Tracking Error Performance of Tracking Filters Based on IMM for Threatening Target to Navel Vessel

Tae Hyun Fang and Jae Weon Choi*

Abstract: Tracking error performance is investigated for the typical maneuvering pattern of the anti-ship missile for tracking filters based on IMM filter in both clear and cluttered environments. Threatening targets to a navel vessel can be categorized into having three kinds of maneuvering patterns such as Waver, Pop-Up, and High-Diver maneuvers, which are classified according to launching platform or acceleration input to be applied. In this paper, the tracking errors for three kinds of maneuvering targets are represented and are investigated through simulation results. Studying estimation errors for each maneuvering target allows us to have insight into the most threatening maneuvering pattern and to construct the test maneuvering scenario for radar system validation.

Keywords: Error monitoring IMM filter, maneuvering patterns, naval vessel.

1. INTRODUCTION

Naval vessel protection is carried out by well-armed weapon systems, which cope with conflict using various potential attack systems such as submarines, aircraft, and other ships. Hence, it is required that a naval vessel have an on-board searching system for detection of potential attack. The observed threats are assigned first to long-range missile systems (area defense) if sufficiently distant, and then those are assigned to surface-to-air missile (SAM) systems or to artillery fire (point defense) if at shorter ranges [1]. When a ship-to-ship engagement takes place, the main threat to a ship is the anti-ship sea-skimming missile, because of its high execution probability and its low detectability [1]. The anti-ship missiles are often called sea skimmers because they attempt to fly at an extremely low elevation where search radars do not see well.

In this paper, maneuvering patterns of an anti-ship missile, which is the main threat to a naval vessel, are considered as having three kinds of maneuvering patterns: Waver, Pop-Up, and High-Diver maneuvers. Waver maneuver is a typical pattern of the missile from a submarine to a ship. Pop-Up maneuver is a typical pattern of the missile from a ship to a ship, and usually, many anti-ship missiles contain both sea-skimming and Pop-Up maneuvers. The High-Diver maneuver is the typical pattern of the missile from an aircraft to a ship. Furthermore the missile launched from a ship may contain all three maneuvers in order to evade the tracker, to change the course, or to re-attack the target ship.

The above mentioned maneuvering patterns of an anti-ship missile are basically considered for a target motion to evaluate the estimation characteristics of the employed tracking filters according to the maneuvering patterns.

For its recursive structure and capability of sequential data processing, Kalman filter is widely used as a tracking filter to estimate position, velocity, and acceleration of a target in real-time. However, since the maneuver of a target may occur unexpectedly, a single Kalman filter of fixed structure is not appropriate to tracking a maneuvering target. The Interacting Multiple Model (IMM) filter was developed as the suboptimal hybrid filter using multiple model approach and has been commonly applied to target the tracking area and other various industrial areas [2-6] because of its cost-effectiveness. The modified IMM filter, which is capable of detecting abrupt change, was proposed to cope with the radical maneuver [7]. In a cluttered environment, we consider Interacting Multiple Model with Probabilistic Data Association (IMMPDA) filter [8], which combines the Probabilistic Data Association (PDA) filter [9] with the IMM filter. In this paper, in

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Tae Hyun Fang is with the WIG Craft Research Division, Korea Ocean Research and Development Institute, 171 Jang-dong, Yuseong-gu, Daejeon 305-343, Korea (e-mail: thfang@moeri.re.kr).

Jae Weon Choi is with the School of Mechanical Engineering, Pusan National University, San 30, Jangjeon-dong, Geumjeong-gu, Busan 609-735, Korea (e-mail: choijw@pusan.ac.kr).

* Corresponding author.

order to investigate the performance of the tracking error for the multiple model approach, we employ the IMM and IMMPDA filters, and the modified IMM and IMMPDA filters.

Estimation characteristics according to the maneuvering patterns of a target are analyzed by examining the root mean square errors of the target position and speed through simulations using the IMM filter and other filters stemmed from the IMM filter.

