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A Study of the PDCA and CAPD Economic Designs of the $\bar{x}$ Control Chart

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Abstract. The PDCA (Plan, Do, Check and Act) cycle is often used in the field of quality management. Recently, business environments have become more competitive, and the due time of products has shortened. In a short production run process, to increase efficiency of management, the necessity for distinguishing the PDCA design that starts with PLAN and the CAPD design that starts with CHECK has been clarified. Starting from Duncan (1956), there have been a number of papers dealing with the economic design of control charts from the viewpoint of production run. Some authors (Gibra, 1971; Ladany and Bedi, 1976; etc.) have studied the economic design for finite-length runs; other authors (Crowder, 1992; Del Castillo and Montgomery, 1996; etc.) have studied the economic design for short runs. However, neither the PDCA nor the CAPD design of control charts has been considered. In this paper, both the PDCA and CAPD designs of the $\bar{x}$ chart are defined based on Del Castillo and Montgomery's design (1996), and their mathematical formulations are shown. Then from an economic viewpoint, the optimal values of the sample size per each sampling, control limits width, and the sampling interval of the two designs are studied. Finally, by numerically analyzing the relations between the key parameters and the total expected cost per unit time, the comparisons between the two designs are considered in detail.

Keywords: Short Production Run, Optimal Design of the Control Chart, Economic Evaluation.

1. INTRODUCTION

The $\bar{x}$ control chart is widely used to monitor the process of manufacturing as an online control tool in industrial management systems. Since Duncan's pioneering work (1956), many studies have been developed to serve different purposes for the economic design of control charts. From the viewpoint of the production run, Jones and Case (1981), Saniga (1989) considered the economic statistical design of the $\bar{x}$ control chart for the infinite-length horizon; Crowder (1992) proposed an optimization design for the control of a process in the short run case; Del Castillo and Montgomery (1996) considered the economic design of the control chart for use in either repetitive or job-shop production processes.

The concept of the PDCA cycle was originally developed by Walter Shewhart, the pioneering statistician who developed the statistical process control in Bell Laboratories in the US during the 1930s. It is often referred to as 'the Shewhart Cycle'. It was taken up and promoted very effectively from the 1950s on by a famous quality management authority, W. Edwards Deming, and is consequently known by many as 'the Deming Wheel'.

Recently, business environments have become more competitive, and the due time of products has shortened. In order to increase efficiency in the industry, more attention has been paid to the PDCA and CAPD processes of quality management (Shiba and Walden, 2001; Ikezawa,

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1985; Matsui, 2005; Sun, Tsubaki and Matsui, 2005).

The general definitions of the control chart's PDCA procedures can be found in Amasaka *et al*. (2003), Takahashi (1999) and Miyakawa (2000). Because of its connection to daily management, the evaluation of the economy of this control chart's PDCA procedures has become a new problem for the manager (Amasaka *et al*., 2003). In this paper, we proposed a PDCA design of the $\bar{x}$ chart based on Del Castillo and Montgomery's design (1996). In Del Castillo and Montgomery's paper (1996), the economic design of the $\bar{x}$ control chart for a short process run has been studied. However, the stage of deciding the control lines was not considered. Because of expanding multi-item, small-sized production and improving the process repeatedly, renewing the control lines has been frequently required. Hence, in this paper, we consider the control chart's PDCA design which is based on the case that starts from deciding the control lines.

On the other hand, when the $\bar{x}$ control chart is used in a short run production process, there is a case that starts from (Check) searching the assignable cause which occurs passively in the out-of-control state. In such a case, to clarify the true problem and treat it quickly, the necessity of considering the CAPD (Check, Act, Plan and Do) model which starts from searching the production process (Check) has been clarified.

However, the model starting from the out-of- control state has not been considered explicitly. Therefore, in this paper, we also propose the CAPD model of the $\bar{x}$ control chart based on the above case.

Economically designed control charts have been considered to serve different purposes and are very useful references for high-quality production. Since Reynolds *et al*. (1988) proposed an $\bar{x}$ control chart with variable sampling intervals (VSI), Bai and Lee (1998) developed an economic design for a VSI $\bar{x}$ control chart; Chen (2003) developed an economic-statistical design for a VSI $\bar{x}$ control chart under non-normality; Yu and Wu (2004) considered an economic design for a VSI moving average control chart. Also, other authors considered the economic design of multivariate control charts (Lowry and Montgomery, 1995; Chou *et al*., 2002; etc.).

This paper is organized as follows: First, the PDCA design of the $\bar{x}$ chart is proposed based on Del Castillo and Montgomery's design. Next, the CAPD design of the $\bar{x}$ chart is proposed based on cases which start from searching an assignable cause in the out-of-control state, and its mathematical formulation is shown. Then from an economic viewpoint, the optimal values of sample size per each sampling n_i, control limits width k_i and sampling interval v_i of the two designs are studied. Finally, by numerically analyzing the relations between the key parameter of λ_i^{-1} (mean time of the in-control period), δ_i (the size of the quality shift in the mean), c_2 (cost of per unit time for the nonconformities), c_4 (cost of restoring an in-control state) and C_t (the total expected cost per unit time), the comparisons between two designs are considered in detail.

2. THE ASSUMPTION AND THE NOTATION

The assumptions of the designs in this paper are as follows:

(1) The production run length T is short, and the process is repetitive (Lowry and Montgomery, 1995).
(2) The random variables of the in-control interval and out-of-control interval are exponentially distributed with the mean λ_i^{-1} and μ_i^{-1}.
(3) The quality shift occurs in the middle of an interval between samples (Ladany and Bedi, 1976)

The notation used is as follows:

C_p	expected cost of PLAN per unit time
C_d	expected cost of DO per unit time
C_c	expected cost of CHECK per unit time ($C_{c(in)}$ and $C_{c(out)}$)
C_a	expected cost of ACT per unit time
C_t	expected total cost per unit time
n_i	the sample size per each sampling (n_1 is the sample size per each sampling of the PDCA design; n_2 is the sample size per each sampling of the CAPD design)
v_i	the sampling interval (v_1 is the sampling interval of the PDCA design; v_2 is the sampling interval of the CAPD design)
T	production run length
T'	the interval of PLAN
I_1	period of in-control state of the PDCA design (the time from the start of the production run to the occurrence of an assignable cause)
O_1	period of out-of-control state of the PDCA design (The time elapsed from the occurrence of an assignable cause to its detection)
I_2	period of in-control state of the CAPD design (the time from the start of an in-control state to occurrence of an assignable cause)
O_2	period of out-of-control state of the CAPD design (the time elapsed from the occurrence of an assignable cause to its detection)
c_{p0}	fixed sampling cost of PLAN
c_{p1}	variable sampling cost of PLAN
c_0	fixed sampling cost of DO
c_1	variable sampling cost of DO
c_2	cost of per unit time for the nonconformities (the cost of CHECK in an out-of-control state)
c_3	cost of a false alarm (the cost of CHECK in the in-control state)
c_4	cost of restoring an in-control state (the cost of ACT)
f_i	number of samples taken during T-T' (f_1 belongs to the PDCA design; f_2 belongs to the CAPD design)

f_i' number of samples taken during T' (f_1' belongs to the PDCA design; f_2' belongs to the CAPD design)
δ_i size of the quality shift in the mean (δ_1 belongs to the PDCA design; δ_2 belongs to the CAPD design)
ρ probability that the past control lines are extended
λ_1^{-1} mean of the I_1 period in the PDCA design
μ_1^{-1} mean of the O_1 period in the PDCA design
λ_2^{-1} mean of the I_2 period in the CAPD design
μ_2^{-1} mean of the O_2 period in the CAPD design
α type *I* error probability
k_i control limits width (k_1 belongs to the PDCA design; k_2 belongs to the CAPD design)
P_o power
E[cycle] expected cycle length

3. THE PDCA DESIGN OF THE $\bar{x}$ CONTROL CHART

In a short run production process, the PDCA design is set up based on the case which starts from deciding the control lines of the $\bar{x}$ chart, and then it maintains the process with them.

3.1 The definition of the PDCA design

In this paper, the procedures (Plan, Do, Check and Act) of the PDCA design of the $\bar{x}$ control chart are defined respectively as shown in Figure 1.

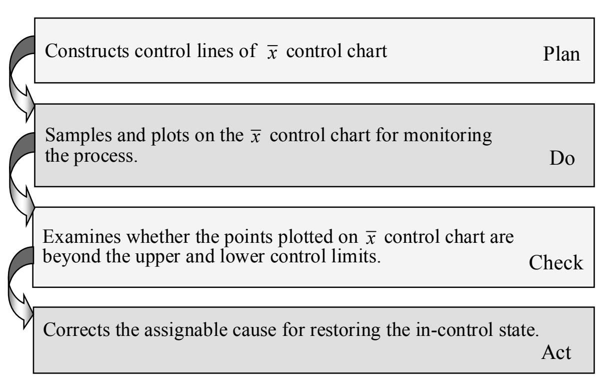

Figure 1. The procedures of the $\bar{x}$ chart's PDCA design

Assume the production process is monitored by the $\bar{x}$ control chart from now on. PLAN is defined as constructing the control lines for future management (center line and upper and lower control limits). DO is defined as sampling and plotting on the $\bar{x}$ control chart for monitoring the process quality with the decided control lines in the PLAN procedure.

CHECK is defined as judging whether the process is an in-control state by the result of the comparison between the point plotted on the $\bar{x}$ chart and the control limits (upper and lower control limits). Finally, ACT is defined as correcting the assignable cause for restoring the in-control state.

3.2 The mathematical formulations of the PDCA design

The evaluation function of the PDCA design is the expected total cost per unit time as follows:

$$C_{t(PDCA)} = \frac{E[cost\ per\ cycle]}{E[cycle\,(PDCA)]} = \frac{E[cost\ per\ cycle]}{E[\min(T'+I_1+O_1,T)]} \quad (1)$$
$$= C_p + C_d + C_{c(in)} + C_{c(out)} + C_a\ .$$

It includes the cost of PLAN, DO, CHECK, ACT which is considered in detail in Appendix 1. Figure 2 shows some of the time variables used in the PDCA design. At the start of the PDCA design, PLAN for deciding the control lines is made in T' time. Therefore, it is thought that the PDCA design starts from the in-control state, because the process is managed by these control lines. Let the process start at the point of *Q*, and let *S* be the point in time at which the quality characteristic shifts to an out-of-control state as shown in Figure 2. At the point of *C*, an assignable cause is detected. Here, the random variables I_1 and O_1 represent the interval from *Q* to *S* and the interval from *S* to *C*. Then the time from the start of the production process until removing the assignable cause is equal to $T'+I_1+O_1$.

Depending on the production process, the production run time *T* can be smaller than *C*, which means that the production run ends before the assignable cause is detected. Therefore, the mean cycle is defined to be equal to $E[\min(T'+I_1+O_1,T)]$.

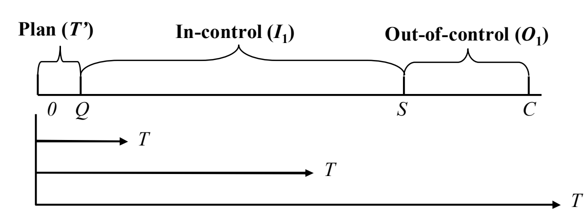

Figure 2. Some of the time variables used in the PDCA design

Based on Del Castillo and Montgomery's design (1996), the PDCA model's mathematical formulations are shown in Appendix 1.

Combining (7)-(11) in Appendix 1, the expected total cost per unit time of the PDCA design is shown as follows:

$$\begin{aligned} C_{t(PDCA)} &= [(c_{p0}+c_{p1}n_1)f_1'/T] + [(c_0+c_1n_1)f_1/T] \\ &+\{c_3\alpha E[\min(I_1,T-T')]f_1/(T-T') \\ &+c_2E[(T-T'-I_1)^+ - (T-T'-I_1-O_1)^+] \\ &+c_4\Pr\{I_1+O_1<T-T'\}\}/E[\min(I_1+O_1+T',T)]. \end{aligned}$$
(2)

If $T'=0$ is assumed, (2) is reduced to (6) of Del Castillo and Montgomery's model (1996).

