International Journal of Reliability and Applications
Vol. 8, No. 2, pp. 183-197, 2007

Optimal Service Contract Policies for Outsourcing Maintenance Service of Assets to the Service Providers

Anisur Rahman*
Griffith School of Engineering, Griffith University Gold Coast Campus, PMB 50, GCMC, Q 9726 Australia,

Gopinath Chattopadhyay
Faculty of Science, Engineering, and Health, Central Queensland University, Gladstone, Q 4680, Australia

Abstract. There is a growing trend for asset intensive industries to outsource maintenance services of their complex assets since outsourcing through service contract reduces upfront investments in infrastructure, expertise and specialised maintenance facilities. Estimation of costs for such contracts is complex and it is important to the user and the service providers for economic variability. The service provider's profit is influenced by many factors such as the terms of the contract, reliability of asset, and the servicing strategies, costs of resources needed to carryout maintenance. There is a need to develop mathematical models for understanding future costs to build it into the contract price. Three policies for service contracts are proposed in this paper considering the concepts of outsourcing maintenance service of assets to the service providers. Conceptual models are developed for estimating servicing costs of outsourcing through service contracts by considering time dependent failure mode.

Key Words: *Outsourcing, Service Contracts, Policies, Cost models*

1. INTRODUCTION

For expensive and complex system/assets (e.g. power generation plants, rail networks used by large industries such as Mining, Jute and Sugarcane, and Steel plants for transportation of material over wide geographically distant areas etc), the maintenance services need to have expertise and specialised maintenance facilities. Often it is found expensive for the owner of such asset/systems to have well built infrastructure, specialised maintenance facilities and maintenance specialists in house. Outsourcing reduces upfront investments in infrastructure, expertise and specialised maintenance facilities (Murthy and

* Corresponding Author.
E-mail address: a.rahman@griffith.edu.au

Ashgarizadeh, 1995). This results in a growing trend for the owner of asset intensive industries to outsource the management of maintenance activities of these assets to external agencies. The service providers for such service can be one of their asset operators or manufacturers of the asset or independent third parties, interested in investing for asset infrastructures.

Estimation of costs for these contracts is complex and it is also important to the users/owners and the service providers. There is a need to develop mathematical models for understanding future costs to build it into the contract price. Failure to do so may result in loss to the service provider or the user/owner because of uncertainties associated with system failures and their implication on business. These costs depend on the reliability of the asset and the servicing strategies (e.g. corrective maintenance, planned preventive maintenance, and/or inspection procedures) to be considered during the contract period. Servicing strategy can be developed by understanding the reliability and its analysis. Failure data are in many cases time or usage dependent for certain conditions. In a probabilistic sense, asset/system failures are functions of usage and/or age.

In this paper, various service contract policies and conceptual models are developed to estimate the costs for service contracts. Parameters of the models are estimated with an application of non-homogeneous Poisson process since most of the real life products are time or usage dependent.

The outline of this paper is: in Section 1, an introduction of outsourcing and service contracts are provided. Section 2 discusses different servicing strategies that deal with both corrective maintenance and planned preventive maintenances. Service contracts together with various servicing policies are discussed in Section 3. In section 4 cost models for service contracts are proposed. Numerical examples of the developed models are provided in this section. In the final section, the summaries and scope for future work are discussed.

2. SERVICING STRATEGIES UNDER CONTRACT

A service contract is the outsourcing of maintenance actions where defects/failures are rectified by an external agent for an agreed period of time. The agent in turns charges a price for such service. The service provider's profit is influenced by many factors such as the terms of the contract, reliability of the asset, servicing strategies, costs of resources needed to carryout maintenance and to provide such services. Blischke and Murthy (2000) proposed a policy for service contract with scope for negotiation. In the recent years service contract has received significant attention due increased profit through selling those services and reduction of risk from owners due to better maintainability provided by the experts in the trade. Murthy and Yeung (1995) proposed stochastic models for expected profit. Murthy and Ashgarizadeh (1995) developed a model to characterise the optimal strategies for a single customer and single service provider. Ashgarizadeh and Murthy (2000) extended this to multiple customers. Rinsaka and Sandoh (2006) proposed mathematical model for setting suitable charge of service contract in the case where a manufacturer offers an additional warranty service under which the failed system is replaced by a new one for its first failure, but minimal repairs are carried out to the system for its succeeding failures before the contract expires. All these models considered corrective maintenance (CM) only (rectification only on failure) as servicing strategy and they ignored to include planned preventive maintenance (PM) actions during the contract. This type of contract is dangerous for some cases such as service contract for outsourcing rail maintenance. If a derailment is occurred due to the rail break/failure, it will cause not only a loss of billion dollars but it will also cause loss of valuable lives. Inclusion of preventive maintenance in the servicing strategy may prevent this type of accident in most of the cases since a planned preventive maintenance can prolong the reliability arising from proper inspection and on time maintenance. Therefore, for real life situations, servicing strategies for repairable systems should involve both corrective maintenance (CM) and planned preventive maintenance (PM). Corrective maintenances are unscheduled actions intended to restore the system/asset to its operational state through corrective actions after the occurrence of failures. In contrast, planned preventive maintenance actions are carried out to reduce the likelihood of failures or to prolong the life of the asset/system and/or to reduce the risk of failures (Murthy and Jack, 2003). Both CM and PM take into account different types of servicing actions which can be used based on the failure mode and type. These actions are classified as per degree of restorability as shown in Figure 2.1.

