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The Use of Particle Swarm Optimization for Order Allocation Under Multiple Capacitated Sourcing and Quantity Discounts

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Abstract. The selection of suppliers and the determination of order quantities to be placed with those suppliers are important decisions in a supply chain. In this research, a non-linear mixed integer programming model is presented to select suppliers and determine the order quantities. The model considers the purchasing cost which takes into account quantity discount, the cost of transportation, the fixed cost for establishing suppliers, the cost for holding inventory, and the cost of receiving poor quality parts. The capacity constraints for suppliers, quality and lead-time requirements for the parts are also taken into account in the model. Since the purchasing cost, which is a decreasing step function of order quantities, introduces discontinuities to the non-linear objective function, it is not easy to employ traditional optimization methods. Thus, a heuristic algorithm, called particle swarm optimization (PSO), is used to find the (near) optimal solution. However, PSO usually generates initial solutions randomly. To improve the PSO solution quality, a heuristic procedure is proposed to find an initial solution based on the average unit cost including transportation, purchasing, inventory, and poor quality part cost. The results show that PSO with the proposed initial solution heuristic provides better solutions than those with PSO algorithm only.

Keywords: Supply Chain Management, Supplier Selection, Order Allocation, Quantity Discount, Particle Swarm Optimization

1. INTRODUCTION

A firm's sourcing strategy is a key driver of an effective supply chain. Sourcing from a single supplier who could supply the entire demand requirements is easier to manage order receipts and can receive discount price. Single-sourcing dependency, however, exposes the buying firm to a greater risk of supply interruption. Having multiple sources ensures a degree of competition and also the possibility of a backup should a source fail to deliver. Thus, the supplier selection and order allocation decisions are important in purchasing department.

The focus of this paper is to decide which suppliers to be selected and how many to order from the selected suppliers in the presence of multiple sourcing with multiple criteria, alternative supplier pricing discount, and supplier capacity limitations. More specifically, this paper examines supplier selection and quantity allocation

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decisions simultaneously for the acquisition of a firm's total demands for multiple products from a pool of qualified suppliers who offer a variety of products and pricing discounts. As such, we explicitly assume that the firm has already established an adequate supplier base and the total quantity requirement for each item in a single period is known by the firm. The price discount provided by the suppliers is all-units quantity discount. A mixed non-linear integer programming model is developed. This model takes into account not only the price, but also the shortage of suppliers' capacity and the buyers' requirements on quality and service.

The remainder of this paper is organized as follows. In the next section, we review the relevant literature. We develop the mixed integer programming model of total cost and propose a heuristic for obtaining the initial solution in Section 3. In Section 4, the particle swarm optimization (PSO) algorithm is presented. We analyze the performance of the heuristic through a numerical example in Section 5. This is followed by the conclusion in Section 6.

2. RELEVANT LITERATURE

The vast majority of research works dealt with the purchasing of materials and addressed two kinds of situations: single or multiple sourcing. For the single sourcing case, it is assumed that all suppliers can fully meet the buyer's requirements. The only decision concerned is the selection of the "best" supplier in which multiple criteria are taken into consideration. Dickson (1966) identified 23 different criteria to be evaluated in the vendor selection process. A review of criteria used in 74 articles in the vendor selection process since 1966 was presented in Weber *et al*. (1991). The authors discussed the complexity of the procurement process from an operations research perspective. Net price, delivery, quality, and vendor capacity were the most often used criteria in these articles. Those models being reviewed differ on the criteria considered and on the methods used to derive partial scores and weights for the criteria. These ranged from simple rating systems and equal weights on the criteria (Timmerman, 1986), to more sophisticated techniques based on pairwise comparisons, such as the analytic hierarchy process (Narasimhan, 1983) and the evaluation technique MACBETH (Oliveira and Lourenço, 2002).

Multiple sourcing is adopted either when none of the suppliers can ensure the supply reliability of a manufacturer's demand requirements or when procurement strategies aim at avoiding dependency on a single supplier. In this context, the buyer faces two decisions: selecting right suppliers among qualified vendors and allocating orders among selected suppliers. Among all the methods used to solve the problem, mathematical programming is the most frequently used approach. Moore and Fearon (1973), Anthony and Buffa (1977), Kingsman (1986), Pan (1989) and Ghodsyport and O'Brien (1998) adopted linear programming formulations; Gaballa (1974), Bender *et al.* (1985), Narasimhan and Stoynoff (1986), Turner (1988), Chaudry *et al.* (1993), Sadrian and Yoon (1994), Rosenthal *et al.* (1995), Kasilingam and Lee (1996), Degraeve and Roodhooft (1999), Jayaraman *et al.* (1999) and Ghodsypour and O'Brien (2001) developed mixed integer programming models; Pirkul and Aras (1985), Benton (1991), Hong and Hayya (1992), Ghodsyport an O'Brien (2001), and Crama *et al.* (2004) used nonlinear programming; multiple objective and goal programming were adopted by Buffa and Jackson (1983), Sharma *et al.* (1989), Weber and Current (1993) and Yahya and Kingsman (2002).

