International Journal of Reliability and Applications
Vol. 8, No. 1, pp. 67-82, 2007

Performance Analysis of a Parallel System Having a Cold Standby Unit

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Abstract. This paper deals with the effectiveness analysis of an engineering system, which has two units of different strengths in parallel and one unit as a cold standby unit. Failure times for all the units have negative exponential distribution whereas their repair times have general distribution. Single server caters the need for the system. The effectiveness analysis of the system is done by using regenerative point technique. The different measures of effectiveness such as mean sojourn time, mean time to system failure, availability, busy period, etc, are derived. Cost factors also taken into consideration.

Key Words: *engineering system, parallel system, standby unit, negative exponential distribution, general distribution, effectiveness analysis, availability, cost factor*

1. INTRODUCTION

Researchers in the field of reliability have studied two unit reparable redundant systems. Systems under different set of conditions have been analyzed and obtained the

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reliability characteristics using the theory of Semi Markov process, regenerative process Markov renewal process and supplementary variable techniques. Kapoor and Kapoor (1975a, 1975b) discussed two unit systems with preventive maintenance and delayed repair. Kumar (1977) and Kumar et al (1978, 1980) considered stochastic behavior of two unit systems with two switching failure modes, repair efficiency and intermittently available repairs.

Kodama et al (1976) analyzed a two unit model with warm standby which is dissimilar to the operative unit. Nakagawa and Osaki (1976) analyzed a two unit parallel redundant system with repair maintenance. Goel and Sharma (1986, 1989) and Sharma and Goel (1985) analyzed two unit systems under different set of conditions using regenerative process technique. Goel et al (1985, 1986) also analyzed a two unit (dissimilar) parallel system with inspection and bivariate exponential life times.

The effectiveness of two unit parallel systems will enhance if these are supported by a standby unit. The present study deals with the system where two units (dissimilar) are online initially and third one is in cold standby. Using regenerative point technique in Markov renewal process we find the different measures of effectiveness and the expected profit incurred in (0, t].

2. ASSUMPTIONS AND NOTATIONS

The following assumptions of the system are made:
(i) The system has three units. Two of them are of a particular kind and the third one is different, (ii) the two dissimilar units (i.e. unit -1 and unit -2) share the load and the third one which is similar to unit – 2. That is failure rate of this unit is same as of unit-2, is kept as cold standby, (iii) Upon failure of unit -1 or unit -2 which are initially online, the unit in standby becomes operative and repair of failed unit starts, (iv) There is a single repair facility always available, (v) Unit -1 is the priority unit and preferred over unit -2 in operations. That is if standby unit (which is similar to unit -2) is working in place of unit -1 and means while unit -1 becomes available then it will take over the operation and standby unit will be back to its initial position, i.e., cold standby, (vi) Unit -1 gets priority in repair in such a way that if unit -1 and unit-2 both are waiting for repair, the repair of unit -1 will be commenced first irrespective of the order of failure, (vii) Switchover time is instantaneous, (viii) Repairs are perfect and restore the unit as new, (ix) Failure rates of units are constant and repair time distributions are general.

Notations

The following notations are used through out the paper:
λ_1, λ_3 : Constant failure rates of unit-1 when (i) unit-2 is in working order and (ii) unit-2 is not in working order, λ_2, λ_4 : Constant failure rates of unit-2 when (i) unit-1 is in working order and (ii) unit-1 is not in working order, f_1 (.), F_1(.): pdf and cdf of repair time of unit-1, f_2(.), F_2(.): pdf and cdf of repair time of unit-2, p_{ij} : $\lim_{t\to\infty} Q_{ij}(t)$, E: set of regenerative states, π_i(.): cdf of time to system failure when S_i □ E, ~: Symbol for Laplace-Stieltjes

transform, *: Symbol for Laplace transform, m_{ij}: contribution to mean sojourn time in state S_i [] E and non-regenerative state, if it occurs before transiting to S_j [] E, m_i: $\sum_j m_{ij}$,

©: Symbol for ordinary convolution,: ◊: Symbol for Stieltjes convolution.

The possible states are given as follows:

State	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9
Unit-1	O	O	r	wr	O	R	wr	R	r	O
Unit-2	O	r	O	R	R	wr	R	wr	O	r
Standby	S	O	O	O	wr	O	wr	wr	wr	wr

Where O: operating status of unit, S: standby, r: unit just entered repair after failure, R: repair of unit is continued from some previous state, wr: unit waiting for repair after failure as the repair facility is busy, Up states: S_0, S_1, S_2, S_3, S_4, S_5, S_8, S_9, Down states: S_6 and S_7, Regenerative states: S_0, S_1, S_2 , S_8, S_9, Non–regenerative states: S3, S_4, S_5, S_6, and S_7. A typical diagram of such a system is given in Figure 1.