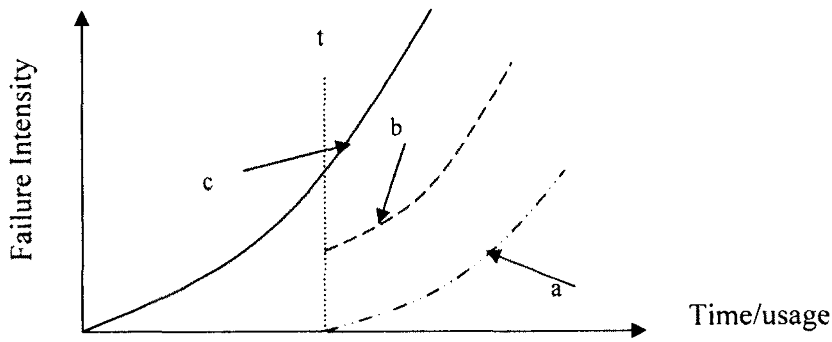


Figure 2.1. Failure rate with effect of various maintenance actions (Chattopadhyay and Rahman, 2004)

The system/asset fails at point *t* due to the malfunctioning of one or more components. And various servicing actions can adopt at this point to restore the functionality of the system/asset. The probable servicing strategies applicable for service contracts are:

(i) *Replacement*: the failed system can be replaced with a new identical system or with a used but good one. This turns failure rate of the item to origin if replaced with new one (see curve 'a' in Figure 2.1). This implies that a replacement with new and identical system restores the full reliability and turned failure rate to zero. If replaced with used, good one, it restores a part reliability and the failure rate falls in any point in between 'as good as new' and 'as bad as old' depending on the age and usage condition of the replaced item/system.

(ii) *Overhauling or perfect repair* is a restorative maintenance action that enables the system to be "as good as new" condition as it turns failure rate close zero (See curve 'a' or close to curve 'a').

(iii) *Imperfect repair* restores a substantial portion and like replacement with used, good item, the failure rate falls in between "as good as new" and "as bad as old (see curve 'b') depending on the type and quality of the repair works.

(iv) *A minimal repair* is the repair/replacement of only the failed component/s and other components of the item/system remain untouched. This makes insignificant improvement of reliability and the condition after maintenance is called "as bad as old" (curve 'c' in), since the failure rate of other components remain unchanged (Barlow and Hunter, 1960).

3. MODELLING POLICIES FOR SERVICE CONTRACTS

In modelling cost of service contracts, three different service contract policies are proposed where rectifications take into account both corrective maintenance and planned preventive maintenance. The corrective maintenance could be replacement of whole system in case of complete failure or the system is beyond economic repair, or a minimal/major repair. Preventive maintenance actions could be planned at constant intervals which retains the system reliability to some extent. Three service contract policies are:

Service contract policy 1: *Under this policy, the contract terminates when contract period reaches to a time or usage level L or a renewal of the system is essential due to the complete failure of the system whichever comes first. Under this policy renewal of the system is not included. Here, the system life is normally considered longer than the contract period. Planned preventive maintenances actions at constant intervals and corrective maintenances in the form of minimal or major repairs must be provided by the service provider.* Please see Figure 3.1.

According to this policy replacement is not included during the contract period which implies that $R \geq L$. where L and R are the contract period and the first replacement (renewal) time respectively (Figure 3.1 represents this policy graphically). Preventive maintenance (PM) are planned to carry out at constant intervals. Between two successive preventive maintenances there could be one or more minimal corrective actions. This type of service contracts is mostly suitable for complex repairable systems with longer life.