2. TARGET TRACKING FILTERS

Consider a dynamic system (target model) of standard and linear discrete time-invariant measurement equation [10]

$$x^t(k+1) = F^t x^t(k) + w^t(k), \tag{1}$$

$$z(k) = Hx(k) + v(k), \tag{2}$$

where $x^t(k)$ is a state vector at time k for model $t(t=1,2,3)$, $z(k)$ is a measurement vector at time k, and $w(k)$ and $v(k)$ are zero-mean, mutually independent white Gaussian noise vectors with covariance matrices Q_k and R_k, respectively.

The basic assumption of the IMM filter is that the system corresponds to one of a finite number of models with known parameters, and model switches occur as a Markov chain with known transition probabilities. In this approach, the state estimate is computed under each possible model hypothesis, with as many filters as the number of the assumed models and with each filter using a different combination of the previous model conditioned estimates.

$M_t(k)$ is the event that model M_t with $t=1, 2, \cdots, N$ is effective during the k-th sampling period, and Z^k is the cumulative set of measurements. Then the basic equations of the IMM filter are as follows:

$$\hat{x}^t(k \mid k) = E\left\{x(k) \mid M_t(k), Z^k\right\}, \tag{3}$$

$$P^t(k \mid k) = E\Big\{\left[x(k) - \hat{x}^t(k \mid k)\right] \times \left[x(k) - \hat{x}^t(k \mid k)\right]' \mid M_t(k), Z^k\Big\}, \tag{4}$$

$$\mu_t(k) = p\left\{M_t(k) \mid Z^k\right\}, \tag{5}$$

where the prime denotes transpose transformation and the predicted state is denoted by $\hat{x}^t(k \mid k-1)$.

The estimate of the state and its covariance are obtained by using the total probability theorem with respect to the model events as follows:

$$\hat{x}(k \mid k) = \sum_{t=1}^{N} \hat{x}^t(k \mid k)\mu_t(k), \tag{6}$$

$$P(k \mid k) = \sum \mu_t(k)\Big\{P^t(k \mid k) + \left[\hat{x}^t(k \mid k) - \hat{x}(k \mid k)\right]\left[\hat{x}^t(k \mid k) - \hat{x}(k \mid k)\right]'\Big\}. \tag{7}$$

The reader may refer to [2,3,5] for details.

In considering the cluttered environment with single target, the IMMPDA filter can be employed for tracking a target instead of the IMM filter. Under the fixed hypothesis assumption in the IMM, the estimates for the given model are calculated by using Kalman filter. The IMMPDA filter can be obtained by simply replacing the Kalman filter by the PDA filter. In this paper in order to investigate the performance of the state estimation error for a possible maneuvering pattern of an anti-ship missile, IMM and IMMPDA filters are employed to track a target in both a cluttered and a clear environment, respectively.

Furthermore, in order to compare the estimation results, we employ the modified IMM and IMMPDA filters proposed by Choi *et al.* [7]. Filters of Choi provide good performance for the estimation error in tracking a target with a high maneuvering motion. When a target is in a high maneuvering motion, the assumption of IMM that the target dynamic systems obey the one of a finite number of the predetermined models does not usually match with the target motion even though the predetermined model set includes the transition model such as a constant acceleration model with high intensity of process noise. Therefore, all of the predicted states obtained from the model set include the error caused by the high maneuver.

From the assumption of Gaussian noise in (1) and (2), it can be seen that each predicted state constructs an uncertainty ellipsoid whose origin is the predicted state and whose shape is formed by the associated covariance matrix. That is, it can be considered that the real target exists almost within the uncertainty ellipsoid and the probability that the target is beyond the uncertainty ellipsoid is very low. For a given bound of the uncertainty ellipsoid, if all of the uncertainty ellipsoid doesn't have a common intersection between the predicted state and the measurement, it can be seen that the state propagation provides the wrong information. The existence of common intersection is investigated for the boundary c^t through the following test:

$$\left[\hat{x}^t(k \mid k) - \hat{x}^t(k \mid k-1)\right]' P^t(k \mid k-1)^{-1} \times \left[\hat{x}^t(k \mid k) - \hat{x}^t(k \mid k-1)\right] \le c^t \quad \text{for } t = 1,2,3. \tag{8}$$

