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Analysis of Optical Transfer Function and Phase Error of the Modified Triangular Interferometer

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Abstract

We synthesize and analyze the optical transfer function(OTF) of the modified triangular interferometer(MTI) using two-pupil synthesis method, and we present the optimal MTI, which can obtain any bipolar function by combining a wave plate and a linear polarizer. Also, we analyze its potential phase error sources caused by polarization components.

Key Words : Optical Transfer Function, Modified Triangular Interferometer(Mti), Two-Pupil Synthesis, Phase Error

1. Introduction

In a conventional incoherent scanning or imaging system, limitations exist on image processing that are due to the resulting nonnegative intensity spread function[point-spread function(PSF)], which, in turn, imposes severe constraints on both the amplitude and the phase of the optical transfer function(OTF)[1]. Such limitations can be circumvented by introducing a two-pupil system that is characterized by a great flexibility in pupil-function specification for a desired synthesized PSF[2-3]. Any bipolar impulse response can be synthesized by using two-pupil methods as long as the pupil function can be arbitrarily specified. Two-pupil systems are usually implemented by separating the responses(i.e., separating the interactive term and the noninteractive term on the basis of the spatial or temporal carriers)[1, 4-6].

The pupils are created by either amplitude or wave-front divisions[1, 3-4, 6].

The synthesis methods divide into two important classes which are distinguished by the mathematical structure of the resultant transfer function. There are basically two kinds of syntheses possible: nonpupil interaction synthesis and pupil interaction synthesis[1].

Recently, two-pupil synthesis by the MTI was reported[7]. A simple two-pupil interaction system was implemented by adding two wave plates and a linear polarizer to Cochran's triangular interferometer[8]. However, the principle of proposed system was described, but in the viewpoint of two-pupil synthesis, the analysis of the proposed system was not described. In this paper, we introduce two-pupil synthesis of OTF in the MTI, and then we demonstrate that removal of

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bias and conjugate image of the incoherent hologram is possible through OTF synthesis based on two-pupil synthesis, and we present the optimal MTI, which can obtain any bipolar function by combining a wave plate and a linear polarizer.

The phase term of complex hologram in the MTI is obtained from four intensity patterns by phase-shifting technique. A phase shift or modulation in the phase-shifting techniques can be induced by moving a mirror, tilting a glass plate, moving a grating, rotating a half-wave plate or analyzer[9-12].

In MTI, a phase shift is implemented by the combination of polarization components. In the extraction of phase term using the combination of polarization components, the phase error occurs. In this paper, we analyze its potential error sources caused by polarization components.

2. Two-pupil synthesis of OTF in the MTI

2.1 One-pupil synthesis of the MTI

Figure 1 shows the MTI obtained by modification of the triangular interferometer. In Fig. 1, PBS represents a polarizing beam splitter. Lens1 and lens2 are lenses with focal lengths f1 and f2, respectively.

If a linear polarizer and wave plates are removed and the PBS is replaced with an ordinary beam splitter, the system is the same as Cochran's triangular interferometer[8].

Figure 2 shows double afocal systems of the light that travels clockwise and counterclockwise in the MTI. The polarizations of the light that travels clockwise and counterclockwise are vertical and horizontal, respectively.

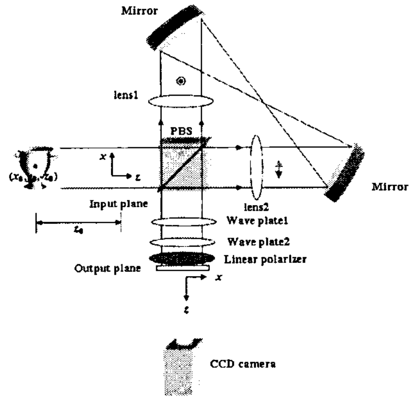

Fig. 1. Modified triangular interferometer

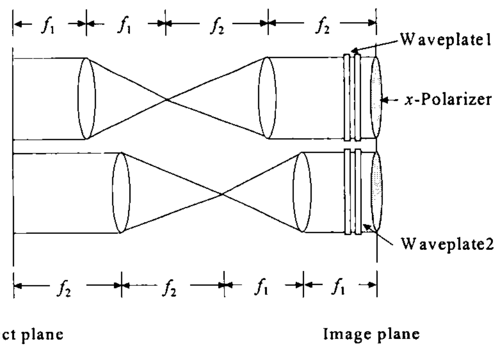

Fig. 2. Double afocal systems for the two beams circulating in opposite directions