3. TRANSITION PROBABILITIES AND MEAN SOJOURN TIME

Note that

$$Q_{ij}(t) = P\,[X_{n+1}=S_j\,;\; T_{n+1}-T_n \le t \mid X_n = S_i] \tag{3.1}$$

as the semi-Markov kernel over E (the set of all regenerative states), where T_0, T_1 ... denote entry into states S_i [] E and X_n is the state visited at epoch T_n. {X ,T} is a Markov renewal process with state space E. The stochastic matrix of imbedded Markov chain is

$$P \equiv [p_{ij}] \equiv [Q_{ij}(\infty)] = Q(\infty) \tag{3.2}$$

The non-zero elements of $[p_{ij}]$ are given as follows

$$P_{01} = \frac{\lambda_2}{\lambda_1+\lambda_2}; \quad P_{02} = \frac{\lambda_1}{\lambda_1+\lambda_2};$$

$$P_{10} = \tilde{F}_2(\lambda_1+\lambda_2); \quad P_{12}^{(3)} = \frac{\lambda_1}{\lambda_1+\lambda_2-\lambda_4}\left\{\tilde{F}_2(\lambda_4) - \tilde{F}_2(\lambda_1+\lambda_2)\right\};$$

$$P_{11}^{(4)} = \frac{\lambda_2}{\lambda_1+\lambda_2-\lambda_3}\left\{\tilde{F}_2(\lambda_3) - \tilde{F}_2(\lambda_1+\lambda_2)\right\}$$

$$P_{18}^{(3,6)} = \frac{\lambda_1}{\lambda_1 + \lambda_2 - \lambda_4}\left\{1 - \tilde{F}_2(\lambda_4)\right\} - \frac{\lambda_1\lambda_4}{(\lambda_1 + \lambda_2 - \lambda_4)(\lambda_1 + \lambda_2)}\left\{1 - \tilde{F}_2(\lambda_1 + \lambda_2)\right\}$$

$$P_{18}^{(4,6)} = \frac{\lambda_2}{\lambda_1 + \lambda_2 - \lambda_3}\left\{1 - \tilde{F}_2(\lambda_3)\right\} - \frac{\lambda_2\lambda_3}{(\lambda_1 + \lambda_2 - \lambda_3)(\lambda_1 + \lambda_2)}\left\{1 - \tilde{F}_2(\lambda_1 + \lambda_2)\right\}$$

$$P_{20} = \tilde{F}_1(2\lambda_2); \quad P_{21}^{(5)} = \frac{2\lambda_2}{2\lambda_2 - \lambda_4}\left\{\tilde{F}_1(\lambda_4) - \tilde{F}_1(2\lambda_2)\right\};$$

$$P_{29}^{(5,7)} = \frac{2\lambda_2}{2\lambda_2 - \lambda_4}\left\{1 - \tilde{F}_1(\lambda_4)\right\} - \frac{\lambda_4}{(2\lambda_2 - \lambda_4)}\left\{1 - \tilde{F}_1(2\lambda_2)\right\}$$

$$P_{81} = \tilde{F}_1(\lambda_4); \quad P_{89}^{(7)} = 1 - \tilde{F}_1(\lambda_4)$$

$$P_{91} = \tilde{F}_2(\lambda_3); \quad P_{98}^{(6)} = 1 - \tilde{F}_2(\lambda_3)$$

it can be verified that

$$P_{01} + P_{02} = P_{10} + P_{12}^{(3)} + P_{11}^{(4)} + P_{18}^{(3,6)} + P_{18}^{(4,6)} = P_{20} + P_{21}^{(5)} + P_{29}^{(5,7)} = 1$$

and

$$P_{81} + P_{89}^{(7)} = P_{91} + P_{98}^{(6)} = 1$$

Mean Sojourn Time

The mean sojourn time in state $S_i \in E$ is defined as the average time of stay of the system in state S_i before before transiting to any other state (regenerative or non-regenerative). If T_i denotes the sojourn time in state S_i, the mean sojourn time in state S_i is given by

$$\mu_i = E(T_i) = \int_0^\infty P(T_i > t)dt$$

Thus,

$$\mu_0 = \frac{1}{\lambda_1 + \lambda_2}; \quad \mu_1 = \frac{1}{\lambda_1 + \lambda_2}\{1 - \widetilde{F}_2(\lambda_1 + \lambda_2)\}$$

$$\mu_2 = \frac{1}{2\lambda_2}\{1 - \widetilde{F}_1(2\lambda_2)\}$$

$$\mu_8 = \frac{1}{\lambda_4}\{1 - \widetilde{F}_1(\lambda_4)\}$$

$$\mu_9 = \frac{1}{\lambda_3}\{1 - \widetilde{F}_2(\lambda_3)\}$$

Let m_i be the mean time taken by the system in regenerative state S_i to any regenerative state S_j when the time is counted from the epoch of entrance into the state S_i. Mathematically,

$$m_i = \int_0^\infty t dQ_i(t) = \int_0^\infty \overline{Q}_i(t)dt$$

where $Q_i(t)$ is the probability of transition of the system from regenerative state S_i to some regenerative or non regenerative state before or at time t and

$$Q_i(t) = \sum_j \left[Q_{ij}(t) + \sum_k Q_{ij}^{(K)}(t) + \sum_k \sum_l Q_{ij}^{(k,l)}(t) \right]$$

