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REDUCTION OF HIGH FREQUENCY EXCITATIONS IN A CAM PROFILE BY USING MODIFIED SMOOTHING SPLINE CURVES

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ABSTRACT–High frequency excitation terms in a cam profile can excite vibration of a cam follower system. In this paper, modified smoothing spline curves are used to reduce the high frequency terms. The essential difference between the proposed method and other existing approaches is its ability to make the principal cam motions smooth while still exactly satisfying boundary conditions of follower displacement, velocity and acceleration. The boundary values usually depend on the ramp properties of a cam. Our method, thus, allows designers to smooth the existing cam motion without any damages on its ramp areas. Because the ramp height, velocity and acceleration are maintained exactly, more radical smoothing is possible. An example shows that the proposed method can be a powerful tool of cam profile smoothing, which removes high frequency components in the cam profile excitations without any changes in ramp properties.

KEYWORDS : Cam profile smoothing, Modified smoothing spline, Automotive engine cam, Low vibration cam

1. INTRODUCTION

Certain types of cam follower systems, such as the automotive engine valve trains, usually employ clearance (valve lash) during the base-circle-dwell period. This ensures the sealing of the combustion chamber by transferring the spring force to the valve seat. The clearance, however, inevitably produces impulsive valve opening and closing, which is an important source of valve train noise. To reduce the magnitude of the impact at the beginning and ending of the valve event, the ramp period is incorporated into the cam profile. According to Stoddant (1953) and Norton *et al.* (1999), the ramp's functions are to compensate for the clearance and the static follower deflections. Effect of ramp properties on the valve train dynamics are studied by Norton *et al.* (1999) and An and Kim (2006). Because the opening and closing valve events must be precisely controlled, not only displacement but also velocity and acceleration of the ramp profile must be rigorously designed.

On the other hand, dynamic behavior of a valve train system is one of the critical concerns in designing automotive engines. Abnormal valve train behaviors such as jumping, bouncing, excessive spring surging, etc, must be prevented throughout the whole range of engine operating speeds. Because a follower motion excites valve train dynamics, fine-tuning of the cam profile is necessary in the early stage of valve train design. If the engine operating speed is constant, cam profile excites the valve train system periodically. Therefore, cam excitation force can be assumed to be a series of harmonic functions. Many researches and experiments, presented by Seidlitz (1989), Norton *et al.* (1999, 2002) and others, have proved that there are strong correlations between the valve train dynamics and the magnitude of cam profile harmonic components. They also showed that, in most automotive engines, significant spring surging is correlated to the 9th~13th harmonic amplitudes of the cam motion. Higher order components such as 20th~50th usually excite the follower vibrations that depend on the natural frequencies of the cam follower system. Even though low order components of the cam profile harmonics cannot be reduced without changing timing or lift of cam event, the amplitude of high order components can be modified without meaningful changes in the cam displacement. This fact allows cam designers to design a dynamically stable valve train system without degrading engine performance.

Data smoothing technique using spline curves is widely used to reduce the amplitude of high order harmonic components in cam profiles. The first method that is worthy of mention is the smoothing technique using spline functions. It was introduced by Reinsch (1963) and improved by Craven and Wahba (1979). Eilers and Marx (1996), Kano *et al.* (2005) presented smoothing methods based on B-spline curves. Since the 1960s, some other smoothing methods that are based on generic spline curves have also been proposed (Hardle, 1994). Because notice-

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able changes in velocity and acceleration curves result from very tiny modifications in displacement curve, the existing cam profile smoothing techniques do not allow significant changes in ramp displacement. This fact usually limits the level of benefit from the profile smoothing.

This paper proposes a new method to smooth existing cam profiles. It is based on modified smoothing spline curves that constrain displacement, velocity and acceleration at boundaries. Therefore, the smoothed cam profile can be connected exactly with the original opening and closing ramps up to the second order derivatives. This particularity of the proposed method enables designers to smooth cam profiles without worrying about the damage to ramps. Because high frequency components in the cam profile excitation are minimized effectively, NVH (Noise, Vibration and Harshness) characteristics of cam follower systems can also be improved without any negative effect.

2. CAM PROFILE SMOOTHING METHOD

2.1. Dynamic Excitation of Cam Profile

In a valve train system, the follower motion is normally driven by cam motion through their contact forces.

