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Posbist Reliability Analysis of Typical Systems

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Abstract. Posbist reliability of typical systems is preliminarily discussed in Cai (1991). In this paper, we focus on the posbist reliability analysis of some typical systems in depth. First, the lifetime of the system is dealt as a fuzzy variable defined on the possibility space (U, Φ, P_{oss}) and the universe of discourse is expanded from $(0, +\infty)$ to $(-\infty, +\infty)$. Then, a concrete possibility distribution function of the fuzzy variable is given, i.e., a Gaussian fuzzy variable. Finally, posbist reliability of typical systems (series, parallel, series-parallel, parallel-series, cold redundant system) is deduced. The expansion makes the proofs of some theorems straightforward and allows us to easily obtain the posbist reliability of typical systems. To illustrate the method a numerical example is given.

Key Words: *Posbist reliability, gaussian fuzzy variable, system lifetime, fuzzy reliability.*

1. INTRODUCTION

Conventional reliability theory is based on the probability assumption and the binary-state assumption (Cai, Wen and Zhang (1993). Although probability theory is a dominant tool in dealing with many conventional reliability problems, yet it is not true in all cases. For example, when the failure probabilities of components are very small (10^{-7}) or when systems are lack of sufficient statistical information,

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and so on. These fundamental problems of conventional reliability theory have led researchers to look for new models or new reliability theories which do not have the shortcomings of the classical probabilistic definition of reliability. Among others, we mention Tanaka et al (1983), Singer (1990), Onisawa (1988), Cappelle and Kerre (1993), Cremona and Gao (1997), Utkin and Gurov (1996) and Cai et al (1991, 1993, 1995) who have all tried to define reliability in terms other than probabilistic ones. Therefore, fuzzy methodology is inevitable introduced to accounting for uncertainty.

With the development of about 10 years, the main categories of fuzzy reliability theory are Cai (1996a):

(1) profust reliability theory: it is based on the probability assumption and the fuzzy-state assumption.

(2) posbist reliability theory: it is based on the possibility assumption and the binary-state assumption.

(3) posfust reliability theory: it is based on the possibility assumption and the fuzzy-state assumption.

We can find several forms of fuzzy reliability theories, including profust reliability theory Cai (1993), posbist reliability theory Cai (1991), and posfust reliability theory, are proposed using new assumptions, such as the possibility assumption and the fuzzy-state assumption, in place of the probability assumption or the binary-state assumption. For some very large complex systems, equipment and some components, it is very difficult to obtain necessary statistical data Huang (1996). Furthermore, their parameters, which defined the occurrence properties of variables, are of no statistical behavior of probability. Thus, subjective evaluation generally by experts based on their engineering judgement is more significant than objective statistics for their reliability behavior. Posbist reliability theory demonstrates its advantages over conventional reliability theory Utkin et al (1996), Cai et al (1991,1995). Cai et al (1991,1995) used the mathematical notions of possibility and fuzzy variable to develop a theory of posbist reliability and posbist reliability of typical systems, such as series, parallel, k-out-of-n and fault-tolerant systems, has been preliminarily discussed. However, Cai's work is confined to nonrepairable systems. Utkin et al (1996) proposed a general approach to formalize the reliability analysis on the basis of a system of functional equations according to Cai's theory. Cooman (1996) introduced the notion of possibilistic structure function based on the concept of classical, two-valued structure function and studied possibilistic uncertainty about the states of a system and its components. Cremona et al (1997) presented a new reliability theory measuring and analyzing structural possibilistic reliability similar in its methodology to the probabilistic reliability theory based on the principles of possibility theory. Moller et al (1999) applied possibility theory to safety assessment of structures considering non-stochastic uncertainties and subjective estimates of objective values by experts. Savoia (2002) presented a method of structural reliability analysis on the basis of possibility theory and fuzzy number approach. Guo et al (2002) developed a new model of structural possibilistic reliability based on possibility theory and fuzzy interval analysis.