Hence,

$$m_0 = \frac{1}{\lambda_1 + \lambda_2}; \quad m_1 = \int_0^\infty \overline{F}_2(t)dt = m_9 \quad and \quad m_1 = \int_0^\infty \overline{F}_1(t)dt = m_8$$

4. MEAN TIME TO SYSTEM FAILURE

Time to system failure can be regarded as the first passage time to a failed state and its mean is called mean time to system failure. Let T_i be the time to system failure for the first time when the system initially starts from regenerative state S_i and $\pi_i(t)$ =P[$T_i \leq t$] be the distribution function of the time to system failure with starting state S_i. We regard the failed states as absorbing state.

Thus,

$$\pi_0(t) = Q_{01}(t) \Diamond \pi_1(t) + Q_{02}(t) \Diamond \pi_2(t) \tag{4.1}$$

$$\pi_1(t) = Q_{10}(t) \Diamond \pi_0(t) + Q_{12}{}^{(3)}(t) \Diamond \pi_2(t) + Q_{11}{}^{(4)}(t) \Diamond \pi_1(t) + Q_{16}{}^{(3)}(t) + Q_{16}{}^{(4)}(t) \tag{4.2}$$

$$\pi_2(t) = Q_{20}(t) \Diamond \pi_0(t) + Q_{21}{}^{(5)}(t) \Diamond \pi_1(t) + Q_{27}{}^{(5)}(t) \tag{4.3}$$

$$\pi_8(t) = Q_{81}(t) \Diamond \pi_1(t) + Q_{87}(t) \tag{4.4}$$

$$\pi_9(t) = Q_{91}(t) \Diamond \pi_1(t) + Q_{96}(t) \tag{4.5}$$

Taking Laplace –Satietjes transform of (4.1-4.5) and solving for $\tilde{\pi}_0(s)$, we have (Omitting the argument "s" for brevity)

$$\tilde{\pi}_0(s) = \frac{\left\{\tilde{Q}_{16}^{(3)} + \tilde{Q}_{16}^{(4)}\right\}\left\{\tilde{Q}_{01} + \tilde{Q}_{02}\tilde{Q}_{21}^{(5)}\right\} + \left\{\tilde{Q}_{01}Q_{12}^{(3)} + \tilde{Q}_{02}\left(1 - Q_{11}^{(4)}\right)\right\}\tilde{Q}_{27}^{(5)}}{1 - \tilde{Q}_{11}^{(4)} - \tilde{Q}_{12}^{(3)}\tilde{Q}_{21}^{(5)} - \tilde{Q}_{01}\left\{\tilde{Q}_{10} + \tilde{Q}_{12}^{(3)}\tilde{Q}_{20}\right\} - \tilde{Q}_{02}\left\{\tilde{Q}_{21}^{(5)}\tilde{Q}_{10} + \tilde{Q}_{20}\left(1 - \tilde{Q}_{11}^{4}\right)\right\}}$$

$$= \frac{N_1(s)}{D_1(s)} \tag{4.6}$$

Using the results, we have $N_1(0)=D_1(0)$. Thus, $\tilde{\pi}_0(0) = 1$. This shows that $\tilde{\pi}_0(t)$ is a proper cdf. Therefore mean time to system failure, when the system initially starts from state S_0, is

$$E(T) = -\left.\frac{d\tilde{\pi}_0(s)}{ds}\right|_{s=0} = \frac{D_1'(0) - N_1'(0)}{D_1(0)} \tag{4.7}$$

We know that m_{ij} is the mean elapsed time by the system in state S_i before transiting to state $S_j \in E$ .Mathematically

$$m_{ij} = -\tilde{Q}_{ij}'(0) = \int_0^\infty \overline{Q}_{ij}(t)dt \tag{4.8}$$

Similarly we get the expression for $m_{ij}^{(k)}$ and it can be verified that

$$\sum_j \left[m_{ij} + \sum_k m_{ij}^{(k)} \right] = m_i \tag{4.9}$$

To obtain the numerator of (4.7) we collect the coefficients of relevant m_{ij} and $m_{ij}^{(k)}$ in $D_1'(0) - N_1'(0)$. Thus we have

$$MTSF = \frac{A}{B}, \tag{4.10}$$

where $A = m_0 X_0 + m_1 X_1 + m_2 X_2$ (4.11)