Figure 1 shows a typical example of automotive engine valve train, which employs a disk cam driving a flat-faced follower and a return spring. In the ideal case, the follower motion is precisely followed by the cam's pre-programmed motion. Unfortunately, this is not possible in practice because of valve train internal vibrations. Many researches on dynamic responses and their influences on cam follower systems have been presented. In these researches, many different types of dynamic models have been introduced to simulate valve train dynamics. Reviews about these models can be found in Norton (2002) or Rothbart (2004). In this paper, the linear models will be employed to avoid redundant complications. However, even if any model is employed, the motion equation of the cam follower system can be written as the matrix form in Equation (1). Dimension of the mass, damping and stiffness matrixes, i.e. **M**, **C** and **K** representatively, depends on the DOF of the used model. Values of the matrixes' components evidently depend on physical parameters of the cam follower system.

$$\mathbf{M}\cdot\ddot{\mathbf{s}}+\mathbf{C}\cdot\dot{\mathbf{s}}+\mathbf{K}\cdot\mathbf{s}=\mathbf{F}(t) \quad (1)$$

Here, **s** is a vector of the lumped mass motions in the model. **F**(*t*) is a vector of external forces that correlate directly with the cam profile curve and its derivatives as follows

$$\mathbf{F}(t)=\mathbf{F}_1\cdot y(t)+\mathbf{F}_2\cdot\dot{y}(t)+\mathbf{F}_3\cdot\ddot{y}(t) \quad (2)$$

Also, $y(t)$, $\dot{y}(t)$, and $\ddot{y}(t)$ are lift, velocity and acceleration of the valve motion respectively. The constant vectors $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ depend totally on physical parameters of the valve train system. This could be confirmed through experimental results that have been presented in Norton (2002) and Rothbart (2004). Alternatively, the cam displacement curve *y*, that is a function of the cam rotation angle *x*, can be approximated by a Fourier series in order domains. If the camshaft rotation speed is denoted to be ω in *rad*/sec, the external force at ω can be rewritten as

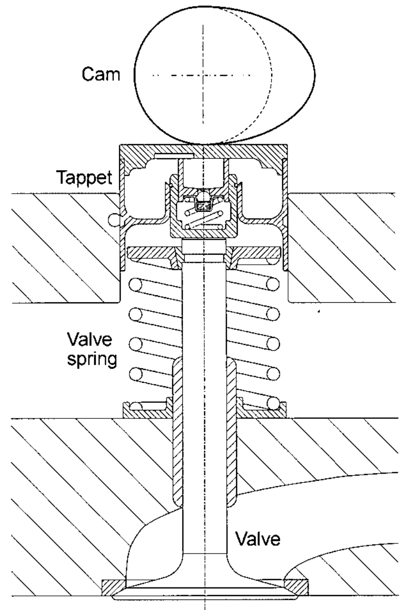


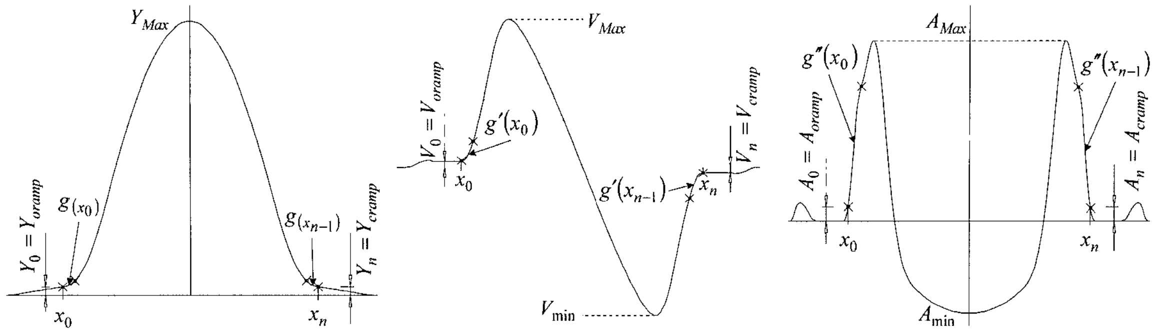
Figure 1. A typical direct acting type OHC valve train.

$$\mathbf{F}(t)=\sum_{\kappa=1}^{n}\mathbf{F}^{\kappa}\alpha^{\kappa}e^{j\kappa\omega t} \quad (3)$$

Here, $\mathbf{F}^{\kappa}=\mathbf{F}_1+j\kappa\omega\mathbf{F}_2-(\kappa\omega)^2\mathbf{F}_3$ is the Fourier coefficient vector of the external force at the *k*-th harmonic order and camshaft rotation speed is ω. α^{κ} is the Fourier coefficient of the cam displacement curve at κ-th harmonic order. And *n* is number of harmonic components employed to analyze the system while $j^2=-1$.

Considering steady state response of Equation (1), large response may occur when one of the excitation frequencies in Equation (3) coincides with (or is multiple of) the natural frequency of the system. This is called resonance, which occurs at the critical speed of the respective harmonic number. Furthermore, the magnitude of the response at resonance depends primarily on the Fourier coefficient α^{κ} of the cam profile curve. As a result, a harmonic analysis of the cam profile curve will give an indication of the performance that is to be expected.