The fixed size window is employed in order to determine the state propagation failure. Using (8) and the window length g, the state propagation error caused by the unpredicted target motion can be detected by checking the equality of the following

$$\sum_{i=k-g+1}^{k}\sum_{t=1}^{N}U\Big\{\Big[\hat{x}^t(i\,|\,i)-\hat{x}^t(i\,|\,i-1)\Big]' \times P^t(i\,|\,i-1)^{-1}\Big[\hat{x}^t(i\,|\,i)-\hat{x}^t(i\,|\,i-1)\Big]-1\Big\}=gN, \tag{9}$$

where $U\{\cdot\}$ denotes a unit step function.

If the state propagation turns out to be incorrect, it is desirable to put a weight on the measurement for better estimation. In order to keep the advantage of IMM and put a weight on the measurement, the simple formulation is given by

$$\hat{x}^a(k\,|\,k)=\hat{x}(k\,|\,k) + G^a(k)\{z(k)-H\hat{x}(k\,|\,k)\}, \tag{10}$$

$$P^a(k\,|\,k)=\Big\{I-G^a(k)H\Big\}P(k\,|\,k), \tag{11}$$

$$G^a(k)=P(k\,|\,k)H'\times\{HP(k\,|\,k)H'+R(k)\}^{-1}. \tag{12}$$

The detection equation of (9) is extended into the IMMPDA by simply introducing the validated measurements. Extending (10)-(12) into IMMPDA requires the minimum mean-square estimates of measurement in order to replace the measurement of (10) with the fused measurement. In order to distinguish the modified filters from the conventional IMM and IMMPDA filters, we hereinafter name the modified filters including the process of (8)-(12) as EIMM (Error monitoring and recovery IMM) and EIMMPDA (Error monitoring and recovery IMMPDA), respectively, to signify error monitoring capability.

3. TARGET MANEUVERING PATTERNS

Maneuvering patterns of the anti-ship missile, which is the main threat to naval vessels, are consistent of three types: Waver, Pop-Up, and High-Diver maneuvers. These maneuvering patterns are illustrated in Fig. 1. It is assumed that the origin of the coordination system is located at the defense ship with NWU frame where the axes are north (N), west (W), and up (U). The reasonable trajectories of a missile can be constructed by the parameters of the below description.

The initial conditions for target trajectories are determined by considering anti-ship missiles such as the Harpoon. The typical trajectory of the Harpoon includes pop-up, waver, high-diver maneuvers, and sea skimming during flight until striking the target ship after launching from the platform. Although the air speed of the Harpoon is about Mach 0.9, we set up about Mach 1.4 for the considered target because the air speed of the anti-ship missile is usually between Mach 0.6 and Mach 2.0. Therefore, the initial speeds of the target are determined as the vectors whose magnitudes are about 500m/s. In addition, we have employed the assumption that when the missile changes the course for the evasive maneuvering the air speed will not exceed Mach 1.4.

The Pop-Up maneuver is the typical pattern of the missile from ship to ship, and usually, many anti-ship missiles perform both sea-skimming and Pop-Up maneuvers. The initial position of the missile is at about 9 km distance apart from the origin. During ship-to-ship engagement, the trajectory of the anti-ship missile consists of the cruising phase and the final phase. It is assumed in simulation that the pop-up maneuver happens in the final phase with 4 km distance after the sea-skimming flight is maintained in the cruising phase with 5 km distance. Fig. 2 shows the generated target trajectory, the air speed, and the acceleration input to generate the trajectory. The initial position and velocity are [−8891 4103 15]m and [454 -209 0]m/s in Cartesian coordinates. The acceleration inputs are [0 0 14]m/s^2 between 11.2 sec and 12.56sec and [-0.2 0 -6.5]m/s^2 between