$$X_0 = 1 - \frac{\lambda_2 \widetilde{F}_2(\lambda_3)}{c} + \frac{2\lambda_1\lambda_2\widetilde{F}_2(\lambda_4)}{bd}\left\{\widetilde{F}_1(2\lambda_2) - \widetilde{F}_1(\lambda_4)\right\}$$
$$+ \lambda_2 \widetilde{F}_2(\lambda_1 + \lambda_2)\left\{\frac{1}{c} - \frac{2\lambda_1\widetilde{F}_1(2\lambda_2)}{bd} + \frac{2\lambda_1\widetilde{F}_1(\lambda_4)}{bd}\right\}$$
$$X_1 = \frac{\lambda_2}{a} + \frac{2\lambda_1\lambda_2}{ad}\left\{\widetilde{F}_1(\lambda_4) - \widetilde{F}_1(2\lambda_2)\right\}$$
$$X_2 = \frac{\lambda_1}{a} + \frac{\lambda_1\lambda_2}{ab}\widetilde{F}_2(\lambda_4) - \frac{\lambda_1\lambda_2}{ac}\widetilde{F}_2(\lambda_3) + \frac{\lambda_1\lambda_2}{abc}(\lambda_3 - \lambda_1)\widetilde{F}_2(\lambda_1 + \lambda_2)$$
$$a = \lambda_1 + \lambda_2; \quad b = \lambda_1 + \lambda_2 - \lambda_4$$
$$c = \lambda_1 + \lambda_2 - \lambda_3; \quad d = 2\lambda_2 - \lambda_4$$

and

$$B = 1 - p_{11}^{(4)} - p_{12}^{(3)} p_{21}^{(5)} - p_{01}\left\{p_{10} + p_{12}^{(3)} p_{20}\right\} - p_{02}\left\{p_{21}^{(5)} p_{10} + p_{20}\left(1 - p_{11}^{(4)}\right)\right\} \tag{4.12}$$

5. AVAILABILITY ANALYSIS

Let $M_i(t)$ is the probability that system starting from up state $S_i \in E$ is up at epoch t without transiting to any regenerative state or returning to itself.

$$M_0(t) = e^{-(\lambda_1 + \lambda_2)t} \tag{5.1}$$
$$M_1(t) = \overline{F}_2(t)\left[\frac{\lambda_1 e^{-\lambda_4 t}}{b} + \frac{\lambda_2 e^{-\lambda_3 t}}{c} + e^{-(\lambda_1+\lambda_2)t}\left\{\frac{\lambda_3\lambda_4 - \lambda_1\lambda_4 - \lambda_2\lambda_3}{bc}\right\}\right] \tag{5.2}$$
$$M_2(t) = \overline{F}_1(t)\left[\frac{\lambda_2 e^{-\lambda_4 t}}{d} + \frac{(\lambda_2 - \lambda_4)e^{-2\lambda_2 t}}{d}\right] \tag{5.3}$$
$$M_8(t) = \overline{F}_1(t)e^{-\lambda_4 t} \tag{5.4}$$
$$M_9(t) = \overline{F}_2(t)e^{-\lambda_3 t} \tag{5.5}$$

Define A_i (t) as the probability that the system is up at epoch t given that initially it was in state $S_i \in E$. We have

$$A_0(t) = M_0(t) + q_{01}(t) \copyright A_1(t) + q_{02}(t) \copyright A_2(t) \tag{5.6}$$
$$A_1(t) = M_1(t) + q_{10}(t) \copyright A_0(t) + q_{11}{}^{(4)}(t) \copyright A_1(t)$$

$$+q_{12}^{(3)}(t) © A_2(t) + q_{18}^{(3,6)}(t) © A_8(t) + q_{18}^{(4,6)}(t) © A_8(t) \tag{5.7}$$

$$A_2(t) = M_2(t) + q_{20}(t) © A_0(t) + q_{21}^{(5)}(t) © A_1(t) + q_{29}^{(5,7)}(t) © A_9(t) \tag{5.8}$$

$$A_8(t) = M_8(t) + q_{81}(t) © A_1(t) + q_{89}^{(7)}(t) © A_9(t) \tag{5.9}$$

$$A_9(t) = M_9(t) + q_{91}(t) © A_1(t) + q_{98}^{(6)}(t) © A_8(t) \tag{5.10}$$

Taking Laplace transform of the above set of equations and solving for $A_0^*(s)$, we have

$$A_0^*(s) = \frac{N_2^{(s)}}{D_2^{(s)}} \tag{5.11}$$

where

$$N_2(s) = M_0^* d_0 + M_1^* d_1 + M_2^* d_2 + M_8^* d_3 + M_9^* d_4 \tag{5.12}$$

$$\begin{aligned} D_2(s) = {} & \{1 - q_{89}^{*(7)} q_{98}^{*(6)}\}\{(1 - q_{20}^* q_{02}^*)(1 - q_{11}^{*(4)} - q_{10}^* q_{01}^*) \\ & - (q_{12}^{*(3)} + q_{10}^* q_{02}^*)(q_{21}^{*(5)} + q_{20}^* q_{01}^*)\} \\ & - q_{29}^{*(5,7)}\{q_{91}^* + q_{98}^{*(6)} q_{81}^*\} - \{q_{18}^{*(3,6)} + q_{18}^{*(4,6)}\}\{q_{81}^* + q_{89}^{*(7)} q_{91}^*\} \end{aligned} \tag{5.13}$$