In general, with any periodic function that has a finite number of singular points, we can always find a convergent Fourier series to represent it. In other words, magnitude of the coefficients α^{κ} will decrease as harmonic



(a) Cam displacement (b) Cam velocity (c) Cam acceleration

Figure 2. Opening and closing ramps in a typical automotive engine cam profile.

order increases. However, as shown in the formula of $\mathbf{F}^{\kappa}$, acceleration terms in the excitation force are proportional to the square of both the harmonic order (κ) and the camshaft rotational speed (ω). Therefore, it is clear that high order components in the cam profile are also important source of valve train vibrations. In order to design a dynamically stable cam follower system, we must reduce the amplitude of the high order harmonic components. Base on Darboux's principle that can be found in Boyd (2000), the smoother the function y is, the faster the convergence of its spectra series α^{κ} will be. This provides a strong motivation to smooth cam profiles.

2.2. Modified Smoothing Spline Curve

In design stage, cam designers usually work with some requirements on displacement, velocity and acceleration curves. Figure 2 shows displacement, velocity and acceleration curves of a typical cam profile that is used in an automotive engine. The main event of the cam profile is connected with the opening and closing ramps. To prevent impulsive valve opening and closing, ramp properties must be precisely controlled in consideration of dynamics of valve trains. In the cam profile smoothing process, it is desirable that only the principal cam motion is modified without any changes in ramp properties. In other words, principal profile must be exactly connected with the original ramps up to the acceleration at the connection points (x_0, and x_n in Figure 2). It is assumed that Y_i are a set of given displacement of the original cam profile at corresponding camshaft rotation angles x_i with i=0, ..., n and $x_0 < x_1 < \cdots < x_n$. To estimate the deviation between the smoothed and original cam profiles, the mathematical concept of "*residuals*" is introduced, which is the sum of the squares of offsets of the points from the curve. If the smoothed cam profile curve is defined as $g(x)$, the deviation can be bounded as follows.

$$c(x)=\sum_{i=0}^{n}\left(\frac{g(x_i)-Y_i}{\delta Y_i}\right)\leq S \tag{4}$$

Here, $\delta Y_i > 0$ controls the weighting of the given control points in displacement. $S \geq 0$ is named redundant constant that controls the allowable deviation between the smoothed curve and the original cam displacement. By adjusting the weight factors and the redundant constant, designers can tighten or loosen the local constraints in the smoothed cam profile.

As mentioned above, smoothness of the designed cam profile is also important for better performance in high-speed cam follower systems. As reviewed in Reinsch (1967), Craven and Wahba (1979), the *roughness penalty function*, which is defined in Equation (5), estimates the roughness of the spline curve $g(x)$.

$$f(x)=\int_{x_0}^{x_n}[g^{(r)}(x)]^2 dx \rightarrow \min \tag{5}$$

Here, superscript (r) is the derivative order. Because the roughness penalty function contains r-th derivative terms, the function $g(x)$ must be the piecewise polynomial of degree N=2r–1. Therefore, degree of the spline that is used in this method must always be odd. Additionally, the curve must satisfy continuous derivative conditions up to the order of (2r–2).

$$g \in C^{2r-2}[x_0, x_n] \tag{6}$$

On the other hand, the smoothed cam profile must also satisfy exactly all the boundary conditions at x_0, and x_n. We denote that in Figure 2, Y_0, V_0, A_0 represent the opening ramp height, velocity and acceleration, respectively. Similarly Y_n, V_n, and A_n are the closing ramp height, velocity and acceleration. The boundary conditions of the smoothed cam profile at the connection points with the opening and closing ramps can be expressed as follows.

$$\begin{aligned} &g(x_0)_+=Y_0;\ g(x_n)_-=Y_n \\ &g'(x_0)_+=V_0;\ g'(x_n)_-=V_n \\ &g''(x_0)_+=A_0;\ g''(x_n)_-=A_n \end{aligned} \tag{7}$$

Although smoothing methods using B-spline or P-spline (an inheritance of B-spline) may satisfy smoothness and fit conditions, it is not appropriate to apply these methods for smoothing an existing cam profile. One of the problems is that the boundary values of the resulting curves and/or their derivatives are not accurate, due to the characteristics of B-spline (Eilers, 1996 and Kano, 2005). On the other hand, because modifications of control points of B-spline only affect some neighboring segments, the resulting curve based on B-spline may hold some uninvited extraordinary points when boundary conditions are exactly constrained. References can be found in Sandgren and West (1989) or Tsay and Huey Jr. (1993). When the continuity of basic spline curves, which are in the form of $g(x)=\sum_{i=0}^{n-1}\sum_{k=0}^{N} a_{i,j}(x-x_i)^k$, is considered, it is found that the velocity boundary values are dependent on the boundary conditions of displacement and acceleration curve (see in Reinsch, 1967). Therefore, to satisfy the boundary conditions (7), some additional terms must be added to the first and last segments of the basic spline curve. The additional terms may be in any form as long as they satisfy the following condition: the velocity value at boundaries become independent of the boundary conditions of displacement and acceleration curves. However, by considering smoothness property and convenience of calculation, a modified smoothing spline of N degree, $g(x)$, is chosen in the form of Equation (8).

