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On Testing Exponentiality Against HNRBUE Based on Goodness of Fit

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Abstract. Based on goodness of fit new testing procedures are derived for testing exponentiality against harmonic new renewal better than used in expectation (HNRBUE). For this aging properties, a nonparametric procedure (U-statistic) is proposed. The percentiles of this test statistic are tabulated for sample sizes n=5(1)30(10)50.The Pitman asymptotic efficiency (PAE) of the test is calculated and compared with, the (PAE) of the test for new renewal better than used (NRBU) class of life distribution [see Mahmoud et al (2003)]. The power of this test is also calculated for some commonly used life distributions in reliability. The right censored data case is also studied. Finally, real examples are given to elucidate the use of the proposed test statistic in the reliability analysis.

Key words: *harmonic new renewal better than used in expectation (HNRBUE), U-statistic, goodness of fit testing, efficiency, relative efficiency, Monte Carlo methods, power and censored data.*

1. INTRODUCTION

In many reliability applications, various classes of life distributions and their dual have been introduced to describe several criteria of aging. These classes have been considered by different authors as Bryson and Siddiqui (1969), Rolski (1975), Barlow and Prochan (1981), Klefsjo (1982), and Abouammoh (1988) among others. Among the most important families of aging are the increasing failure rate (IFR), increasing failure rate average (IFRA),new better than used (NBU), new better than used in expectation (NBUE),

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harmonic new better than used in expectation (HNBUE) and decreasing mean remaining life (DMRL).The null distribution for all of the above families is the exponentiality. Thus, we often encounter testing H_0 : a life distribution is exponential versus H_1 : a life distribution belongs to an aging family. Dealing with this problem seems to have started in the work of Proschan and Pyke (1967) for the IFR class and was followed by many, including Barlow and Proschan (1969), Bickel and Doksum (1969), and Ahmed (1975). For the NBU class, the work started by Hollander and Proschan (1972), followed by many, including Koul (1977), Kumazawa (1983) and Ahmad (1994), among others. Testing for the NBUE class originated in Hollander and Proschan (1975), while testing for HNBUE began in Kleffsjo (1983) and Ahmad et al (1999).

Let X be a nonnegative random variable with distribution function F and survival function $\overline{F} = 1 - F$. Assume also that X is absolutely continuous with probability density function f and has mean μ and variance σ^2.

Consider a device with life length X and life distribution F. The device is replaced instantly upon failure by a sequence of mutually independent devices. These devices are independent of the first unit and identically distributed with the same life distribution F. When the renewal of the system is continued indefinitely, the stationary life distribution of a device in operation at time t is $W_F(t) = \frac{1}{\mu}\int_0^t \overline{F}(u)du, \ t \geq 0.$

The survival of a device in operation at any time $t \geq 0$ is given by the stationary renewal distribution,

$$\overline{W_F}(t) = \mu_F^{-1}\int_t^{\infty} \overline{F}(u)du, \qquad \text{for } 0 \leq t < \infty,$$

where $\mu_F = \int_0^{\infty} \overline{F}(x)dx < \infty$, is the mean of life distribution F.

Abouammoh et al (2000) introduced the NRBU,NRBUE and HNRBUE classes of life distributions and studied the relation between them. Montasser (2002) established the test statistic for testing exponentiality versus HNBRUE classes of life distribution based on U-test statistic by using expected value method.

Hendi and Abouammoh (2001) investigated two test statistics for testing exponentiality versus NBRUE, HNBRUE classes of life distribution based on U-statistic. Mahmoud et al (2002) investigated the test statistic for NRBU based on U-statistic. In (2003), Mahmoud et all studied two test statistics for NRBU and RNBU classes of life distributions as alternatives based on the moment inequalities.

In fact, stochastic comparison between the random variable T with distribution F and its renewal random variable T_{W_F} with life distribution W_F, for which $W_F(0-) = 0$, density function w_F and renewal survival function $\overline{W_F}$, leads to the following Definition.

Definition 1.1 A random variable T with finite mean μ is called harmonic new renewal better than used parent in expectation, denoted by HNRBUE, if $T_W \leq^{st} T_G$,where T_G is exponential random variable with the same mean μ_w.