Using Jones vector representation[13], the pupil functions of the light that travels clockwise and counterclockwise in one-pupil system of Fig. 2, respectively, are given by

$$P_{mcw} = P_x R(-\psi) W_2 R(\psi) R(-\psi) W_1 R(\psi) P_{cw} = W_{mcw} P_{cw} \quad (1)$$

$$P_{mccw} = P_x R(-\psi) W_2 R(\psi) R(-\psi) W_1 R(\psi) P_{ccw} = W_{mccw} P_{ccw} \quad (2)$$

where P_{mcw} and P_{mccw} denote the pupil functions of the light that travels clockwise and counterclockwise, respectively, and P_x, $R(\psi)$, W_1, and W_2 are defined as

$$P_x=\begin{pmatrix}1&0\\0&0\end{pmatrix},\quad R(\psi)=\begin{pmatrix}\cos\psi&\sin\psi\\-\sin\psi&\cos\psi\end{pmatrix},$$

$$W_1=e^{-i\phi}\begin{pmatrix}e^{-i\Gamma_1/2}&0\\0&e^{i\Gamma_1/2}\end{pmatrix},$$

$$W_2=e^{-i\phi}\begin{pmatrix}e^{-i\Gamma_2/2}&0\\0&e^{i\Gamma_2/2}\end{pmatrix},$$

$$\phi=\frac{1}{2}(n_s+n_f)\frac{\omega l}{c},$$

where P_x represents Jones matrix of a linear polarizer, ψ represents the azimuth angle of slow axis of wave plate with respect to the x axis. Γ_1 and Γ_2 represent the phase retardations of wave plates 1 and 2, respectively. n_s and n_f represent the refractive index of the slow and fast components of wave plates, respectively, and ω, l, c represent the frequency of the light beam, the thickness of wave plate, and the velocity of the light in the vacuum, respectively. And, the pupil functions of the light that travels clockwise and counterclockwise in the triangular interferometer, respectively, are given by[7]

$$P_{cw}(x,y)=\frac{ik}{2\sqrt{2\pi z_0}}\exp\left[-i\frac{k}{2z_0}(\alpha x-x_0)^2+(\alpha y-y_0)^2\right]=a\exp(-i\theta_{cw}) \quad (3)$$

$$P_{ccw}(x,y)=\frac{ik}{2\sqrt{2\pi z_0}}\exp\left[-i\frac{k}{2z_0}(\beta x-x_0)^2+(\beta y-y_0)^2\right]=b\exp(-i\theta_{ccw}) \quad (4)$$

where P_{cw} and P_{ccw} denote the pupil functions of the light that travels clockwise and counterclockwise, respectively, k is the wavenumber, and $\alpha\equiv-f_1/f_2$, $\beta\equiv-f_2/f_1$.

The OTFs corresponding to Eqs. (3) and (4) are given by

$$OTF_{mcw}=P_{mcw}(f_x,f_y)\otimes P_{mcw}(f_x,f_y) \quad (5)$$

$$OTF_{mccw}=P_{mccw}(f_x,f_y)\otimes P_{mccw}(f_x,f_y) \quad (6)$$

where f_x and f_y denote the spatial frequencies.

From the above equations, we see that any bipolar function cannot be produced by the one-pupil system of the MTI.