$$g(x)=\begin{cases} \sum_{k=0}^{N} a_{0,k}(x-x_0)^k+e_0(x-x_0)(x-x_1)^N & \text{with } x_0 \le x < x_1 \\ \sum_{k=0}^{N} a_{i,k}(x-x_i)^k,\ x_i \le x < x_{x+1},\ i=1\ldots n-2 & \\ \sum_{k=0}^{N} a_{n-1,k}(x-x_{n-1})^k+e_{n-1}(x-x_{n-1})^N(x-x_n) & \text{with } x_{n-1} \le x < x_n \end{cases} \quad (8)$$

The smoothed cam profile $g(x)$ is a solution of the optimization problem in Equation (5), which satisfies the inequality constraint in Equation (4), the continuity conditions in Equation (6) and the boundary conditions in Equation (7). To solve this problem, the Lagrange multiplier method that converts the constrained problem into an unconstrained one is employed.

As reviewing Boyd and Vandenberghe (2004), we denote the estimative function $\hat{f}(x, \lambda)$ of Equation (5) under constraints (4), and Lagrangian multiplier λ as follows

$$\hat{f}(x, \lambda)=\int_{x_0}^{x_n}[g^{(r)}(x)]^2dx+\lambda\cdot\left\{\sum_{i=0}^{n}\left(\frac{g(x_i)-Y_i}{\delta Y_i}\right)^2-S\right\} \quad (9)$$

The curve $g(x)$ reaches the smoothest profile when $\hat{f}(x, \lambda) \to \min$. The optimal solution may be obtained by the standard methods of the calculus of variations that can be found in Stone (2002). If $h=(x_n-x_o)/n$ and $h_i=x_{i+1}-x_i$ are defined, the variation function of the function $\hat{f}(x, \lambda)$ will be as follows

$$\Psi=[g^{(r)}(x)]^2+\frac{\lambda}{h}\left(\frac{g(x_i)-y}{\delta y}\right)^2 \quad (10)$$

Here, δy is the positive weighting function that corresponds to discrete weighting factors δY_i. Thereby, the necessary condition for the extreme value of function (9) to exist becomes

$$[\Psi]_{g(x)}=\frac{\partial\Psi}{\partial g(x)}+\ldots+(-1)^r\frac{d^r}{dx^r}\left(\frac{\partial\Psi}{\partial g^{(r)}(x)}\right)=0 \quad (11)$$

From continuity condition (6), boundary conditions (7), and necessary condition (11), we can determine the optimal curve $g(x)$. As previously mentioned, the proposed approach aims to be applied in high-speed cam follower systems. Because of physical essence of cam mechanisms, the curve needs to satisfy continuity up to the second order. The optimal curve $g(x)$, which is a smoothed cam profile, needs to be a modified cubic or quintic function.

If operating speed is not very high, a smoothing spline that is based on the modified cubic spline curve can be used. However, because the modified cubic spline is less flexible than modified quintic spline, its smoothness property is sometime unsatisfactory. If cam profiles are operated at high speeds, not only a low peak value but also a continuous jerk curve is required. This property exceeds the ability of the modified cubic spline curves and necessitates the use of a modified quintic spline curve. Therefore, the modified quintic spline would be chosen to present in this paper. This algorithm is presented in Section 3.1.

3. ALGORITHM AND NUMERICAL SOLUTION

3.1. Modified Smoothing Quintic Spline Method

Modified smoothing quintic spline is a set of piecewise polynomials in the form of Equation (8) with N=5. As a result, Equation (11) now becomes as follows

$$\frac{\lambda}{h}\frac{g(x)-y}{\delta y^2}-g^{(6)}(x)=0 \quad (12)$$

Because the sixth order derivative of the curve can be approximated by its fifth order derivative terms, we can obtain the following relation from Equation (12).

$$-2\lambda\frac{g(x_i)-Y_i}{\delta Y_i^2}+g^{(5)}(x_i)_+-g^{(5)}(x_i)_-=0 \quad (13)$$

From the continuity condition (6), this curve must satisfied continuity up to the fourth order. If derivative order is denoted as k=0,..., N−1, the polynomial coefficients can be obtained by a system of equations as follows

$$g^{(k)}(x_i)_- - g^{(k)}(x_i)_+ = \begin{cases} 0, \; k=0,1,2,3,4; \; i=1...(n-1) \\ -\dfrac{2\lambda}{\delta Y_i^2}(g(x_i)-Y_i), \; k=5; \; i=1...(n-1) \end{cases} \quad (14)$$

Because the modified quintic spline in the form of Equation (8) have (6n+2) unknown coefficients in total and Equations (7, 14) have only 6n equations, two more equations are required. Because jerk values at the boundaries are dependent on the other properties (see Craven and Wahba, 1979), derivative of jerk at the boundaries are assigned to be zero.

$$g^{(4)}(x_0)_+ = g^{(4)}(x_n)_- = 0 \quad (15)$$

Therefore, (6n+2) polynomial coefficients of the modified quintic spline can be obtained by solving the system of (6n+2) Equations (7, 14, 15). These coefficients are now functions of the Lagrange multiplier. If Lagrange multiplier is determined, the set of piecewise polynomials in Equation (8), which represents a smoothed cam profile, is determined.