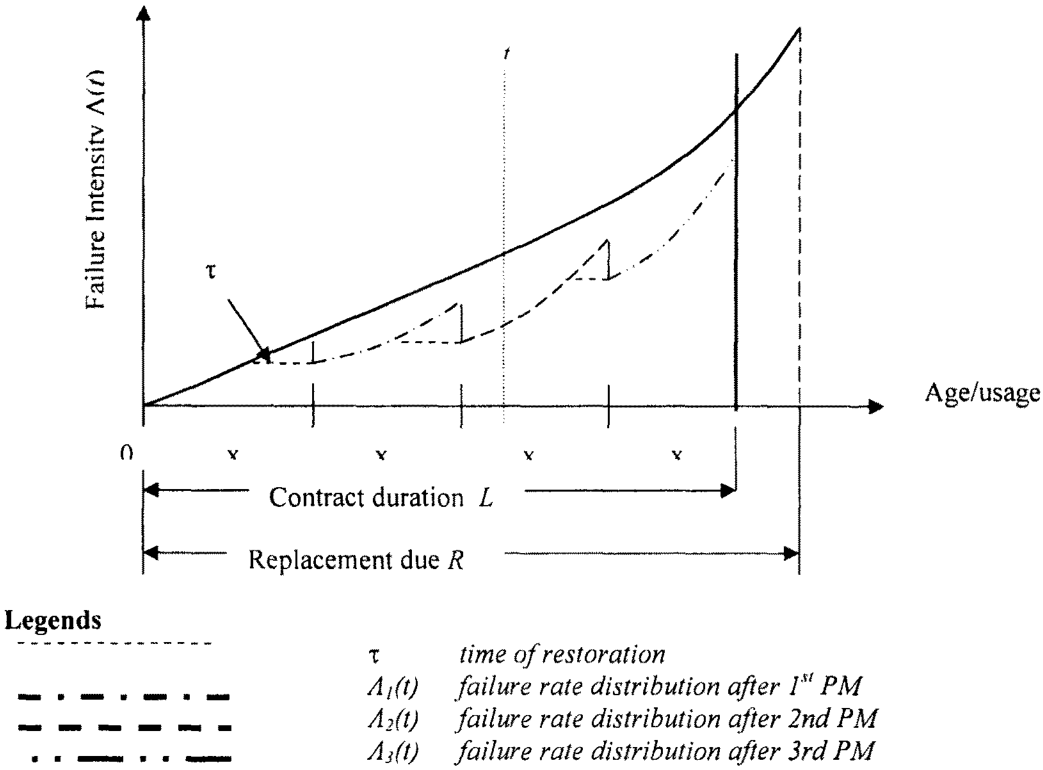


Figure 3.1. Graphical representation of the service contract Policy model 1

Service contract policy 2: *Under this policy, the contract period is up to the first replacement (renewal) of the system due to complete failure of System or economically beyond any service.* Please see Figure 3.2.