2.2 Two-pupil synthesis of the MTI

We describe the OTF synthesis of the MTI based on two-pupil synthesis[2, 14]. The effective pupil function of the MTI in Fig. 1 is given by

$$P(f_x,f_y)=\exp\left(-i\frac{\Gamma_1}{2}\right)\left[\cos\frac{\Gamma_2}{2}P_{ccw}(f_x,f_y)-i\exp(i\Gamma_1)\sin\frac{\Gamma_2}{2}P_{cw}(f_x,f_y)\right] \quad (7)$$

In the case of the incoherent system, the OTF of Eq. (7) is given by

$$\begin{aligned}OTF=&\cos^2\frac{\Gamma_2}{2}P_{ccw}(f_x,f_y)\otimes P_{ccw}(f_x,f_y)\\&+\sin^2\frac{\Gamma_2}{2}P_{cw}(f_x,f_y)\otimes P_{cw}(f_x,f_y)\\&+i\exp(-i\Gamma_1)\cos\frac{\Gamma_2}{2}\sin\frac{\Gamma_2}{2}P_{ccw}(f_x,f_y)\otimes P_{cw}(f_x,f_y)\\&-i\exp(i\Gamma_1)\cos\frac{\Gamma_2}{2}\sin\frac{\Gamma_2}{2}P_{cw}(f_x,f_y)\otimes P_{ccw}(f_x,f_y)\end{aligned} \quad (8)$$

The PSF corresponding to Eq. (8) is given by

$$\begin{aligned}h(x,y;\Gamma_1,\Gamma_2)=&\cos^2\frac{\Gamma_2}{2}|P_{ccw}(x,y)|^2+\sin^2\frac{\Gamma_2}{2}\frac{\Gamma_2}{2}|P_{cw}(x,y)|^2\\&+i\exp(-i\Gamma_1)\cos\frac{\Gamma_2}{2}\sin\frac{\Gamma_2}{2}P_{ccw}(x,y)P^*_{cw}(x,y)\\&-i\exp(i\Gamma_1)\cos\frac{\Gamma_2}{2}\sin\frac{\Gamma_2}{2}P_{cw}(x,y)P^*_{ccw}(x,y)\end{aligned} \quad (9)$$

We have to synthesize the OTFs to produce the complex hologram without bias and the conjugate image. Through the OTF synthesis using the combination of the phase retardation of wave plates, we can obtain the cosine and sine functions.

First, in the case in which $\Gamma_1=\pi/2$ and $\Gamma_2=\pm\pi/2$, through the OTF synthesis using Eq. (9), we can obtain the cosine component.

$$h_r(x,y)=\frac{1}{2}\left\{h\left(x,y;\frac{\pi}{2},\frac{\pi}{2}\right)-h\left(x,y;\frac{\pi}{2},-\frac{\pi}{2}\right)\right\} \\ =|P_{cw}(x,y)P_{ccw}(x,y)|\cos\theta(x,y) \quad (10)$$

where $\theta(x,y)=\theta_{cw}(x,y)-\theta_{ccw}(x,y)$. And, in the case in which $\Gamma_1=0$ and $\Gamma_2=\pm\pi/2$, through the OTF synthesis using Eq. (9), we can obtain the sine component.

$$h_i(x,y)=\frac{1}{2}\left\{h\left(x,y;0,-\frac{\pi}{2}\right)-h\left(x,y;0,\frac{\pi}{2}\right)\right\} \\ =|P_{cw}(x,y)P_{ccw}(x,y)|\sin\theta(x,y) \quad (11)$$

We can obtain the complex hologram without bias and the conjugate image using Eqs. (10) and (11). The derived complex hologram using the OTF synthesis is equivalent to the previous result[7] and the complex hologram without bias and without conjugate image is described as follows.

$$H(x,y)=|P_{cw}(x,y)P_{ccw}(x,y)|[\cos\theta(x,y)\pm i\sin\theta(x,y)] \\ =|P_{cw}(x,y)P_{ccw}(x,y)|\exp[\pm i\theta(x,y)] \quad (12)$$

Using the analysis based on the OTF synthesis using two-pupil method, we present another combination of phase retardation of wave plates that can obtain the cosine and sine functions. Examples are described in the following.

