Journal of the Society of Korea Industrial and Systems Engineering
Vol, 30, No. 2, pp.75－82, June 2007.

공급업체의 불확실성하에서 하향리스크 제약을 고려한 신문팔이문제

김형태* · 고성석**† · 김주철***

*삼성SDS SCM그룹
**건국대학교 산업공학과
***연세대학교 경제학과

Newsvendor Problem with Downside-risk Constraint under Unreliable Supplier

Hyoung-Tae Kim* · Sung-Seok Ko**† · Joo-Cheol Kim***

*SCM Business Group, Smsung SDS
**Department of Industrial Engineering, Konkuk University
***Department of Economics, Yonsei University

이 논문의 공급업체의 불확실성하에서의 신문팔이 문제를 다루고 있다. 즉 공급업체의 공급량이 소매업체가 주문한 양을 충족하기 못하는 상황을 고려한 것이다. 여기서 우리는 기대 이익을 최대화하는 최적의 주문량을 구하였고, 기존의 연구와 달리 소매업체의 위험 성향을 고려하기 위해 하향 리스크에 대한 제약 조건을 추가하여, 위험 성향이 높을수록 더 많은 주문량을 사용한다는 결과를 사례 연구를 통해 확인할 수 있었다.

Keywords : Newsvendor Problem, Uncertain Supply, Risk-aversion

1. Introduction

Even though stochastic demand assumption with deterministic supply is adopted in most of the single-period inventory model, in many real-life situations, one can easily observe random gaps between the originally placed order quantity and the actually achieved quantity. Increased popularity of global sourcing is one big reason for generating less than perfect supply processes from suppliers to retailers. For example, to reduce purchase costs and attract a larger base of customers, retailers such as Wal-Mart, Home Depot and Dollar General are constantly seeking suppliers with lower prices and finding them at greater and greater distances from their distribution centers (DCs) and stores. Consequently, a significant proportion of shipped products from overseas suppliers is susceptible to defects. Reasons for defects include missing parts, misplaced products (at DCs, stores) or mistakes in orders and shipments. A similar example could be a typical production line where the production yield assumes less than 100% resulting in a different number of goods manufactured than originally planned. In these situations, the problem is how to choose the size of an order or how many parts to begin production to meet a one time fixed demand. White [17] solved the problem of determining the optimal initial lot size under uncertain production yield and provided a critical value

† 교신저자 ssko@konkuk.ac.kr

much like the classical newsvendor solution. One of the earliest papers on the uncertain supply under economic order quantity (EOQ) framework was written by Silver [14]. He studied two cases, in the first case, the standard deviation of the amount received is independent of the lot size, while in second case the standard deviation is proportional to the lot size. One of the interesting results among his findings was that the optimal order quantity depends only on the mean and the standard deviation of the amount received. Shih [13] studied the optimal ordering schemes in a case where the proportion of defective products in the accepted lots has a known probability distribution. The yield rate is thus between 0 and 1 and is assumed to be independent of the lot size. Similar to Silver's results he showed that the optimal order quantity depends only on the model parameters and the first two moments of the underlying yield distribution. As expected, the optimal order quantity is greater than that of the certain yield case, but it was less intuitive that the optimal lot size decreases when the variance of the yield rate distribution increases. Noori and Keller [9] extended Silver's model to obtain an optimal production quantity when the amount of products received at stores assumes probability distributions such as uniform, normal and gamma. They showed that for a uniform demand case the optimal ordering policy is independent of the yield distribution, but Rekik et al. [11] found out that the Noori and Keller's result is not always valid for all system parameters. They identified several cases defined by certain ranges of system parameters to investigate the validity of Noori and Keller's previous results. For the detailed survey of random yield literature, including exclusive random yield models, Yano and Lee [18] is the most popular reference.

Most of the previous research on the single-period inventory problem focused on finding the optimal size of order which maximizes the expected total profits, but if we consider the risk preferences, especially risk aversion of the decision maker, those expected value approaches may lead to suboptimal solutions. Suppose a situation is tried where the decision maker can make two choices. With the first choice, he can make either 1 million dollars or 0 dollars with 50% chance each, and with the second choice he receives 0.4 million dollars for certain. Even though the expected value for the first choice is 0.5 million, which is greater than that of the second choice, few will choose the first option.