3.2. Determination of Lagrange Multiplier

Polynomial coefficients $a_{i,0}$ of the smoothed spline curves is defined in the form of vector $\mathbf{a}=\{a_{1,0}..a_{i,0}..a_{n-1,0}\}^T$. The initially given cam displacement is also defined as $\mathbf{y}=\{Y_1..Y_i..Y_{n-1}\}_T$. Therefore, an additional equation for the objective function to be minimum can be written in a matrix form as

$$\frac{\partial f(x,\lambda)}{\partial \lambda} = \sum_{i=0}^{n}\left(\frac{g(x_i)-Y_i}{\delta Y_i^2}\right)^2 - S = (\mathbf{a}-\mathbf{y})^T\mathbf{D}^{-2}(\mathbf{a}-\mathbf{y}) - S = 0 \quad (16)$$

Here $\mathbf{D}$ is a diagonal matrix, whose diagonal terms are equal to the weighting factors of control points. Lagrange multiplier λ must be determined to obtain the coefficients of the piecewise polynomials in the smoothing spline curves from Equations (7, 14, 15). Because the coefficients of the spline are functions of λ, the left-side of Equation (16) is also a function of the Lagrange multiplier. Therefore, the Lagrange multiplier can be obtained from Equation (16). However, analytic solutions to Equation (16) are almost impossible to find. Accordingly, numerical solution can be regarded as a realistic option. In this study, the Newton-Raphson method was chosen for the problem. It is easy to verify from Equation (16) that λ=0 is one of its solutions. However, this is a trivial solution, which makes the spline curve a straight line.

If we define $G(\lambda)=\|(\mathbf{a}-\mathbf{y})^T\mathbf{D}^{-2}(\mathbf{a}-\mathbf{y})\|_2$, it is a function of λ. Equation (16) is now rewritten as $G^2(\lambda)=S$. Because $G(\lambda)$ is a function of Lagrange multiplier only, we can find derivative of $G^2(\lambda)$ as

$$G(\lambda)\frac{dG(\lambda)}{d\lambda} = (\mathbf{a}-\mathbf{y})^T\mathbf{D}^{-2}\frac{d\mathbf{a}}{d\lambda} \quad (17)$$

$G(\lambda)$, $\mathbf{a}$, and $d\mathbf{a}/d\lambda$ are completely evaluated for each value of λ. Therefore, at the i-th step, $d\lambda_i$ can be determined as follows from Equation (17).

$$d\lambda_i = \frac{G^2(\lambda_i) - \sqrt{S}\cdot G(\lambda_i)}{(\mathbf{a}|_{\lambda=\lambda_i} - \mathbf{y})^T \cdot \mathbf{D}^{-2} \cdot (d\mathbf{a}/d\lambda)|_{\lambda=\lambda_i}} \quad (18)$$

As a result, the value of λ at the next step in the Newton-Raphson method is updated as

$$\lambda_{i+1} = \lambda_i - d\lambda_i \quad (19)$$

4. ILLUSTRATIVE EXAMPLE

This section presents an example to illustrate the application of our proposed method to smooth an existing cam profile. The results are compared with that obtained by Reinsch's algorithm. A typical asymmetric DRRD (Dwell-Rise-Return-Dwell) cam profile, which is used in an automotive engine valve train, is considered. Physical parameters of the cam profile are summarized in Table 1.

Figure 3 compares displacement, velocity, acceleration and jerk curves of the three cam profiles. One is the original cam profile before smoothing and the others are cam profiles smoothed by Reinsch's algorithm and the proposed algorithm. Because the cam profile modification by smoothing is very small, it is difficult to distinguish between the original and the smoothed cam profiles in displacement and velocity curves (Figure 3a, 3b). This fact ensures that cam profile smoothing has no meaningful influence on the intake and exhaust flow efficiency.

Table 1. Design parameters of a cam profile.