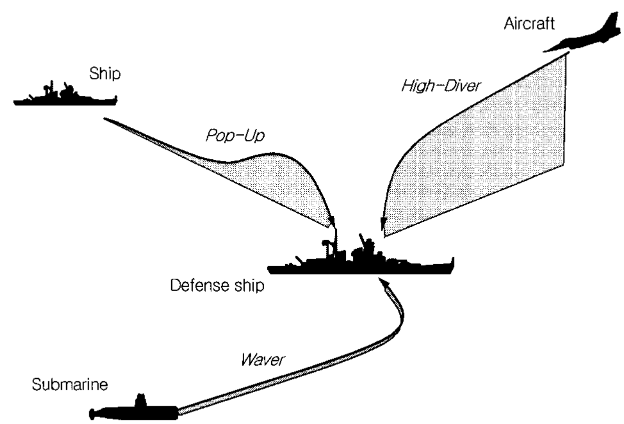

Fig. 1. Representative maneuvering patterns of anti-ship missiles.

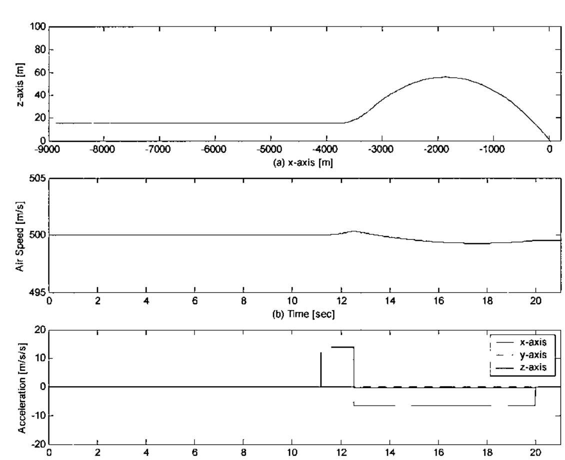

Fig. 2. Pop-Up maneuver: (a) target trajectory, (b) air speed, (c) acceleration input.

12.56sec and 20sec.

Waver maneuver is an evasive maneuvering pattern of an anti-ship missile or a maneuvering pattern of the torpedo from submarine to ship. In this study since the tracking problem for the missile is considered, its motion is given by Fig. 3. The initial position and velocity are [-4106 -5891 15]m and [354 354 0] m/s in Cartesian coordinates. It is assumed that the missile turns with acceleration input [-50 20.7 0] m/s^2 between 8.8sec and 15.16sec so that it passes through the origin.

The High-Diver maneuver is the typical pattern of the missile from aircraft to ship. The specified trajectory in Fig. 4 is generated to evaluate the performance of tracking filters. The initial position is determined by [8891 4103 1000]m in Cartesian coordinates, for which the origin is located at about a 10km distance from there. The initial velocity is [-454 -209 -10]m/s. The acceleration inputs are [0 0 -35]m/s^2 between 12.8sec and 14sec and [12 0 -28]m/s^2 between 14sec and 19sec.

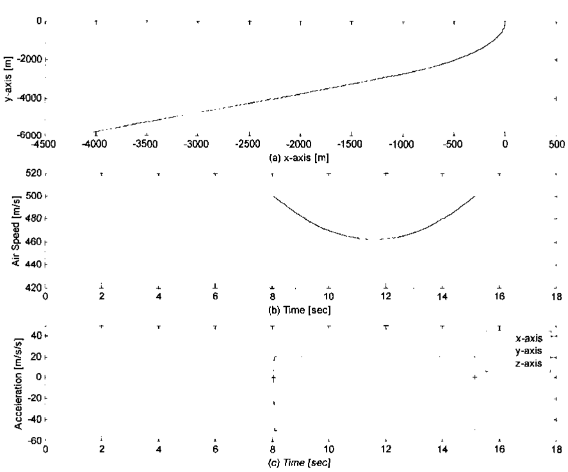

Fig. 3. Target trajectory for Waver maneuvering.