To overcome this 'flaw of average' in solving the single-period inventory problem many researchers studied the behavior of Risk Averse Newsboy. These include Lau [7], Spulber [15], Bouakiz and Sobel [2], Eeckhoudt et al. [5], Agrawal and Seshadri [1], Chen and Federgruen [3], Seifert et al. [12], Chen et al. [4], Haksoz and Seshadri [6]. Lau [7] examined newsvendor solutions which maximize expected utility. He also investigated the new objective function of maximizing the probability of achieving a budgeted profit. Eeckhoudt et al. [5] examined the risk and risk aversion in a single-period inventory problem where demand is stochastic while supply is deterministic. They show that the optimal order quantity decreases as decision maker's risk-aversion increases because a lower order amount definitely reduces the inherent risks of the outcome. In Bouakiz and Sobel [2], they explored the newsvendor problem with the exponential utility and showed that a base-stock policy is optimal when a multi-period newsvendor problem is optimized with an exponential utility criterion. Agrawal and Seshadri [1] also investigated the newsvendor problem with the objective being maximizing the expected utility. In their problem setting, both price and order quantity are decision variables for the risk-averse retailer.

In our paper, we extend these researches to the case of uncertain supply situation, but instead of adopting utility function and finding the optimal solution, which maximizes the expected utility as in Eeckhoudt et al. [5], we introduce a constraint, so called 'Value-at-Risk', into the given model to reflect the decision makers risk preferences. Then we investigate the various impacts of risk preference, which is specified by parameters of 'Value-at-Risk', on the optimal order quantity using relevant numerical examples.

This paper is organized as follows: in Section 2, we describe the basic newsvender model under the assumption of unreliable supplier. We review results from the previous literature on the concavity of the objective function followed by the optimality condition which is the necessary condition for any order quantity to be optimal. Then, we analyze the impact of the mean and variance of the yield distribution on the resulting optimal solution when the uncertain yield rate follows Beta and Uniform distributions respectively. In Section 3, we add 'Value-at-Risk' constraint into the basic model and discuss the analytical solution procedure for this new problem. Numerical studies are conducted to discuss the impact of the constraint on the resulting solutions. Finally, Section 4 concludes the paper.

2. Optimal Order Quantity

Consider a newsvendor problem with unreliable supplier instead of uncertain demand. We assume the amount of demand,

θ, is known, but if retailer's order quantity is Q, then the amount of arrived product at retailer is YQ, where Y represents random proportion of Q with distribution function $G(y)$ (p.d.f. $g(y)$). Let's define p as retail price and w as wholesale price. And all unsold products are returned to the supplier at salvage price s. Then, we can define the profit function as follows

$$Z_y(Q) = p\min(\theta, YQ) - wYQ + s(YQ-\theta)^+$$

where $x^+ = \max(x, 0)$ or equivalently,

$$Z_y(Q) = \begin{cases} (p-w)YQ & if \;\; YQ \le \theta \\ (p-s)\theta - (w-s)YQ & otherwise \end{cases}$$

Since $Z_y(Q)$ is concave function in Q, and expectation operator preserves the concave property, $E[Z_y(Q)]$ is concave function in Q. If Q is integer, we can obtain optimal order quantity Q^*, which maximizes the expected profit, satisfying the following equation.

$$Q^* = sup_Q\{Q | E[Z_y(Q+1)] - E[Z_y(Q)] \ge 0\}$$

For more detail analysis, we assume Q to be real. Then the expected profit is

$$\begin{aligned} E[Z_y(Q)] &= \int_0^{\frac{\theta}{Q}} Z_y(Q)dG(y) + \int_{\frac{\theta}{Q}}^1 Z_y(Q)dG(y) \\ &= \int_0^{\frac{\theta}{Q}} (p-w)yQdG(y) \\ &\quad + \int_{\frac{\theta}{Q}}^1 ((p-s)\theta - (w-s)yQ)dG(y) \\ &= (p-s)Q\int_0^{\frac{\theta}{Q}} ydG(y) + (p-s)\theta(1-G(\frac{\theta}{Q})) \\ &\quad -(w-s)QEY \cdots\cdots (2.1) \end{aligned}$$

The expected profit function is concave in Q. This concavity is guaranteed because the second derivative of $E[Z_y(Q)]$ is negative

$$\frac{\partial^2 E[Z_y(Q)]}{\partial Q^2} = -(p-s)\frac{\theta^2}{Q^3}g(\frac{\theta}{Q}) \le 0$$

Thus this concavity allows us to rely on the first order condition to find the optimal order quantity that maximizes the expectation profit function. The first derivative of $E[Z_y(Q)]$ is

$$\frac{\partial E[Z_y(Q)]}{\partial Q} = (p-s)\int_0^{\frac{\theta}{Q}} ydG(y) - (w-s)EY$$

It can be shown that the optimal order quantity, Q^*, satisfies

$$\int_0^{\frac{\theta}{Q^*}} ydG(y) = \frac{w-s}{p-s}EY \cdots\cdots (2.2)$$

Once the distribution of Y and the demand level, θ are specified, the corresponding solution can be computed using the above equation. And, the expected profit at the optimal order quantity, i.e, $E[Z_y(Q^*)]$, is simplified to

$$E[Z_y(Q^*)] = (p-s)\theta(1-G(\frac{\theta}{Q^*}))$$

If the supplier is very greedy and has more power over the retailer, he wants to set wholesale price as large as possible $w = p$. In this case the equation (2.2) gives us $Q^* = \theta$, since $(w - s) = (p - s)$. This implies that retailer orders only the amount of demand, and he does not gain any profit. It sounds quite natural because if the retailer orders beyond the amount of demand, there exist possibilities that he loses the money because of the overstock resulting in cheap salvage profits. Therefore, though the retailer has no gain, it should order as much as possible without any loss to keep the relationship with customer.