This definition means that,

$$\overline{W}(t) \le \overline{G}(t), \qquad \forall\, t \ge 0, \tag{1.1}$$

Where $G(t) = 1 - \exp(-t/\mu)$.

In fact (1.1) is equivalent to the form

$$\int_0^t \frac{\overline{F}(x)}{\int_x^\infty \overline{F}(u)du} dx \ge \int_0^t \frac{1}{\mu_w} du, \qquad t \ge 0.$$

i.e.

$$\int_t^\infty \overline{F}(u)du \le \mu_F \exp\{-t/\mu_{W_F}\} \qquad \text{for } t \ge 0. \tag{1.2}$$

where $\mu_{W_F} = \int_0^\infty \overline{W}_F(u)du = \frac{1}{2\mu}\mu_{(2)}$.

The corresponding dual class harmonic new renewal worse than used parent in expectation, denoted by HNRWUE, is defined by reversing the inequality sign of (1.1).

The main theme of this paper is the problem of testing $H_0 : F$ is exponential against $H_1 : F$ is HNRBUE class of life distribution and not exponential with non constant failure rate.

In the following section we derive non parametric test for testing exponentiality against HNRBUE properties using goodness of fit approach.

2. TESTING AGAINST HNRBUE CLASS ALTERNATIVE IN THE COMPLETE SAMPLE.

Suppose $X_1, X_2, \ldots, X_n$ represents a sample from a population with distribution F. We need to test the null hypothesis $H_0 : F$ is exponential, against $H_1 : F \in$ HNRBUE.

We use the following measure of departure from H_0

$$\delta_F = E\left(\mu_F \exp\left\{-x/\mu_{W_F}\right\} - \int_x^\infty \overline{F}(u)du\right) \ge 0$$

$$= \left\{\int_0^\infty \mu_F \exp\left\{-x/\mu_{W_F}\right\} - \int_x^\infty \overline{F}(u)du\right\} dF_0(x). \tag{2.1}$$

We can take $F_0(x) = 1 - e^{-x}, \quad x \ge 0$.

The following lemma is essential for the development of our test statistic.

Lemma 2.1: Let X be a random variable with distribution F. Then

$$\delta_F = (2\mu + \mu_2)E(1-e^{-x}) - 2\mu^2 \quad (2.2)$$

Proof: Clearly from (1.2) F is **HNRBUE** iff,

$$\left(\mu_F \exp\left\{ -x \Big/ \mu_{W_F} \right\} - \int_x^\infty \bar{F}(u)du \right) \geq 0 \quad .$$

Take the integral with respect to $F_0(x)$,
then,

$$\int_0^\infty \mu_F \exp\left\{ -x \Big/ \mu_{W_F} \right\} dF_0(x) = \mu\left[\frac{\mu_2}{2\mu + \mu_2} \right].$$

Thus, the result follows directly.

Note that δ_F =0 under H_0 and δ_F >0 under H_1.

2.1 Empirical test statistic for HNRBUE alternative.

To estimate δ_F, Let $X_1, X_2, \ldots, X_n$ be a random sample from F. Let $\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} X_i$ and $E(1-e^{-x}) = \frac{1}{n}\sum_{j=1}^{n}(1-e^{-x_j})$ respectively.

Then the estimate of δ_F in (2.2) is given by

$$\hat{\delta}_{F_n} = (2\hat{\mu} + \hat{\mu}_2)E(1-e^{-x}) - 2\hat{\mu}^2.$$

And the empirical form of δ_F can be written as:

$$\hat{\delta}_{F_n} = \frac{1}{n^2}\sum_{j=1}^{n}\sum_{i=1}^{n}\left[(2X_i + X_i^2)(1-e^{-X_j}) - 2X_iX_j\right]. \quad (2.3)$$

If we define $\Phi(X_1, X_2) = (2X_1 + X_1^2)(1-e^{-X_2}) - 2X_1X_2$,
and define the symmetric Kernel

$$\psi(X_1, X_2) = \frac{1}{2!}\sum_R \Phi(X_{i1}, X_{i2}).$$

Where the summation is over all arrangements of ,then $\hat{\delta}_{F_n}$ in (2.3) is equivalent to the U_n-statistic given by

$$U_n = \frac{1}{\binom{n}{2}}\sum_{i<j}\Psi(X_i, X_j).$$

The following Theorem summarizes the asymptotic normality of $\hat{\delta}_{F_n}$ or U_n.

