system in a different situation.

The experimental data of the spring stiffness is shown in Figure 3. Spring stiffness can have nonlinearity; however, the nonlinear terms are neglected because the ERR of a nonlinear term of the spring is below 1%. As a result, the calculated value of spring stiffness is about 37,121 N/m.

To develop a damper model, the experimental data shown in Figure 5 is used. The equation of motion of a damper can change depending on the exciting frequency. In this study, a random excitation between 0 and 50 Hz was used to reflect an actual road excitation. The initial model of the damper was made on the basis of the Wallaschek model (Wallaschek, 1990) and the effect of the gas stiffness (Choi, Y. T. *et al.*, 2000) and mass density of the fluid, $\dot{x}\,\mathrm{sgn}(\dot{x})$, and friction term, $\mathrm{sgn}(\dot{x})$, were included because the damper was an oil damper filled with nitrogen gas. Wallaschek assumed that all the terms of a damper model were linear. In this study, the nonlinear terms were added to improve the accuracy of the model, where L is equal to 5. The final model is expressed by Equation (2).

The model selection process continues until $1-\sum_{i=1}^{n}[ERR]_i \leq 0.01$ is satisfied. Finally, the nonlinear damper

Table 1. Estimated system parameters.

Par.	Value	Par.	Value
k_1	−1,935 (N/m)	c_3	2,145 (N(s/m)^3)
k_3	16,740,666 (N/m^3)	h	−2,170 (N(s/m)^2)
c_1	2,814 (Ns/m)	F_{fric}	41 (N)
c_2	1,468 (N(s/m)^2)	–	–

model presented in Equation (3) has been estimated. The parameters for the final model set were estimated by using the least square method. The estimated parameters are shown in Table 1.

$$D(x,\dot{x})=\sum_{n=1}^{L} k_n x^n+\sum_{n=1}^{L} c_n \dot{x}^n +h(\dot{x})^2\mathrm{sgn}(\dot{x})+F_{fric}\mathrm{sgn}(\dot{x}) \quad (2)$$

$$D(x,\dot{x})=k_1x+k_3x^3+c_1\dot{x}+c_2\dot{x}^2 +c_3\dot{x}^3+h(\dot{x})^2\mathrm{sgn}(\dot{x})+F_{fric}\mathrm{sgn}(\dot{x}) \quad (3)$$

4. NONLINEAR SUSPENSION MODEL

The nonlinear suspension model is composed of the spring and the damper model developed in the previous chapter. And then, to investigate the ride quality, an equation of motion of the suspension including the nonlinear terms is derived for a quarter car model, where the stiffness and the damping of a tire were not considered. Also, to validate the developed nonlinear suspension model, the experimental data of the Belgian road (Choi and Park, 2003) were used for numerical integration of the model.

4.1. Nonlinear Quarter Car Model

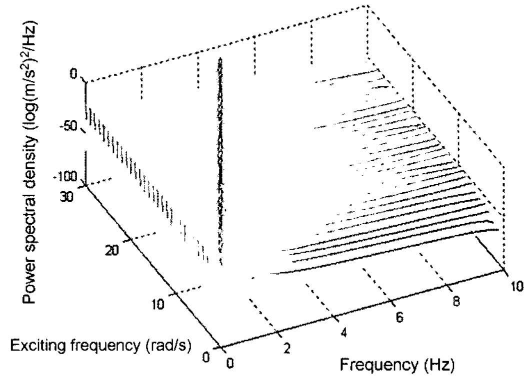
For analysis of the ride quality of the suspension system, a nonlinear quarter car model is set up. For the system under an actual road excitation, the nonlinear equation of motion is defined as Equation (4), where m is the quarter car mass, k_0 is spring stiffness, the other terms represent the characteristics of the damper, $f(t)$ is the measured wheel force using a wheel force transducer during running., x is the relative displacement between the response of car body and the road roughness.

$$m\ddot{x}+(k_0+k_1)x+k_3x^3+c_1\dot{x}+c_2\dot{x}^2+c_3\dot{x}^3 +h(\dot{x})^2\mathrm{sgn}(\dot{x})+F_{fric}\mathrm{sgn}(\dot{x})=f(t) \quad (4)$$