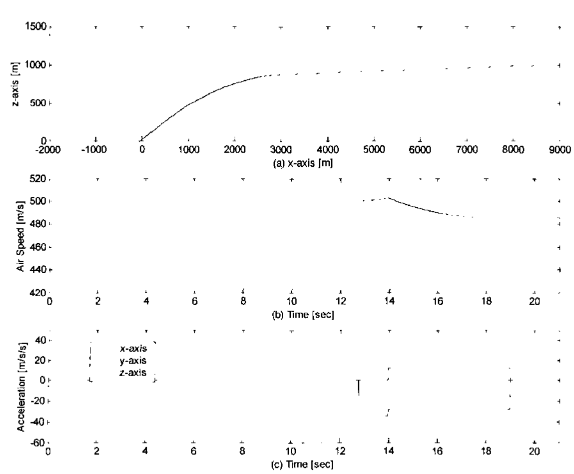

Fig. 4. Target trajectory for High-Diver maneuver.

4. TRACKING ERRORS TO MANEUVERING PATTERNS

In order to investigate the performance of tracking filters according to the maneuvering patterns the simulations of 300 Monte-Carlo runs are fulfilled for the IMM, IMMPDA, EIMM and EIMMPDA filters.

For (1) and (2) the state vector $x \in R^{9\times1}$ consists of target position, velocity, and acceleration for each axis, and the measurement vector $z \in R^{3\times1}$ consists of target position for each axis. For the simulations of the sampling time $T = 0.1s$. the measurement noise covariance $R_{11} = R_{22} = R_{33} = 144\text{m}^2$ and $R_{12} = R_{13} = R_{23} = 9\text{m}^2$ are used. The constant speed model M^1 has process noise with a standard deviation 1m/s. The constant acceleration model M^2 has process noise with a standard deviation of 3m/s^2 and the third model M^3 has process noise with a standard deviation of 10m/s^2. The assumed model switching probabilities (Markov chain transition probabilities) are indicated in Table 1.

The detection probability that depends on the signal-to-noise ratio is set by 0.98 in a cluttered environment. The number of clutter points is generated by a Poisson random process with clutter density of 4 per 1 kilometer. Therefore the number of clutter may be as much as 64 within 1 cubic kilometer. The validation gate of size is given by 6. The value of the effective window length g of (9) is taken to be 2 for EIMM and EIMMPDA filters.

Numerical results for the simulations are shown in Table 2 in the terms of the mean and the standard deviation for the RMS (root mean square) of the estimation errors. Each measure is calculated at both areas of position estimation error (p.e. in Table 2) and velocity estimation error (v.e. in Table 2) for all of the considered filters. Since the magnitudes of the estimation errors are dependent on the involved acceleration input, comparing the measures between the considered filters can provide the insight for the tracking performance. IMM (IMMPDA) and EIMM (EIMMPDA) have the almost identical estimation error for the non-maneuvering trajectories since EIMM (EIMMPDA) enables the estimation errors to be decreased in maneuvering intervals. In addition,

Table 1. Model switching probabilities.

	$M^1(k+1)$	$M^2(k+1)$	$M^3(k+1)$
$M^1(k)$	0.85	0	0.15
$M^2(k)$	0	0.85	0.15
$M^3(k)$	0.33	0.33	0.34

Table 2. Mean and standard deviation of the RMS errors for 300 simulations (p.e., v.e., and std mean position error, velocity error, and standard deviation respectively).

IMM				
		Waver	Popup	Highdiver
p.e.	mean	9.7643	7.0047	8.1353
	std	6.4697	3.1337	4.7495
v.e.	mean	13.8289	6.8233	9.4272
	std	12.6411	4.0074	8.2481
EIMM				
p.e.	mean	9.868	7.0305	8.1448
	std	6.5479	3.0984	4.7396
v.e.	mean	13.9247	6.8834	9.4047
	std	12.6943	3.9951	8.1905
IMMPDA				
p.e.	mean	10.7447	7.3566	8.6006
	std	7.8067	3.3306	5.4871
v.e.	mean	16.2116	6.8293	10.0386
	std	15.427	4.0459	9.3215
EIMMPDA				
p.e.	mean	10.1289	7.2222	8.3512
	std	6.9857	3.2147	5.0112
v.e.	mean	15.0949	6.7819	9.7064
	std	14.0975	3.9951	8.7072

the fused estimates and its covariance matrix of (6), (7) are not used to produce the model probability of (5) in the revised manuscript. Since EIMM and EIMMPDA update the fused estimates and its covariance matrix by using the additional information from the inconsistency detection of the multiple models, the model probabilities of the IMM (EIMMPDA) is totally equal to those of IMM (IMMPDA).