Pirkul and Aras (1985) analyzed the problem of determining order quantities for multiple items with all-units discounts. They formulated the problem as a nonlinear programming model that minimized the sum of aggregate purchasing costs, inventory carrying costs, and ordering costs and developed a heuristic algorithm using Lagrangian relaxation. Benton (1991) also devellopped a nonlinear programming model and a Lagrangian relaxation heuristic for the similar problem while storage and investment limitations were considered as constraints in the paper. Rosenthal *et al.* (1995) presented a mixed integer linear programming model to solve the problem for the case in which the vendor can sell items individually or as part of a bundle. Each vendor offered only one type of bundle, and the buyer could purchase at most one bundle per vendor.

Turner (1988) formulated the problem as a linear programming model that minimized the total contract cost, with constraints addressing demand satisfaction, vendor capacities, minimum and maximum order quantities, and geographic region purchasing restrictions. The discount types offered by the vendors were: deferred rebates based on the total value of the order, deferred rebates based on the order quantity, and marginal discounts based on the total value of the order. Weber and Current (1993) presented a multiple objective approach to analyze the inherent tradeoffs involved in multicriteria vendor selection problems and demonstrated with real world single-item data.

Bender *et al.* (1985) developed a commercial computerized model for vendor selection at IBM. They used mixed integer programming to minimize the sum of purchasing with quantity discount, transportation and inventtory costs by considering multiple items, multiple time periods, vendors' quality, delivery and capacity, but no mathematical formulations were presented. Quantity discount models involved distinct price breaks for each ingredient and supplier. Chaudry *et al.* (1993), in particular, considered a supplier selection problem involving multiple side-constraints: capacity, delivery performance, ingredient quality, etc. They proposed a linear and mixed integer programming formulation to minimize the purchasing costs for each ingredient separately.

Sadrian and Yoon (1994) proposed a mixed integer programming model to optimize the total cost of purchases in the presence of quantity discount. Their model did not consider inventory costs and other time-dependent parameters. It was solved using a commercial mathematical programming package. Kasilingam and Lee (1996) proposed a mixed integer programming model that considered stochastic demand, parts quality, purchasing and transportation cost, lead-time, cost of bad quality parts, and fixed cost for establishing vendors. Jayaraman *et al.* (1999) proposed a mixed integer programming model considering the presence of capacity constraints for suppliers and quality and delivery requirements for buyers. They solved the problem using a standard software package, GAMS.

Ghodsyport an O'Brien (2001) formulated a mixed integer nonlinear programming model to solve the multiple sourcing problem, which took into account the total cost of purchasing, storage, transportation and ordering costs. Buyer limitations on budget, quality, service, etc. were considered as side constraints. Crama *et al.* (2004) formulated a nonlinear mixed 0-1 programming model to solve the procurement decisions for multi-plant and multi-product with total quantity discounts problem. Kawtummachai and Van Hop (2005) proposed an algorithm that was based on the predetermined policy and assigned orders to multiple suppliers to minimize the total purchase cost while maintaining the minimum on-time delivery requirement. Burke *et al.* (2007) analyzed the sourcing decision based on three supplier pricing schemes, linear discounts, incremental units discounts, and all units discounts. Heuristic algorithms were developed to identify a quantity allocation decision for the buyer.

In the context of this paper, the modeling of supplier pricing schemes using fixed setup costs plus the concave all-units discount components. Thus, the primary difference in our work as compared to the previous research is that we analyze the buyer's problem under more realistic quality, inventory and delivery consideration. In the next section, we develop our modeling approach and a heuristic to obtain the initial solution.

3. MODEL FORMULATION

There are multiple criteria considered in selecting suppliers. Weber *et al.* (1991) concluded that the most often used criteria are price, delivery, quality, and vendor capacity. We thus take into account these four purchasing criteria in our model. We formulate this model to take into account the "all-units" discount pricing. It is assumed that the buyer is willing to pay a slightly higher price in order to achieve better aggregate quality, lead-time, and service. In Figure 1, Q_1 and Q_2 represent the first two cutoff points of all-units quantity discount for a supplier. The slope of the line segments OA and BC are specified by the discount rates of two quantity segments, respectively. If the ordered amount Q is within the discount break, $Q_1 \le Q < Q_2$, then the unit price for each of the Q units is p_2 (= c_2/Q_2) where $p_2 < p_1$ (= c_1/Q_1).

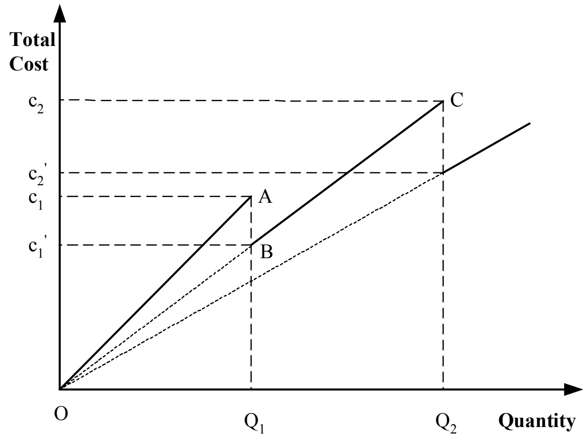

Figure 1. Suppliers' "all-units" discount cost function

3.1 Notations

The following notations are used throughout the paper.