In this paper, based on posbist reliability theory, the lifetime of the system is dealt as a fuzzy variable defined on the possibility space (U, Φ, P_{oss}) and the universe of discourse is expanded to $(-\infty, +\infty)$. In most practical cases, the possibility distribution function (i.e., the membership function) $\mu_x(x)$ can be approximately by two functions $L(x)$ and $R(x)$ with a point of intersection $max\mu(x) = 1$, i.e., $L - R$ type possibility distribution function. For more details, see Dubios and Prade (1979). So, we consider the lifetime of the system as a Gaussian fuzzy variable, which is a kind of special $L - R$ type fuzzy variable. On the basis of these, posbist reliability design of typical systems (series, parallel, series-parallel, parallel-series, cold redundant systems) is deeply discussed. In section 3, we can see that the expansion from $(0, +\infty)$ to $(-\infty, +\infty)$ doesn't influence the nature of the problems we will solve. On the contrary, it makes the proofs in Cai (1991,1995) straightforward and the complexity of calculation is greatly weakened.

2. POSBIST RELIABILITY THEORY

Posbist reliability theory assumes that (1) the system failure behavior is fully characterized in the context of possibility theory and that (2) at any time the system is in one of the two crisp states: perfect functioning state or complete failed state Cai (1991).

2.1 Basic concepts in the possibility context

The concept of posbist reliability theory is introduced in detail in Cai (1991). Here we simply consider the basic definitions related to this theory.

Definition 2.1. (Cai (1991)) A fuzzy variable X is a real valued function defined on a possibility space (U, Φ, P_{oss})

$$X : U \to R = (-\infty, +\infty)$$

Its membership function μ_x Its membership function R to the unit interval $[0, 1]$ with

$$\mu_x(x) = P_{oss}(X = x), x \in R$$

Thus a fuzzy set $\widetilde{X}$ can be induced on R

$$\widetilde{X} = \{x, \mu_x(x)\}$$

Based on $\widetilde{X}$, we may induce the distribution function of possibility associated to X such that

$$\pi(x) = \mu_x(x)$$

Definition 2.2. (Cai (1991)) The possibility distribution function of a fuzzy variable X, denoted by π_x, is a mapping from R to the unit interval $[0, 1]$ such that

$$\pi(x) = \mu_x(x) = P_{oss}(X = x), x \in R$$

Definition 2.3. (Cai (1991)) Given a possibility space (U, Φ, P_{oss}), the sets $A_1, A_2, \cdots, A_n \subset \Phi$ are said to be mutually unrelated if for any permutation of the set $\{1, 2, \cdots, n\}$, denoted by $\{i_1, i_2, \cdots, i_k\}(1 \leq k \leq n)$, there holds

$$p_{oss}(A_{i_1} \cap A_{i_2} \cdots \cap A_{i_3}) = min(P_{oss}(A_{i_1}), P_{oss}(A_{i_2}), \cdots P_{oss}(A_{i_k}))$$

Definition 2.4. (Cai (1991)) Given a possibility space (U, Φ, P_{oss}), the fuzzy variables $X_1, X_2, \cdots, X_n$are said to mutually unrelated if for any permutation of the set $\{1, 2, \cdots, n\}$, denoted by $\{i_1, i_2, \cdots, i_k\}(1 \leq k \leq n)$, the sets

$$\{X_{i_1} = x_1\}, \{X_{i_2} = x_2\}, \cdots, \{X_{i_k} = x_k\}$$

are unrelated $(x_1, x_2, \cdots, x_k \in R)$.

2.2 Lifetime of the system

Failure of a system based on binary-state assumption is defined precisely. However, the instant of a system failure occurrence is uncertain so that we can't determine it accurately, and it is characterized in the context of possibility measures. According to the existence theorem of possibility space Utkin et al (1996), we can reasonably assume there exists a single possibility space (U, Φ, P_{oss}) to characterize all the failure uncertainty of the system and its components. Accordingly, lifetimes of the system and its components should be dealt as Nahmias' fuzzy variables defined on the possibility space.