If the supplier and the retailer are in equivalent position, the supplier should set wholesale price less than retail price $p > w$. In this case, we can obtain $Q^* \ge \theta$ and the retailer can expect more profit than if order quantity is θ. From equation (2.2), we can derive

$$E[Z_y(Q^*)] = (p-s)\theta(1-G(\frac{\theta}{Q^*})) \ge (p-w)\theta EY$$

The above shown expression makes sure that optimal order quantity(Q^*) gives us more expected profit than when order quantity is θ, since the right term is the expected profit when order quantity is θ.

Now we consider how the optimal order quantity(Q^*) changes as the variance of Y changes while we fixed the expected value EY at a constant. If variance of Y is close to 0, then, as would be expected, the optimal order quantity becomes θ/EY, because with no variability we know the exact amount will be received. Thus we only need to order θ/EY to meet the demand.

To further investigate the impact of supply variability on the resulting optimal order quantity we consider two examples in the following examples. In these examples we use the fixed demand $\theta = 100$ and two different probability distributions for Y, which are $Beta(\alpha, \beta)$ and $Uniform(L, U)$ respectively.

<Table 1> and <Table 2> summarize results from the first example when $Y \sim Beta(\alpha, \beta)$ with $EY = 0.8$ and $EY = 0.9$

respectively. As shown in both Tables, reduction in supply variability results in higher expected payoff regardless of different parameter settings. Readers who want detailed discussion on this are referred to Lin and Hou [8]. Based on similar findings shown in our examples, they investigated trade-off between the inventory savings from variability reduction and the capital investment for reducing the yield variability.

In <Table 1>, when the cost vector (p, w, s) is equal to (10, 4, 2), we can see that the optimal order quantity is not always increasing as the variance decreases. The optimal order quantity initially increases as the variance decreases from 0.071 to 0.015. But, once the variance decreases below 0.015, the optimal quantity is not further increasing but starts to decrease and finally converges to $\theta/EY = 125$ as the variability becomes 0. <Table 2> shows similar results as in <Table 1>. With the cost vector (p, w, s) set at (10, 4, 2), the optimal order quantity, Q^* increases until the variance reduction reaches 0.002, and after passing 0.002 the order quantity steadily converges on $\theta/EY = 111.11$ as the variance goes to 0.

When $\tilde{Y} \sim Uniform(L, U)$, after some manipulation, optimal order quantity (Q^*) can be obtained as follows,

$$(Q^*)^2 = \frac{\theta^2}{\frac{w-s}{p-s}U^2 + \frac{p-w}{p-s}L^2} \quad \text{(2.3)}$$

Since Y is uniform random variable, we can set $U = EY + I$ and $L = EY - I$. Substituting these expressions in (2.3), then we can obtain the following formula:

$$(Q^*)^2 = \frac{\theta^2}{h(I)} \quad \text{(2.4)}$$

where

$$h(I) = (I + kEY)^2 + 4\frac{(p-w)(w-s)}{(p-s)^2}(EY)^2$$
$$k = \frac{2w-(p+s)}{p-s}$$

In this formula, we know that since $h(I)$ is a convex function in I, when $I \le -kEY$, $h(I)$ is decreasing and Q^* is increasing, and when $I \ge -kEY$, $h(I)$ is increasing and Q^* is decreasing; therefore, the value of k plays an important role in characterizing optimal order quantity Q^*. Note that there exists a positive relationship between $Var(Y)$ and I, since $Var(Y) = I^2/3$

Theorem 2.1 *Under the above shown setting, there exist the following properties in* Q^*.

$$\begin{cases} if\ k \ge 0 & Q^*\ is\ increasing\ as\ I\ is\ decreasing \\ if\ \frac{EY-1}{EY} \le k \le 0 & Q^*\ have\ maxiμm\ value\ at\ I = -kEY \\ if\ \frac{EY-1}{EY} \ge k & Q^*\ is\ decreasing\ as\ I\ is\ decreasing \end{cases}$$

Proof. The proof of Theorem 2.1 is substituted by explanation of the equation $h(I)$ shown above together with the fact that $h(I)$ is a parabola and $I \le EY$.