Parameters:

a_{ij} : unit cost of receiving a defect part of item *i* from supplier *j*

b_{ij}^{k} : cutoff point of item *i* associated with segment *k* of supplier *j*

c_{ij}^{k} : unit price of item *i* associated with segment *k* of supplier *j*, c_{ij}^{1} is the original price

D_i : aggregate demand for item *i* over the planning horizon

d_{ij} : unit transportation cost of item *i* provided by supplier *j*

e_{ij} : fixed cost of receiving defect item *i* from supplier *j*

f_j : fixed cost of selecting supplier *j*

i : item index, $i = 1, 2, 3, \ldots, n$

j : supplier index *j*, $j = 1, 2, 3, \ldots, m$

K_{ij} : number of discount segments of item *i* provided by supplier *j*, $k = 1, 2, \ldots, K_{ij}$

L_i: maximum allowable lead-time of item *i*

l_{ij} : lead-time of item *i* provided by supplier *j*

m: number of suppliers

n : number of items

$p_{ij}(x_{ij})$: unit price of item *i* provided by supplier *j* at quantity of x_{ij}

Q_i : minimum quality requirement of item *i*

q_{ij} : percentage of good parts of item *i* provided by supplier *j*

r_i : inventory carrying rate of item *i*

V_{ij} : capacity of item *i* from supplier *j*

Decision variables:

x_{ij}: quantity of item *i* provided by supplier *j*

$$Y_j = \begin{cases} 1 & \text{if supplier } j \text{ is selected} \\ 0 & \text{otherwise} \end{cases}$$

$$Z_{ij} = \begin{cases} 1 & x_{ij} > 0 \\ 0 & \text{otherwise} \end{cases}$$

3.2 Assumptions

The assumptions of this research are as follows.

1. Each supplier offers "all-units" discount price for each item he provides. This implies that the unit cost of purchasing x_{ij} units of item *i* from vendor *j*, is defined to be

   $$p_{ij}(x_{ij}) = c_{ij}^{k} \qquad b_{ij}^{k-1} \le x_{ij} < b_{ij}^{k},\ k = 1, \cdots, K_{ij} \qquad (1)$$

   where c_{ij}^{k} is the unit price of item *i* from supplier *j* for discount segment *k*. $0 = b_{ij}^{0} < b_{ij}^{1} < \cdots < b_{ij}^{K_{ij}} = \infty$ is the sequence of quantities at which price cutoffs occur, and K_{ij} is the number of quantity segments in vendor *j*'s price discount for item *i*.
2. The demand during the planning horizon is known.
3. Each supplier needs to meet a certain maximum lead-time L_i in which to fulfill an order for item *i*.
4. Each supplier has its production capacity.
5. Each supplier needs to satisfy the minimum quality requirement of item *i*.
6. The buyer has fixed cost associated with establishing a supplier and fixed cost due to receiving defect parts for items.

3.3 The Model

The objective of the model is to minimize the fixed cost of establishing vendors, the fixed cost due to receiving poor quality parts, purchasing cost, transportation cost, cost of defect parts, and inventory cost during the planning period. The model may be formulated as follows:

$$Min \quad \sum_{j=1}^{m} f_j Y_j + \sum_{i=1}^{n}\sum_{j=1}^{m} e_{ij} Z_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{m}\left[p_{ij}(x_{ij})x_{ij} + x_{ij}d_{ij} + x_{ij}a_{ij}\left(1-q_{ij}\right) + \frac{1}{2}x_{ij}p_{ij}(x_{ij})r_i \right] \qquad (2)$$

$$s.t. \quad \sum_{j=1}^{m} x_{ij}q_{ij} \ge D_i \qquad \forall I \quad (3)$$

$$x_{ij} \le V_{ij}Y_j \qquad \forall i,j \quad (4)$$

$$l_{ij}Z_{ij} \le L_i \qquad \forall i,j \quad (5)$$

$$q_{ij}Z_{ij} \ge Q_i \qquad \forall i,j \quad (6)$$

$$Z_{ij} \le Y_j \qquad \forall i,j \quad (7)$$

$$x_{ij} \ge 0 \qquad \forall i,j \quad (8)$$

$$Z_{ij} = 0,1 \qquad \forall i,j \quad (9)$$

$$Y_j = 0,1 \qquad \forall j \quad (10)$$

The objective function in equation (2) represents the total costs that include six components: the fixed costs incurred to employ the suppliers, fixed costs due to receiving poor quality parts, purchasing costs with price quantity discounts, transportation costs, costs of defect parts, and inventory carrying costs. Constraint (3) ensures that the quantity ordered for each item meets the quantity demanded during the planning horizon. Constraint (4) ensures that the order quantity placed with the supplier does not exceed its capacity. Constraints (5) and (6) are the product lead-time and quality constraints. These two constraints will help to reduce the number of decision variables and will not be explicitly included while solving the formulation. Constraint (7) requires that a vendor be selected before orders are placed with that vendor. Constraint (8) ensures that the order allocation should be greater than or equal to 0 while constraints (9) and (10) are the integrality restrictions.

If we let

$$F_{ij}(x_{ij}) = \left[\begin{array}{ll} c_{ij}^{1}(x_{ij} + \frac{r_i}{2}x_{ij}) & 0 \le x_{ij} < b_{ij}^{1} \\ c_{ij}^{2}(x_{ij} + \frac{r_i}{2}x_{ij}) & b_{ij}^{1} \le x_{ij} < b_{ij}^{2} \\ \vdots & \vdots \\ c_{ij}^{K_{ij}}(x_{ij} + \frac{r_i}{2}x_{ij}) & b_{ij}^{K_{ij}-1} \le x_{ij} < b_{ij}^{K_{ij}} \end{array} \right.$$

then the objective function can be modified as:

$$Min \quad \sum_{j=1}^{m} f_j Y_j + \sum_{i=1}^{n}\sum_{j=1}^{m} e_{ij} Z_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{m}[F_{ij}\left(x_{ij}\right) + x_{ij}d_{ij} + x_{ij}a_{ij}\left(1-q_{ij}\right)] \qquad (2')$$

Due to the presence of quantity discount and the discontinuities in $F_{ij}(.)$, it is not possible to find the solution as there were no price breaks. We will develop an algorithm to find a good initial solution for further search by particle swarm optimization proposed in Section 3.