Figs. 5, 6, and 7 represent the estimation errors of the four filters (IMM, IMMPDA, EIMM and EIMMPDA) for the Pop-up, Waver, and High-Diver maneuvers defined in Section 3, respectively. The IMMPDA filter and EIMMPDA filter are carried out in the cluttered environment with non-unitary detection probability. In the simulation results, the vertical axis of the figures represents the RMS estimation errors for the position and the velocity. Therefore those are equal to distance error and air speed error respectively. For the figures, the dashed line and the dashed line with dot denote the IMM and IMMPDA respectively, and the solid line and the solid line with dot denote the EIMM and EIMMPDA respectively.

Fig. 5 indicates the estimation errors of the IMM, IMMPDA, EIMM and EIMMPDA filter when the

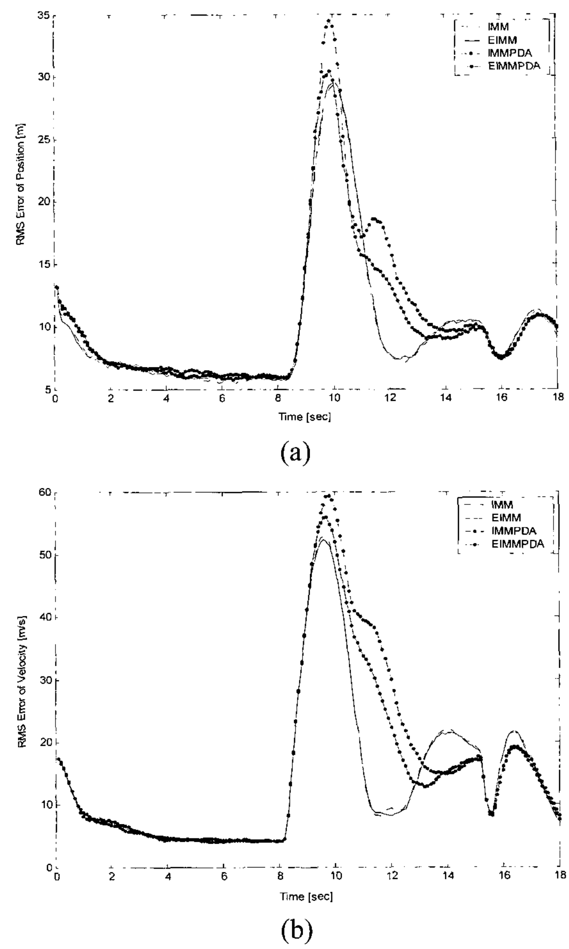

Fig. 5. RMS errors of IMM, EIMM, IMMPDA, and EIMMPDA for Waver maneuver.

target performs Waver maneuver. As seen from Fig. 5, after the target starts to turn with acceleration at 8.8 sec, the estimation errors for the considered filters increase drastically at some interval. However, when the IMM filter adapts to the target maneuver and the inner dominant dynamic model becomes a constant acceleration model, the errors begin to decrease. When the target stops maneuvering, the same results are found because the estimation error depends on the change of the acceleration input. In other words, until the dominant dynamic model becomes a constant speed model, the estimation errors increase. The steady-state errors in acceleration mode are a little higher than those in constant speed mode. However, because of the short interval of the acceleration mode, it is difficult to identify the steady-state error in acceleration mode.