Theorem 2.1: As $n \to \infty$, $n^{\frac{1}{2}}(U_n - \delta_F)$ is asymptotically normal with mean 0 and variance σ^2 given in (2.5). Under H_0, the variance σ^2 reduces to,

$$\sigma_0^2 = Var\left[\frac{1}{2}X^2 - 4e^{-X} - 3X + 4\right] = \frac{103}{3}.$$

Proof: Using the standard Theorem of U-statistic [see Lee (1989)]. We only need to evaluate the asymptotic variance σ^2 where,

$$\sigma^2 = \text{var}\left\{\sum_{i=1}^{2} \eta_i\right\}. \tag{2.4}$$

With $\eta_i = E(\phi(X_1, X_2 \mid X_i))$.

Recall the definition of $\Phi(X_1, X_2)$, from above, so it is not difficult to show that

$$\eta_1 = 2X_1 \int_0^\infty (1 - u - e^{-u})dF(u) + X_1^2 \int_0^\infty (1 - e^{-u})dF(u),$$

and

$$\eta_2 = 2\int_0^\infty udF(u) + \int_0^\infty u^2 dF(u) - 2X_2 \int_0^\infty udF(u) + e^{-X_2} \int_0^\infty (-2u - u^2)dF(u).$$

Substitute in (2.4)

Hence (2.5) holds.

$$\sigma^2 = \text{var}\left\{2X \int_0^\infty (1 - u - e^{-u})dF(u) + X^2 \int_0^\infty (1 - e^{-u})dF(u) + 2\int_0^\infty udF(u) + \int_0^\infty u^2 dF(u) - 2X \int_0^\infty udF(u) + e^{-X} \int_0^\infty (-2u - u^2)dF(u)\right\}. \tag{2.5}$$

3. MONTE CARLO CRITICAL POINTS for $\hat{\delta}_{F_n}$ STATISTIC.

A FORTRAN program is used to compute $\hat{\delta}_{F_n}$ statistic, based on 5,000 simulated samples with sample size n= 5(1)30(10)50. Table 3.1 gives upper percentile points of the statistic of $\hat{\delta}_{F_n}$.

3.1 The power estimates

The power of the test statistic $\hat{\delta}_{F_n}$ is considered at significance level $\alpha = 0.05$ and for commonly used distributions in reliability modeling. These distributions are

Table 3.1 critical values for the upper percentiles of $\hat{\delta}_{F_n}$.

n	%90	%95	%98	%99
5	.0052	.0017	.0154	.0666
6	.0028	.0195	.1050	.2452
7	.0017	.0295	.1397	.2356
8	.0041	.0554	.1812	.3352
9	.0132	.0803	.2210	.3389
10	.0251	.0859	.2229	.3489
11	.0222	.0977	.2437	.3779
12	.0303	.1007	.2399	.3744
13	.0376	.1150	.2449	.3603
14	.0482	.1240	.2730	.4119
15	.0430	.1104	.2190	.3220
16	.0550	.1401	.2662	.3838
17	.0528	.1279	.2542	.3729
18	.0533	.1262	.2555	.3829
19	.0542	.1322	.2498	.4060
20	.0494	.1106	.2306	.3485
21	.0604	.1272	.2348	.3706
22	.0643	.1407	.2642	.3572
23	.0615	.1248	.2406	.3254
24	.0675	.1393	.2426	.3459
25	.0653	.1316	.2443	.3479
26	.0598	.1260	.2402	.3512
27	.0585	.1172	.2171	.3243
28	.0596	.1205	.2163	.2943
29	.0627	.1317	.2546	.3350
30	.0676	.1354	.2435	.3417
40	.0708	.1248	.2177	.2927
50	.0691	.1171	.1916	.2645

(i) Linear failure rate family, $\overline{F}(x) = \exp(-x - \theta x^2/2)$ for $x \geq 0, \theta \geq 0$

(ii) Pareto family, $\overline{F}(x) = (1 - \theta x)^{\frac{1}{\theta}}$ for $x \geq 0, \theta \geq 0$

and

(iii) Weibull family, $\overline{F}(x) = \exp(-x^{\theta})$ for $x \geq 0, \theta \geq 0$

For the previous alternatives, the powers for the proposed test are tabulated in Table 3.2, using 5,000 replications for sample size n=10, 20 and 30, and parameter values $\theta = 1$, 2 and 3.