In this paper, both the random variables I_1 and O_1 are assumed to be independently and exponentially distributed with mean λ_1^{-1}, μ_1^{-1}, then (2) is

$$\begin{aligned}C_{t(PDCA)} &= [(c_{p0}+c_{p1}n_1)f_1'/T]+[(c_0+c_1n_1)f_1/T]\\ &+\{c_3\alpha\frac{1}{\lambda_1}(1-e^{-\lambda_1(T-T')})f_1/(T-T')\\ &+c_2[\frac{1}{\mu_1}+\frac{1}{\lambda_1-\mu_1}(e^{-\lambda_1(T-T')}-\frac{\lambda_1}{\mu_1}e^{-\mu_1(T-T')})]\\ &+c_4[1+\frac{1}{\lambda_1-\mu_1}(\mu_1 e^{-\lambda_1(T-T')}-\lambda_1 e^{-\mu_1(T-T')})]\}/\\ &[\frac{1}{\lambda_1-\mu_1}\{\frac{\mu_1}{\lambda_1}(e^{-\lambda_1(T-T')}-1)-\frac{\lambda_1}{\mu_1}(e^{-\mu_1(T-T')}-1)\}+T'].\end{aligned} \tag{3}$$

Where type I error probability (α) and the out-of-control period(μ_1^{-1}) are explained in Appendix 3.

4. THE CAPD DESIGN OF THE $\bar{x}$ CONTROL CHART

In a short run production process, the CAPD design is set up based on the case that starts from (Check) searching the assignable cause which occurs passively in the out-of-control state.

4.1 The definition of the CAPD design

In this paper, we assume that the CAPD design starts from the out-of-control state by an assignable cause. The procedure (Check, Act, Plan, and Do) of the CAPD design is defined respectively as shown in Figure 3.

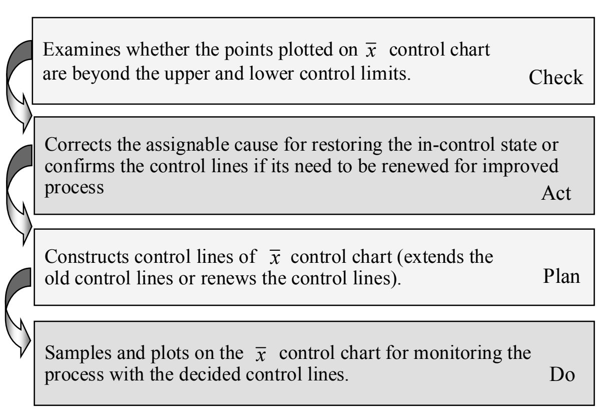

Figure 3. The procedures of the $\bar{x}$ chart's CAPD design

An assignable cause is defined as one that occurs in the production process, but the manager does not know it until it is detected on the $\bar{x}$ control chart. This means that the manager understands for the first time that the process has shifted to the out-of-control state by searching for the process when the plotted point is beyond the control limits (Check).

The ACT procedure is defined as correcting the assignable cause for restoring the in-control state or as confirming the control lines if they need to be renewed for having been improved process. The PLAN procedure is defined as constructing the control lines of the $\bar{x}$ control chart (extending the old control lines or renewing the control lines). The period of PLAN is defined in detail in Appendix 2.3. Finally, in the DO procedure, the quality of the process is controlled by using the decided control lines.

4.2 The mathematical formulations of the CAPD design

The evaluation function of the CAPD design is the expected total cost per unit time as follows:

$$\begin{aligned}C_{t(CAPD)} &= \frac{E[cost\ \ per\ cycle]}{E[cycle\ (CAPD)]}=\frac{E[cost\ \ per\ cycle]}{E[\min(O_2+T'+I_2,T)]}\\ &= C_{c(in)}+C_{c(out)}+C_a+C_p+C_d.\end{aligned} \tag{4}$$

Figure 4 shows some of the time variables used in the CAPD design. In this paper, the CAPD design is defined as starting from the out-of-control state (at point 0 (zero)) by an assignable cause. However, the manager does not understand it until the process is searched for when the plotted point is beyond the control limits.

At point C', let the assignable cause be detected for the first time by the $\bar{x}$ control chart, which can be corrected instantly (or it is confirmed that the old control lines need to be renewed.). During T', the control lines are determined (the past control lines are extended or renewed which we will describe in detail in Appendix 2.3). From point Q', the process is monitored by the examined control lines which correspond to a new in-control state.

The random variables O_2 and I_2 represent the interval from 0 to C' and the interval from Q' to S'.

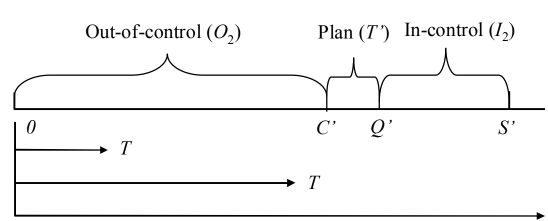

Figure 4. Some of the time variables used in the CAPD design

The CAPD model's mathematical formulations are shown and explained in Appendix 2.

Combining (12)-(16) in Appendix 2, the expected total cost per unit time of the CAPD design is shown as follows:

$$C_{t(CAPD)} = (1-\rho)(c_0 + c_1 n_2) f_2'/T + (c_0 + c_1 n_2) f_2 / T$$
$$+\{c_3 \alpha\, E[(T-T'-O_2)^+ - (T-T'-O_2-I_2)^+] f_2/(T-T')$$
$$+c_2 E[\min(O_2, T)^+]$$
$$+c_4 \Pr\{O_2 < T\}\}/E[\min(O_2+T'+I_2, T)]. \quad (5)$$

In this paper, both the random variables I_2 and O_2 are independently and exponentially distributed with mean λ_2^{-1} and μ_2^{-1}, then,

$$C_{t(CAPD)} = (1-\rho)(c_0 + c_1 n_2) f_2'/T$$
$$+(c_0 + c_1 n_2) f_2 / T$$
$$+\{c_2 \frac{1}{\mu_2}(1-e^{-\mu_2 T}) + c_3 \alpha [\frac{1}{\lambda_2} + \frac{1}{\lambda_2 - \mu_2}$$
$$(\frac{\mu_2}{\lambda_2} e^{-\lambda_2 (T-T')} - e^{-\mu_2 (T-T')})] f_2/(T-T') + c_4(1-e^{-\mu_2 T})\}/$$
$$[\frac{1}{\lambda_2 - \mu_2}\{\frac{\mu_2}{\lambda_2}(e^{-\lambda_2 (T-T')} - 1) - \frac{\lambda_2}{\mu_2}(e^{-\mu_2 (T-T')} - 1)\} + T']. \quad (6)$$

Where type *I* error probability (α) and the out-of-control period (μ_2^{-1}) are explained in Appendix 3.