In <Table 3>, when the cost vector is (10, 8, 2), Q^* is increasing as I is decreasing since $k = 2 > 0$ and when $(p, w, s) = (10, 4, 2)$, Q^* is decreasing as I is decreasing since $k = -0.5 < (EY - 1)/EY = -0.25$. When $(p, w, s) = (10, 5.5, 2)$, Q^* has the maximum value at $(L, U) = (0.70, 0.90)$, since $k = -1/8$. <Table 4> show the results when $EY = 0.9$. We can also observe that the properties of the optimal order quantity, Q^*, satisfy the conditions specified in Theorem 2.1.

As the summary of these two examples we can assert the following for the newsvendor problem with uncertain supply :

- The reduced variability in supply always results in increased profits ;
- The optimal order quantities are not necessarily increasing or decreasing upon reduced supply variability (it depends on the distribution of Y and model parameters (p, w, s)) ;
- As the variability vanishes, the optimal order quantity converges on θ/EY

3. Downside Risk Constraint

In this section, by defining a constraint known as 'Downside Risk' or 'Value-at-Risk', we study the optimal order quantity involving a constraint limiting downside risk.

We consider downside risk of the retailer as the probability that his realized profit is less than or equal to its specified target profit. Let α be the target profit, then the downside risk of the retailer is defined to be the probability that its probability is no greater than α, i.e.

$$P(Z_y(Q) \le \alpha)$$

The retailer wants to choose an order quantity Q which maximizes its expected profit while satisfying the fact that its prespecified downside risk does not exceed a specified probability(β). With this scenario the newsvendor problem now becomes,

$$Max_{Q \geq 0} \quad E[Z_y(Q)] \quad \cdots\cdots (3.5)$$
$$s.t \quad P(Z_y(Q) \leq \alpha) \leq \beta \quad \cdots\cdots (3.6)$$

Note that this problem can be thought of as a risk-aversion problem. That is, for risk-aversion pairs (α_1, β_1) and (α_2, β_2) if $(\alpha_1 \geq \alpha_2)$ and $(\beta_1 \leq \beta_2)$. then the second pair represents a higher risk-aversion to the given risk than does the first.

Now we can solve problems (3.5) and (3.6). We assume that $\alpha \leq (p-w)\theta$, since $Z_y(Q) \leq (p-w)\theta$ for any Q. This means that if $\alpha > (p-w)\theta$, there exists no feasible solution for any β, because downside risk probability is always 1.

The following proposition characterizes the downside risk probability.

Proposition 3.1 *For any target profit level $\alpha \leq (p-w)\theta$, the downside-risk probability becomes,*

$$P(Z_y(Q) \leq \alpha) = \begin{cases} 1 & if\ Q \leq Q_a \\ G(Q_a/Q) & if\ Q_a < Q \leq Q_b \\ 1 + G(Q_a/Q) - G(Q_b/Q) & if\ Q > Q_b \end{cases} \cdots\cdots (3.7)$$

where $Q_a = \alpha/(p-w)$ *and* $Q_b = \dfrac{(p-s)\theta}{w-s}$.

Proof. The downside risk in equation (3.6) can be rephrased as

$$\begin{aligned} P(Z_y(Q) \leq \alpha) &= P(Z_y(Q) \leq \alpha | Y < \frac{\theta}{Q}) P(Y < \frac{\theta}{Q}) \\ &+ P(Z_y(Q) \leq \alpha | Y \geq \frac{\theta}{Q}) P(Y \geq \frac{\theta}{Q}) \\ &= P(Y \leq \min(\frac{\alpha}{(p-w)Q}, \frac{\theta}{Q})) \\ &+ P(Y \geq \max(\frac{(p-s)\theta - \alpha}{(w-s)Q}, \frac{\theta}{Q})) \end{aligned}$$