and d_i's are the cofactors of the elements of first column of

$$\underset{\sim}{q} = \begin{bmatrix} 1 & -q_{01}^* & -q_{02}^* & 0 & 0 \\ -q_{10}^* & 1 - q_{11}^{*(4)} & -q_{12}^{*(3)} & -(q_{18}^{*(3,6)} + q_{18}^{*(4,6)}) & 0 \\ -q_{20}^* & -q_{21}^{*(5)} & 1 & 0 & -q_{29}^{*(5,7)} \\ 0 & -q_{81}^* & 0 & 1 & -q_{89}^{*(7)} \\ 0 & -q_{91}^* & 0 & -q_{98}^{*(6)} & 1 \end{bmatrix}$$

which are

$$d_0 = \{1 - q_{11}^{*(4)} - q_{12}^{*(3)} q_{21}^{*(5)}\}\{1 - q_{89}^{*(7)} q_{98}^{*(6)}\} - q_{12}^{*(3)} q_{29}^{*(5,7)}\{q_{91}^{*} + q_{89}^{*(6)} q_{81}^{*}\}$$
$$- \{q_{18}^{*(3,6)} + q_{18}^{*(4,6)}\}\{q_{81}^{*} + q_{89}^{*(7)} q_{91}^{*}\} \quad (5.14)$$

$$d_1 = \{1 - q_{89}^{*(7)} q_{98}^{*(6)}\}\{q_{01}^{*} + q_{02}^{*} q_{21}^{*(5)}\} + q_{29}^{*(5,7)}\{q_{91}^{*} + q_{98}^{*(6)} q_{81}^{*}\} \quad (5.15)$$

$$d_2 = \{1 - q_{89}^{*(7)} q_{98}^{*(6)}\}\{q_{01}^{*} q_{12}^{*(3)} + q_{02}^{*}(1 - q_{11}^{*(4)})\}$$
$$- q_{02}^{*}\{q_{18}^{*(3,6)} + q_{18}^{*(4,6)}\}\{q_{81}^{*} + q_{89}^{*(7)} q_{91}^{*}\} \quad (5.16)$$

$$d_3 = q_{29}^{*(5,7)} q_{98}^{*(6)}\{q_{01}^{*} q_{12}^{*(3)} + q_{02}^{*}(1 - q_{11}^{*(4)})\}$$
$$+ \{q_{18}^{*(3,6)} + q_{18}^{*(4,6)}\}\{q_{01}^{*} - q_{21}^{*(5)} - q_{29}^{*(5,7)} q_{91}^{*}\} \quad (5.17)$$

and

$$d_4 = q_{29}^{*(5,7)}\{q_{01}^{*} q_{12}^{*(3)} + q_{02}^{*}(1 - q_{11}^{*(4)})\}$$
$$- \{q_{18}^{*(3,6)} + q_{18}^{*(4,6)}\}\{q_{01}^{*} - q_{21}^{*(5)}) q_{89}^{*(7)} + q_{29}^{*(5,7)} q_{81}^{*}\} \quad (5.18)$$

Note that

$$q_{ij}^{*}\Big|_{s=0} = p_{ij} \text{ and } q_{ij}^{*'}(0) = -m_{ij}$$

$$q_{ij}^{*(k)}\Big|_{s=0} = p_{ij}^{(k)} \text{ and } q_{ij}^{*'(k)}(0) = -m_{ij}^{(k)}$$

Thus we find that $D_2(0) = 0$.

The steady state availability of the system with the initial state S_0;
is obtained by

$$A_0(\infty) = \lim_{s\to\infty} sA_0^{*}(s) = \frac{N_2(0)}{D_2'(0)}$$

$$A_0(\infty) = \lim_{s\to\infty} sA_0^{*}(s) = \frac{N_2(0)}{D_2'(0)}\ as\ \frac{s}{D_2(s)}\Big|_{s=0} \text{ is indeterminate form.}$$

To obtain $D'_2(0)$ we collect the relevant coefficient of $m_{ij} = -q_{ij}^{*'}(0)$ and $m_{ij}^{(k)} = -q_{ij}^{*'(k)}(0)$ in $D'_2(0)$ which are as follows

coefficient of m_{01} = coefficient of m_{02}

$$
\begin{aligned}
&= \{\tilde{F}_2(\lambda_3) + \tilde{F}_1(\lambda_4) - \tilde{F}_1(\lambda_4)\tilde{F}_2(\lambda_3)\} \\
&\quad [\{\frac{\lambda_1}{b}\tilde{F}_2(\lambda_4) - \frac{\lambda_1\lambda_4}{ab}\tilde{F}_2(\lambda_1+\lambda_2)\}\tilde{F}_1(2\lambda_2) \\
&\quad + \{1 - \frac{\lambda_1}{a}\tilde{F}_1(2\lambda_2)\}\tilde{F}_2(\lambda_1+\lambda_2)] \\
&= Y_0.
\end{aligned}
\tag{5.19}
$$