First, in the case in which $\Gamma_1=\pm\pi/2$ and $\Gamma_2=\pi/2$, through the OTF synthesis using Eq. (9), we can obtain the cosine component.

$$h_r(x,y)=\frac{1}{2}\left\{h\left(x,y;\frac{\pi}{2},\frac{\pi}{2}\right)-h\left(x,y;-\frac{\pi}{2},\frac{\pi}{2}\right)\right\} \\ =|P_{cw}(x,y)P_{ccw}(x,y)|\cos\theta(x,y) \quad (13)$$

And, in the case in which $\Gamma_1=\pi, 0$ and $\Gamma_2=\pi/2$, through the OTF synthesis using Eq. (9), we can obtain the sine component.

$$h_i(x,y)=\frac{1}{2}\left\{h\left(x,y;\pi,\frac{\pi}{2}\right)-h\left(x,y;0,\frac{\pi}{2}\right)\right\} \\ =|P_{cw}(x,y)P_{ccw}(x,y)|\sin\theta(x,y) \quad (14)$$

We can obtain the complex hologram without bias and conjugate image using Eqs. (13) and (14).

From the above results, we can see that there are several kinds of the OTF synthesis to obtain the complex hologram without bias and conjugate image. Therefore, we can see that there are several kinds of pupil functions to implement the complex hologram in the MTI.

2.3 The optimization of the MTI

In Section 2, we can obtain the complex hologram without bias and without conjugate image by changing the pupil function by the combination of the phase retardation of wave plates in the MTI. But to obtain the sine function, we must obtain the intensity patterns in the MTI without wave plate 1. This means that we need two operation modes to obtain the complex hologram without bias and conjugate image in the MTI. Two operation modes are inconvenient in the case of using the MTI. For example, in obtaining the complex hologram, we must use the MTI with two wave plates to obtain the cosine function, and use the MTI with one wave plate to obtain the sine function. Accordingly, to solve the problem, we need to obtain the cosine function in the MTI with one wave plate. We describe the method of obtaining the cosine function in the MTI with one wave plate in the following.

First, the effective pupil function of the MTI with a linear polarizer oriented at an azimuth of 45 degrees in Fig. 1 is given by

$$P_{45}(f_x,f_y)=\frac{1}{2}\exp\left(-i\frac{\Gamma_1}{2}\right)\times \\ \left\{\left[\left(\cos\frac{\Gamma_2}{2}-i\sin\frac{\Gamma_2}{2}\right)P_{ccw}(f_x,f_y)\right.\right.$$

$$+\exp(i\Gamma_1)\left(\cos\frac{\Gamma_2}{2}-i\sin\frac{\Gamma_2}{2}\right)P_{cw}(f_x,f_y)\Big]\,\hat{x}$$
$$+\Big[\left(\cos\frac{\Gamma_2}{2}-i\sin\frac{\Gamma_2}{2}\right)P_{ccw}(f_x,f_y)$$
$$+\exp(i\Gamma_1)\left(\cos\frac{\Gamma_2}{2}-i\sin\frac{\Gamma_2}{2}\right)P_{cw}(f_x,f_y)\Big]\,\hat{y}\Big\} \quad (15)$$

In the case of the incoherent system, the OTF of Eq. (15) is given by

$$OTF=\frac{1}{2}\left(\cos^2\frac{\Gamma_2}{2}+\sin^2\frac{\Gamma_2}{2}\right)$$
$$\times\{P_{ccw}(f_x,f_y)\otimes P_{ccw}(f_x,f_y)+P_{cw}(f_x,f_y)\otimes P_{cw}(f_x,f_y)$$
$$+\ \exp(-i\Gamma_1)P_{ccw}(f_x,f_y)\otimes P_{cw}(f_x,f_y)$$
$$+\exp(i\Gamma_1)P_{cw}(f_x,f_y)\otimes P_{ccw}(f_x,f_y)\} \quad (16)$$

The PSF corresponding to Eq. (16) is given by

$$h_{45}(x,y;\Gamma_1,\Gamma_2)=\frac{1}{2}\left(\cos^2\frac{\Gamma_2}{2}+\sin^2\frac{\Gamma_2}{2}\right)\{|P_{ccw}(x,y)|^2+|P_{cw}(x,y)|^2$$
$$+\ \exp(-i\Gamma_1)P_{ccw}(x,y)P^*_{cw}(x,y)+\exp(i\Gamma_1)P_{cw}(x,y)P^*_{ccw}(x,y)\} \quad (17)$$