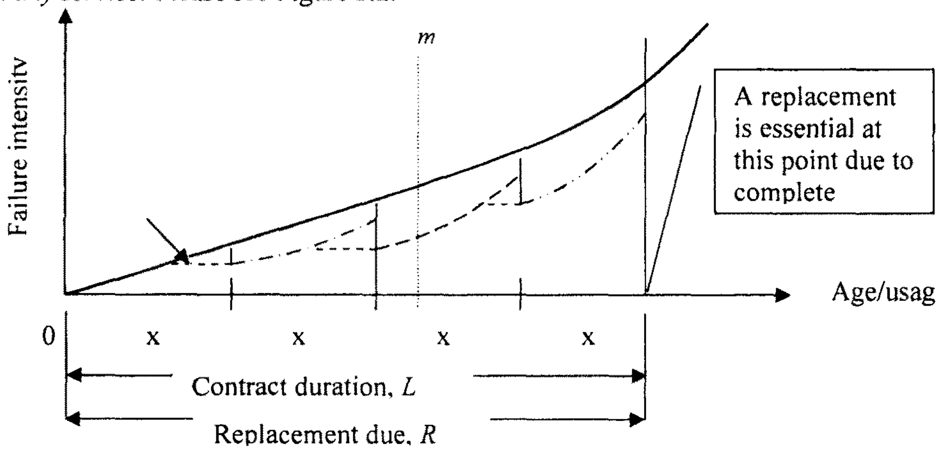
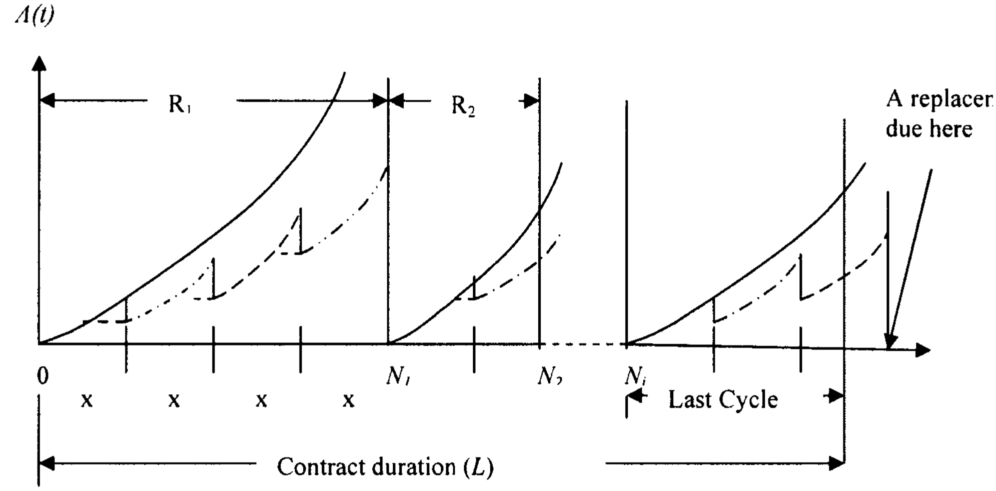


Figure 3.2. Graphical representation of the Service contract Policy model 2.

***Service contract policy** 3: Under this policy, the servicing strategy covers any rectification action including replacement (renewals)/s of the System due to complete failure during the contract period* (Figure 3.3 represents this policy graphically).



Legends

- - - - - - - -	τ	*time of restoration*
— · — · — · —	$\Lambda_1(t)$	*failure rate distribution after 1st PM*
— — — — — —	$\Lambda_2(t)$	*failure rate distribution after 2nd PM*
· · —— · · ——	$\Lambda_3(t)$	*failure rate distribution after 3rd PM*

Figure 3.3. Graphical representation of Service contract policy Model 3

In between two consecutive replacements, there may be one or more constant interval PM actions and there may also be a number of minimal repairs in between two successive PMs. Replacement or renewal time R is random variable This policy is applicable for long term service contracts.

4. MODELLING COSTS OF SERVICE CONTRACT FOR DIFFERENT POLICIES

Total costs of service contract depends on the type of policies defined in the previous section and may include the cost of planned preventive maintenance and corrective maintenance in the form of minimal repairs and failure replacement, cost of inspections and condition monitoring, cost of risks, and penalties for failure to meet agreed safety, reliability and availability of requirements.

The total expected costs of service contract

$$C_T = C_s + C_i + C_d + C_r + C_p$$

C_s= *total expected costs of maintenance services*
C_i = *expected costs of inspection*
C_d = *costs of downtime*
C_r = *cost of risks associated*
C_p = *penalty for failure to meet contract agreements.*

In this paper, we are aiming to develop conceptual cost models for service contracts that focus only on the maintenance services during the contract period. These models can be further extended by including the provisions for inspection, downtime, and risks associated with such contracts and incurred penalties due to failure to meet contract agreement.

Formulation of Models:

In modelling service contracts, we consider a number of assumptions for simplification purpose. These models can be made more realistic in future by relaxing one or more of these assumptions.

4.1 Assumptions

The following assumptions are made for all the cases.

- All corrective rectifications other than replacement are minimal repairs.
- Preventive maintenance actions are taken at constant interval (x) till each replacement throughout the contract period.
- PM restores life of life to some extent.
- The level of restoration depends on the type and quality of the maintenance performed.
- Age restoration, after each preventive maintenance (PM) is constant.
- An item is replaced only when it fails completely otherwise minimal repairs are preferred.
- All replacement is made with new and identical rail.
- All cost factors are constant over the contract period.
- Money discount is 1 throughout the contract period (this assumption is true when contract period is short).

4.2 Notations and Reliability Preliminaries

Failure intensity (failure rate) $\Lambda_{pm}(t) = \Lambda(t - k\tau)$ (4.1)

$\Lambda_{pm}(t)$: Failure intensity at time t, with maintenance.
$\Lambda(t)$: Original failure intensity at t when no maintenance is performed.
N: Number of times the planned servicing is performed during the contract period
N_i: Number of times the planned servicing is performed during the ith replacement (Policy 3) and $i = 1, 2, 3,\ldots\ldots$
M: Number of replacements corrective actions.
L: Duration (length) of service contract
k: Number of times PM is carried up to t.