3.4 Initial Solution Heuristic

The initial solution heuristic is based on the supplier's weighted average unit cost which takes into account supplier's capacity, all-units discount price, inventory cost, defect cost, and transportation cost. The rationale for this is to allocate the quantity to the supplier who can provide lower cost. For simplicity, the initial solution is described according to the algorithm steps as shown in Figure 2.

Step 1: Compute the net demand of each item based on the demand and inventory on hand.
Step 2: Determine the supplier bases on the predetermined lead-time and quality requirements. $i = 1$.
Step 3: Compute the average cost based on the net demand of item i and sort the average cost in ascending order.
Step 4: Select the supplier, say j, with the lowest average cost of item i. If the net demand of item i is less than the capacity of j, assign all the net demand to j and go to Step 6. Otherwise, assign j up to its capacity and update the net demand (net demand = net demand - V_{ij}).
Step 5: Repeat Step 4 until net demand of item i equals to 0.
Step 6: $i = i + 1$. If $i \leq n$, go to Step 3; otherwise, stop.

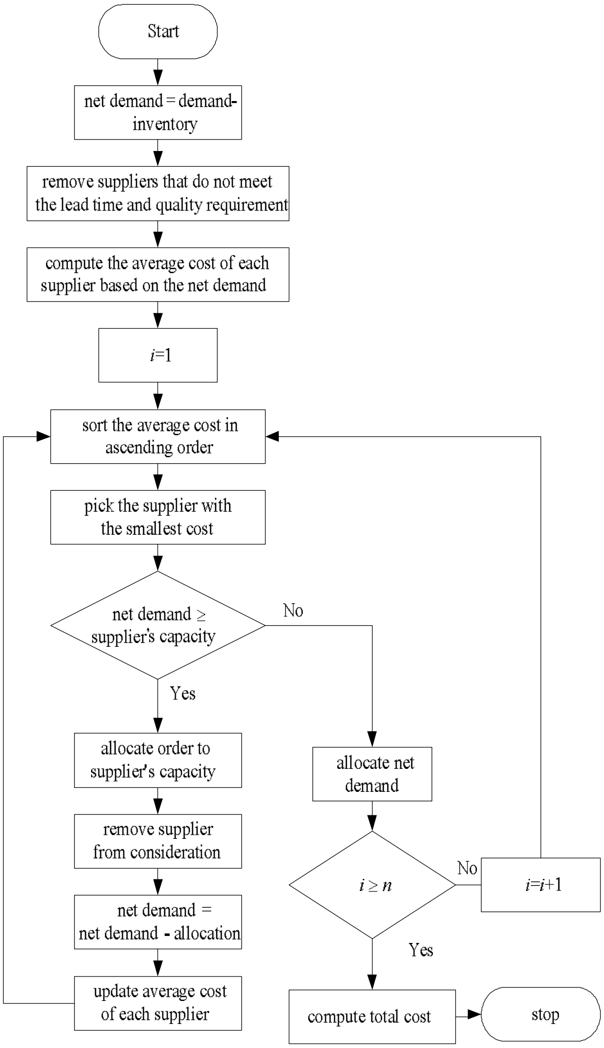

Figure 2. Flowchart of finding the initial solution

The initial solution found here will be then used as an input for the particle swarm optimization approach proposed in next section.

4. PARTICLE SWARM OPTIMIZATION

Particle swarm optimization (PSO), inspired by the social behaviour of bird flocking or fish schooling, is a population-based stochastic optimization technique developed by Kennedy and Eberhart in 1995. PSO is initialized with a population of random solutions and the potential solutions, called "particles", in PSO fly through the searching space. Each particle is also assigned a randomized velocity. The positions of individual particles are adjusted (via changing the velocity) according to its own flying experience, i.e., previous best (*pBest*), and its companions' flying experiences, i.e., global best (*gBest*), as shown in Figure 3. Velocity changing is weighted by a random term, with separate random numbers being generated for acceleration toward *pBest* and *gBest* locations. The general procedure of PSO is as follows.

1. Initialization. Randomly generate a population of potential solutions, called particles, and each particle is assigned a randomized velocity. The population size, b, is problem-dependent and suggested to be between 20 and 40 by Hu and Eberhart (2002).
2. Velocity Update. The particles are flown through hyperspace by updating their own velocities. The velocity update of a particle is dynamically adjusted, subject to its own past path and those of its companions. The particle updates its velocity and positions with following equation (11) and equation (12).

$$V_{id}^{new} = W \times V_{id}^{old} + c_1 \times rnd_1 \times \left(P_{id} - X_{id}^{old}\right) + c_2 \times rnd_2 \times \left(P_{gd} - X_{id}^{old}\right) \quad (11)$$

$$X_{id}^{new} = X_{id}^{old} + V_{id}^{new} \quad (12)$$

where V_{id}^{new} is the particle new velocity, V_{id}^{old} is the current particle velocity, and W is the inertia weight, c_1 and c_2 are learning factors. Eberhart and Shi (2001) suggested $c_1 = c_2 = 2$ and $W = 0.5 + (\text{rand}()/2)$. P_{id} is the *pBest* value, while P_{gd} is the *gBest* value. X_{id}^{new} is the new particle (solution) and X_{id}^{old} is the current particle. rnd_1 and rnd_2 are random numbers between (0, 1).