coefficient of m_{10} = coefficient of $m_{11}^{(4)}$ = coefficient of $m_{12}^{(3)}$
= coefficient of $m_{18}^{(3,6)}$ = coefficient of $m_{18}^{(4,6)}$

$$
= \{\tilde{F}_2(\lambda_3) + \tilde{F}_1(\lambda_4) - \tilde{F}_1(\lambda_4)\tilde{F}_2(\lambda_3)\}\left\{1 - \frac{\lambda_1}{a}\tilde{F}_1(2\lambda_2)\right\} = Y_1. \tag{5.20}
$$

coefficient of m_{20} = coefficient of $m_{21}^{(5)}$ = coefficient of $m_{29}^{(5,7)}$

$$
\begin{aligned}
&= \{\tilde{F}_2(\lambda_3) + \tilde{F}_1(\lambda_4) - \tilde{F}_1(\lambda_4)\tilde{F}_2(\lambda_3)\} \\
&\quad \left\{\frac{\lambda_1}{b}\tilde{F}_2(\lambda_4) - \frac{\lambda_1\lambda_4}{ab}\tilde{F}_2(\lambda_1+\lambda_2)\right\} = Y_2.
\end{aligned}
\tag{5.21}
$$

coefficient of m_{81} = coefficient of $m_{89}^{(7)}$

$$
\begin{aligned}
&= \left\{1 - \frac{2\lambda_2}{d}\tilde{F}_1(\lambda_4) + \frac{\lambda_4\tilde{F}_1(2\lambda_2)}{d}\right\}\{1 - \tilde{F}_2(\lambda_3)\} \\
&\quad \left\{\frac{\lambda_1}{b}\tilde{F}_2(\lambda_4) - \frac{\lambda_1\lambda_4}{ab}\tilde{F}_2(\lambda_1+\lambda_2)\right\} \\
&\quad + \left\{1 - \frac{\lambda_1}{b}\tilde{F}_2(\lambda_4) + \left(\frac{\lambda_1\lambda_4}{ab} + \frac{\lambda_2\lambda_3}{ac}\right)\tilde{F}_2(\lambda_1+\lambda_2) - \frac{\lambda_2}{c}\tilde{F}_2(\lambda_3)\right\} \\
&\quad \left\{1 - \frac{\lambda_1}{a}\tilde{F}_1(2\lambda_2)\right\} = Y_3.
\end{aligned}
\tag{5.22}
$$

coefficient of m_{91} = coefficient of $m_{98}^{(6)}$

$$= \left\{1 - \widetilde{F}_1(\lambda_4)\right\}\left\{1 - \frac{\lambda_1}{a}\widetilde{F}_1(2\lambda_2)\right\}$$

$$\left\{1 - \frac{\lambda_1}{b}\widetilde{F}_2(\lambda_4) + \left(\frac{\lambda_1\lambda_4}{ab} + \frac{\lambda_2\lambda_3}{ac}\right)\widetilde{F}_2(\lambda_1 + \lambda_2) - \frac{\lambda_2}{c}\widetilde{F}_2(\lambda_3)\right\}$$

$$+\left\{1 - \frac{2\lambda_2}{d}\widetilde{F}_1(\lambda_4) + \frac{\lambda_4\widetilde{F}_1(2\lambda_2)}{d}\right\}\left\{\frac{\lambda_1}{b}\widetilde{F}_2(\lambda_4) - \frac{\lambda_1\lambda_4\widetilde{F}_2(\lambda_1 + \lambda_2)}{ab}\right\} = Y_4 \quad (5.23)$$

Thus,

$$D_2'(0) = m_0 Y_0 + m_1 Y_1 + m_2 Y_2 + m_8 Y_3 + m_9 Y_4 \quad (5.24)$$

$$N_2(0) = M_0^*(0)d_0(0) + M_1^*(0)d_1(0) + M_2^*(0)d_2(0) + M_8^*(0)d_3(0) + M_9^*(0)d_4(0) \quad (5.25)$$

6. BUSY PERIOD ANALYSIS

Let $W_i(t)$ denote the probability that the system starting from state S_i ⁞ E does not transit to any other state . Mathematically,

$$W_1(t) = e^{-(\lambda_1+\lambda_2)t}\overline{F}_2(t) \quad (6.1)$$

$$W_2(t) = e^{-2(\lambda_2)t}\overline{F}_1(t) \quad (6.2)$$

$$W_8(t) = e^{-(\lambda_4)t}\overline{F}_1(t) \quad (6.3)$$

and

$$W_9(t) = e^{-(\lambda_3)t}\overline{F}_2(t) \quad (6.4)$$

Let $B_i(t)$ be the probability that the repair facility is busy at epoch t given that the system starts from regenerative state S_i at t=0. By using the probabilistic arguments we have the recursive relations in $B_i(t)$ as follows