5. NUMERICAL EXPERIMENTS

In this section, we present the following example to illustrate the PDCA and CAPD designs of the $\bar{x}$ chart. The model parameters in this example are directly borrowed from Montgomery (1985) and Chou *et al*. (2001).

Example A manufacturer produces non-returnable glass bottles for packaging a carbonated soft drink beverage. The wall thickness of the bottles is a key quality characteristic. The manufacturer uses an $\bar{x}$ chart to monitor the process, and it is estimated that the production run length T is 4 days. Based on an analysis of the salaries of quality-control technicians and the costs of test equipment, it is estimated that the fixed cost of taking a sample is \$1 (i.e. $c_0 = 1$). The estimated variable cost of sampling is estimated to be \$0.10 per bottle (i.e., $c_1 = 0.10$), and it takes approximately 45 min (i.e., $v_i = 0.0316$ day) to measure and record the wall thickness of a bottle. Process shifts occur at random with a frequency of about one every 20 h of operation (i.e., $\lambda_i^{-1} = 0.833$ day). On average, when the process goes out of control, the magnitude of the shift is approximately two standard deviations (i.e., $\delta_i = 2.0$). The cost of correcting an assignable cause is \$25, while the cost of investigating a false alarm is \$50 (i.e., $c_4 = 25$ and $c_3 = 50$). The manufacturer estimates that the cost for the nonconformities in the out-of-control state for 1 day is \$2400 (i.e., $c_2 = 2400$). In this paper, we also consider the PLAN stage which determines the control lines, therefore, it is estimated that the fixed cost and variable cost of taking the sample is \$2.5, \$0.25 (i.e., $c_{p0} = 2.5$, $c_{p1} = 25$).

When an $\bar{x}$ chart is used in the process, selection of the sample size per each sampling (n_i), the control limits width (k_i) and the sample interval (v_i) is usually called the design of the control chart, which is a very important responsibility for the manager. Therefore, in this section, we first investigate the optimal solution (n_i, k_i, v_i) to minimize C_t of the PDCA and CAPD designs, respectively.

Next, to compare the PDCA and CAPD designs, the key parameters of λ_i^{-1} (mean time of the in-control period), δ_i (the size of the quality shift in the mean), c_2 (cost for the nonconformities per unit time), c_4 (cost of restoring an in-control state) and C_t (the expected total cost per day) are investigated in detail.

The key difference of our PDCA design from that of Del Castillo and Montgomery's is that it considers the PLAN stage (the process of deciding the control lines). Therefore, we first consider the relation between C_t (the expected total cost per day) and T' (the interval of PLAN) of the PDCA design with the result of the above example (the optimal values $k = 2.99$, $n = 5$, $v = 0.0313$ day in Figure 9-1 of Montgomery (1985)).

From Figure 5, we can note that Ct is at a minimum level at $T'=1$. This is because a longer T' decreases $C_{c(out)}$ by decreasing the ratio of the out-of-control period at the mean cycle, while a longer T' increases samples costs.

Because sample size n_1 per each sampling is the main parameter of the PLAN stage, we also show the relation of C_t, n_1 and T' of the PDCA design in Figure 6.

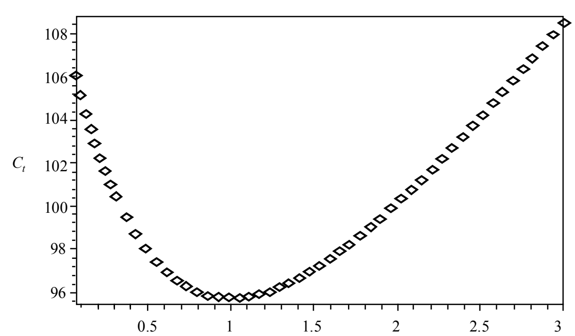

Figure 5. Relation between C_t and T' of the PDCA design

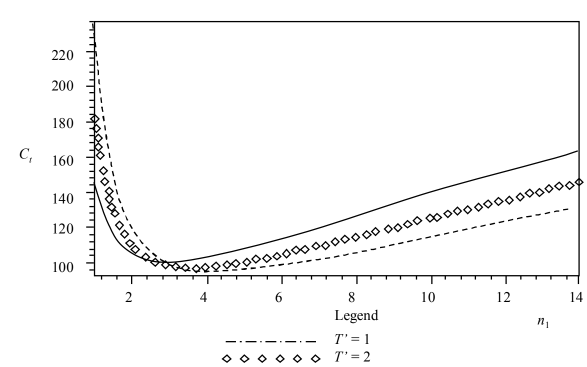

Figure 6. Relation between C_t, T' and n_1 of the PDCA design

From Figure 6, we can note that the optimal n_1 decreases with the increasing of T'. This is because C_p increases with an increase of T' (because of an increase in the number of samples), therefore, as a result, the optimal value of n_1 becomes small to decrease the effect on the total expected cost of C_t.

Below, we investigate the optimal values (n_i, k_i, v_i) to minimize C_t of the PDCA ($i = 1$) and CAPD ($i = 2$) designs based on the result ($T' = 1$) of Figure 5.