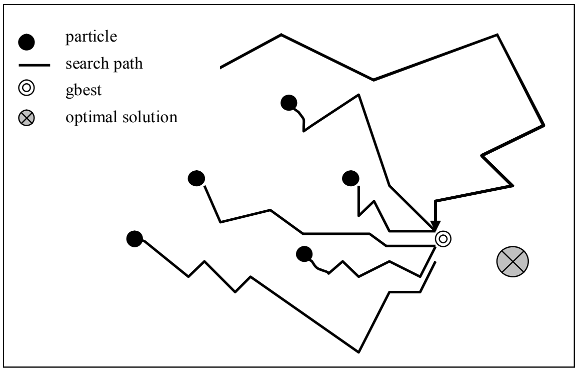

Figure 3. Particle Swarm Optimization

Particles' velocities on each dimension are clamped to a maximum velocity V_{max}, a parameter specified by the user. If the updated velocity exceeds V_{max}, then the velocity on that dimension is limited to V_{max}. Eberhart and Shi (2001) suggested V_{max} being set at about 10~20% of the dynamic range of the variable on each dimension.

In PSO, only *gBest* gives out the information to others. It is a one-way information sharing mechanism. The evolution only looks for the best solution. Compared with the genetic algorithm, all the particles tend to converge to the best solution quickly even in the local version in most cases. There are two key steps when applying PSO to optimization problems: the representation of the solution and the fitness function. One of the advantages of PSO is that PSO can take real numbers as particles.

The population size selected was problem-dependent. The number of particles most commonly used is in the typical range of 20~40 (Hu and Eberhart, 2002). The dimension of particles is determined by the problem to be optimized. The range of particles is also determined by the problem to be optimized, the user can specify different ranges for different dimension of particles. Learning factors, c_1 and c_2, usually equal to 2. However, other settings were also used in different papers. But usually c_1 equals to c_2 and ranges from [0, 4]. The stop condition is based on the maximum number of iterations the PSO execute and the minimum error requirement.

The PSO algorithm we used in the research is described as follows based on an example of 2 items and 2 suppliers. Figure 4 shows the flowchart for the algorithm.

Step 1: Compute the initial solution as described in section 3.4.
Step 2: Randomly generate 10 particles (solutions).
Step 3: Replace the first solution by the initial solution found in step 1.
Step 4: Compute the objective function (fitness) of each solution and find the minimum value and set it as *gBest* and *pBest*.
Step 5: $k = 1$.
Step 6: Update each particle velocities and positions by equations (11) and (12), respectively.
Step 7: Compute the fitness value of each particle. Find the smallest one and set it as *pBest*.
Step 8: Compare the fitness value of *pBest* (F_p) and *gBest* (F_g). If $F_p < F_g$, *gBest* = *pBest*.
Step 9: If $k > N$ (maximum number of iterations = 100 in this paper), then stop. Otherwise, $k = k + 1$ and go to Step 6.

5. NUMERICAL EXAMPLE

The PSO algorithm was coded in Visual Basic 6.0, and run on a PC with an AMD XP-1800 processor and 512 MB RAM, under the Windows XP operating system. An example problem with solutions is presented in this section to illustrate the proposed model. To determine the best values of parameters, a series of pilot experiments were conducted. The best values for our problem are as follows: $b = 20$, $c_1 = c_2 = 2$, $N = 100$, and $W = 0.9$. The example is run for 10 times with different random seeds. There are four different items and five suppliers. Table 1 shows the inventory carrying rates (r_i), the acceptable lead times (L_i), the minimum quality requirements (Q_i), the unit costs of a defect part (a_{ij} for all suppliers), the production capacities (V_{ij} for all suppliers), and the total demand (D_i) for the four part types. The variable costs of receiving a defect part are $0.1, $0.12, $0.14, and $0.18, respectively, for the four part types for all suppliers. The available capacities are 700, 700, 1000, and 800, respectively, for the four part types for all suppliers. Table 2 presents the lead times (l_{ij}), the percentage of good parts (q_{ij}), and the transportation cost (d_{ij}) for the four parts received from the five suppliers. The fixed costs associated with the selected suppliers (f_j) are $20, $18, $22, $20, and $21, respectively, for the five suppliers, while the fixed cost due to receiving defect parts (e_{ij}) are $4, $3.8, $3.5, $3, and $3.1, respectively, for each item *i* from the five suppliers as shown in Table 3.