Definition 2.5. (Cai (1991)) Given a possibility space (U, Φ, P_{oss}), lifetime of a system is a non-negative real-valued fuzzy variable

$$X : U \to R^+ = (0, +\infty)$$

with possibility distribution function

$$\mu_x(x) = P_{oss}(X = x), x \in R^+$$

Then posbist reliability of a system is the possibility that the system performed its assigned functions properly during a predefined exposure period under a given environment.

$$\begin{aligned} R(t) &= P_{oss}(X > t) \\ &= \sup_{u>t} P_{oss}(X = u) \\ &= \sup_{u>t} \mu_x(u), t \in R^+ \end{aligned} \tag{2.1}$$

In real-life problems, considering the requirement of simplifying operation, we may expand the universe of discourse about system lifetimes from $(0, +\infty)$ to $(-\infty, +\infty)$ i.e.

$$X : U \to R = (-\infty, +\infty)$$

However, in the next section, we will see that the expansion doesn't influence the nature of the problems we will solve. On the contrary, it makes the proofs in Cai (1991,1995) straightforward and the complexity of calculation is greatly weakened

3. POSBIST RELIABILITY ANALYSIS OF TYPICAL SYSTEMS

First, we assume that the components composing a system are mutually unrelated. That is to say, the lifetimes of the components, denoted by $X_1, X_2, \cdots, X_n$, are mutually unrelated. Furthermore, we assume the possibility distribution function of X_i is a Guassian fuzzy variable. Its characteristic of distribution is illustrated in Fig 3.1.

$$\mu_{x_i}(x) = \begin{cases} \exp\left(-\left(\frac{m_i - x}{b_i}\right)^2\right), & x \leq m_i \\ \exp\left(-\left(\frac{x - m_i}{b_i}\right)^2\right), & x > m_i \end{cases} \tag{3.1}$$

where $m_i, b_i > 0, i = 1, 2, \cdots, n$.

3.1 Posbist reliability of series systems

Consider a series system consisting of n components. Suppose x is the system lifetime and $X_i (1 \leq i \leq n)$ is the lifetime of component i . There holds

$$X = min(X_1, X_2, \cdots, X_n). \tag{3.2}$$

Theorem 3.1. (Cai (1991)) Consider a series system of two components. Let the system lifetime be X with possibility distribution function μ_X and X_1, X_2 be the lifetimes of the two components respectively defined on possibility space (U, Φ, P_{oss}). If we assume that X_1, X_2 are mutually unrelated and each a normal, strictly convex fuzzy variable, with continuous possibility distribution functions $\mu_{X_1}(x)$ and $\mu_{X_2}(x)$ respectively. Then there exists a unique pair $(a_1, a_2), a_1, a_2 \in R^+$, such that

$$\mu_x(x) = \begin{cases} max(\mu_{X_1}(x), \mu_{X_2}(x)), & x \leq a_1 \\ \mu_{X_1}(x), & a_1 < x \leq a_2 \\ min(\mu_{X_1}(x), \mu_{X_2}(x)), & x > a_2 \end{cases} \tag{3.3}$$

Since we assume that X_1, X_2 are mutually unrelated, normal and strictly convex fuzzy variable, with continuous possibility distribution functions, posbist reliability of the series system of two components is

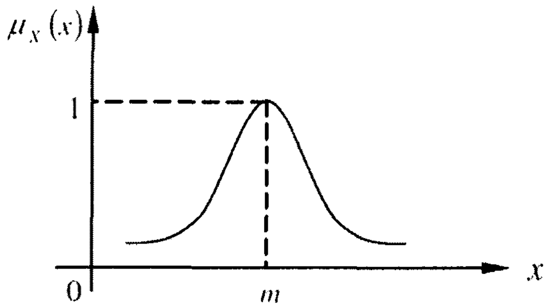

Figure 3.1. Possibility distribution function of X_i.

$$
\begin{aligned}
R(t) &= \sup_{u>t} \mu_X(u) \\
&= \begin{cases} 1, & t \leq a_1 \\ \mu_{X_1}(t), & a_1 < t \leq a_2 \\ min(\mu_{X_1}(t), \mu_{X_2}(t)), & t > a_2 \end{cases} \\
&= \begin{cases} 1, & t \leq m_1 \\ \exp\left(-\left(\frac{m_1 - t}{b_1}\right)^2\right), & m_1 < t \leq m_2 \\ min\left(\exp\left(-\left(\frac{t-m_1}{b_1}\right)^2\right), \exp\left(-\left(\frac{t-m_2}{b_2}\right)^2\right)\right), & t > m_2 \end{cases}
\end{aligned} \tag{3.4}
$$