Maximum cam lift	Y_{max}=9.85 (mm)
Maximum velocity	V_{max}=0.285 (mm/deg)
Minimum velocity	V_{min}=−0.280 (mm/deg)
Max. acceleration (rise)	A_{max1}=0.0147 (mm/deg^2)
Max. acceleration (return)	A_{max2}=0.0148 (mm/deg^2)
Minimum acceleration	A_{min}=−0.0092 (mm/deg^2)
Angles of ramp connection	[x_0, x_n]=[−0.62°, 62.0°]
Opening ramp height	Y_0=0.05 (mm)
Closing ramp height	Y_n=0.15 (mm)
Opening ramp velocity	V_0=0.0151 (mm/deg)
Closing ramp velocity	V_n=−0.0151 (mm/deg)
Opening ramp acceleration	A_0=7.8×10^{-4} (mm/deg^2)
Closing ramp acceleration	A_n=3.4×10^{-4} (mm/deg^2)

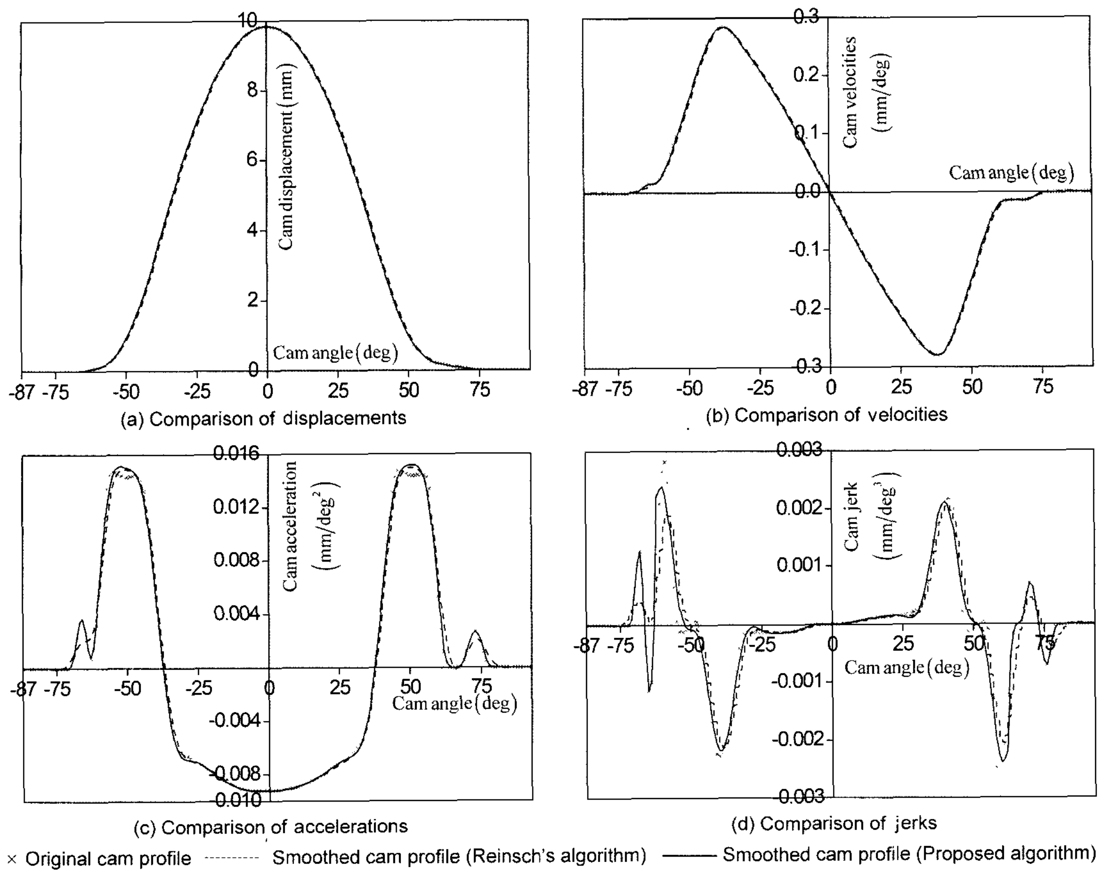


Figure 3. Comparison of the original and the smoothed cam profiles.

However, cam profile changes by smoothing are noticeable in the acceleration and jerk curves (Figure 3c, 3d). Because of the inherent characteristics of the smoothing spline, both algorithms generate smooth acceleration curves (Figure 3c). Peak values of the jerk curves are significantly reduced by the cam profile smoothing (Figure 3d), which can improve dynamic performance of high-speed valve train system.

As long as we are concerned with the main event of the cam profile, there is no big difference between Reinsch's algorithm and the proposed algorithm. Both the algorithms can generate relatively smoother acceleration curves and reduce jerk levels. However, the most critical problem of Reinsch's algorithm is that ramp properties are damaged