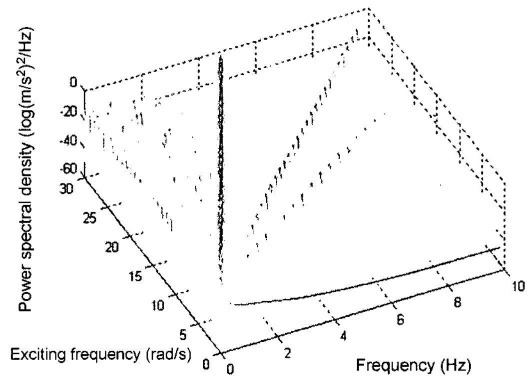
4.2. Nonlinear Characteristics

To predict the frequency response of the developed nonlinear model, displacement is calculated according to the excitation force as the exciting frequency increases. The characteristics of the nonlinear model is compared with those of the linear model. The linear model can be expressed by Equation (5), where c_1 is the viscous damping coefficient of the damper and k_0 is the linear stiffness of the spring from the Equation (4).

Figures 7(a) and (b) are the waterfall diagrams of the linear and nonlinear suspension systems with increasing excitation frequency. The nonlinear model shows high frequency components at the low excitation frequency range. The higher frequency component is due to the nonlinearity of the suspension system. Therefore, this characteristic of the nonlinear model can affect the ride quality of a car.



(a) Linear model



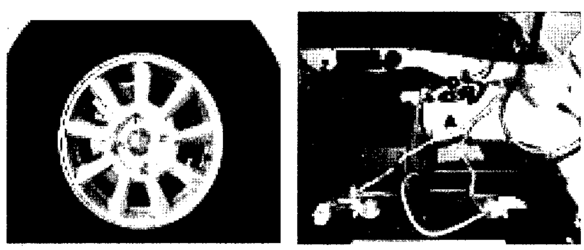
(b) Nonlinear model

Figure 7. Waterfall diagrams of the models.

$$m\ddot{x}+c_1\dot{x}+k_0x=f(t) \quad (5)$$

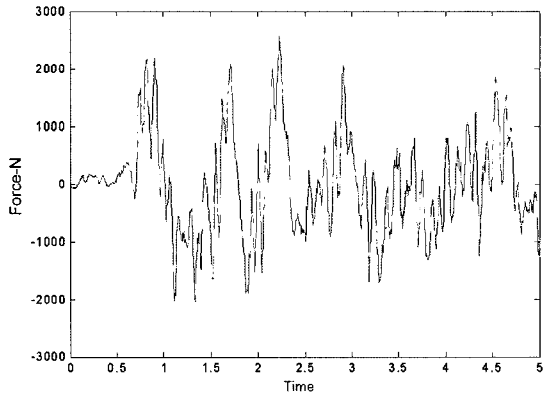
4.3. Model Verification

To validate the nonlinear quarter car model, a driving test was performed as shown in Figure 8. The force exerted from the road was measured by using a wheel force transducer as shown Figure 8(a), and the car body motion was measured by using accelerations that were attached to the mount part of the suspension system as shown Figure 8(b).

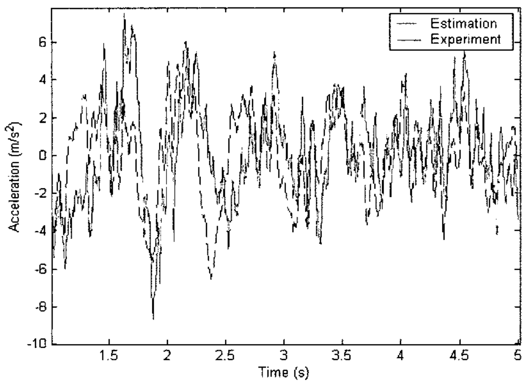


(a) (b)

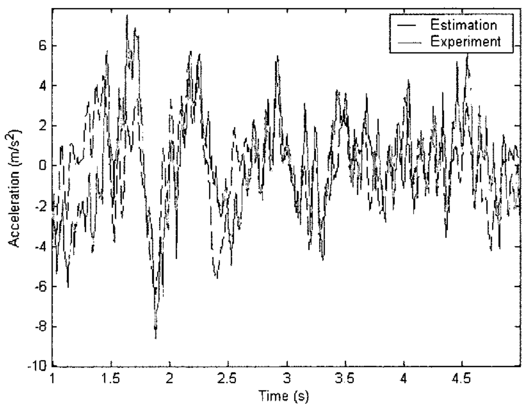
Figure 8. Sensor location for the driving test.



(a) Measured wheel force on Belgian road at 20 km/h



(b) Linear model



(c) Nonlinear model

Figure 9. Model verification.