Second, the effective pupil function of the MTI with a linear polarizer oriented at an azimuth of -45 degree in Fig. 1 is given by

$$P_{-45}(f_x,f_y)=\frac{1}{2}\exp\left(-i\frac{\Gamma_1}{2}\right)$$
$$\times\Big\{\Big[\left(\cos\frac{\Gamma_2}{2}+i\sin\frac{\Gamma_2}{2}\right)P_{ccw}(f_x,f_y)$$
$$-\exp(i\Gamma_1)\left(\cos\frac{\Gamma_2}{2}+i\sin\frac{\Gamma_2}{2}\right)P_{cw}(f_x,f_y)\Big]\,\hat{x}$$
$$+\Big[-\left(\cos\frac{\Gamma_2}{2}-i\sin\frac{\Gamma_2}{2}\right)P_{ccw}(f_x,f_y)$$
$$+\exp(i\Gamma_1)\left(\cos\frac{\Gamma_2}{2}+i\sin\frac{\Gamma_2}{2}\right)P_{cw}(f_x,f_y)\Big]\,\hat{y}\Big\} \quad (18)$$

In the case of the incoherent system, the OTF of Eq. (18) is given by

$$OTF=\frac{1}{2}\left(\cos^2\frac{\Gamma_2}{2}+\sin^2\frac{\Gamma_2}{2}\right)\{P_{ccw}(f_x,f_y)\otimes P_{ccw}(f_x,f_y)$$
$$+P_{cw}(f_x,f_y)\otimes P_{cw}(f_x,f_y)$$
$$-\ \exp(-i\Gamma_1)P_{ccw}(f_x,f_y)\otimes P_{cw}(f_x,f_y)$$
$$-\exp(i\Gamma_1)P_{cw}(f_x,f_y)\otimes P_{ccw}(f_x,f_y)\} \quad (19)$$

The PSF corresponding to Eq. (19) is given by

$$h_{-45}(x,y;\Gamma_1,\Gamma_2)=\frac{1}{2}\left(\cos^2\frac{\Gamma_2}{2}+\sin^2\frac{\Gamma_2}{2}\right)\{|P_{ccw}(x,y)|^2+|P_{cw}(x,y)|^2$$
$$-\ \exp(-i\Gamma_1)P_{ccw}(x,y)P^*_{cw}(x,y)-\exp(i\Gamma_1)P_{cw}(x,y)P^*_{ccw}(x,y)\} \quad (20)$$

Then, in the case in which $\Gamma_1=0$ and $\Gamma_2=\pi/2$, through the OTF synthesis using Eqs. (17) and (20), we can obtain cosine component.

$$h_i(x,y)=\frac{1}{2}\left\{h_{45}\left(x,y;0,\frac{\pi}{2}\right)-h_{-45}\left(x,y;0,\frac{\pi}{2}\right)\right\}$$
$$=|P_{cw}(x,y)P_{ccw}(x,y)|\cos\theta(x,y) \quad (21)$$

From Eq. (21), we can see that we obtain the cosine function using two intensity patterns obtained by rotating a linear polarizer by 45 degrees and -45 degrees, respectively, with respect to x axis in the MTI with one wave plate. Accordingly, we can obtain the complex hologram without bias and conjugate image using MTI with one wave plate.

Table 1. Intensity patterns by combination of a wave plate and a linear polarizer

azimuth angle of a linear polarizer	phase retardation of a wave plate	PSF
0	$\Gamma_2=\pi/2$	$1/2\,[1-\sin\theta(x,y)]$
	$\Gamma_2=\pi/2$	$1/2\,[1+\sin\theta(x,y)]$
45	$\Gamma_2=\pi/2$	$1/2\,[1+\cos\theta(x,y)]$
-45	$\Gamma_2=\pi/2$	$1/2\,[1-\cos\theta(x,y)]$

Table 1 shows the PSFs by the combination of the azimuth angle of a linear polarizer and the phase retardation of one wave plate in the MTI. We can see that the complex hologram without bias and without conjugate image is obtained by combining four PSFs in Table 1 electronically.