Since $\alpha \leq (p-w)\theta$, the following relationship exists

$$Q_a \leq \theta \leq Q_b$$

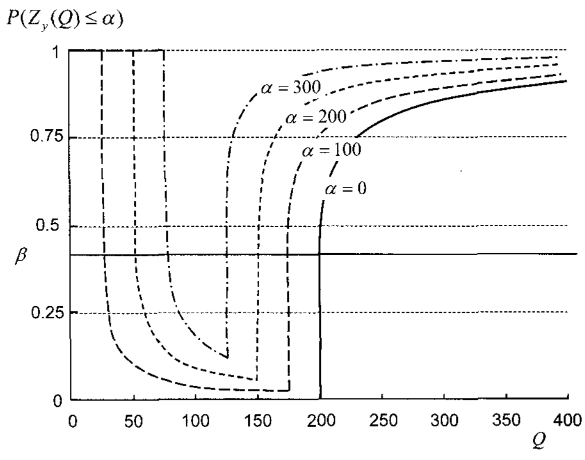
Together with the relationship, we have

$$P(Z_y(Q) \leq \alpha) = P(Y \leq \frac{Q_a}{Q}) + P(Y \geq \frac{Q_b}{Q})$$

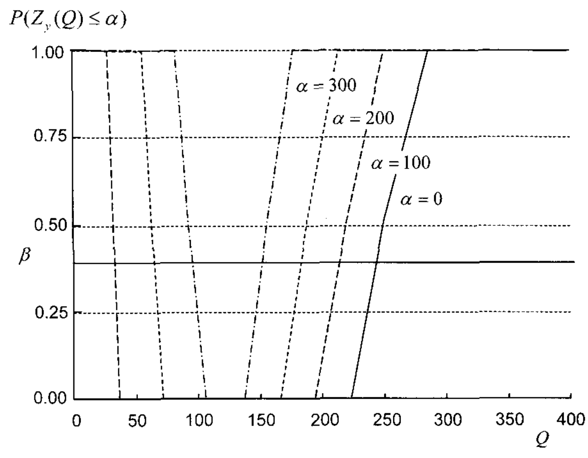
When $Q \leq Q_a$, then the first term has to be 1 and the second term vanishes; when $Q_a \leq Q \leq Q_b$ the first term takes a value less than 1 while the second term is still zero. These observations yield downside-risk probability as in equation (3.7).

If the distribution of Y is specified and parameters of downside-risk (α, β) are determined, we can calculate the downside-risk probability and corresponding optimal order quantity satisfying the objective function (3.5) with the given constraint (3.6).

Assuming the cost vector (p, w, s) is (10, 6, 2), <Figure 1> and <Table 2> exhibit the shapes of the downside-risk probabilities when $Y \sim Beta(1, 0.25)$ and $Y \sim U(0.70, 0.90)$, respectively. These figures illustrate that the range of the feasible solution for constraints are shrinking as α increases. It can be shown that all feasible order quantities are described by a closed and bounded interval. Moreover the downside-risk probability has the minimum value at $Q = Q_b$. Denoted by S, the feasible solution set has the form of $S = \{Q | Q_L \leq Q \leq Q_U\}$. Then the optimal solution (Q_d^*) can be computed by comparing the bounded values (Q_L and Q_U). If $Q^* \in S$, where Q^* is unconstrained solution obtained in (2.2), then $Q_d^* = Q^*$. Otherwise, if $Q^* \leq (\geq) Q_L(Q_U)$, then $Q_d^* = Q_L(Q_U)$ by concavity property of $E[Z_y(Q)]$.



<Figure 1> $Y \sim Beta(1, 0.25)$



<Figure 2> $Y \sim U(0.7, 0.9)$

<Table 5> show the numerical examples when $Y \sim Beta(1, 0.25)$ and $(p, w, s) = (10, 6, 2)$ under various combinations of downside-risk parameter values (α, β). In <Table 5>, *N/F* indicates that there exists no feasible solution set. When $\alpha = 200$ and $\beta = 0.1$, since the lower bound is greater than Q^*, the optimal order quantity (Q_d^*) is the same as the lower bound (Q_L) by concavity of $E[Z_y(Q)]$. In almost all cases of our example, the interval S contains the optimal order quantity (Q^*) of the unconstrained problem, resulting that the constrained optimal solution is, $Q_d^* = Q^*$. As can be seen in <Table 5>, higher values of α may induce no feasible solution set.

As explained in the beginning of this section, larger α and smaller β constitutes the higher risk aversion pair. From <Table 5>, the optimal order quantity increases as α increases while β is fixed at values 0.1, 0.2, 0.3. Similarly, for fixed values of α the optimal order quantity increases as β decreases. From these results we can conclude that in uncertain supply problems the higher risk aversion increases the size of the order to compensate for the variability in the supply amount. It is interesting to compare these results with the regular newsvendor model where only the uncertainty is in the demand variability. In Eeckhoudt et al. [5], they adopted the concave transformation of the utility function to express the higher risk aversion and solved the optimization problem where the optimal order quantity maximizes the expected utility. One of their results is that the retailer orders less and less as his degree of risk aversion increases, which is in contrast to our result. Under the demand uncertainty, the retailer with severe risk aversion does not order even a single product for fear of losing its investments. On the contrary, under the supply uncertainty, if there exists a certain demand amount, the retailer with higher risk aversion increases its order, making sure to meet the given demand. Even though methods of describing the risk aversion are not exactly the same between ours and Eeckhoudt's [5] the fundamental reasoning is identical, and this fact entitles our finding meaningful.

