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Testing NBUCA Class of Life Distribution Using U-Test

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Abstract. In this paper, testing exponentiality against new better than used in convex average and denote by (NBUCA) , or its dual (NWUCA) is investigated through the U-test. The percentiles of these tests are tabulated for samples sizes $n = 5(1)40$. The power estimates of the test are simulated for some commonly used distributions in reliability. Pitman's asymptotic efficiency of the test is calculated and compared. Data of 40 patients suffering from blood cancer disease (Leukemia) is considered as a practical application of the proposed test in the medical sciences.

Key Words : *Life distributions, positive ageing, U-statistic, asymptotic normality, power estimates, asymptotic relative efficiency, testing exponentiality*

1. INTRODUCTION

The aging life is usually characterized by a nonnegative random variable $X \geq 0$ with distribution function (cdf) F and survival function (sf) $\bar{F} = 1 - F$. Associated with X is the notion of "random remaining life" at age t, denoted by X_t, where X_t has an sf as

$$\bar{F}_t(x) = \frac{\bar{F}(x+t)}{\bar{F}(t)}, \quad x, t \geq 0. \tag{1.1}$$

Note that $X_t \overset{st}{=} X$, or $\bar{F}_t(x) = \bar{F}(x)$ (st denotes the stochastic ordering) if and only if $\bar{F}$ is an exponential distribution. Comparing X and X_t in various forms and types create classes of aging useful in many biomedical, engineering and statistical studies, see Barlow and Proschan (1981). It is well known that the relation $X_t \overset{st}{\leq} X$ or $\bar{F}_t(x) \leq \bar{F}(x)$ defines the class of new better than used (NBU). On the other hand, the relation $E(X_t) \leq E(X)$ defines the class of new better than used in

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expectation (NBUE), and decreasing mean residual lifetime (DMRL). Testing exponentiality against anyone of these classes forms a vast literature pool. Most of the testing procedures, are based on developing empirical estimates of departure from exponentiality in favor of the alternative class. The result test statistics are mainly version of U-statistics. For this vast literature we refer the reader to the surveys by Doksum and Yandell(1984); Loh(1984) and Hendi et al (1998). In the present paper, comparing between the life distribution of a new unit with that of the remaining life or a used unit in increasing convex average order leads us to introduce a new class of life distribution. This new class is larger and perhaps more practical than the $NBUC$ class introduced by Cao and Wang (1991) and further by several authors, including Hendi, Mashour and Montaseer (1993), Li,Li and Jing(2000), Hu and Xie (2002), and Ahmad and Mugdadi (2003) among others. Our new class compares a new life to that is used (of age t) in a new or ordering sense which we call "increasing convex average" ordering.

The paper is organized as fillows: Section 2 contains, notations and basic properties which are used to intoduce the class of the " new better than used in the increasing convex average" (denoted by NBUCA) In Section 3 of this paper, we use U-statistic to test $H_0 : F$ is exponential(μ) versus $H_1 : F \in NBUCA$ and not exponential, where $\mu = \int_0^\infty \bar{F}(u)du$. Also, we simulate the critical points for the statistic used in the test through Monte Carlo methods for sample sizes $n = 5(1)40$. Next,in section 4 we calculate the power of the test based on some other alternative life distributions, including, the linear failure rate, Gamma and Weibull distributions. To show the efficiency of our results, we calculate Pitman asymptotic efficiency and compare our result by this given by Kango (1993). Finally, we apply our test to real practical data in medical science given by Abouammah et al. (1994) in section 6.

On the other hand, an ordeing of life variable that proved useful in producing classes of life distributions is due to Stoyan(1983), cf. Bhattacharjee(1991) for definition and properties.

2. DEFINITIONS

In this section we present definitions, notations and basic properties used throughout the paper. We also give the definition of the new better than used in the increasing convex average class of life distributions. We use increasing in place of "non-decreasing" and "decreasing" in place of "non-increasing".

Let X and Y be non-negative random variables with distribution function $F(x)$ and $G(x)$ respectively, and survival function $\bar{F}(x)$ and $\bar{G}(x)$. We say that X is smaller than Y in the

(i) the usual stochastic order(denoted by $X \leq_{st} Y$), if $\bar{F}(x) \leq \bar{G}(x)$;

(ii) the increasing convex order (denoted by $X \leq_{icx} Y$), if

$$\int_x^\infty \bar{F}(u)du \leq \int_x^\infty \bar{G}(u)du \texttt{ for all x;}$$

(iii) the increasing convex average order (denoted by$X \leq_{icxa} Y$),if

$$\int_0^\infty \int_x^\infty \bar{F}(u)dudx \leq \int_0^\infty \int_x^\infty \bar{G}(u)dudx \texttt{ for all x.}$$

See Ahmad et al.(2006).

In economic theory, these orders are known as *first-order stochastic dominance* denoted by $X\ FSD\ Y$, *second-order stochastic dominance* denoted by $X\ SSD\ Y$, and *weak third order stochastic dominance*, denoted by $X\ WTSD\ Y$. For more details, see Deshpande et al.(1986) or Kaur et al. (1994).