Proof. Without loss of generality, we assume $a_1 \leq a_2$. Since

$$X = min(X_1, X_2),$$

We have

$$
\begin{aligned}
R_s(t) &= P_{oss}(X > t) \\
&= P_{oss}((X_1 > t) \cap (X_2 > t)) \\
&= min(P_{oss}(X_1 > t), P_{oss}(X_2 > t))
\end{aligned} \tag{3.5}
$$

Further

$$
\begin{aligned}
P_{oss}(X_1 > t) &= \sup_{u>t} \mu_{X_1}(u) \\
&= \begin{cases} 1, & x \leq a_1 \\ \mu_{X_1}(x), & x > a_1 \end{cases}
\end{aligned} \tag{3.6}
$$

and

$$P_{oss}(X_2 > t) = \sup_{u>t} \mu_{X_2}(u) = \begin{cases} 1, & x \le a_1 \\ \mu_{X_2}(x), & x > a_2 \end{cases} \tag{3.7}$$

Then

$$R_s(t) = \sup_{u>t} \mu_X(u) = \begin{cases} 1, & x \le a_1 \\ \mu_{X_1}(t), & a_1 < t \le a_2 \\ min(\mu_{X_1}(t), \mu_{X_2}(t)), & t > a_2 \end{cases} \tag{3.8}$$

Since we assume X_1, X_2 are normal fuzzy variable, the Eq. (3.4) becomes evident. QED.

For a series system of n components, if $X_1, X_2, \cdots, X_n$ are mutually unrelated and

$$\mu_{X_1}(x) = \mu_{X_2}(x) \cdots = \mu_{X_n}(x)$$

Then according to Theorem 1, we can easily arrive at

$$\mu_X(x) = \mu_{X_1}(x) \tag{3.9}$$

So posbist reliability of the series system of n components

$$\begin{aligned} R_s(t) &= \sup_{u>t} \mu_X(u) \\ &= \sup_{u>t} \mu_{X_1}(u) \\ &= \begin{cases} 1, & t \le a_1 \\ \mu_{X_1}(t), & t > a_1 \end{cases} \\ &= \begin{cases} 1, & t \le m_1 \\ \exp\left(-\left(\frac{t-m_1}{b_1}\right)^2\right), & t > m_1 \end{cases} \end{aligned} \tag{3.10}$$

3.2 Posbist reliability of parallel systems

Consider a parallel system consisting of n components. Suppose X is the system lifetime and $X_i (1 \le i \le n)$ is the lifetime of component i. There holds

$$X = max(X_1, X_2, \cdots, X_n) \tag{3.11}$$

Theorem 3.2. (Cai (1991)) Consider a parallel system of two components. Let the system lifetime be X with possibility distribution function μ_X and X_1, X_2 be the lifetimes of the two components respectively defined on possibility space (U, Φ, P_{oss}). If we assume that X_1, X_2 are mutually unrelated and each a normal, strictly convex fuzzy variable, with continuous possibility distribution functions $\mu_{X_1}(x)$ and $\mu_{X_2}(x)$ respectively. Then there exists a unique pair $(a_1, a_2), a_1, a_2 \in R^+$, such that

$$\mu_X(x) = \begin{cases} min(\mu_{X_1}(x), \mu_{X_2}(x)) & x \le a_1 \\ \mu_{X_2}(x) & a_1 < x \le a_2 \\ max(\mu_{X_1}(x), \mu_{X_2}(x)) & x > a_2 \end{cases} \tag{3.12}$$

Since we assume that X_1, X_2 are mutually unrelated, normal and strictly convex fuzzy variable, with continuous possibility distribution functions, posbist reliability of the parallel system of two components is

$$\begin{aligned} R_p(t) &= \sup_{u>t} \mu_X(u) \\ &= \begin{cases} 1, & t \le a_2 \\ max\left(\mu_{X_1}(t), \mu_{X_2}(t)\right), & t > a_2 \end{cases} \\ &= \begin{cases} 1, & t \le m_2 \\ max\left(\exp\left(-\left(\frac{t-m_1}{b_1}\right)^2\right), \exp\left(-\left(\frac{t-m_2}{b_2}\right)^2\right)\right), & t > m_2 \end{cases} \end{aligned} \tag{3.13}$$