The acceleration was measured at the mount of the suspension during running on the Belgian road at 20 km/h. Figure 9 shows the measured wheel force and the comparison between the experimental result and the numerical integration results after removing the transient. Figure 9(a) is the measured wheel force using wheel force transducer. Figure 9(b) is the comparison of the results obtained from the linear model and Figure 9(c) is the comparison of the results obtained from the nonlinear model. MSE (mean square error) between two models is almost double, where the MSE of the linear model is 6.19 m/s^2, and the MSE of the nonlinear is 4.82 m/s^2. Also, the MSE's of the linear and nonlinear model on the Belgian road at 30 km/h. are 8.04 m/s^2 and 6.70 m/s^2, respectively. As the numerical integration results of a nonlinear model provided better agreement than those of the linear model provided, it can be said that the nonlinear model is more accurate and useful than the linear model for ride quality analysis.

5. RIDE QUALITY

For analysis of the ride quality, nonlinear quarter car model excited by white noise between 0 and 50 Hz was used. As an index of the ride quality, K factor (ISO 2631, 1985), was used, which is useful for evaluating a wheel vibration model (VDI 2057, 1987).

5.1. Road Excitation

For road random excitation, Monte Carlo simulation (Shinozuka and Deodatis, 1991) was carried out. The power spectral density for a random road excitation is the Gaussian power spectrum density function.

From the power spectral density function of a road, the road shape function in time domain can be defined as Equation (6) using the combination of sine functions. In this study, the random excitation on the road was made by 1,000 random numbers for 70 seconds. The cut-off frequency was set to 50 Hz and the RMS value of the input force was set to 1,000 N, which was obtained from experimental data (Choi *et al.*, 2003).

$$d(t)=\sum_{i=0}^{N-1} \sqrt{4D \cdot (f_{i+1}-f_i)} \cdot \cos(2\pi f_i t + \phi_i) \tag{6}$$

D : Gaussian power spectrum density function
N : No. of frequency to be considered
f_i : Considered frequency between 0 and cut-off fre-

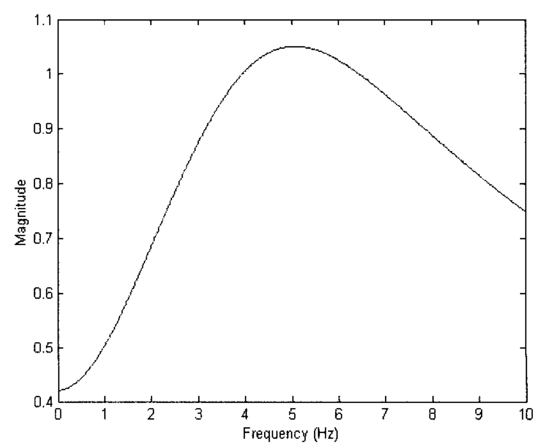


Figure 10. Magnitude of transfer function.

quency

θ_i : Independent random phase angles uniformly distributed between 0 and 2π

5.2. Criterion for Ride Quality

The ride quality of a car can indicate how a person feels the vibration of a car. If acceleration and power spectral density can be expressed by a vibration model, the ride quality can be expressed by using the K factor.

ISO 2631 defines the discretion factor for human vibration.

In ISO 2631, the transfer function of the human body, G(f), is expressed as Equation (7). The magnitude is shown as Figure 10. As shown in Figure 10, the sensitive frequency for a human is between 3 and 8 Hz.

$$G(f)=\frac{49.42(i\omega)+465.76}{(i\omega)^2+48.97(i\omega)+1{,}108.95} \quad (7)$$

If the magnitude of the power spectrum density for acceleration is defined as $a(f)$, the effective human vibration becomes

$$a_m(f)=|G(f)|\cdot a(f) \quad (8)$$

Then, a_m is the acceleration of the human body in the frequency domain. The magnitude of the acceleration can be expressed as follows

$$a_m=(\Sigma a_m^2(f))^{1/2} \quad (9)$$

K factor, as an overall ride quality factor, can be expressed as

$$K=20a_{tm} \quad (10)$$

5.3. Evaluation of Ride Quality

Ride quality is estimated by using K factor. The difference of the ride quality between the linear and the nonlinear model is investigated by comparison of the ride quality estimationfor the experimental data between these two models. To verify the validity of the nonlinear model, the experimental datas shown in Figure 8 were used. The K factor for the experimental data was 7.11. The K factor of the nonlinear model was 6.93. However, the K factor of the linear model was 5.22. The result shows that the K factor for the nonlinear model was close to the experimental results. Also, it tells us that the nonlinear model estimated the ride quality better than the linear model, which means that nonlinear model is more useful for ride quality analysis.