In our examples, we could luckily obtain the closed and bounded feasible solution set together with the optimal order quantity, but generally it is difficult to obtain the closed form of S, the feasible solution set. This is due to the fact that the downside-risk probability depends on distribution of Y and thus we cannot guarantee the monotonicity or the convexity of the downside risk in terms of Q. In other words, it is possible to have two or more discrete intervals representing parts of the feasible solution set. To overcome this complication and derive the closed form solution of S we sacrifice some generalities by restricting the value of α to be zero in the rest of this section. But, luckily, one of the natural selections of α is zero because if α is zero it implies a specified scenario where the retailer wants to avoid the situation if his return on investment (ROI) becomes negative; in other words, if he becomes bankrupt. Denoted by $Q_0 = (p-s)\theta/(w-s)$, the downside-risk probability becomes

$$P(Z_y(Q) \le 0) = P(Y \ge \frac{Q_0}{Q})$$

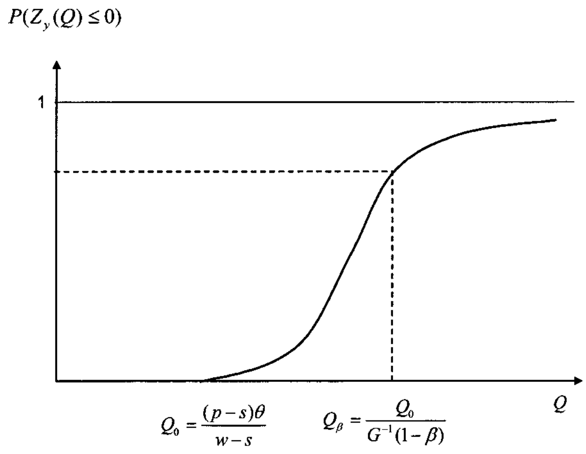
When $Q \le Q_0$, downside-risk probability is equal to zero, since $Y \in [0, 1]$. And as Q goes to infinity, the corresponding downside-risk probability has a monotonically increasing property and converges to 1. <Figure 3> illustrates this. By the above shown properties of downside-risk probability and concavity of $E[Z_y(Q)]$, we can conclude the following theorem.

Theorem 3.2 *For target profit level* $\alpha = 0$, *the optimal solution for the problem (3.5) and (3.6) is*

$$Q_d^* = \min(Q^*, Q_\beta)$$

where $Q_\beta = Q_0 / G^{-1}(1-\beta)$

Proof. The proof of Theorem 3.2 is substituted by <Figure 3>.



<Figure 3> Downside-Risk Probability when $\alpha = 0$

4. Summary and Conclusion

We have first reviewed the previous results of uncertain supply problems. The decrease in supply variability does not necessarily increase the optimal order amount in both cases where the yield ratio Y follows *Beta* and *Uniform* distributions respectively. The reduction in supply variability, however, always provides the retailer with increased profits.

With the downside-risk constraint the considered optimization problem could reflect the decision maker's risk attitude. The downside-risk parameter pair (α_1, β_1) exhibits higher risk aversion than another pair (α_2, β_2) whenever $(\alpha_1 \geq \alpha_2)$ and $(\beta_1 \leq \beta_2)$. Under the fixed demand, examples show that the increased risk aversion increases the retailer's order quantity whenever the supply amount is uncertain. This is the exact opposite result compared to the one from traditional newsvendor problems where the only uncertainty resides in the demand side. Even though there exist vast amount of research on inventory problems under supply uncertainty, few have considered the impact of the risk aversion on the resulting solutions. Our approach should be the first to consider the risk aversion and the uncertain supply problem.

In section 3, we assume α to be zero to make the presentation easier. It will be a challenging task to find the closed form of S with general values of α. Furthermore, the way the risk aversion generally works in uncertain supply problem is worthy of future study. One approach is to adopt the utility function as in Eeckhoudt et al. [5] under fixed demand and uncertain supply assumptions.

References

[1] Agrawal, V. and Seshadri, S.; "Impact of uncertainty and risk aversion on price and order quantity in the newsvendor problem," *Manufactruing & Service Operational Management*, 2 : 410-423, 2000.

[2] Bouakiz, M. and Sobel, M. J.; "Inventory control with an exponential utility criterion," *Operations Research*, 40 : 603-608, 1992.

[3] Chen, F. and Federgruen, A.; "Mean-variance analysis of basic inventory models," Working Paper, Columbia University, New York, New York, 2000.

[4] Chen, X., Sim, M., Simchi-levi, D., and Sun, P.; "Risk aversion in inventory management," Working Paper, MIT, Cambridge, Massachusetts, 2003.

[5] Eeckhoudt, L., Gollier, C., and Schlesinger, H.; "The risk-averse (and prudent) newsboy," *Management Science*, 41 : 786-794, 1995.

[6] Haksoz, C. and Seshadri, S.; "Supply chain operations in the presence of spot market : A review with discussion," Working Paper, *Stern School of Business*, New York University, New York, New York, 2005.