Proof. The proof is analogous to that of posbist reliability of series systems. QED.
.1in

Similarly, for a parallel system of n components, if $X_1, X_2, \cdots, X_n$ are mutually unrelated and

$$\mu_{X_1}(x) = \mu_{X_2}(x) \cdots = \mu_{X_n}(x)$$

Then we can easily arrive at posbist reliability of this parallel system of n components

$$\begin{aligned} R_p(t) &= \sup_{u>t} \mu_X(u) \\ &= \sup_{u>t} \mu_{X_1}(u) \\ &= \begin{cases} 1, & t \le a_1 \\ \mu_{X_1}(t), & t > a_1 \end{cases} \\ &= \begin{cases} 1, & t \le m_1 \\ \exp\left(-\left(\frac{t-m_1}{b_1}\right)^2\right), & t > m_1 \end{cases} \end{aligned} \tag{3.14}$$

3.3 Posbist reliability of series-parallel systems

Consider a series-parallel system which is a series system of m subsystems, and where every subsystem is a parallel system of n components.

Let $X_i (1 \leq i \leq n)$ be the lifetime of component i, let X be the system lifetime. Further, we assume $X_1, X_2, \cdots, x_{n+m}$ are unrelated and every a normal, strictly convex fuzzy variable, with the same continuous possibility distribution function

$$\mu_{X_1}(x) = \mu_{X_2}(x) \cdots = \mu_{X_{n+m}}(x) \tag{3.15}$$

Then for every subsystem, which is a parallel system, we can arrive at posbist reliability of it according to Eq.(3.14)

$$\begin{aligned} R_{pi}(t) &= \sup_{u>t} \mu_{X_1}(u) \\ &= \begin{cases} 1, & t \leq a_1 \\ \mu_{X_1}(t), & t > a_1 \end{cases} \\ &= \begin{cases} 1, & t \leq m_1 \\ \exp\left(-\left(\frac{t-m_1}{b_1}\right)^2\right), & t > m_1 \quad 1 \leq i \leq n \end{cases} \end{aligned} \tag{3.16}$$

Further, according to Eq.(3.10), we can arrive at posbist reliability of the series-parallel system

$$\begin{aligned} R_{sp}(t) &= R_{pi}(t) \\ &= \sup_{u>t} \mu_{X_1}(u) \\ &= \begin{cases} 1, & t \leq m_1 \\ \exp\left(-\left(\frac{t-m_1}{b_1}\right)^2\right), & t > m_1 \end{cases} \end{aligned} \tag{3.17}$$

3.4 Posbist reliability of parallel-series systems

Consider a parallel-series system which is a parallel system of m subsystems, and where every subsystem is a series system of n components

Let $X_i (1 \leq i \leq n)$ be the lifetime of component i, let X be the system lifetime. Further, we assume $X_1, X_2, \cdots, x_{n+m}$ are unrelated and every a normal, strictly convex fuzzy variable, with the same continuous possibility distribution function

$$\mu_{X_1}(x) = \mu_{X_2}(x) \cdots = \mu_{X_{n+m}}(x) \tag{3.18}$$

Then for every subsystem, which is a series system, we can arrive at posbist reliability of it according to Eq.(3.14)

$$\begin{aligned} R_{si}(t) &= \sup_{u>t} \mu_{X_1}(u) \\ &= \begin{cases} 1, & t \le a_1 \\ \mu_{X_1}(t), & t > a_1 \end{cases} \\ &= \begin{cases} 1, & t \le m_1 \\ \exp\left(-\left(\frac{t-m_1}{b_1}\right)^2\right), & t > m_1 \quad 1 \le i \le n \end{cases} \end{aligned} \tag{3.19}$$

Further, according to Eq.(3.10), we can arrive at posbist reliability of the parallel-series system

$$\begin{aligned} R_{ps}(t) &= R_{si}(t) \\ &= \sup_{u>t} \mu_{X_1}(u) \\ &= \begin{cases} 1, & t \le m_1 \\ \exp\left(-\left(\frac{t-m_1}{b_1}\right)^2\right), & t > m_1 \end{cases} \end{aligned} \tag{3.20}$$

To sum up, it is obvious that series, parallel, series-parallel and parallel-series systems consisting of the components with the same possibility distribution function of the lifetime have the identical posbist reliability.