τ: Age restoration after each PM. $\tau = \alpha x$, *where,* α is the quality of the maintenance, α ranges from 0 to 1.

When $\alpha = 1$ signifies– 'as good as new" and $\alpha = 0$ is 'as bad as old'.

C_{re} Cost of replacement

C_{mr} Cost for each minimal repair.

C_{pm} Cost for each PM

C_{cl} Expected cost for the last cycle.

4.3 Modelling Cost for Service Contract Policy 1

According to this policy, the contract terminates at a predetermined time L or at a time/usage level when renewal is essential due to the complete failure of the item/asset, which comes first. This implies that replacement is not covered in this policy. That is $M = 0$. The Figure 3.1 is the graphical representation of the model of the policy 1. Here, the expected cost of service contract per unit time for repairable item/asset with corrective servicing at failure and planned preventive maintenance is given by the total sum of expected costs of all minimal repairs and the expected costs of all the planned preventive over the contract period divided by the length of service contract (L or R which comes first). Expected cost of service contract per unit time

= ((Expected costs of all minimal repairs

+ Expected cost of all planned preventive maintenances) during the contract period) / Length of contract duration.

Expected cost of all minimal repairs over the contract period can be given by

$$C_{mr}\sum_{k=0}^{N+1}\int_{kx}^{(k+1)x}\Lambda_{pm}(t)dt \tag{4.2}$$

Now substituting value of $\Lambda(t)$ from the Equation (4.1) in Equation (4.2), the expected cost of minimal repair can be given by the Equation (4.3)

Expected cost of minimal repair

$$C_{mr}\sum_{k=0}^{N+1}\int_{kx}^{(k+1)x}\Lambda(t-k\tau)dt \tag{4.3}$$

Expected cost of preventive maintenance during the contract

$$NC_{pm} \tag{4.4}$$

The total expected cost per unit time $C(L,x, N)$ can therefore be expressed as

$$C(L,x,N)=\frac{1}{x\times(N+1)}\left[C_{mr}\sum_{k=0}^{N+1}\int_{kx}^{(k+1)x}\Lambda(t-k\tau)dt+N\ C_{pm}\right] \tag{4.5}$$

4.4 Modelling Cost for Service Contract Policy 2

According to this policy, duration of contract is randomly variable and the contract is terminated any time at which a renewal is mandatory due the complete failure of the asset. This implies $L= R$. Please see Figure 3.2.

Here, expected total cost per unit time of service contract can be estimated as follows
Expected total cost per unit time
= (expected total cost of all the minimal repairs + expected total cost of all planned preventive maintenance over life)/ (asset/product's lifetime)

Which is similar to the Equation 4.5. The only difference is the use of a randomly variable contract period instead of a fixed contract period since the contract terminates due to the complete failure or discard of the item due technological, commercial or economic reasons and therefore, an optimal contract period L^* is needed to be modelled. This is the product of optimal number of preventive maintenance (N^*) and optimal interval of PM (x^*). That is

$$L^* = \left(N^* + 1\right)^* x^*$$

The final model for the strategy 2 can be given by

$$C\left(L^*\right) = \frac{1}{L^*}\left[C_{mr}\sum_{k=0}^{N+1}\int_{kx}^{(k+1)x}\Lambda(t - k\tau)dt + N\ C_{pm}\right] \tag{4.6}$$

4.5 Modelling Cost for Service Contract Policy 3

This policy is appropriate for long term service contracts. Under the conditions of this policy, one or more replacements or renewals due to complete failure of the item/asset are covered over the prefixed contract period. We denote the total number of replacement M. Before each replacement constant interval preventive maintenances and minimal repairs as corrective actions are carried out as in the policies 1 and 2. Between the last replacements (renewals) and the end of contract duration there may still be some excess times where one or more Preventive maintenances as well as some minimal repairs need to carry out. We named this excess period as the last cycle. We assume the nature and behaviour of last cycle is similar to that of the policy 1. Please see Figure 3.3. Let R_1, R_2, ... represent the renewal times respectively.

Here, the expected cost per unit time
= (expected total costs of all minimal repairs up to the last cycle
+ expected total cost of all preventive maintenances up the last cycle + expected total cost of replacements + expected cost of last cycle) / Length of service contract.