5.1 The optimal values (n_i, k_i, v_i) of the PDCA and CAPD designs

First, in order to find the optimal values (n_i, k_i, v_i) to minimize C_t of the PDCA model, we evaluate a wide range of possible values used in this case, and show the results of nearing the optimal solution in Table 1. Moreover, to clarify the change of this PDCA design's minimum value of C_t according to the change in parameters n_1, k_1, v_1, we also show the results in Figures 7-9 as follows:

Table 1. Detailed analysis of the values (n_i, k_i, v_i) of the PDCA design

n_2	k_2	v_2	P_o	α	$C_{t(PDCA)}$
5	3.00	0.060	0.92951	0.00270	81.84201
5	3.00	0.070	0.92951	0.00270	82.54037
5	3.00	0.080	0.92951	0.00270	84.47467
5	3.10	0.060	0.91499	0.00194	82.54272
5	3.10	0.070	0.91499	0.00194	83.45332
5	3.10	0.080	0.91499	0.00194	85.58594
5	3.20	0.060	0.89834	0.00137	83.52220
5	3.20	0.070	0.89834	0.00137	84.66257
5	3.20	0.080	0.89834	0.00137	87.01378
6	3.00	0.060	0.97122	0.00270	81.34163
6	3.00	0.070	0.97122	0.00270	81.26537
6	3.00	0.080	0.97122	0.00270	82.51201
6	3.10	0.060	0.96399	0.00194	81.47930
6	3.10	0.070	0.96399	0.00194	81.52566
6	3.10	0.080	0.96399	0.00194	82.88217
6	3.20	0.060	0.95534	0.00137	81.80581
6	3.20	0.070	0.95534	0.00137	81.97823
6	3.20	0.080	0.95534	0.00137	83.45102
7	3.00	0.060	0.98903	0.00270	82.51019
7	3.00	0.070	0.98903	0.00270	81.92621
7	3.00	0.080	0.98903	0.00270	82.74892
7	3.10	0.060	0.98579	0.00194	82.38195
7	3.10	0.070	0.98579	0.00194	81.87803
7	3.10	0.080	0.98579	0.00194	82.76852
7	3.20	0.060	0.98176	0.00137	82.39266
7	3.20	0.070	0.98176	0.00137	81.96428
7	3.20	0.080	0.98176	0.00137	82.92113

The expected total cost of C_t per day associated with the use of the PDCA design is given by equation (3). Type I error probability α and power Po are shown in equations (17) and (18) in Appendix 3.

In Table 1, we can note that the minimum C_t of the PDCA design is \$81.26537 per day, and the economically $\bar{x}$ chart would use a sample size per each sampling of $n_1 = 6$, the control limits would be located at $\pm k_1\sigma$ with $k_1 = 3$, and samples would be taken at the interval of $v_1 = 0.07$ day. The type I error probability α is 0.0027, and the power Po of the test is 0.97122.

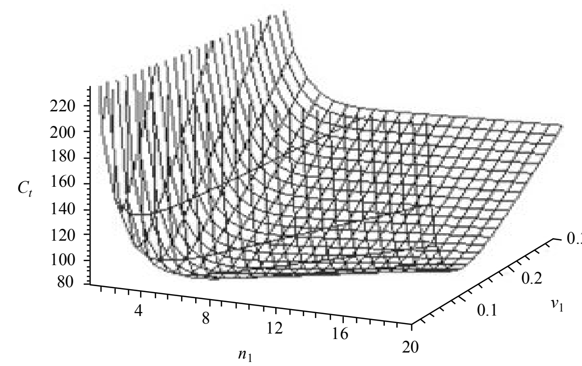

Figure 7. The effects of n_1 and v_1 on the minimum C_t of the PDCA design ($k_1 = 3$)

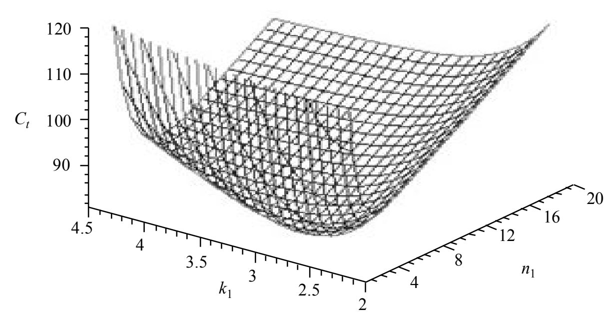

Figure 8. The effects of n_1 and k_1 on the minimum C_t of the PDCA design ($v_1 = 0.07$)

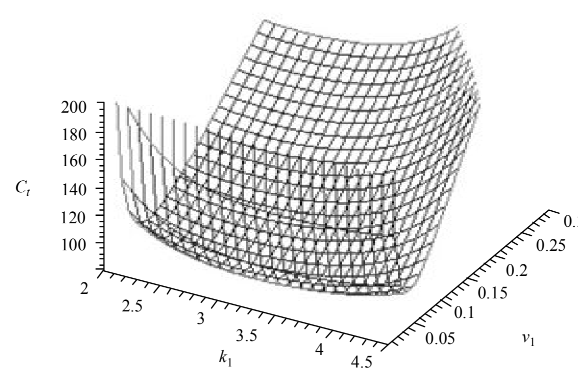

Figure 9. The effects of k_1 and v_1 on the minimum C_t of the PDCA design ($n_1 = 6$)

Next, in order to find the optimal values (n_2, k_2, v_2)

to minimize C_t of the CAPD model, we also evaluate a wide range of possible values used in this case, and show the results of nearing the optimal solution in Table 2. Moreover, to clarify the change of this CAPD design's minimum value of C_t according to the change in parameters n_2, k_2, v_2, we also show the results in Figures 10-12 as follows:

Table 2. Detailed analysis of the values (n_2, k_2, v_2) of the CAPD design

n_2	k_2	v_2	P_o	α	$C_{t(PDCA)}$
5	2.90	0.04	0.94204	0.00373	79.43875
5	2.90	0.05	0.94204	0.00373	77.22562
5	2.90	0.06	0.94204	0.00373	77.80179
5	3.00	0.04	0.92951	0.00270	79.40785
5	3.00	0.05	0.92951	0.00270	77.48556
5	3.00	0.06	0.92951	0.00270	78.30599
5	3.10	0.04	0.91499	0.00194	79.67284
5	3.10	0.05	0.91499	0.00194	78.03614
5	3.10	0.06	0.91499	0.00194	79.10691
6	2.90	0.04	0.97719	0.00373	80.39294
6	2.90	0.05	0.97719	0.00373	77.23246
6	2.90	0.06	0.97719	0.00373	77.04223
6	3.00	0.04	0.97122	0.00270	80.00498
6	3.00	0.05	0.97122	0.00270	77.04843
6	3.00	0.06	0.97122	0.00270	77.06644
6	3.10	0.04	0.96399	0.00194	79.85369
6	3.10	0.05	0.96399	0.00194	77.08151
6	3.10	0.06	0.96399	0.00194	77.19971
7	2.90	0.04	0.99161	0.00373	82.38552
7	2.90	0.05	0.99161	0.00373	78.53136
7	2.90	0.06	0.99161	0.00373	77.82597
7	3.00	0.04	0.98903	0.00270	81.83366
7	3.00	0.05	0.98903	0.00270	78.14342
7	3.00	0.06	0.98903	0.00270	77.55662
7	3.10	0.04	0.98579	0.00194	81.48620
7	3.10	0.05	0.98579	0.00194	77.93245
7	3.10	0.06	0.98579	0.00194	77.44843

The expected total cost of C_t per day associated with the use of the CAPD design is given by equation (6). Type *I* error probability α and power *Po* are also shown in equations (17) and (18).