Table 1. Input data for each item

Item	r_i	L_i	Q_i	a_{ij} (for all j)	V_{ij} (for all j)	D_i
1	0.2	2.5	0.8	0.1	700	1165
2	0.25	3	0.7	0.12	700	1397
3	0.3	2	0.8	0.14	1000	2329
4	0.35	4	0.8	0.18	800	1747

Table 3. Fixed cost and fixed defect cost for suppliers

Supplier	f_j	e_{ij} (for all part types)
s1	20	4
s2	18	3.8
s3	22	3.5
s4	20	3
s5	21	3.1

Table 2. Input data for different items and suppliers

Item	l_{ij}					q_{ij}					d_{ij}				
	s1	s2	s3	s4	s5	s1	s2	s3	s4	s5	s1	s2	s3	s4	s5
1	2	2	2.5	2.5	2.5	0.8	0.9	0.9	0.8	0.85	0.6	0.65	0.78	0.55	0.5
2	3	3	2.5	3	3	0.7	0.75	0.7	0.9	0.95	1.6	1.7	1.4	1.5	1.8
3	2	2	2	2	2	0.85	0.85	0.9	0.9	0.8	2.7	2.7	2.8	2.4	2.5
4	4	4	3	3.5	3	0.9	0.8	0.85	0.9	0.9	3.6	3.4	3.5	3.6	3.7

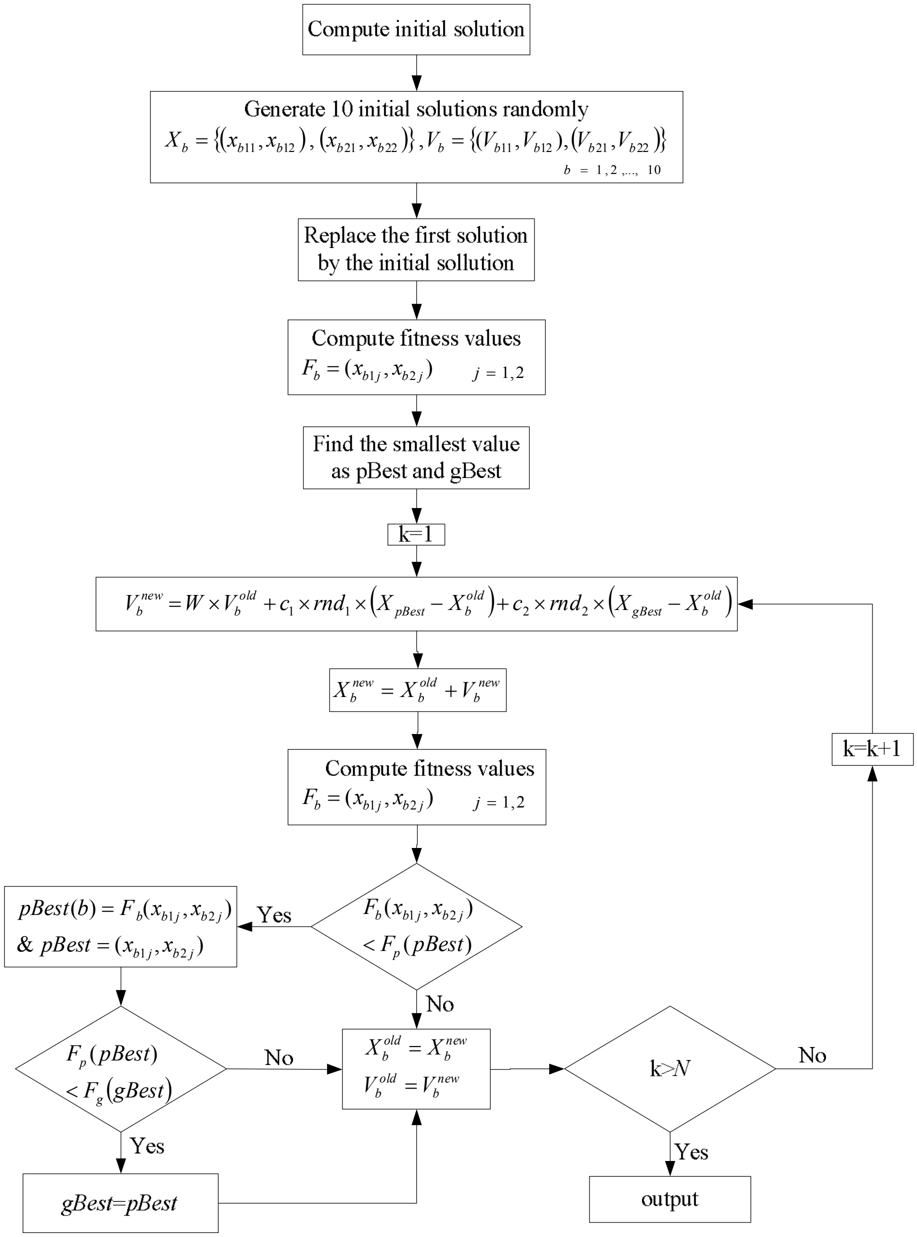

Figure 4. Flowchart for the proposed PSO algorithm

The quantity discount schedules for the four part types and five suppliers are shown in table 4. Here we assume that the "all-units" discount prices are different among different suppliers and different part types. Different discount schedules can be applied for the proposed model with some modification on the price break function.

Table 5 presents the initial solution based on the algorithm proposed in Section 3.4 and the best randomly generated initial solution among all 10 runs. The total costs are \$34107.9 and \$31472.05 for best randomly generated initial solution and proposed initial solution heuristic, respectively. The proposed heuristic allocates the orders based on the average unit cost which takes into account the fixed cost, inventory carrying cost, defect part cost as well as the price breaks of each part type/supplier. Four suppliers are selected in this example. Since we assign orders to the suppliers up to their capacity, each item has one or two suppliers receiving full capacity orders. This indicates that the heuristic can generated better initial solution than the one randomly generated.