3. Phase error analysis

The main potential sources of error in the optimized MTI are imperfections of the polarization elements and azimuth angle error of the polarization elements. We shall analyze the effects of these potential error sources one by one.

3.1 Imperfections of polarization elements

We shall deal mainly with the errors that are introduced by imperfections in the λ/4 plate. In this case, we assume that the azimuth angle error of wave plate is zero. In Fig. 3, the Jones matrix of output beam in the output plane is given by

$$E_{out}=A(\varphi_2)WP(\varphi_1)E_{in} \tag{22}$$

where E_{in} represents input optical wave, and $A(\varphi_2)$, $\mathcal{W}(\varphi_1)$ represent Jones matrices of a polarizer and a wave plate, respectively. The Jones matrices of polarization components are given by, respectively,

$$E_{in}=\begin{pmatrix} P_{ccw} \\ P_{cw} \end{pmatrix}=\begin{pmatrix} be^{-j\theta_{ccw}} \\ ae^{-j\theta_{cw}} \end{pmatrix} \tag{23}$$

$$A(\varphi_2)=\begin{pmatrix} \cos^2\varphi_2 & 1/2\sin 2\varphi_2 \\ 1/2\ \sin 2\varphi_2 & \sin^2\varphi_2 \end{pmatrix} \tag{24}$$

$$WP(\varphi_1)=\begin{vmatrix} 2i\sin^2\varphi_1\sin\frac{\Gamma}{2}+e^{-i\frac{\Gamma}{2}} & -i\sin 2\varphi_1\ \sin\frac{\Gamma}{2} \\ -i\sin 2\varphi_1\ \sin\frac{\Gamma}{2} & -2i\sin^2\varphi_1\sin\frac{\Gamma}{2}+e^{i\frac{\Gamma}{2}} \end{vmatrix} \tag{25}$$

where $\mathcal{W}(\varphi_1)$ represents the Jones matrix for λ/4 plate. φ_2, φ_1 represent the azimuth angle of a linear polarizer and a wave plate, respectively.

The phase error introduced by imperfections in the λ/4 plate can be obtained from four intensity patterns as follows.

(1) Intensity for $\varphi_1=\pi/4,\ \varphi_2=0$

$$I_2=\frac{1}{2}a^2(1-\cos\Gamma)+\frac{1}{2}b^2(1+\cos\Gamma)-ab\sin\phi\sin\Gamma \tag{26}$$

(2) Intensity for $\varphi_1=-\pi/4,\ \varphi_2=0$

$$I_4=\frac{1}{2}a^2(1-\cos\Gamma)+\frac{1}{2}b^2(1+\cos\Gamma)+ab\sin\phi\sin\Gamma \tag{27}$$

(3) Intensity for $\varphi_1=\pi/4,\ \varphi_2=\pi/4$

$$I_1=\frac{1}{2}a^2+\frac{1}{2}b^2+ab\cos\phi \tag{28}$$

(4) Intensity for $\varphi_1=\pi/4,\ \varphi_2=-\pi/4$

$$I_3=\frac{1}{2}a^2+\frac{1}{2}b^2-ab\cos\phi \tag{29}$$

From Eqs. (26)~(29), the phase difference ϕ' of optical waves P_{cw} and P_{ccw} is given by

$$\tan\phi'=\frac{I_4-I_2}{I_1-I_3}=\tan\phi\sin\Gamma \tag{30}$$

where ϕ' includes the error introduced by a wave plate. For a nonideal λ/4 plate, we have

$$\Gamma=\pi/2+\gamma \tag{31}$$

where γ is the error in the relative retardation introduced by λ/4 plate. Substituting Eq. (31) into Eq. (30), we obtain

$$\tan\phi'=\tan\phi\cos\gamma \tag{32}$$

Since

$$\tan\phi'=\tan(\phi+\Delta\phi)\approx\tan\phi+\Delta\phi\sec^2\phi \tag{33}$$

we can calculate the error from Eqs. (32) and (33) as follows:

$$\Delta\phi=-1/4\sin(2\phi)\gamma^2 \tag{34}$$

3.2 Azimuth angle error

We shall deal with the errors that are introduced by the azimuth angle error in the $\lambda/4$ plate. In this case, we assume that a wave plate is ideal. In discussing the azimuth angle error we assume that the azimuth angle errors of all polarization elements are zero except a wave plate and the azimuth angle of a wave plate is considered as variable including phase error.