In Table 2, we can note that the minimum C_t of the CAPD design is \$77.0484 per day, and the economically $\bar{x}$ chart would use a sample size per each sampling n_2 = 6, the control limits would be located at $\pm k_2\sigma$ with k_2 = 3.0, and samples would be taken at the interval of v_2 = 0.05 day. The type *I* error probability α is 0.0027 and power *Po* of the test is 0.9712.

From the results obtained from Table 1 and Table 2, we also note that the optimal values n_i (= 6) and k_i (= 3) of the two designs are the same, Type *I* error probability α (=0.0027) and power P_o (=0.97122) are the same, as well. The only difference is the optimal values v_i (PDCA design's is 0.07 and CAPD design's is 0.05).

Because the C_t of the CAPD design (\$77.0484 per day) shows the lowest cost in the results of Tables 1 and Table 2, we can know that the CAPD design (n_2, k_2, v_2) = (6, 3.0, 0.05) is more suitable for this case.

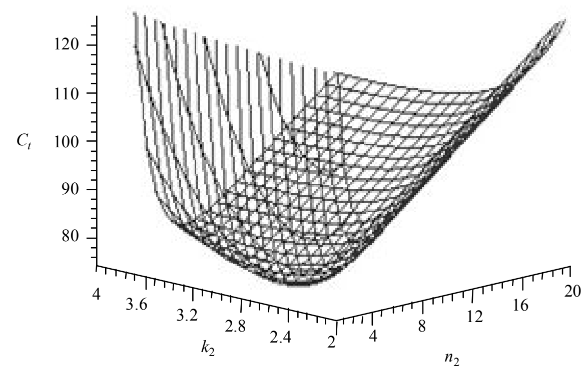

Figure 10. The effects of n_2 and v_2 on the minimum C_t of the CAPD design (k_2 = 3)

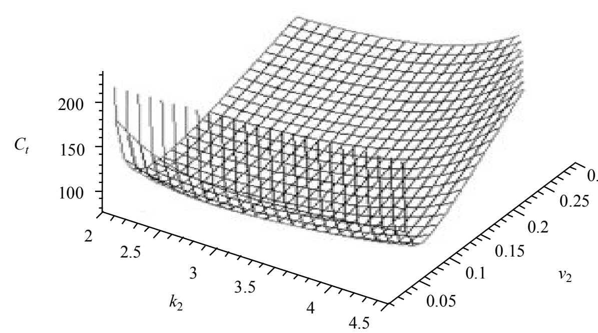

Figure 11. The effects of n_2 and k_2 on the minimum C_t of the CAPD design (v_2 = 0.05)

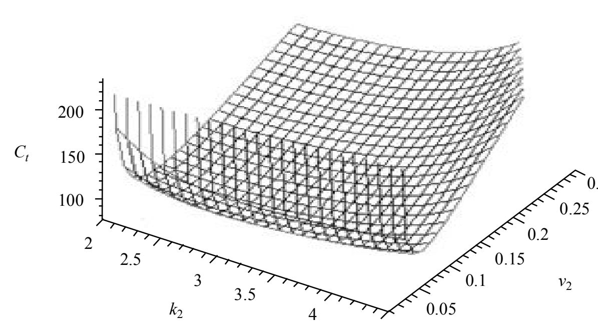

Figure 12. The effects of k_2 and v_2 on the minimum C_t of the CAPD design (n_2 = 6)

5.2 Comparison between the PDCA and CAPD designs

When the $\bar{x}$ chart is used in the short production run process, the mean of the period of in-control λ_i^{-1} and the size of the quality shift in the mean δ_i are important influential elements on the total expected cost of C_t, which changes from the change of the process situation (workers, raw materials, machines and a change in the inspection method or standards, etc.). Therefore, to compare the two designs, we first study the relations between λ_i^{-1}, δ_i and C_t based on the result $((n_i, k_i, v_i) = (6, 3.0, 0.05))$ of §5.1.

From Table 3, we can note that C_t of the PDCA design is cheaper than C_t of the CAPD design when λ_i is small (λ_i^{-1} is long), while C_t of the CAPD design becomes cheaper than C_t of the PDCA design with an increase of λ_i (decrease of λ_i^{-1}).

In other words, when the assignable cause occurs frequently (i.e., the interval of the in-control state is short), the CAPD design is more economical; and when this is not the case (i.e., when the interval of the in-control state is long), the PDCA design is more economical.

From Table 4, we can note that C_t of the two designs decreases by the increasing of the size of the quality shift in the mean δ_i. This is because a larger δ_i decreases $C_{c(out)}$ by the increasing of power. Also, we can note that although C_t of the PDCA design is cheaper than C_t of the CAPD design when δ_i is small, C_t of the CAPD model becomes cheaper than C_t of the PDCA model with an increase of δ_i.

In addition, the cost for the nonconformities per hour (c_2) and the cost of ACT (c_4) are the key parameters of the total expected cost of C_t. We also examined the relation between c_2 and C_t, c_4 and C_t, respectively.

From Table 5, we can note that C_t of the CAPD design is cheaper than C_t of the PDCA design when c_2 is small, while C_t of the PDCA model becomes cheaper than C_t of the CAPD model with an increase of c_2.

From Table 6, we can note that C_t of the CAPD design is cheaper than C_t of the PDCA design when c_4 is small, while C_t of the PDCA model becomes cheaper than C_t of the CAPD model with an increase of c_4.