We run the PSO with 10 randomly generated initial solutions and the PSO with one initial solution replaced by our proposed heuristic in Section 3.4. The best results of the PSO approach with both initial solution approaches are presented in Table 6. The total costs after PSO are \$31573.42 and \$31403.75, respectively. This indicates that the proposed initial solution heuristic can provide better results than the random approach.

To ensure that our algorithm can find (near) optimal solution, we use LINGO (2000) to solve the mixed integer programming model developed in Section 3 based on the number of suppliers found by PSO. The optimal total cost found by LINGO is $31399.22 as shown in Table 6. The selected suppliers found by LINGO

Table 4. Discount break for different items and suppliers

Item 1									
s1		s2		s3		s4		s5	
Range	Price	Range	Price	Range	Price	Range	Price	Range	Price
0~250	1.18	0~300	1.05	0~550	0.9	0~400	0.95	0~350	1
251~500	1.12	301~450	0.97	>550	0.75	400~650	0.85	351~600	0.92
>500	0.97	>450	0.89			>650	0.76	>600	0.82

Item 2									
s1		s2		s3		s4		s5	
Range	Price	Range	Price	Range	Price	Range	Price	Range	Price
0~350	2	0~250	1.8	0~300	2.1	0~550	1.9	0~400	2
351~600	1.86	251~500	1.7	301~450	1.97	>550	1.7	400~650	1.85
>600	1.64	>500	1.5	>450	1.78			>650	1.63

Item 3									
s1		s2		s3		s4		s5	
Range	Price	Range	Price	Range	Price	Range	Price	Range	Price
0~500	3.1	0~700	3.1	0~350	3.2	0~600	2.8	0~450	2.9
501~850	2.85	>700	2.9	351~650	3	601~900	2.5	451~750	2.69
>850	2.57			651~950	2.56	>900	2.29	>750	2.46
				>950	2.24				

Item 4									
s1		s2		s3		s4		s5	
Range	Price	Range	Price	Range	Price	Range	Price	Range	Price
0~250	3.7	0~450	3.8	0~350	4	0~450	3.9	0~400	4.1
251~500	3.51	451~700	3.53	351~700	3.6	451~750	3.51	400~750	3.73
501~750	3.14	>700	3.15	>700	3.32	>750	3.12	>750	3.28
>750	2.77								

Table 5. Initial solution generated by random approach and proposed heuristic

Item	Randomly generated solution					Proposed heuristic				
	s1	s2	s3	s4	s5	s1	s2	s3	s4	s5
1	292	128	291	195	259	0	0	0	700	465
2	368	101	367	194	367	0	700	0	697	0
3	229	700	187	700	513	0	329	0	1000	1000
4	729	706	99	42	171	800	800	0	147	0
Total Cost	34107.90					31472.05				

Table 6. Results of PSO with different initial solution approaches and LINGO

Item	PSO with random solution					PSO with proposed heuristic					LINGO solution				
	s1	s2	s3	s4	s5	s1	s2	s3	s4	s5	s1	s2	s3	s4	s5
1	0	465	0	0	700	0	0	0	465	700	0	0	0	465	700
2	0	0	700	697	0	699	698	0	0	0	697	700	0	0	0
3	0	329	0	1000	1000	0	0	951	927	451	0	0	951	927	451
4	800	800	0	147	0	800	800	147	0	0	800	800	0	147	0
Total cost	31573.42 (0.55%)*					31403.75 (0.01%)					31399.22				

*: percentage deviation from the LINGO solution

are the same as those found by PSO except for item 4 and the order allocations are only slightly different between two results. The total cost found by PSO with proposed initial solution heuristic is only 0.01% deviated from that of LINGO while the initial solution is 0.2% away from the solution found by LINGO. This indicates that both our initial solution and PSO algorithms could find near optimal solution.

6. CONCLUSION

The decisions on supplier selection and order placement among the selected suppliers are important for purchasing managers. Such decisions may greatly affect a firm's ability to compete in the supply chain as they frequently account for a large portion of production cost. This paper presents a nonlinear mixed integer programming model that can be used in the simultaneous determination of the number of suppliers to utilize and the purchase quantity allocations among suppliers in a multiple sourcing system, multiple items circumstance. The model takes into account the quantity discount price, quality and delivery requirements, transportation costs, and inventory carrying cost. Both fixed and variable costs due to receiving poor quality items are explicitly modeled in the objective function. Due to the presence of quantity discount, an algorithm to find an initial solution is proposed and the particle swarm optimization (PSO) approach is applied to further improve the initial solution. Based on the numerical example, the proposed heuristic can find a solution within 0.01% of the optimum. We believe the model proposed can be used to help the purchasing managers under a wide variety of operation conditions.

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REFERENCES

Anthony, T. F, and Buffa, F. P. (1977), Strategic purchase scheduling, *Journal of Purchasing and Materials Management*, **13**, 27-31.

Bender, P. S., Brown, R. W., Isaac, M. H. and Shapiro, J. F. (1985), Improving purchasing productivity at IBM with a normative decision support system, *Interfaces*, **15**, 106-115.

Benton, W. C. (1991), Quantity discount decisions under conditions of multiple items, multiple suppliers and resource Limitations, *International Journal of Production Research*, **29**, 1953-1961.

Buffa, F. P. and Jackson, W. M. (1983), A goal programming model for purchase planning, *Journal of Purchasing and Materials Management*, **18**, 27-34.