Expected total cost of minimal repairs up to the last cycle

$$= C_{mr}\left\{\sum_{i=0}^{M}\sum_{k=0}^{N_i+1}\int_{kx}^{(k+1)x}\Lambda(t - k\tau)dt\right\} \tag{4.7}$$

Expected total cost of PM up to the last cycle

$$= \sum_{i=0}^{M} N_i C_{pm} \tag{4.8}$$

where $i = 1,2,3,\ldots..$

Total cost of replacement = MC_{re} (4.9)

Therefore the total cost of service contract up to the last cycle can be given by adding equations

$$=\left[C_{mr}\left\{\sum_{i=0}^{M}\sum_{k=0}^{N_i+1}\int_{kx}^{(k+1)x}\Lambda(t-k\tau)dt\right\}+\sum_{i=0}^{M}N_iC_{pm}+MC_{re}\right] \tag{4.10}$$

Expected total cost of the last cycle (C_{cl}) can be expressed as

$$C_{cl}=C_{mr}\left\{\sum_{k=0}^{N_{cl}}\int_{k_{cl}x}^{(k_{cl}+1)x}\Lambda(t-k\tau)dt\right\}+(N_{cl})C_{pm} \tag{4.11}$$

N_{cl} is the number of PMs in the last cycle

Therefore, the total expected cost per unit time $C(L, x, N_i, M)$ can be expressed by adding Equations 4.10, and 4.11 divided by the contract length and that is given by Equation 4.12

$$C(L,x,N_i,M)=\frac{1}{L}\left[C_{mr}\left\{\sum_{i=0}^{M}\sum_{k=0}^{N_i+1}\int_{kx}^{(k+1)x}\Lambda(t-k\tau)dt\right\}+\sum_{i=0}^{M}N_iC_{pm}+C_{cl}+MC_{re}\right] \tag{4.12}$$

4.6. Model Analysis and Parameter Estimation

For analysis purpose, it is assumed that the failure rate increases with time /usage which implies asset/product failures follows Non homogeneous Poisson process (NHPP). This is the case for most of the real life complex assets/ product.

Let the failure intensity function of the asset with life distribution $F(t)$ having density function $f(t)=\frac{d[F(t)]}{dt}$ be defined by $\Lambda(t)=\frac{f(t)}{1-F(t)}$

When it follows NHPP this can be modelled as:

$$F(t)=1-\exp(-(\lambda t)^{\beta}) \tag{4.13}$$

and

$$f(t)=\lambda\beta(\lambda t)^{\beta-1}\exp(-(\lambda t)^{\beta}) \tag{4.14}$$

With the parameters β (known as shape parameter of the distribution) > 0 and λ (Known as inverse of characteristic life parameter for the distribution) > 0 (Chattopadhyay and Murthy, 2000).

β greater than 1 indicates an increasing failure rate of the item under study and ageing is predominant in failure mechanism. Then the failure intensity function $\Lambda(t)$ derived from (4.13) and (4.14) can be given by

$$\Lambda(t)=\frac{f(t)}{1-F(t)}=\frac{\lambda\beta(\lambda t)^{\beta-1}\exp(-(\lambda t)^{\beta})}{1-(1-\exp(-(\lambda t)^{\beta})}=\lambda\beta(\lambda t)^{\beta-1} \tag{4.14}$$

For Service contract policy 1, the expected cost of minimal repair strategy when it follows NHPP can be expressed as

$$= C_{mr}\left\{\sum_{k=0}^{N+1} \lambda^{\beta} \beta \int_{kx}^{(k+1)x} (t - k\tau)^{\beta-1} dt\right\}$$

$$= C_{mr}\left\{\sum_{k=0}^{N+1} \lambda^{\beta} x^{\beta} \left[(k - k\alpha + 1)^{\beta} - (k - k\alpha)^{\beta}\right]\right\} \tag{4.15}$$

where, $\tau = x$, where, α is the quality of PM .

Therefore, the total expected cost per unit time $C(L, x, N)$ can therefore be expressed as

$$C(L, x, N) = \frac{1}{L}\left[C_{mr}\left\{\sum_{k=0}^{N} \lambda^{\beta} x^{\beta} \left[(k - k\alpha + 1)^{\beta} - (k - k\alpha)^{\beta}\right]\right\} + N\, C_{pm}\right] \tag{4.16}$$

Now an optimum x, optimum number of PM (N) and minimal total expected cost per unit time can be obtained by differentiating the Equation (4.16) with respect to x and equating it to zero. These optimal values can be obtained by programming in any mathematical software such as MATLAB, MAPLE. There could be excess age for the item with some trade off or salvage value has shown in the Figure 3.1. This could be subject of interest in future.