The nonlinear suspension model was used to analyze ride quality according to the variations of the parameters. As shown Figure 11, Variations of the parameters were considered to be within ±5%, ±10%, ±15%, ±20% of the original value of Table 1. Therefore, as the spring stiffness (k_1), the dynamic viscosity of the fluid (c_1), and the friction between piston and cylinder (F_{fric}) of a damper increased, and as the mass density of the fluid (h) decreased, the ride quality improved. Among these parameters, the dynamic viscosity of the fluid was the most influential parameter of ride quality. The second influential parameter was the suspension stiffness and the third parameter, the mass density of the fluid, and the last parameter, the friction between the piston and cylinder. Consequently, in the design of a passenger car, the viscosity of the fluid in a damper should be considered first when trying to achieve good ride quality of a car.

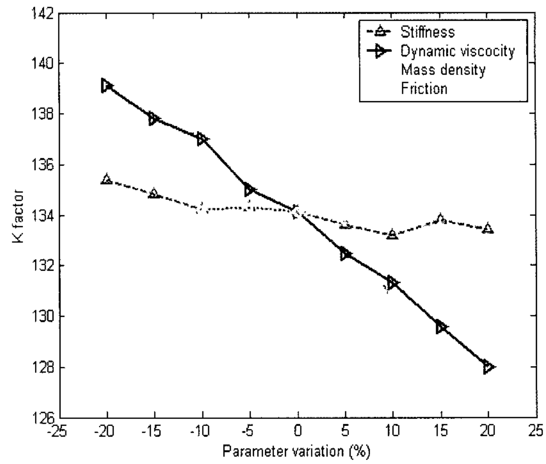


Figure 11. K factor on random excitation.

6. CONCLUSION

To develop nonlinear models of a spring and damper, which is an oil damper filled with nitrogen gas, a nonlinear system identification method is used for the experimental data. The results showed that the spring is almost linear in the actual use range, but the damper has nonlinearity due to gas stiffness, mass density of the fluid, and the friction between the piston and cylinder.

And the spring and damper model was used to make a single DOF nonlinear suspension model. The numerical integration results of the nonlinear suspension model gave better agreement with the results of the drive test than those of the linear model did.

Using the developed nonlinear suspension model, the ride quality, defined by the K factor on random road excitation, was calculated. The K factor for the nonlinear model was close to the experimental results. The nonlinear model estimated the ride quality better than the linear model, which means that the nonlinear model is more useful for ride quality analysis. The parameter study also showed that the dynamic viscosity of the fluid is the most influential parameter of ride quality.

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REFERENCES

Association of German Engineers (1987). Effect of mechanical vibrations on human being. VDI 2057, *Association of German Engineers.*

Billings, S. A. (1999). A direct approach to identification of nonlinear differential models from discrete data. *Mechanical System and Signal Processing,* **13**, 739–755.

Chen, S., Billings, S. A. and Luo, W. (1989). Extended model set, global data and threshold model identification of severely non-linear system. *Int. J. Control,* **50**, 1897–1923.

Choi, Y. T., Choi, S. B. and Park, D. W., (2000). A sliding mode control of a full-car ER suspension via hardware-in-the-loop-simulation. *J. Dynamic System, Measurement and Control,* **122**, 114–121.

Choi, Y. S. and Park, J. P. (2003). Nonlinear parameter estimation of suspension system. *Proc. Spring Conf., Korean Society for Noise and Vibration Engineering,* 281–286.

Clark, D. C. (1962). A preliminary investigation into the dynamic behavior of vehicles and highways. *SAE Trans.,* **70**, 447–455.

International Organization for Standardization (1985). Evaluation of human exposure to whole-body vibration - part 1: General requirement, ISO 2631/1. *Int. Organization for Standardization.*

Kohr, R. H. (1961). Analysis and simulation of automobile ride. *SAE Trans.,* **69**, 110–119.

Rao, A. K. and Giannopoulos, F. (1981). Dynamic loads on suspension component using mechanism programs. *SAE Paper No.* 811307.

Reybrouck, K. (1994). A nonlinear parametric model of automotive shock absorber. *SAE Paper No.* 940869.

Shinozuka, M. and Deodatis, G. (1991). Simulation of stochastic processes by spectral representation. *Applied Mechanics Revies,* **44**, 191–204.

Wallaschek, J. (1990). Dynamics of nonlinear automative shock absorber. *Int. J. Nonlinear Mechanics,* **25**, 299–308.