Figs. 5(a) and 5(b) represent the position errors and the velocity errors for the applied four filters, respectively. While the errors for IMM and EIMM have a small difference, the error of EIMMPDA is significantly reduced in comparison with the error of IMMPDA. Therefore, it can be understood that the EIMMPDA can quickly adapt to the target maneuver-

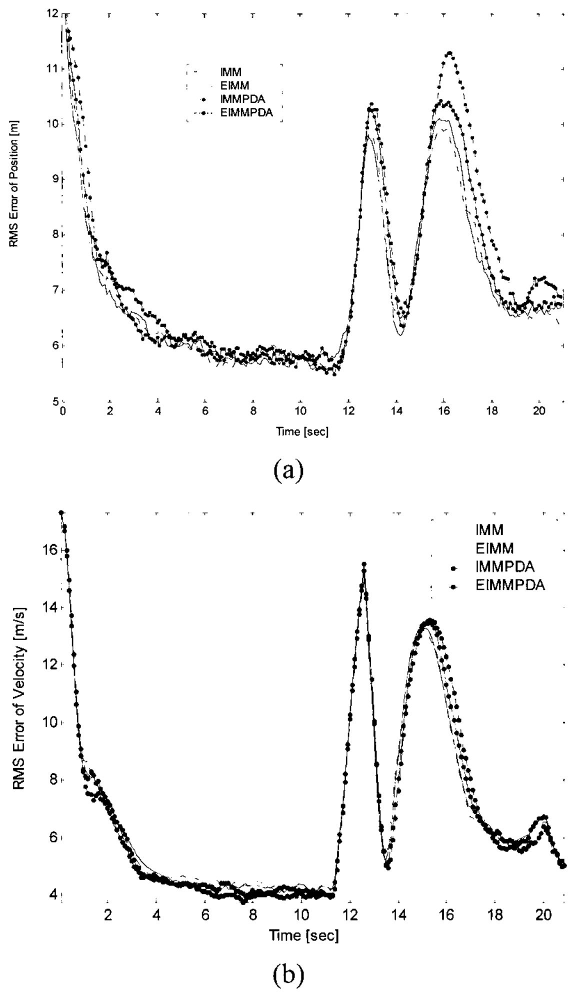

Fig. 6. RMS errors of IMM, EIMM, IMMPDA, and EIMMPDA for Pop-Up maneuver.

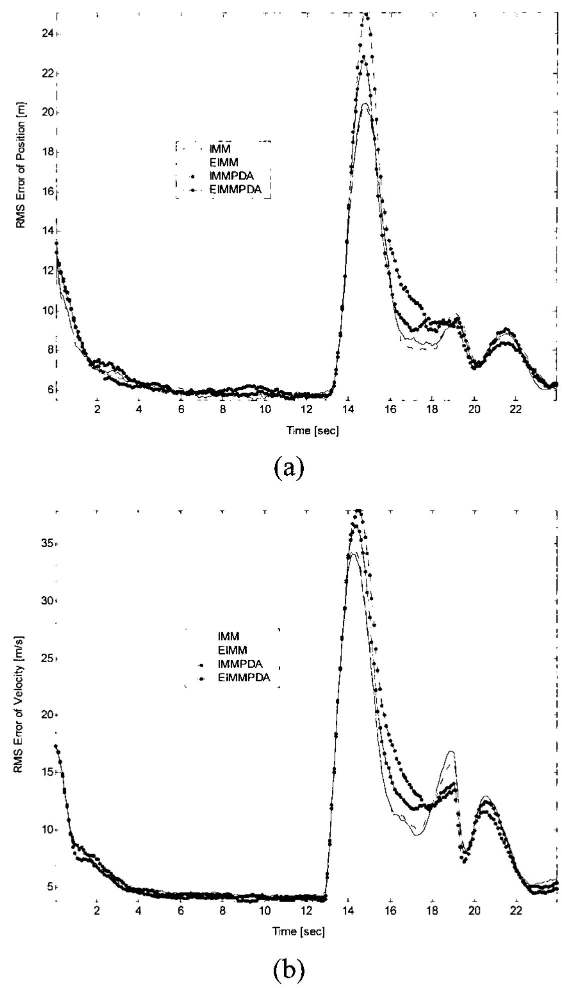

Fig. 7. RMS errors of IMM, EIMM, IMMPDA, and EIMMPDA for High-Diver maneuver.

ing. Missing the target in which the validated region does not include the measurement from the target occurs usually after the abrupt change of the acceleration input because the increased estimation error causes the predicted state to be inaccurate. Then the estimates may converge on the real target again or may diverge as the estimation error increases infinitely. That is why the estimation error fluctuates when it decreases after beginning the maneuver. Additionally, during non-acceleration input and after completing adaptation to maneuvering, the errors for the filters are almost identical to each order.