Similarly, when asset failure follows Non homogeneous Poisson process with shape parameter β and inverse characteristic parameter λ , the Service contract policy 2 model can be expressed by modifying Equation 4.6 as

$$C\left(L, x^{*}, N^{*}\right) = \frac{1}{x^{*} N^{*}}\left[C_{mr}\left\{\sum_{k=0}^{N^{*}} \lambda^{\beta} x^{*\beta} \left[(k - k\alpha + 1)^{\beta} - (k - k\alpha)^{\beta}\right]\right\} + N^{*} C_{pm}\right] \tag{4.17}$$

Similarly, the final format of Service contract policy 3 model can be expressed as

$$C(L, x, N_i, M) = \frac{1}{L}\left[\begin{array}{l} C_{mr}\left\{\sum_{i=0}^{M} \sum_{k=0}^{N_i} \lambda^{\beta} x^{\beta} \left[(k - k\alpha + 1)^{\beta} - (k - k\alpha)^{\beta}\right]\right\} \\ + \sum_{i=0}^{M} N_i C_{pm} + C_{cl} + M C_{re} \end{array}\right] \tag{4.18}$$

Parameter Estimation

Some of the useful methods of parameter estimation are: method of least square, method of Moments, and method of Maximum likelihood. Non-parametric analysis might be used if data requires that approach (Crowder et al, 1991). Suzuki (1985) proposed parametric and non parametric methods of estimating lifetime distribution from field failure data with supplementary information about censoring times obtained from following up a portion of the product that survive contract period.

When failure follows first Weibull distribution, parameters of the developed models can be estimated by using maximum likelihood estimation method and these are given by

$$\hat{\lambda} = \left[\frac{n}{T^{\beta}}\right]^{\frac{1}{\beta}} \tag{4.19}$$

and

$$\hat{\beta} = \frac{n}{n \ln T - \sum_{i=1}^{n} \ln(t_i)} \tag{4.20}$$

where,

t_i time to its *ith* failure

n number of failures over contract period

T is the observation period

Numerical Examples

In this part the developed models are illustrated with numerical examples. For this purpose, it is assumed an asset with the shape parameter β = 2 and the inverse characteristic parameter of failure data λ = 0.189 per year.

Let:

Cost of minimal repair, C_{mr} = \$50

Cost of each preventive maintenance, C_{pm} = \$150

Cost of replacement, C_{re} = \$1000

Quality of each PM, α = .16 that is each Pm is 16% effective.

A MAPLE program is run developed and run which results in the following Table 4.1.

Table 4.1. Numerical example of the Service contract policy Models

Service Contract Policy	Contract Period	Optimal Number of Preventive Maintenance	Optimal Interval of Preventive Maintenances (yrs)	Contract Length (Yrs)	Servicing Cost per Year ($)
Policy 1	Fixed	1	2.5	5	38.40
Policy 2	Random	1	3.9	7.8	25.63
Policy 3	Fixed long term	Intractable using simple MAPLE Program but can be solved applying Simulation approach as in Figure 3.3.			

Example of service contract policy 1

The result implies that under the above contract deal and contract duration of 5 years, the optimal number of planned preventive maintenance with 16% effectiveness is one and this should be carried out at an interval 2.5 years.

Example of service contract policy 2

Under this policy and the above contract deal, and the given failure distribution the optimal number of planned preventive maintenance with 15% effectiveness is 1. This should be carried out at an interval of 3.9 years. This results in a contract period of 7.8 years. Note that the mean time to failure of the item is 5.29 years without any maintenance. But a preventive maintenance action with 16% effectiveness under this policy can extend the life of the item by (7.8 -5.29) or 2.51 years with a minimum cost of $25.63 per year.

Example of service contract policy 3

The model developed for Policy 3 is complex and it is difficult to solve analytically. Therefore a simulation approach is needed to solve this. A flow diagram of the proposed model is presented in the Figure 4.1.