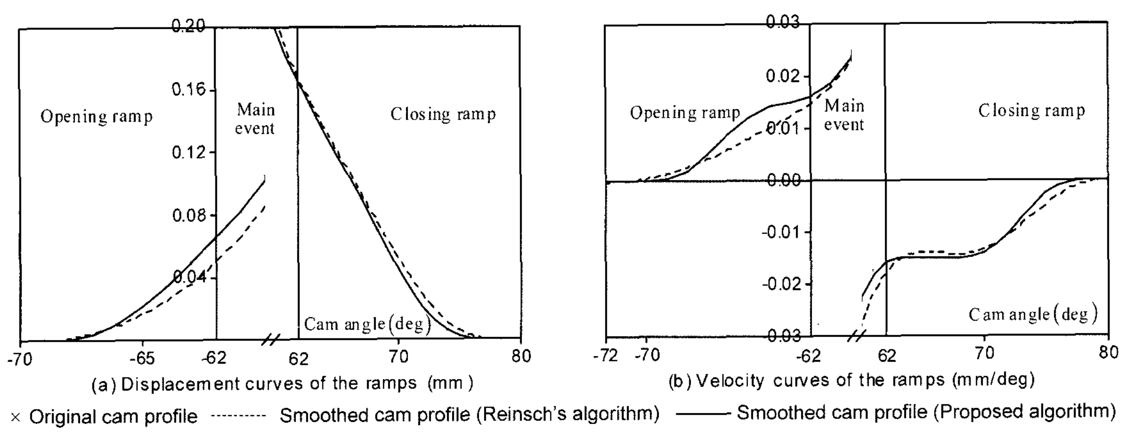


Figure 4. Comparison of the ramp properties of the original and the smoothed cam profiles.

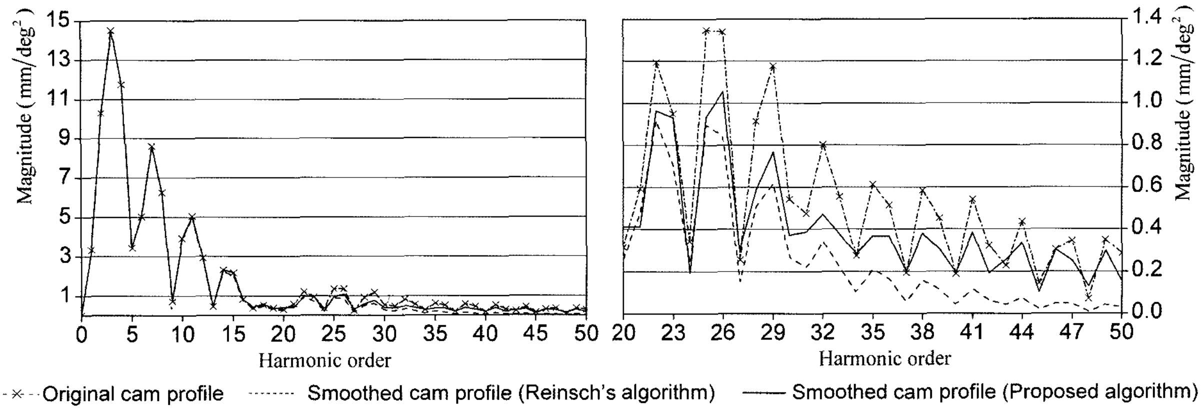


Figure 5. Comparison of harmonic amplitudes of the acceleration of the original and the smoothed cam profiles.

Table 2. Comparison of ramp heights and velocities between original cam profile and smoothed cam profiles.

Rot. Angle (deg)	Displacement (mm)			Velocity (mm/deg)			Rot. Angle (deg)	Displacement (mm)			Velocity (mm/deg)		
	Orig. cam profile	Reinsch method	Our method	Orig. cam profile	Reinsch method	Our method		Orig. cam profile	Reinsch method	Our method	Orig. cam profile	Reinsch method	Our method
(A) Opening ramp							(B) Closing ramp						
-79	0.0000	0.0001	0.0000	0.0000	-0.0000	0.0000	60	0.2001	0.2107	0.2019	-0.0213	-0.0277	-0.0226
-78	0.0000	0.0001	0.0000	0.0000	-0.0000	0.0000	61	0.1814	0.1866	0.1818	-0.0174	-0.0217	-0.0180
-77	0.0000	-0.0000	0.0000	0.0000	-0.0001	0.0000	**62**	**0.1652**	**0.1673**	**0.1652**	**-0.0157**	**-0.0177**	**-0.0157**
-76	0.0000	-0.0001	0.0000	0.0000	-0.0001	0.0000	63	0.1500	0.1512	0.1500	-0.0151	-0.0154	-0.0151
-75	0.0000	-0.0003	0.0000	0.0000	-0.0002	0.0000	64	0.1350	0.1366	0.1350	-0.0150	-0.0142	-0.0150
-74	0.0000	-0.0006	0.0000	0.0000	-0.0003	0.0000	65	0.1200	0.1227	0.1200	-0.0150	-0.0139	-0.0150
-73	0.0000	-0.0009	0.0000	0.0000	-0.0003	0.0000	66	0.1050	0.1088	0.1050	-0.0150	-0.0140	-0.0150
-72	0.0000	-0.0012	0.0000	0.0000	-0.0003	0.0000	67	0.0900	0.0947	0.0900	-0.0150	-0.0142	-0.0150
-71	0.0000	-0.0015	0.0000	0.0000	-0.0001	0.0000	68	0.0750	0.0805	0.0750	-0.0150	-0.0142	-0.0150
-70	0.0000	-0.0014	0.0000	0.0000	0.0004	0.0000	69	0.0600	0.0663	0.0600	-0.0148	-0.0139	-0.0148
-69	0.0000	-0.0006	0.0000	0.0004	0.0012	0.0004	70	0.0455	0.0526	0.0455	-0.0140	-0.0133	-0.0140
-68	0.0007	0.0011	0.0007	0.0019	0.0025	0.0019	71	0.0321	0.0398	0.0321	-0.0124	-0.0121	-0.0124
-67	0.0037	0.0043	0.0037	0.0049	0.0041	0.0049	72	0.0206	0.0284	0.0206	-0.0102	-0.0105	-0.0102
-66	0.0105	0.0094	0.0105	0.0088	0.0061	0.0088	73	0.0117	0.0188	0.0117	-0.0075	-0.0086	-0.0075
-65	0.0214	0.0164	0.0214	0.0123	0.0081	0.0123	74	0.0056	0.0112	0.0056	-0.0048	-0.0065	-0.0048
-64	0.0351	0.0255	0.0351	0.0143	0.0101	0.0143	75	0.0021	0.0057	0.0021	-0.0026	-0.0046	-0.0026
-63	0.0500	0.0367	0.0500	0.0151	0.0123	0.0151	76	0.0005	0.0021	0.0005	-0.0010	-0.0029	-0.0010
-62	**0.0652**	**0.0500**	**0.0652**	**0.0159**	**0.0147**	**0.0159**	77	0.0000	-0.0001	0.0000	-0.0002	-0.0016	-0.0002
-61	0.0817	0.0661	0.0822	0.0180	0.0180	0.0185	78	0.0000	-0.0011	0.0000	-0.0000	-0.0006	-0.0000
-60	0.1012	0.0861	0.1029	0.0223	0.0226	0.0234	79	0.0000	-0.0014	0.0000	0.0000	-0.0001	0.0000