(1) Intensity for $\varphi_1=\pi/4, \varphi_2=0$

$$I_2=\frac{1}{2}a^2\sin^2 2\varphi_1+b^2(\cos^4\varphi_1+\sin^4\varphi_1) + ab\sin 2\varphi_1\{\cos\phi\cos 2\varphi_1-\sin\phi\} \tag{35}$$

(2) Intensity for $\varphi_1=-\pi/4, \varphi_2=0$

$$I_4=\frac{1}{2}a^2\sin^2 2\varphi_1'+b^2(\cos^4\varphi_1+\sin^4\varphi_1') + ab\sin 2\varphi_1'\{\cos\phi\cos 2\varphi_1'-\sin\phi\} \tag{36}$$

(3) Intensity for $\varphi_1=\pi/4, \varphi_2=\pi/4$

$$I_1=\frac{1}{2}[a^2\{\frac{1}{2}\sin^2 2\varphi_1-\frac{1}{2}\sin 4\varphi_1+\sin^4\varphi_1+\cos^4\varphi_1\} + b^2\{\frac{1}{2}\sin^2 2\varphi_1+\frac{1}{2}\sin 4\varphi_1+\sin^4\varphi_1+\cos^4\varphi_1\} + 2ab\{\cos\phi\sin^2 2\varphi_1+\sin\phi\cos 2\varphi_1)\}] \tag{37}$$

(4) Intensity for $\varphi_1=\pi/4, \varphi_2=-\pi/4$

$$I_3=\frac{1}{2}[a^2\{\frac{1}{2}\sin^2 2\varphi_1+\frac{1}{2}\sin 4\varphi_1+\sin^4\varphi_1+\cos^4\varphi_1\} + b^2\{\frac{1}{2}\sin^2 2\varphi_1-\frac{1}{2}\sin 4\varphi_1+\sin^4\varphi_1+\cos^4\varphi_1\} - 2ab\{\cos\phi\sin^2 2\varphi_1+\sin\phi\cos 2\varphi_1)\}] \tag{38}$$

From Eqs. (35)~(38), the phase difference ϕ' of optical waves P_{cw} and P_{ccw} is given by

$$\tan\phi'=\frac{I_4-I_2}{I_1-I_3} \tag{39}$$

For $\varphi_1=\pi/4$ and $\varphi_1'=-\pi/4$, we assume that azimuths of a wave plate including phase error are $\varphi_1=\pi/4+\varepsilon_1$ and $\varphi_1'=-\pi/4+\varepsilon_1'$, and ϵ_1 and ε_1' represent the azimuth angle errors in a wave plate. Substituting $\varphi_1=\pi/4+\varepsilon_1$ and $\varphi_1'=-\pi/4+\varepsilon_1'$ into Eq. (39), we obtain the phase error as

$$\triangle\phi=(1+\sin^2\phi-2\cot 2\beta\sin\phi)\varepsilon_1-\cos^2\phi\varepsilon_1' \tag{40}$$

where $\cot 2\beta=(b^2-a^2)/2ab$.

3.3 Azimuth angle error of a linear polarizer

We shall deal with the errors that are introduced by the azimuth angle error in the linear polarizer. In this case, we assume that a linear polarizer is ideal. In discussing the azimuth angle error we assume that the azimuth angle errors of all polarization elements are zero except a linear polarizer.