Table 3. The comparison between two designs by λ_i

C_t	λ_i														
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5
PDCA (days)	52.3	58	63.2	68	72.3	76.1	79.5	82.6	85.4	87.8	90.1	92.1	93.9	95.5	97
CAPD (days)	60.5	63	65.4	67.7	70	72.2	74.4	76.4	78.3	80.1	81.8	83.4	84.9	86.3	87.6

Table 4. The comparison between two designs by δ_i

C_t	δ_i														
	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75
PDCA (days)	1070	768	386	198	127	99	88	84	82.355	82.093	82.053	82.048	82.04793	82.04791	82.04790
CAPD (days)	1927	1031	423	204	125	94	82	77	75.738	75.452	75.407	75.403	75.4023	75.40224	75.40223

Table 5. The comparison between two designs by c_2

C_t	c_2														
	50	100	150	200	250	300	350	400	450	500	550	600	650	700	750
PDCA (days)	70	84	97	111	125	138	152.0	166	179	193	207	220	234	248	262
CAPD (days)	62	77	92	107	122	137	151.8	167	182	197	212	227	241	256	271

Table 6. The comparison between two designs by c_4

C_t	c_4														
	20	70	120	170	220	270	320	370	420	470	520	570	620	670	720
PDCA (days)	81	103	124	146	168	189	211	232	254	275	297	318	340	361	383
CAPD (days)	75	98	122	145	169	192	216	239	263	286	310	333	357	380	404

6. CONCLUSIONS

In this paper, we proposed the PDCA design of the $\bar{x}$ chart based on Del Castillo and Montgomery's design.

Then, based on the cases which start from the check stage, we also proposed the CAPD design of the $\bar{x}$ chart and showed its mathematical formulation.

Following this, from an economic viewpoint, we studied the optimal values of the sample size per each sampling n_i, control limits width k_i and sampling interval v_i of the two designs.

Finally, by numerically analyzing the relations between the key parameters of λ_i^{-1} (mean time of the in-control period), δ_i (the size of the quality shift in the mean), c_2 (cost of per unit time for the nonconformities), c_4 (cost of restoring an in-control state) and C_t (the total expected cost per unit time), some important conclusions may be drawn:

1. The CAPD design is less expensive for the process in which the assignable cause occurs frequently (the mean period of the in-control state is short); and the PDCA design is less expensive for the process in which the mean period of the in-control is long.
2. The CAPD design is less expensive for the process in which the assignable cause of magnitude δ_i which results in a shift in the mean is large; and the PDCA design is less expensive for the process in which the assignable cause of magnitude δ_i is small.
3. The CAPD design is less expensive for the process in which the risk of the nonconformities is small; and the PDCA design is less expensive for the process in which the risk of the nonconformities is large.
4. The CAPD design is less expensive for the process in which the cost of ACT is small; and the PDCA design is less expensive for the process in which the cost of ACT is large.

We expect that the examinations of the two designs of the $\bar{x}$ control chart will become useful references for quality maintenance and improvement of activity in industrial management systems.

APPENDIX

1. The mathematical formulations (the PDCA design)

The PDCA model's mathematical formulations are explained in detail as follows:

1.1 The PLAN cost of the PDCA design

Because PLAN is defined as constructing the control lines, sampling is necessary, therefore, the expected cost of PLAN per cycle is calculated by (7). Where f_1' denotes the number of samples taken during T', c_{p0} is the fixed sampling cost of PLAN and c_{p1} is the variable sampling cost of PLAN

$$C_p = [(c_{p0} + c_{p1}n_1)f_1'/T]E[cycle]. \tag{7}$$

1.2 The DO cost of the PDCA design

DO is defined as sampling and plotting on the $\bar{x}$ control chart every interval v_1 for monitoring the process. If f_1 denotes the number of samples taken during T - T' (> 0), then the expected cost of DO per unit time is

$$C_d = [(c_0 + c_1 n_1)f_1/T]E[cycle]. \tag{8}$$

1.3 The CHECK cost of the PDCA design

CHECK is defined as examining whether the points plotted on the $\bar{x}$ control chart are beyond the upper and lower control limits. We consider that the cost of CHECK in the PDCA design includes the costs of the in-control state based on the risk of type I error and the costs of the out-of-control state based on the risk of the nonconforming goods. The CHECK period of the out-of-control state (denoted as O_1) is from point S to point C. But T can be smaller than S and C, therefore, the mean of the CHECK period in the out-of-control state is calculated by $E[(T-T'-I_1)^+ - (T-T'-I_1-O_1)^+]$, where the notation $(T-T'-I_1)^+ = \max(0, T-T'-I_1)$ is used to denote the positive part of $T-T'-I_1$. Because the false alarm can take place only when the process is in the in-control state, the CHECK period of the in-control state (denoted as I_1) is from Q to S. But T can be smaller than S, which means that the production run time ends before detecting the assignable cause, therefore, the CHECK period of the in-control state is calculated by $E[\min(I_1, T-T')]$. Therefore, the expected cost of CHECK per cycle is calculated by (9) and (10).

$$C_{c(in)} = c_3 \alpha E[\min(I_1, T-T')]f_1/(T-T'), \tag{9}$$

$$C_{c(out)} = c_2 E[(T-T'-I_1)^+ - (T-T'-I_1-O_1)^+]. \tag{10}$$

1.4 The ACT cost of the PDCA design

It is considered that the process would be not restored back to the in-control state when production T is smaller than C. Therefore, the expected cost per cycle of ACT is calculated by (11), where, c_4 denotes the cost of restoring the in-control state.

$$C_a = c_4 \Pr\{I_1 + O_1 < T - T'\}. \tag{11}$$

2. The mathematical formulations (the CAPD design)

The CAPD model's mathematical formulations are explained in detail as follows:

2.1 The CHECK cost of the CAPD design

CHECK of the CAPD design is defined as examining whether the points plotted on the $\bar{x}$ control chart are beyond the upper and lower control limits.