Table 3.2 power estimates using $\alpha = .05$

Distribution	**Parameter** θ	Sample size		
		10	20	30
L.F.R	1	1.000	1.000	1.000
	2	1.000	1.000	1.000
	3	1.000	1.000	1.000
Pareto	1	.842	.948	.978
	2	.912	.988	.999
	3	.955	.997	1.000
Weibull	1	.959	.948	.952
	2	1.000	.998	.997
	3	1.000	1.000	1.000

Table 3.2 shows that our test has a good power, and the power is getting as smaller as HNRBUE approaches the exponential distribution.

4. ASYMPTOTIC RELATIVE EFFICIENCY (ARE).

Since the above test statistic $\hat{\delta}_{F_n}$ is new and no other test are known for HNRBUE class. We compare our test to the smaller classes and choose NRBU and RNBU classes based on U-statistic that proposed by Mahmoud et al (2003). Our calculation are achieved by using Pitman asymptotic relative efficiency (PARE) which is defined in the following definition.

Definition 4.1 Let T_{1n} and T_{2n} be two test statistics for testing

$$H_0 : F_\theta \in \{F_{\theta_n}\}, \theta_n = \theta + cn^{-1/2},$$

where c is an arbitrary constant, then the asymptotic efficiency of T_{1n} relative to T_{2n} is defined by,

$$e(T_{1n},T_{2n}) = \left[\mu_1'(\theta_0) \Big/ \sigma_1(\theta_0)\right] \Big/ \left[\mu_2'(\theta_0) \Big/ \sigma_2(\theta_0)\right],$$

where $\mu_i(\theta_0) = \left\{\lim_{n\to\infty} \left(\frac{\partial}{\partial\theta} E(T_{in})\right)\right\}_{\theta\to\theta_0}$ and

$\sigma_i^2(\theta_0) = \{\lim_{n\to\infty} \mathrm{var}_\theta(T_{in})\}_{\theta\to\theta_0}, i = 1,2$, is the null variance.

Notice that T_{1n} is more efficient than T_{2n} if $e(T_{1n},T_{2n}) > 1$.

We choose the following alternatives:

(i) Weibull family : $\overline{F_1}(x) = \exp(-x^\theta), x > 0, \theta \geq 1$.

(ii) Linear failure rate family : $\overline{F_2}(x) = \exp(-x - \theta x^2/2), x > 0, \theta \geq 0$.

(iii) Makeham family : $\overline{F_3}(x) = \exp(-x + \theta(x + e^{-x} - 1)), x > 0, \theta \geq 0$.

(iv)Gamma family : $\overline{F_4}(x) = \int_x^\infty e^{-u} u^{\theta-1} du / \Gamma(\theta), x > 0, \theta \geq 0$.

Note that H_0 (the exponential) is attained at $\theta = 1$, in (i) and (iv), and is attained at $\theta = 0$ in (ii) and (iii).

Direct calculations of the asymptotic efficiencies of the HNRBUE class compared with (NRBU) RNBU class (Mahmoud et al, (2003)). Table 4.1 show that our test has much higher asymptotic efficiency.

Table 4.1. Asymptotic relative efficiencies of $\hat{\delta}_{F_n}$ to Mahmoud et al (2003).

Efficiency	Weibull	Linear failure rate	Makeham	Gamma
$\dfrac{\mu'(\theta)}{\sigma(\theta_0)} = \dfrac{\frac{\partial \delta_F}{\partial \theta}\vert_{\theta=\theta_0}}{\sigma(\theta_0)} = \hat{\delta}_{F_n}$	.39299	.08533	.27023	.5073
$\hat{\Delta}_{F_n}$ (Mahmoud et al (2003))	.3571	.7143	.0893	.1191
E($\hat{\delta}_{F_n}$, $\hat{\Delta}_{F_n}$)	1.1005	.119459	3.02609	4.25944