3.5 Posbist reliability of cold redundant systems

The operating mechanism of a cold redundant system is as follows. At any time only one operative component is required and the other operatives are redundant if they are not failed. We suppose the components are activated sequentially in order. Failure and performance deterioration will never occur to components in idle states. A system failure occurs only when no operative component is available. Here we only discuss the instance in which the fail-safe device and the conversion switch are absolutely reliable.

Consider a cold redundant system of n mutually unrelated components, as depicted in Fig 3.2. Let $X_i(1 \le i \le n)$ be the lifetime of component i. Let X be the system lifetime. Further, $X_i(1 \le i \le n)$ and X are fuzzy variables defined on a possibility space (U, Φ, P_{oss}), with possibility distribution functions $\mu_{X_i}(x)$ and $\mu_X(x)$ respectively. We have

$$X = X_1 + X_2 + \cdots + X_n \tag{3.21}$$

Then posbist reliability of the system is

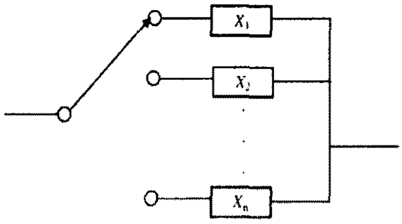

Figure 3.2. Cold redundant system.

$$R(t) = P_{oss}(X > t) = \sup_{u>t} P_{oss}(X = u) = \sup_{u>t} \mu_X(u) \quad (3.22)$$

For a cold redundant system of two components, since X_1, X_2 are $L - R$ type fuzzy numbers, then according to the addition operation of $L - R$ type fuzzy numbers Dubois et al (1979), we have

$$\mu_X(x) = \mu_{(X_1+X_2)}(x) = \begin{cases} \exp\left(-\left(\frac{m_1+m_2-x}{b_1+b_2}\right)^2\right), & x \le m_1 + m_2 \\ \exp\left(-\left(\frac{x-(m_1+m_2)}{b_1+b_2}\right)^2\right), & x > m_1 + m_2 \end{cases} \quad (3.23)$$

Further, we can arrive at posbist reliability of the system

$$R_{cr}(t) = \sup_{u>t} \mu_X(u) = \sup_{u>t} \mu_{(X_1+X_2)}(u) = \begin{cases} 1, & t \le m_1 + m_2 \\ \exp\left(-\left(\frac{t-(m_1+m_2)}{b_1+b_2}\right)^2\right), & t > m_1 + m_2 \end{cases} \quad (3.24)$$

Similarly, for a cold redundant system of n components, we have

$$\mu_X(x) = \mu_{(X_1+X_2+\cdots+X_n)}(u)$$

$$
= \begin{cases} \exp\left(-\left(\frac{\sum_{i=1}^{n} m_i - x}{\sum_{i=1}^{n} b_i}\right)^2\right), & x \le \sum_{i=1}^{n} m_i \\ \exp\left(-\left(\frac{x - \sum_{i=1}^{n} m_i}{\sum_{i=1}^{n} b_i}\right)^2\right), & x > \sum_{i=1}^{n} m_i \end{cases} \tag{3.25}
$$

Then we can easily arrive at posbist reliability of the system

$$
\begin{aligned}
R_{cr}(t) &= \sup_{u>t} \mu_X(u) \\
&= \sup_{u>t} \mu_{(X_1+X_2+\cdots+X_n)}(u) \\
&= \begin{cases} name1, & t \le \sum_{i=1}^{n} m_i \\ \exp\left(-\left(\frac{t - \sum_{i=1}^{n} m_i}{\sum_{i=1}^{n} b_i}\right)^2\right), & x > \sum_{i=1}^{n} m_i \end{cases}
\end{aligned} \tag{3.26}
$$

4. EXAMPLE

Calculate posbist reliability of a series, parallel, cold redundant system (supposing the fail-safe device and the conversion switch absolutely reliable), respectively, and every kind of system consists of two mutually unrelated components. Further, we assume the lifetime of each component is normal fuzzy variable, i.e.

$$
\mu_{x_1}(x) = \begin{cases} exp\left(-\left(\frac{120-x}{40}\right)^2\right), & x \le 120 \\ exp\left(-\left(\frac{x-120}{40}\right)^2\right), & x > 120 \end{cases} \qquad \mu_{x_2}(x) = \begin{cases} exp\left(-\left(\frac{130-x}{50}\right)^2\right), & x \le 130 \\ exp\left(-\left(\frac{x-130}{50}\right)^2\right), & x > 130 \end{cases}
$$

Here $t = 140$.