We consider that the cost of CHECK in the CAPD design includes the costs of the in-control state based on the risk of the first error and the costs of the out-of-control state based on the risk of the nonconforming goods. Therefore, the expected cost of CHECK cycle is calculated as follows:

$$C_{c(out)} = c_2 E[\min(O_2, T)], \tag{12}$$

$$C_{c(in)} = c_3 \alpha\, E[(T - T' - O_2)^+ - (T - T' - O_2 - I_2)^+]\, f_2/(T - T'). \tag{13}$$

2.2 The ACT cost of the CAPD design

In the CAPD design, it is considered that the process would be not restored back to the in-control state when the production T is smaller than C'. Therefore, the expected cost per cycle of ACT is calculated as follows:

$$C_a = c_4 \Pr\{O_2 < T\}. \tag{14}$$

2.3 The PLAN cost of the CAPD design

Because PLAN is defined as constructing the control lines, sampling is necessary, therefore, the expected cost of PLAN per cycle is calculated as follows: However, in the CAPD design, it is thought that the PLAN's cost is not taken into account when the past control lines are extended. Therefore, if ρ denotes the probability of using the past control lines, then

$$C_p = [(c_0 + c_1 n_2) f_2{}'/T](1-\rho)E[cycle]. \tag{15}$$

2.4 The DO cost of the CAPD design

DO is defined as sampling and plotting on the $\bar{x}$ control chart every interval v_2 for monitoring the process. Here, if the number of samples taken during $((T - T') > 0)$ is f_2, then, the expected cost of DO per cycle is

$$C_d = (c_0 + c_1 n_2)(f_2/T)E[cycle]. \tag{16}$$

3. Power, the type I error probability and the out-of-control period

In this paper, the statistical hypothesis is that the mean equals a standard value. When $\Phi(Z) = e^{-Z^2/2}/\sqrt{2\pi}$ is the standard normal density, α (the type I error probability) and P_o (power) of the $\bar{x}$ control chart are given by (Del Castillo and Montgomery, 1996),

$$\alpha = 2\int_{k_i}^{\infty} \Phi(Z)\, dZ, \tag{17}$$

$$P_o = \int_{-\infty}^{-k-\delta_i\sqrt{n_i}} \Phi(Z)\, dZ + \int_{k-\delta_i\sqrt{n_i}}^{\infty} \Phi(Z)\, dZ. \tag{18}$$

Where δ_i is the size of the quality shift in the mean, and k_i is control limits width.

We assume that the out-of-control period O_i is an exponential random variable with the mean μ_i^{-1}. In this paper we use Ladany and Bedi's assumption (1976) that the shift occurs in the middle of an interval between samples and set μ_i^{-1} as follows:

$$\mu_i^{-1} = v_i(1/P_o - 1) + v_i/2 = v_i(1/P_o - 1/2). \tag{19}$$

REFERENCES

Amasaka, K. ed. (2003), *Manufacturing Fundamentals: The Application of Intelligence Control Chart-Digital Engineering for Superior Quality Assurance*, Japanese Standards Association (in Japanese).

Bai, D. S. and Lee, M. K. (1998), An economic design of variable sampling interval $\bar{x}$ control Charts, *International Journal of Production Economics*, **54**, 57-64.

Chen, Y. K. (2003) An economic-statistical designed for a VSI $\bar{x}$ control chart under non-normality, *The International Journal of Advanced Manufacturing Technology*, **22**, 602-610.

Chou, C.-Y., Li, M.-H. C., Wang, P.-H. (2001), Economic Statistical Design of Averages Control Charts for Monitoring a Process under Non-normality, *The International Journal of Advanced Manufacturing Technology*, **17**, 603-609.

Chou, C. Y., Liu, H. R., Chen, C. H., Huang, X. R. (2002), Economic-statistical design of multivariate control charts using quality loss function, *The International Journal of Advanced Manufacturing Technology*, **20**, 916-924.

Crowder, S. V. (1992), An SPC designs for short production run: minimizing expected cost, *Technometrics*, **34**, 64-73.

Del Castillo, E. and Montgomery, D. C. (1996), A General Design for the Optimal Economic Design of $\bar{x}$ Charts Used to Control Short or Long Run Processes, *IIE Transactions*, **28**, 193-201.

Duncan, A. J. (1956), The economic design of charts used to maintain current control of a process, *Journal of the American Statistical Association*, **51**, 228-242.

Gibra, I. N. (1971), Economically Optimal Determination of the Parameters of $\bar{x}$ Control Charts, *Management Science*, **17**, 635-647.

Ikezawa, T. (1985), I have leaned from TQC, *Quality Management*, **36**(1), 6-12 (in Japanese).

Jones, L. L. and Case, K. E. (1981), Economic design of a joint $\bar{x}$ and R Chart, *IIE Transactions*, **13**, 182-

195.

Ladany, S. P. and Bedi, D. N. (1976), Selection of the Optimal Setup Policy, *Naval research Logistics Quarterly*, **23**, 219-233.

Lowry, C. A. and Montgomery, D. C. (1995), A review of multivariate control chart, *IIE Transactions*, **27**, 800-810.

Matsui, M. (2005), A management cycle model: switching control under lot processing and time span, *Journal of Japan Industrial Management Association*, **56**, 256-264.

Miyakawa, M. (2000), *Technologe for Getting Quality-What the Taguchi Method has Brought Us,* Union of Japanese Scientists and Engineers (in Japanese).

Montgomery, D. C. (1985), *Introduction to Statistical Quality Control*, John Wiley & Sons, Inc.

Reynolds, M. R., Amin, R. W., Arnold, J. C., Nachlas, J. A. (1988), $\bar{x}$ Charts with variable sampling intervals, *Technometrics*, **30**, 181-192.

Saniga, E. M. (1989), Economic statistical Design of control Charts with an application to $\bar{x}$ and R Chart, *Technometrics*, **31**, 313-320.

Shiba, S. and Walden, D. (2001), *Four Practical Revolutions in Management: Systems for Creating Unique Organizational Capability (Total Quality Management)*, Productivity Pr.

Sun, J., Tsubaki, M., and Matsui, M. (2005), Economic considerations in CAPD Model of P Control Chart for Quality Improvement. *ICQ'05-Tokyo International Conference on Quality Proceedings*, VI-10.

Takahashi, T. (1999), The basic concept of quality management and SQC/TQM, *Communications of Japan Industrial Management Association*, **9**, 60-63 (in Japanese).

Yu, F. J. and Wu, H. H. (2004), An economic design for variable sampling interval MA control charts, *The International Journal of Advanced Manufacturing Technology*, **24**, 41-47.