5. TESTING AGAINST HNRBUE CLASS for CENSORED DATA.

In this section, a test statistic is proposed to test H_0 versus H_1 with randomly right censored samples. In the censoring model, we observe the pair (Z_j, δ_j), $j = 1......n$ where $Z_j = \min(X_j, Y_j)$ and

$$\delta_j = \begin{cases} 1 & \text{if} \quad Z_j = X_j \quad (j^{th} \text{observn is uncensored}) \\ 0 & \text{if} \quad Z_j = Y_j \quad (j^{th} \text{observn is censored}) \end{cases},$$

where $X_1, X_2, ..., X_n$ denote their true lifetime from distribution F. And $Y_1, Y_2, ..., Y_n$ be (i .i .d.) according to distribution G. Also X's and Y's are independent. Let $Z(0) = 0 < Z(1) < Z(2) < ...... < Z(n)$ denote the ordered Z's and $\delta_{(j)}$ is the δ_j corresponding to $Z_{(j)}$ respectively.

Kaplan and Meier (1958) proposed the product limit estimator of $\overline{F_n}(X)$ as following.

$$\overline{F_n}(X) = 1 - F_n(X) = \prod_{[j:Z_{(j)} \le X]} \{(n-j)/(n-j+1)\}^{\delta_{(j)}},\ X \in [0, Z_{(n)}]. \tag{5.1}$$

Now for testing $H_0 : \delta_F = 0$, against $H_1 : \delta_F > 0$, using the random right censored data, we propose the following test statistic.

$$\hat{\delta}^C_{F_n} = (2\hat{\mu} + \hat{\mu}_2) \int_0^\infty (1 - e^{-x}) dF_n(x) - 2\mu^2 \tag{5.2}$$

For computational purpose, $\hat{\delta}^C_{F_n}$ in (5.2) may be rewritten as

$$\hat{\delta}^C_{F_n} = \left\{ 2\left[\sum_{k=1}^{n} \prod_{m=1}^{k-1} c_m^{\delta_{(m)}} (Z_{(k)} - Z_{(k-1)}) \right] + \left[\sum_{i=1}^{n} Z(i)^2 \left\{ \prod_{p=1}^{i-2} c_p^{\delta_{(p)}} - \prod_{p=1}^{i-1} c_p^{\delta_{(p)}} \right\} \right] \right\} \bullet$$

$$\left\{ \sum_{j=1}^{n} (1 - e^{-X(j)}) \left\{ \prod_{m=1}^{j-2} c_m^{\delta(m)} - \prod_{m=1}^{j-1} c_m^{\delta(m)} \right\} \right\} - 2 \left[\sum_{k=1}^{n} \prod_{m=1}^{k-1} c_m^{\delta_{(m)}} (Z_{(k)} - Z_{(k-1)}) \right]^2 \tag{5.3}$$

where $dF_n(Z_j) = \overline{F_n}(Z_{j-1}) - \overline{F_n}(Z_j)$, $C_K = [n-k][n-k+1]^{-1}$, and

Table 5.1 gives the critical values percentiles of $\hat{\delta}^C_{F_n}$ for sample sizes n=5(1)50, 60(10)81, based on 5,000 replications.

Table 5.1 Critical values for percentiles of $\hat{\delta}^C_{F_n}$

n	%90	%95	%98	%99
5	.9413	1.1573	1.5579	1.8681
6	1.0436	1.3473	1.7919	2.2095
7	1.1808	1.5622	2.2845	2.8322
8	1.2859	1.6711	2.2878	2.7641
9	1.4005	1.8384	2.4698	3.1270
10	1.5036	2.0010	2.9137	3.7670
11	1.5351	2.0817	2.9333	3.5430
12	1.6427	2.2443	3.0402	3.7411
13	1.7278	2.3211	3.3377	4.0931
14	1.7378	2.3196	3.0792	3.9169
15	1.8054	2.3966	3.5115	4.2598
16	1.8280	2.5115	3.4341	4.1351
17	1.9907	2.6888	3.6187	4.3820
18	1.9265	2.5576	3.4385	4.3393
19	2.0129	2.6766	3.7021	4.6844
20	2.0112	2.6985	3.7188	4.6972
21	2.0183	2.6769	3.7391	4.6106
22	2.0536	2.8026	4.0717	4.8740
23	2.1888	2.8922	4.1236	5.2945
24	2.1521	2.8440	3.9777	5.0883
25	2.1734	2.9115	4.0594	5.1219