by the profile smoothing. Reinsch's algorithm does not distinguish between the main event and ramps, therefore, the whole ramp profiles are changed from its original design (Table 2 and Figure 4a, 4b). Because the ramp heights and velocities must be precisely controlled to maintain the valve opening and closing performances, it is difficult to use Reinsch's algorithm in automotive engine cam profile smoothing. On the other hand, the proposed algorithm makes only the main event of the cam profile smooth without any damages on the ramp areas. Displacement, velocity and acceleration of the smoothed cam profile are exactly connected with the original ramps (Table 2). Therefore, the proposed algorithm could be effectively used to smooth automobile engine cam profiles.

Figure 5 shows harmonic amplitudes of the three cam profiles. At low order ranges (1st~2th harmonics), magnitudes of the harmonic components of the three cam profiles are almost identical. This result was already expected because cam profile modification by smoothing is very small. The differences between the harmonic amplitudes are only obvious at high order components (20th ~50th). As shown in Figure 5, magnitude of the harmonic components of the smoothed cam profiles decreases faster than that of the original cam profile. The harmonic amplitudes of the cam profile obtained by the proposed algorithm are smaller than those of the original, but are generally greater than those of Reinsch's algorithm. This result also expected because the proposed algorithm smoothes only the main event while Reinsch's algorithm smoothes the whole period of the cam profile. It is inevitable for the unsmoothed ramp to produce some unwanted high frequency excitation components. However, because actual valve motion does not appear during the ramp periods, these differences do not create any negative effect on the valve train internal vibration.

Harmonic amplitudes of the cam profile obtained by the proposed method are almost 50% reduced in high order components (20th~50th). Because these harmonic components usually coincide with valve train natural frequencies in the operating speed range of automotive engines, reduction of the harmonic amplitudes can significantly improve valve train internal vibration. However, because harmonic amplitude reduction in 9th~13th order is negligible, suppression of valve spring surging cannot be expected by profile smoothing.

5. CONCLUSIONS

In this paper, a new cam profile smoothing method is presented, which uses modified smoothing spline curves. The conclusions are summarized as follows:

(1) Unlike other existing algorithms, the proposed method can smooth a given cam profile without any damages on its ramp areas. Although main event of the cam profile is modified by smoothing, the ramp height, velocity and acceleration are maintained exactly. Therefore, the method can be a powerful tool of cam profile smoothing, which removes high frequency components in the cam profile excitations without any changes in ramp properties.

(2) Even though the proposed method smoothes only the main event of the cam profile, high frequency components in the cam profile excitation can be reduced up to 50% in a typical application to an automotive engine cam.

(3) Because relatively low order (1st-20th) harmonic components in the cam profile excitation cannot be reduced by the profile smoothing, it is not a solution to suppressing valve spring surging.

(4) Cam designers can use this method to improve NVH characteristics of a cam follower system. Because ramp properties are not changed, valve train dynamics in high operating speeds can also be improved without any negative effect on the engine performance.

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