Burke, G. J., Carrillo, J. and Vakharia, A. J. (2008), Heuristics for sourcing from multiple suppliers with alternative quantity discounts, *European Journal of Operational Research*, **186**, 317-s329.

Chaudhry, S. S., Forst, F. G. and Zydiak, J. L. (1993), Vendor selection with price breaks, *European Journal of Operational Research*, **70**, 52-66.

Crama, Y., Pascual, J. R. and Torres, A. (2004), Optimal procurement decisions in the presence of total quantity discounts and alternative product recipes, *European Journal of Operational Research*, **159**, 364-378.

Degraeve, Z. and Roodhooft, F. (1999), Improving the efficiency of the purchasing process using total cost of ownership information: The case of heating electrodes at Cockerill Sambre S. A., *European Journal of Operational Research*, **112**, 42-53.

Dickson, G. W. (1966), An analysis of supplier selection system and decision, *Journal of Purchasing*, **2**, 5-17.

Eberhart, R. and Shi, Y. (2001), Tracking and optimizing dynamic system with particle swarms, *Proceedings of Congress on Evolutionary Computation*, Seoul, Korea, 94-97.

Gaballa, A. A. (1974), Minimum cost allocation of tenders, *Operational Research Quarterly*, **25**, 389-398.

Ghodsypour, S. H. and O'Brien, C. (1998), A decision support system for supplier selection using an integrated analytical hierarchy process and linear programming, *International Journal of Production Economics*, **56-57**, 199-212.

Ghodsypour, S. H. and O'Brien, C. (2001), The total cost of logistics in supplier selection, under conditions of multiple sourcing, multiple criteria and capacity constraint, *International Journal of Production Economics*, **73**, 15-27.

Hong, J. D. and Hayya, J. C. (1992), Just-in-time purchasing: single or multiple sourcing?, *International Journal of Production Economics*, **27**, 175-181.

Hu, X. and Eberhart, R. (2002), Multiobjcetive optimization using dynamic neighborhood particle swarm optimization, *Proceedings of the 2002 Congress on Evolutionary Computation*, Honolulu, U.S.A., 1677-681.

Jayaraman, V., Srivastava, R. and Benton, W. C. (1999), Supplier selection and order quantity allocation: A comprehensive model, *Journal of Supply Chain Management*, **35**, 50-58.

Kasilingam, R. G. and Lee, C. P. (1996), Selection of vendors: A mixed-integer programming approach, *Computers & Industrial Engineering*, **31**, 347-350.

Kawtummachai, R. and Van Hop, N., (2005), Order allocation in a multiple-supplier environment, *International Journal of Production Economics*, **93-94**, 231-238.

Kennedy, J. and Eberhart, R. (1995), Particle swarm optimization, *Proceedings of the IEEE International Conference on Neural Networks*, Perth, W. A., Australia, 1942-1948.

Kingsman, B. G. (1986), Purchasing raw materials with uncertain fluctuating prices, *European Journal of Operational Research*, **25**, 358-372.

Lindo System Inc. (2000), *Optimization Modeling with LINGO*, Third Edition, Lindo Systems Inc., Chicago, Illinois.

Moore, D. L. and Fearon, H. E. (1973), Computer-assisted decision-making in purchasing, *Journal of Purchasing*, **9**, 5-25.

Narasimhan, R. (1983), An analytical approach to supplier selection, *Journal of Purchasing and Materials Management*, **19**, 27-32.

Narasimhan, R. and Stoynoff, L. K. (1986), Optimizing aggregate procurement allocation decisions, *Journal of Purchasing and Materials Management*, **22**, 23-30.

Oliveira, R. C. and Lourenço, J. C. (2002), A multicriteria model for assigning new orders to service supplier, *European Journal of Operational Research*, **139**, 390-399.

Pan, A. C. (1989), Allocation of order quantity among suppliers, *Journal of Purchasing and Materials Management*, **25**, 39-39.

Pirkul, H. and Aras, O. A. (1985), Capacitated multiple item ordering problem with quantity discounts, *IIE Transactions*, **17**, 206-211.

Rosenthal, E. C., Zydiak, J. L. and Chaudhry, S. S. (1995), Vendor selection with bundling, *Decision Sciences*, **26**, 35-48.

Sardian, A. A. and Yoon, Y. S. (1994), A procurement decision support system in business volume discount environments, *Operations Research*, **42**, 14-23.

Sharma, D. Benton, W. C. and Srivastava, R. (1989), Competitive strategy and purchasing decision, *Proceedings of the 1989 Annual Conference of the Decision Sciences Institute*, 1088-1090.

Timmerman, E. (1986), An approach to vendor performance evaluation, *Journal of Purchasing and Materials Management*, **22**, 2-8.

Turner, I. (1988), An independent system for the evaluation of contract tenders, *Journal of Operational Research Society*, **39**, 551-561.

Weber, C. A., Current, J. R. and Benton, W. C. (1991), Vendor selection criteria and methods, *European Journal of Operational Research*, **50**, 2-18.

Weber, C. A. and Current, J. R. (1993), A multiobjective approach to vendor selection, *European Journal of Operational Research*, **68**, 173-184.

Yahya, S. and Kingsman, B. (2002), Modeling a multiobjective allocation problem in a government sponsored entrepreneur development programme, *European Journal of Operational Research*, **136**, 430-448.