Fig. 6 shows the position errors (a) and the velocity error (b) when the target performs Pop-Up maneuver. It can be shown in the figures that the magnitude of the errors is relatively significant comparing the applied acceleration input. Because of the acceleration change with the short interval, there is insufficient time for the filter to cope with the target motion. The EIMM and EIMMPDA produce slightly improved error performance since the modified filters have benefits in case of tracking the highly maneuvering target. The missile strikes the ship right after finishing the Pop-Up maneuvering or in performing the maneuver if it is correctly guided. Then the ship encounters the missile with large estimation error, which causes the target to miss. Therefore, Pop-Up maneuver at the final phase can be very threatening to the ship.

Fig. 7 shows the position error (a) and the velocity error (b) for the High-Diver maneuver. This maneuvering pattern is similar to the Waver maneuver in the sense of the acceleration input. In addition, the estimation errors for both maneuvering patterns have the similar aspect because it is assumed that the noise property of the measurement is defined by the same value for all axes. However, the decreased error of EIMMPDA is not as much as the Waver maneuver. It is caused from the fact that the maximum error is governed by the magnitude of the acceleration input. The maximum error with the relatively small value compared to the Waver maneuver allows the errors to be rapidly decreased in a cluttered environment after the acceleration input is terminated.

5. CONCLUSIONS

The performance of tracking errors for anti-ship missiles is investigated in this paper. The investigation has been carried out by employing the four filters called IMM, IMMPDA, EIMM, and EIMMPDA filters, and three maneuvering patterns known as Waver, Pop-up, and High-diver. The magnitude of the estimation error depends on the applied acceleration input and it can be identified by comparing the simulation results for the three maneuvering patterns. The employed filters based on multiple model show that its estimation error can be rapidly decreased when the assumed model corresponds to the target motion. Despite this fact, comparing the model changing approach with maneuvering detection, the employed filters have low decreasing rate. The aspect of the estimation error provides the distinction between the employed filters and the filters changing model. In case that no information about the tracking filter algorithm is given due to security risks and the performance for the tracking system is required to be measured, the results from this study can provide the insight with constructing the test trajectory for the test flight of the radar systems. Furthermore, the estimation errors for the non-multiple model approach will be analyzed in order to investigate the concrete performance for the tracking system.

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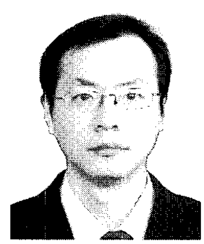

Tae Hyun Fang received the B.S. degree in Mechanical Engineering from Pusan National University, Busan, Korea, in 1994, and the M.S. and Ph.D. degrees in Mechanical and Intelligent Systems Engineering from the same university in 1999 and 2003. He is currently a Senior Research Scientist in the WIG Craft Research Division, Korea Ocean Research and Development Institute, Daejeon, Korea. His current interests include WIG attitude control, sensor fusion techniques, and target-tracking filters.

Jae Weon Choi received the B.S., M.S., and Ph.D. degrees in Control and Instrumentation Engineering from Seoul National University, Seoul, Korea, in 1987, 1989, and 1995, respectively. He is currently a Professor in the School of Mechanical Engineering, Pusan National University, Korea. His current research interests include spectral theory for linear time-varying systems, software enabled control architectures with applications to navigation, guidance and control system design for underwater vehicles, and open-control platform based home networks. He is also interested in target tracking filter design and common control technologies in aircraft and cars. Dr. Choi is a Senior Member of IEEE and AIAA. He is also a Member of both the IFAC Technical Committee on Linear Systems and the Aerospace Committee. Furthermore, he is currently an Editor for the International Journal of Control, Automation and Systems, and an Associate Editor on the Conference Editorial Board of the IEEE Control Systems Society, a position he has held since the year 2000.