$$B_0(t) = q_{01}(t) © B_1(t) + q_{02}(t) © B_2(t) \quad (6.5)$$

$$B_1(t) = W_1(t) + q_{10}(t) © B_0(t) + q_{11}^{(4)}(t) © B_1(t) + q_{12}^{(3)}(t) © B_2(t)$$
$$+ q_{18}^{(3,6)}(t) © B_8(t) + q_{18}^{(4,6)}(t) © B_8(t) \quad (6.6)$$

$$B_2(t) = W_2(t) + q_{20}(t) \copyright B_0(t) + q_{21}^{(5)}(t) \copyright B_1(t) \quad + q_{29}^{(5,7)}(t) \copyright B_9(t) \qquad (6.7)$$

$$B_8(t) = W_8(t) + q_{81}(t) \copyright B_1(t) + q_{89}^{(7)}(t) \copyright B_9(t) \qquad (6.8)$$

$$B_9(t) = W_9(t) + q_{91}(t) \copyright B_1(t) + q_{98}^{(6)}(t) \copyright B_8(t) \qquad (6.9)$$

Taking the Laplace transform of the above set of equations the solution can be expressed in the following matrix form:

$$\left(B_0^*, B_1^*, B_2^*, B_8^*, B_9^*\right)' = q^{-1}\left(0, W_1^*, W_2^*, W_8^*, W_9^*\right)' \text{ (dropping "s" for brevity)}$$

Where q is same as give in availability analysis

Thus,

$$B_0^*(s) = \frac{N_3^{(s)}}{D_2^{(s)}} \qquad (6.10)$$

$$N_3(s) = W_1^* d_1 + W_2^* d_2 + W_8^* d_3 + W_9^* d_4 \qquad (6.11)$$

$D_2(s)$ is given in equation (5.13) and d_i's are as equations (5.14-5.18).

In the long run the fraction of time for which the repair facility is busy is given by:

$$B_0 = \lim_{t \to \infty} B_0(t) = \lim_{s \to 0} sB_0^*(s) = \frac{N_3(0)}{D_2'(0)} \qquad (6.12)$$

Here,

$$N_3(0) = W_1^*(0)d_1(0) + W_2^*(0)d_2(0) + W_8^*(0)d_3(0) + W_9^*(0)d_4(0) \qquad (6.13)$$

7. EXPECTED NUMBER OF VISITS BY THE REPAIR FACILITY

We define $V_i(t)$ as the expected number of visits by the repairman in (0,t] given that the system initially starts from Regenerative state S_i.

Thus,

$$V_1(t) = Q_{10}(t)\Diamond V_0(t) + Q_{12}{}^{(3)}(t)\Diamond V_2(t) + Q_{11}{}^{(4)}(t)\Diamond V_1(t) + \left\{Q_{18}{}^{(3,6)}(t) + Q_{18}{}^{(4,6)}(t)\right\}\Diamond V_8(t) \tag{7.1}$$

$$V_2(t) = Q_{20}(t)\Diamond V_0(t) + Q_{21}{}^{(5)}(t)\Diamond V_1(t) + Q_{29}{}^{(5,7)}(t)\Diamond V_9(t) \tag{7.2}$$

$$V_8(t) = Q_{81}(t)\Diamond V_1(t) + Q_{89}{}^{(7)}(t)\Diamond V_9(t) \tag{7.3}$$

$$V_9(t) = Q_{91}(t)\Diamond V_1(t) + Q_{98}{}^{(6)}(t)\Diamond V_8(t) \tag{7.4}$$

Solving these equations with the use of L.S. transform we get

$$\tilde{V}_0(s) = \frac{N_4^{(s)}}{D_4^{(s)}} \tag{7.5}$$

where (dropping "s" for brevity)

$$N_4(s) = \left(\tilde{Q}_{01} + \tilde{Q}_{02}\right)\left[\left\{1 - \tilde{Q}_{11}{}^{(4)} - \tilde{Q}_{12}{}^{(3)}\tilde{Q}_{21}{}^{(5)}\right\}\left(1 - \tilde{Q}_{98}{}^{(6)}\tilde{Q}_{89}{}^{(7)}\right)\right\} - \left\{\tilde{Q}_{12}{}^{(3)}\tilde{Q}_{29}{}^{(5,7)}\left(\tilde{Q}_{91} + \tilde{Q}_{98}{}^{(6)}\tilde{Q}_{81}\right) - \left(\tilde{Q}_{18}{}^{(3,6)} + \tilde{Q}_{18}{}^{(4,6)}\right)\left(\tilde{Q}_{89}{}^{(7)}\tilde{Q}_{91} + \tilde{Q}_{81}\right)\right\}\right] \tag{7.6}$$

and

$$D_4(s) = \left(1 - \tilde{Q}_{89}{}^{(7)}\tilde{Q}_{98}{}^{(6)}\right)\left\{1 - \tilde{Q}_{20}\tilde{Q}_{02}\left(1 - \tilde{Q}_{11}{}^{(4)} - \tilde{Q}_{10}\tilde{Q}_{01}\right)\right\} - \left(\tilde{Q}_{12}{}^{(3)} - \tilde{Q}_{10}\tilde{Q}_{02}\right)\left(\tilde{Q}_{21}{}^{(5)} - \tilde{Q}_{20}\tilde{Q}_{01}\right) - \left\{\tilde{Q}_{29}{}^{(5,7)}\left(\tilde{Q}_{91} + \tilde{Q}_{98}{}^{(6)}\tilde{Q}_{81}\right) - \left(\tilde{Q}_{18}{}^{(3,6)} + \tilde{Q}_{18}{}^{(4,6)}\left(\tilde{Q}_{81} + \tilde{Q}_{89}{}^{(7)}\tilde{Q}_{91}\right)\right)\right\} \tag{7.7}$$