[7] Lau, H.; "The newsboy problem under alternative optimization objectives," *Journal of the Operational Research Society*, 31 : 525-535, 1980.

[8] Lin, L. and Hou, K. L.; "An inventory system with investment to reduce yield variability and set-up cost," *Journal of the Operational Research Society*, 56 : 67-74, 2005.

[9] Noori, A. Hamid and Keller, Gerald; "One-period order quantity strategy with uncertain match between the amount received and quantity requisitioned," *INFOR*, 24 : 1-11, 1986.

[10] Pasternack, B. A.; "Optimal pricing and return policies for perishable commodities," *Marketing Science*, 4 : 166-176, 1985.

[11] Rekik, Y., Sahin, E., and Dallery, Y.; "A comprehensive analysis of the Newsvendor model with unreliable supply," *OR Spectrum*, 14 : 32-39, 2005.

[12] Seifert, R. W., Thonemann, U. W., and Hausman, W. H.; "Optimal procurement strategies for online spot markets," *European Journal of Operational Research*, 152 : 781-799, 2004.

[13] Shih, W.; "Optimal inventory policies when stockouts result from defective products," *International Journal on Production Research*, 18 : 677-686, 1980.

[14] Silver, E.; "Establishing the order quantity when the amount received is uncertain," *INFOR.*, 14 : 32-39, 1976.

[15] Spulber, D. F.; "Risk sharing and inventories," *Journal of Economic Behavior and Organization*, 6 : 55-68, 1985.

[16] Pasternack, B. A.; "Optimal pricing and return policies for perishable commodities," *Marketing Science*, 4 : 166-176, 1985.

[17] White, J. A.; "On absorbing markov chains and optimum batch production quantities," *AIIE Transactions*, 2 : 82-87, 1970.

[18] Yano, C. and Lee, H.; "Lot sizing with random yields: a review," *Operations Research*, 43 : 311-334, 1995.

Appendix

<Table 1> $Y \sim Beta(\alpha, \beta)$ with $EY=0.8$

(p, w, s)			(10, 8, 2)		(10, 6, 2)		(10, 4, 2)	
α	β	$Var(Y)$	Q^*	$EZ(Q^*)$	Q^*	$EZ(Q^*)$	Q^*	$EZ(Q^*)$
1	0.25	0.071	100.22	155.60	103.01	317.51	117.93	491.14
2	0.50	0.046	101.87	160.81	108.60	328.33	127.06	514.15
3	0.75	0.034	104.03	162.65	112.42	335.10	130.52	527.22
4	1.00	0.027	105.92	164.46	114.87	340.52	131.95	536.10
5	1.25	0.022	107.47	166.16	116.54	344.90	132.57	542.57
6	1.50	0.019	10.8.73	167.69	117.75	348.48	132.82	547.51
7	1.75	0.016	109.78	160.05	118.66	351.47	132.89	551.43
8	2.00	0.015	110.67	170.26	119.37	354.01	132.87	554.64
9	2.25	0.013	111.43	171.35	119.94	356.20	132.79	557.31
10	2.50	0.012	112.08	172.32	120.40	358.11	132.69	559.59

<Table 2> $Y \sim Beta(\alpha, \beta)$ with $EY=0.9$

(p, w, s)			(10, 8, 2)		(10, 6, 2)		(10, 4, 2)	
α	β	$Var(Y)$	Q^*	$EZ(Q^*)$	Q^*	$EZ(Q^*)$	Q^*	$EZ(Q^*)$
1	0.11	0.043	100.01	159.69	100.40	339.98	105.81	523.76
2	0.22	0.028	100.09	175.83	101.44	356.84	109.35	544.65
3	0.33	0.021	100.34	179.12	102.74	361.41	111.61	552.62
4	0.44	0.017	100.75	180.15	103.93	363.86	112.98	557.59
5	0.55	0.014	101.22	180.70	104.90	365.75	113.79	561.26
10	1.11	0.007	103.25	182.90	107.52	372.34	114.93	571.73
20	2.22	0.004	105.35	186.01	109.12	379.00	114.74	580.06
30	3.33	0.003	106.40	187.90	109.81	382.43	114.39	583.86
40	4.44	0.002	107.04	189.18	110.12	384.60	114.10	586.12
50	5.55	0.002	107.48	190.12	110.32	386.13	113.87	587.66