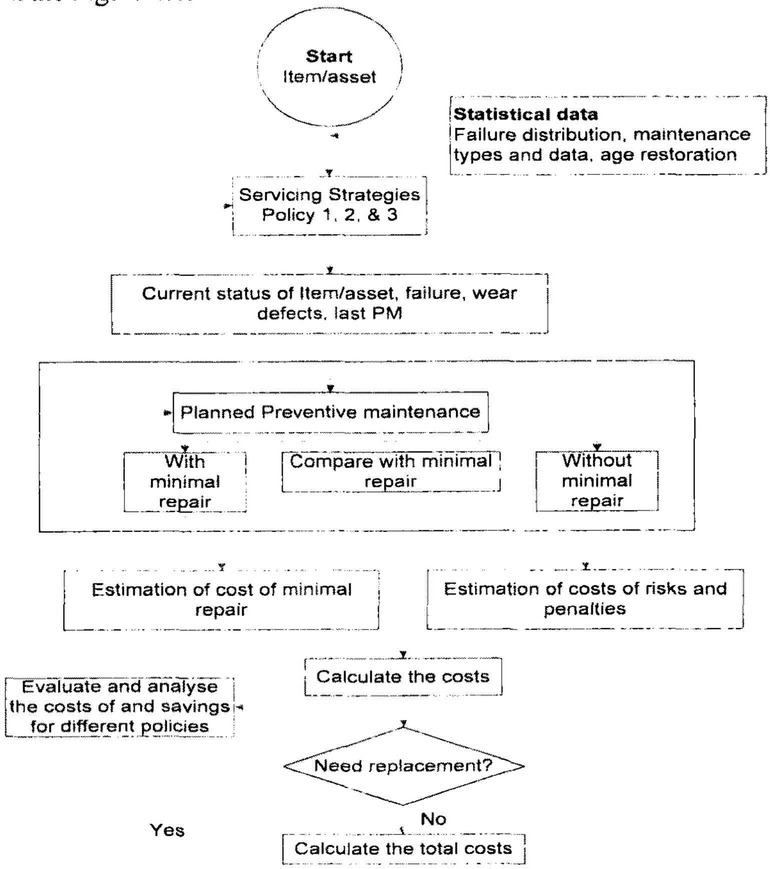


Figure 4.1. Framework for service contract cost model

5. CONTRIBUTION AND FUTURE WORKS

Three strategies (short-term and long-term) for Service contracts are proposed in this paper considering the concepts of outsourcing maintenance of assets to the service providers. Conceptual models were developed in estimating costs. These models were analysed and illustrated with numerical examples considering failures of asset/product follow Non homogeneous Poisson process. Real life rail failure data were used to illustrate the examples. Total costs of alternative strategies and cost per unit of service provided is considered for managerial decision. A simulation model is proposed for complex long-term service contracts. These models can be applicable to outsourcing maintenance service and service contracts for repairable systems. These models can be further extended by including discount rate, provisions for used items, and utility functions for linking customer/manufacturers risk preferences. More complex models could be developed linking risks, downtime and penalties for failure to meet agreed safety, reliability and availability standards.

REFERENCES

Ashgarizadeh, E., Murthy, D.N.P. (2000), Service contract: a stochastic mode, *Mathematical and Computer Modelling*, **31**, pp.11-20.

Barlow, R. E. and Hunter, L.(1960), Optimum preventive maintenance policies, *Operations Research*, **8**, pp. 90-100.

Crowder, M. J., Kimber, A. C.; Smith, R. L., Sweeting, T. J. (1991). *Statistical analysis of reliability data*, Chapman and Hall, UK.

Blischke, W. R., Murthy, D.N.P. (2000), *Warranty and service contracts*, New York, John Willey & Sons Inc.

Chattopadhyay, G. N., Murthy, D. N. P. (2000). Warranty cost analysis for second-hand products, *Mathematical and Computer Modelling*, **31**, pp. 81-88.

Murthy, D.N.P., Ashgarizadeh, E., (1995), Modelling service contracts, Presented at the *INFORMS* Meeting in New Orleans, USA.

Murthy, D.N.P., Jack, N. (2003), *Warranty and maintenance*, in Hoang Pham edit.: Handbook of Reliability Engineering, pp 305-314.

Murthy D.N.P., Yeung, V. (1995), Modelling and analysis of service contracts, *Mathematical and Computer Modelling*, **22**, pp.219-225.

Chattopadhyay, G.N. and Rahman, A. (2004), Optimal maintenance decisions for power supply timber poles, *International Journal of Reliability and Application*, **5**, pp. 115-128.

Rinsaka, K., Sandoh, H., (2006), A stochastic model on an additional warranty service contract, *Computers and Mathematics with Applications*, **51**, pp. 179-188.

Suzuki, K. (1985). Estimation of lifetime parameters from incomplete field data, *Technometrics*, **27**, pp 263-272.