8. COST ANALYSIS

We now obtain the cost function of the system considering mean up time of the system, expected busy period of the repair facility and expected number of visits by repairman
The expected profit incurred in (0, t] is given by

G (t) =expected total revenue in (0, t] –expected total repair cost in (0, t]
-expected cost of visits by repairman in (0, t]
$= c_1\mu_{up}(t) - c_2\mu_b(t)\ c_3V_0(t)$ (8.1)

where

$$\mu_{up}(t) = \int_0^t A_0^*(u)du \tag{8.2}$$

is expected up time of the system

$$\mu^*_{up}(t) = \frac{A_0^\bullet(s)}{s} \tag{8.3}$$

$$\mu_b(t) = \int_0^t B_0^\bullet(u)du \tag{8.4}$$

is the expected busy period of repairman

$$\mu^*_b(t) = \frac{A_0^\bullet(s)}{s} \tag{8.5}$$

c_1 = revenue per unit up time
c_2 = cost per unit time for which the system is under repair
and
c_3 = cost per visit by the repair man

The expected profit per unit time in steady state is

$$G = \lim_{t\to\infty} \frac{G(t)}{t} = \lim_{s\to 0} s^2\widetilde{G}(s) = c_1A_0 - c_2B_0 - c_3V_0 \tag{8.6}$$

9. DISCUSSION

This paper analyses an engineering system of immense practical importance. The system is comprised of three units out of which two are in parallel and one is kept as cold standby unit. In this paper the units in parallel are not similar and have different specifications. The unit which has better specifications is taken as priority unit. The cold standby unit is similar to the non priority unit. On failure of any of the online unit the cold standby unit will be instantaneously operative and repair of the failed unit will start. Priority unit gets priority in both operation and repair. This enhances the performance of the system and reduces the cost also. The failure time distribution for the units is negative exponential with different parameters and repair rates are considered to be general as repairs are time consuming and not instantaneous. Different characteristics of system performance such as mean sojourn time, mean time to system failure and availability are derived for the system under study. The concept of cost is very important one. Hence, in this paper the expected profit incurred is calculated with the help of expected uptime of the system, expected busy period of the repair facility and expected number of visits by the repair facility. We use the method of regenerative stochastic process for finding the different effectiveness characteristics of the system.

For future research we plan to study the effects of switching devices on the performance of this type of systems and to consider the Weibull distribution with different parameters as the failure time distribution for different units. Also, we will conduct simulation studies for the system under study.

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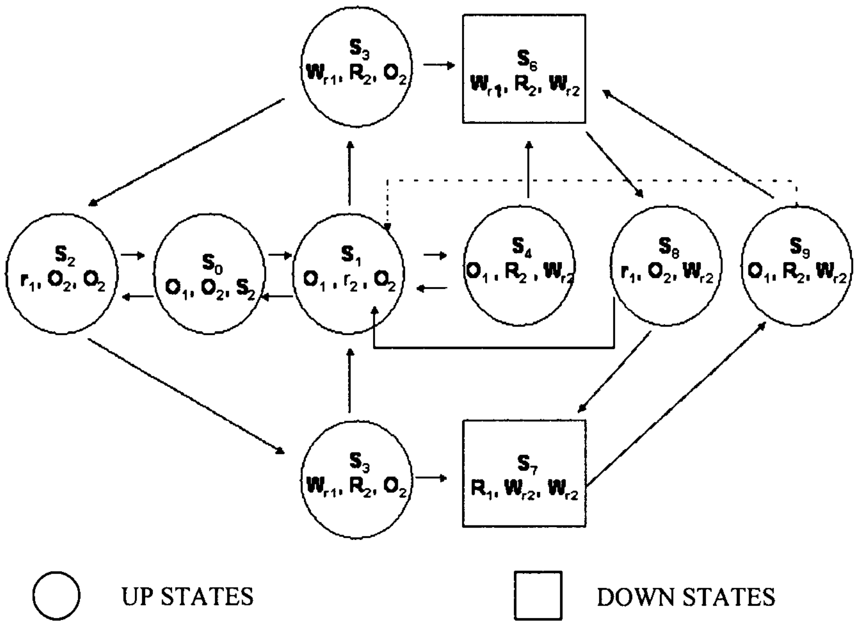

Figure 4.1. Transition diagram for the system