(1) Intensity for $\varphi_1=\pi/4, \varphi_2=0$

$$I_3=\frac{1}{2}(a^2+b^2)+ab\sin(2\varphi_2-\phi) \tag{41}$$

(2) Intensity for $\varphi_1=-\pi/4, \varphi_2=0$

$$I_1=\frac{1}{2}(a^2+b^2)+ab\sin(2\varphi_2+\phi) \tag{42}$$

(3) Intensity for $\varphi_1=\pi/4, \varphi_2=\pi/4$

$$I_4=\frac{1}{2}(a^2+b^2)+ab\sin(2\varphi_2'-\phi) \tag{43}$$

(4) Intensity for $\varphi_1=\pi/4, \varphi_2=-\pi/4$

$$I_2=\frac{1}{2}(a^2+b^2)+ab\sin(2\varphi_2''-\phi) \tag{44}$$

From Eqs. (41)~(44), the phase difference ϕ' of optical waves P_{cw} and P_{ccw} is given by

$$\tan\phi' = \frac{\sin(2\varphi'_2 - \phi) - \sin(2\varphi''_2 - \phi)}{\sin(2\varphi_2 + \phi) - \sin(2\varphi_2 - \phi)} \quad (45)$$

For $\varphi_2 = 0$, $\varphi'_2 = \pi/4$ and $\varphi''_2 = -\pi/4$, we assume that azimuths of polarizer including phase error are $\varphi_2 = 0 + \varepsilon_2$, $\varphi'_2 = \pi/4 + \varepsilon'_2$ and $\varphi'_2 = -\pi/4 + \varepsilon''_2$, and ε_2, ε'_2 and ε''_2 represent the azimuth angle errors in a linear polarizer. Substituting $\varphi_2 = 0 + \varepsilon_2$, $\varphi'_2 = \pi/4 + \varepsilon'_2$ and $\varphi'_2 = -\pi/4 + \varepsilon''_2$ into Eq. (45), we obtain the phase error as

$$\triangle\phi = \cos^2\phi(\varepsilon'_2 + \varepsilon''_2) \quad (46)$$

Total phase error by polarization components is written by

$$\triangle\phi = -1/4\sin(2\phi)\gamma^2 + (1 + \sin^2\phi - 2\cot 2\beta \sin\alpha)\varepsilon_1 - \cos^2\phi(\varepsilon'_1 - \varepsilon'_2 - \varepsilon''_2) \quad (47)$$

If a=b, then $\cot 2\beta = 0$. In this case, total phase is given by

$$\triangle\phi = -1/4\sin(2\phi)\gamma^2 + (1 + \sin^2\phi)\varepsilon_1 - \cos^2\phi(\varepsilon'_1 - \varepsilon'_2 - \varepsilon''_2) \quad (48)$$

In the extraction of phase term using the combination of polarization components, the phase error occurs. The retardation error of the commercially available phase plate make the second-order error very small. Accordingly, phase error in the optimized MTI is mainly due to the azimuth angle errors of the polarization components. The azimuth angle errors can be minimized by using a computer-controlled phase-shifting apparatus, which can accurately control the rotating angle of the polarization components[15].

4. Conclusions

We derived and analyzed the OTFs of the MTI based on the general OTF synthesis. Because we can obtain only the one OTF in one-pupil system of the MTI, we cannot remove bias and conjugate image in the reconstructed image. Hence, we cannot obtain any bipolar function in one-pupil system of the MTI. But, in the MTI with two-pupils, we can remove bias and conjugate image in the reconstructed image, and can obtain two bipolar functions, cosine and sine functions. Also, through the OTF synthesis, we can see that there are several kinds of the combination of the phase retardation of wave plates for removing bias and conjugate image.

However, we need two operation modes to obtain the complex hologram without bias and conjugate image in the MTI. To solve the problem, we proposed the optimized MTI with one wave plate, which can obtain cosine and sine functions by the combination of one wave plate and one linear polarizer.

We also analyzed the potential errors of the optimized MTI. In the extraction of phase term using the combination of polarization components, the phase error occurs. The retardation error of the commercially available phase plate make the second-order error very small. Accordingly, phase error in the optimized MTI is mainly due to the azimuth angle errors of the polarization components.

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