<Table 3> $Y \sim Uniform(L, U)$ with $EY=0.8$

(p, w, s)		(10, 8, 2)		(10, 5.5, 2)		(10, 4, 2)	
L	U	Q^*	$EZ(Q^*)$	Q^*	$EZ(Q^*)$	Q^*	$EZ(Q^*)$
0.60	1.00	109.11	166.67	125.00	400.00	138.68	557.78
0.62	0.98	110.67	169.89	125.35	405.01	137.57	562.42
0.64	0.96	112.25	172.89	125.63	410.03	136.39	566.97
0.66	0.94	113.84	175.98	125.83	415.05	135.14	571.43
0.68	0.92	115.44	179.15	125.95	420.08	133.82	575.79
0.70	0.90	117.04	182.40	125.99	425.10	132.45	580.07
0.72	0.88	118.64	185.74	125.95	430.11	131.04	584.24
0.74	0.86	120.25	189.17	125.83	435.12	129.58	588.32
0.76	0.84	121.84	192.69	125.63	440.10	128.08	592.31
0.78	0.82	123.43	196.30	125.35	445.06	126.55	596.20
0.80	0.80	125.00	200.00	125.00	450.00	125.00	600.00

<Table 4> $Y \sim Uniform(L, U)$ with $EY=0.9$

(p, w, s)		(10, 8, 2)		(10, 5.5, 2)		(10, 4, 2)	
L	U	Q^*	$EZ(Q^*)$	Q^*	$EZ(Q^*)$	Q^*	$EZ(Q^*)$
0.80	1.00	10.4.83	184.24	111.35	407.79	117.04	582.40
0.81	0.99	105.46	185.74	111.39	410.01	116.48	584.24
0.82	0.98	106.10	187.25	111.41	412.24	115.90	586.07
0.83	0.97	106.73	188.78	111.43	414.46	115.32	587.88
0.84	0.96	107.36	190.33	111.42	416.69	114.74	589.66
0.85	0.95	107.99	191.90	111.40	418.91	114.15	591.43
0.86	0.94	108.62	193.48	111.37	421.14	113.55	593.18
0.87	0.93	109.25	195.08	111.33	423.36	112.95	594.92
0.88	0.92	109.87	196.70	111.27	425.57	111.34	596.63
0.89	0.91	110.49	198.34	111.20	427.79	111.73	598.32
0.90	0.90	111.11	200.00	111.11	430.00	111.11	600.00

<Table 5> $Y \sim Beta(1, 0.25)$ and $(p,w,s)=(10,6,2)$

α	$\beta=0.1$				$\beta=0.2$				$\beta=0.3$			
	Q_L	Q_U	Q_d^*	$EZ(Q_d^*)$	Q_L	Q_U	Q_d^*	$EZ(Q_d^*)$	Q_L	Q_U	Q_d^*	$EZ(Q_d^*)$
0	0	200.05	103.01	317.51	0	200.49	103.01	317.51	0	202.09	103.01	317.51
20	14.88	195.04	103.01	317.51	8.63	195.42	103.01	317.51	6.68	196.88	103.01	317.51
40	29.77	190.03	103.01	317.51	17.27	190.36	103.01	317.51	13.36	191.68	103.01	317.51
60	44.65	185.02	103.01	317.51	25.90	185.30	103.01	317.51	20.04	186.48	103.01	317.51
80	59.54	180.02	103.01	317.51	34.53	180.25	103.01	317.51	26.72	181.29	103.01	317.51
100	74.42	175.01	103.01	317.51	43.17	175.20	103.01	317.51	33.40	176.11	103.01	317.51
120	89.30	170.01	103.01	317.51	51.80	170.16	103.01	317.51	40.08	170.94	103.01	317.51
140	104.19	165.00	104.19	317.34	60.43	165.12	103.01	317.51	46.76	165.78	103.01	317.51
160	119.07	160.00	119.07	300.59	69.07	160.09	103.01	317.51	53.44	160.63	103.01	317.51
180	133.96	155.00	133.96	271.75	77.70	155.06	103.01	317.51	60.12	155.49	103.01	317.51
200	148.84	150.00	148.84	237.36	86.33	150.04	103.01	317.51	66.80	150.37	103.01	317.51
220	N/F	N/F	N/F	0	94.97	145.02	103.01	317.51	73.48	145.27	103.01	317.51
240	N/F	N/F	N/F	0	103.60	140.01	103.60	317.46	80.16	140.18	103.01	317.51
260	N/F	N/F	N/F	0	112.24	135.00	112.24	310.67	86.84	135.11	103.01	317.51
280	N/F	N/F	N/F	0	120.87	130.00	120.87	297.54	93.52	130.06	103.01	317.51
300	N/F	N/F	N/F	0	N/F	N/F	N/F	0	100.20	125.02	103.01	317.51
320	N/F	N/F	N/F	0	N/F	N/F	N/F	0	106.88	120.01	106.88	317.47
340	N/F	N/F	N/F	0	N/F	N/F	N/F	0	113.56	115.00	113.56	312.96
360	N/F	N/F	N/F	0	N/F	N/F	N/F	0	N/F	N/F	N/F	0