4.1 The series system

Posbist reliability of it is:

$$
R_S(T) = \sup_{u>t} \mu_X(x)
$$

$$= \begin{cases} 1, & t \leq m_1 \\ exp\left(-\left(\frac{m_1-t}{b_1}\right)^2\right), & m_1 < t \leq m_2 \\ min\left(exp\left(-\left(\frac{t-m_1}{b_1}\right)^2\right), exp\left(-\left(\frac{t-m_2}{b_2}\right)^2\right)\right), & t > m_2 \end{cases}$$

$$= \begin{cases} 1, & t \leq 120 \\ exp\left(-\left(\frac{120-t}{40}\right)^2\right) & t \leq 120 \\ min\left(exp\left(-\left(\frac{t-120}{40}\right)^2\right), exp\left(-\left(\frac{t-130}{50}\right)^2\right)\right), & t > 130 \end{cases} \quad (4.1)$$

4.2 The parallel system

Posbist reliability of it is:

$$\begin{aligned} R_p(t) &= \sup_{u>t} \mu_X(u) \\ &= \begin{cases} 1, & t \leq a_2 \\ max(\mu_{X_1}(t), \mu_{X_2}(t)), & t > a_2 \end{cases} \\ &= \begin{cases} 1, & t \leq m_2 \\ max\left(exp\left(-\left(\frac{t-m_1}{b_1}\right)^2\right), exp\left(-\left(\frac{t-m_2}{b_2}\right)^2\right)\right), & t > m_2 \end{cases} \end{aligned} \quad (4.2)$$

4.3 The cold redundant system

Posbist reliability of it is:

$$\begin{aligned} R_{cr}(t) &= \sup_{u>t} \mu_X(u) \\ &= \sup_{u>t} \mu_{X_1+X_2}(u) \\ &= \begin{cases} 1, & t \leq m_1 + m_2 \\ exp\left(-\left(\frac{t-(m_1+m_2)}{b-1+b_2}\right)^2\right), & t > m_1 + m_2 \end{cases} \\ &= \begin{cases} 1, & t \leq 250 \\ exp\left(-\left(\frac{t-250}{90}\right)^2\right), & t > 250 \end{cases} \end{aligned} \quad (4.3)$$

when $t = 140$,we have:

$$\begin{aligned} R_s(140) &= exp\left(-\left(\frac{140-120}{40}\right)^2\right) \\ &= 0.7788 \end{aligned}$$

$$R_p(140) = exp\left(-\left(\frac{140-130}{50}\right)^2\right) = 0.9608$$

$$R_{cr}(140) = 1$$

In this case, we can arrive at a conclusion as follows:

Posbist reliability of parallel systems is higher than that of series systems, and posbist reliability of cold redundant systems is higher than that of parallel systems.

5. CONCLUSIONS

This paper adds new insights to the important work of Cai (1991,1995) as follow:

(1) Although conventional reliability theory has been a dominant tool to evaluating system safety and analyzing failure uncertainty, yet the uncertainty concerned with the system and its components cannot be always defined in the framework of probability. With the advent of highly complex systems and vast variations of system characteristics, people have realized that probability theory is not a panacea. In this paper, based on posbist reliability theory, the lifetime of the system is considered as a Gaussian fuzzy variable. On the basis of these, posbist reliability of typical systems (series, parallel, series-parallel, parallel-series, cold redundant systems) is deduced.

(2) The universe of discourse about system lifetimes defined in Cai (1991,1995) is expanded from $(0, +\infty)$ to $(-\infty, +\infty)$. However, we can find in section 3 this expansion doesn't influence the nature of the problems we will solve. On the contrary, it makes the proofs in Cai (1991,1995) straightforward and the complexity of calculation is greatly weakened.

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