26	2.3264	2.9699	4.2165	5.0133
27	2.2560	2.9920	4.1306	5.5193
28	2.3601	2.1036	4.2191	5.2993
29	2.3992	3.3992	4.4910	5.5040
30	2.4541	3.2401	4.4618	5.5620
40	2.7278	3.5256	4.7434	5.5943
50	3.1020	4.6395	5.4601	6.5676
60	3.4560	4.6359	6.1886	7.8480
70	3.6888	4.8878	6.3110	7.3191
81	4.00813	5.1409	6.8304	8.4383

5.1 Applications:

Example 1:

Consider the data in Abouammoh et al (1994). These data represent 40 patients suffering from blood cancer from one of Ministry of Health Hospitals in Saudi Arabia and the ordered life time (in days) are recorded:
115,181,255,418,441,461,516,739,743,789,807,865,924,983,1024,1062
1063,1165,1191,1222,1222,1251,1277,1290,1357,1369,1408,1455,
1478,1549,1578,1578,1599,1603,1605,1696,1735,1799,1815,1852.

It was found that the test statistic $\hat{\delta}_{F_n}$ for the data set by using formula (2.3) $\hat{\delta}_{F_n}$ =1064.35215 that we accept H_1 which states that the set of data have **HNRBUE** property under significant level at 95% upper percentile.

Example 2:

Using the data set given in Grubbs (1971), these data have been used in Shapiro et al (1995). The data set are the times between arrivals of 25 customers at a facility:
1.80, 2.89, 2.93, 3.03, 3.15, 3.43, 3.48, 3.57, 3.85, 3.92, 3.98, 4.06, 4.11, 4.13, 4.16, 4.23, 4.34, 4.37, 4.53, 4.62, 4.65, 4.84,4.91, 4.99, 5.17.

It was found that the test statistic for the data set, by using formula (2.3) is $\hat{\delta}_{F_n}$ =-7.8549, which is less than the critical value in Table 3.1 Then we accept the null hypothesis of exponentiality and not **HNRBUE** property at 95% upper percentile.

Example 3:

Consider the data Susarla and Vanryzin (1978), which represent 81 survival times (in weeks) of patients of melanoma. Out of these 46 represents non-censored data and the ordered values are:
13,14,19,19,20,21,23,23,25,26,26,27,27,31,32,34,34,37,38,38,40,46,50,53,54,57,58,59,60, 65,65,66,70,85,90,98,102,103,110,118,124,130,136,138,
141,234.

The ordered censored observations are: 6, 21, 44, 50, 55, 67, 73, 80, 81, 86, 93, 100, 108, 114, 120,124,125,129,130,132,134,140,147,148,151,152, 152,158,181,190,193,194,213,215.

Now, taking into account the whole set of survival data (both censored and uncensored), and computing the statistic from (5.3) censored data, we get $\hat{\delta}^C_{F_n}$ =1306.739 , which is greater than the critical value in Table 5.1 at %95 upper percentile , then , we accept H_1 which states that the set data have **HNRBUE** property.

6. CONCLUSION

It is known that testing exponentiality is widely discussed goodness of fit problems. Many approaches to the problem have been considered, some based on the empirical distributions, while others are based on certain measures of departure from exponentiality, [see Ascher (1990)]. In this paper, we derived a U-Statistic test based on goodness of fit approach for testing exponentiality against life class **HNRBUE**. Selected critical values and the power estimates of the test are calculated via the simulation, and tabulated in Table 3.1 and Table 3.2. The Pitman asymptotic efficiency (PAE) of the test is calculated and compared with, the (PAE) of the test for new renewal better than used **(NRBU)** class of life distribution [see Mahmoud et al (2003)]. It is clear from Table 4.1 that our test is more efficient than NRBU test. Right- censored data problem is also considered to illustrate the theoretical results. Practical applications of our test in the medical sciences are present.

Suggestion for future work.

For testing exponentiality against harmonic new renewal better than used in expectation **(HNRBE)** class of life distributions based on moment inequality method can be studied.

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