On the other hand, in reliability theory, it has been found useful to define non-parametric classes if lifetime distributions by stochastic comparison of survival function of the lifetime of a new one. For example, let $X_t = [X - t|x > t]$ denote to residual lifetime of X at time t, and it is the time to failure of a unit with lifetime t and let X be a non-negative random variable with distribution function F. we say that

(i) X (or$F$)is new better than used (denoted by $X \in NBU$) if

$$X_t \leq_{st} X, \texttt{ for all } t \geq 0,$$

(ii) X (or$F$) is new better than used in the convex order (denoted by $X \in NBUC$) if

$$X_t \leq_{icx} X, \texttt{ for all } t \geq 0;$$

The NBU class was introduced by Bryson and Siddiqqui (1969) and independently by Marsharll and Proschan(1972). It has grown to become one of the most studied classes life distributions. The new class is larger and perhaps more practical than $NBUC$ class introduced by Cao and Wang (1991).

Following the same ideas, we now introduce a new aging class of life distributions by stochastic comparison of the survival function of residual lifetime of a used unit with that of the lifetime of a new one in the increasing convex average order sense. Precisely, we have:

Definition 2.1. A random variable X(or F) is said to be new better than used in the convex average order (denoted by $NBUCA$) if

$$X_t \leq_{icxa} X,$$

equivalently, $X \in NBUCA$ if

$$\int_0^\infty \int_x^\infty \bar{F}(u+t)dudx \leq \bar{F(t)} \int_0^\infty \int_x^\infty \bar{F}(u)dudx \text{ for all t.}$$

or, $X \in NBUCA$ if $X \ WSTSD \ X_t$.

Note that Definition 2.1 is to saying that $2 \int_0^\infty \bar{v}(x+t)dt \leq \mu_2 \bar{F}(t)$, where $\mu_2 = E(X^2)$, assumed finite, and $\bar{v}(x) = \int_x^\infty \bar{F}(u)du$.

In fact, the $NBUCA$ class is related to, but contains and is much large than the $NBUC$ class. The following chain summarizes the implications among some of the previous class

$$IFR \Rightarrow IFRA \Rightarrow NBU \Rightarrow NBUC \Rightarrow NBUCA.$$

3. TESTING EXPONENTIALITY VERSUS NBUCA CLASS

The proposed test depends on a sample $X_1, X_2, \ldots, X_n$ from a population with density function F which will be used to test the null hypothesis $H_0 : \bar{F}(x) = exp(\frac{-x}{\mu})$, $x > 0$, $\mu > 0$ (μ is unknown) versus $H_1 : \bar{F} \in NBUCA$, that is $X_t \leq_{icxa} X$ or equivalently

$$\int_0^\infty \int_x^\infty \bar{F}(u+t)dudx \leq \bar{F}(t) \int_0^\infty \int_x^\infty \bar{F}(u)dudx \text{ for all } x, t \geq 0.$$

In order to test H_0 against H_1, we use the following measure of departure from H_0

$$\delta_{ca} = E[\bar{F}(t) \int_0^\infty \int_x^\infty \bar{F}(u)dudx - \int_0^\infty \int_x^\infty \bar{F}(u+t)dudx] \tag{3.1}$$

simplifying (3.1) we obtain

$$\delta_{ca} = E[\bar{F}(t)\mu_2 - \int_t^\infty (x-t)^2 dF(x)] \tag{3.2}$$

where $\mu_2 = E(X^2)$, assumed finite, there for

$$\delta_{ca} = \int_0^\infty [\bar{F}(t)\mu_2 - \int_t^\infty (x-t)^2 dF(x)]dF(t). \tag{3.3}$$

Note that $\delta_{ca} = 0$ under H_0 and $\delta_{ca} > 0$ under H_1 and $\mu_2 = \sum_{k=1}^n \frac{X_k^2}{n}$By using random sample of size n, the empirical estimate $\hat{\delta}_{ca}$ of δ_{ca}, $\bar{F}_n(x) = \frac{1}{n}\sum_{j=1}^n I(X_j > x)$ denote the empirical survival distribution, $dF_n(x) = \frac{1}{n}$, μ is estimated by the sample mean $\bar{X}$ and μ_2 is estimated by $\hat{\mu}_2 = \sum_{k=1}^n \frac{X_k^2}{n}$. Then $\hat{\delta_{ca}}$, is given by using (3.3) as

$$\hat{\delta}_{ca} = \int_0^\infty [\frac{1}{n}\sum_{j=1}^n I(X_j > x) \sum_{k=1}^n \frac{X_k^2}{n} - \frac{1}{n}\sum_{j=1}^n (X_j - x)^2 I(X_j > x)]dF_n(x) \tag{3.4}$$

i.e.

$$\hat{\delta}_{ca} = \frac{1}{n^3} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} [X_k^2 - (X_j - X_i)^2] I(X_j > X_i) \tag{3.5}$$

where

$$I(x > y) = \begin{cases} 1 & x > y, \\ 0 & \text{otherwise.} \end{cases} \tag{3.6}$$

Thus to make the test statistic $\hat{\delta}_{ca}$ in (3.5) scale invariant we take

$$\hat{\Delta}_{ca} = \hat{\delta}_{ca} / \bar{X}^2 \tag{3.7}$$

where $\bar{X} = \sum_{k=1}^{n} \frac{X_k}{n}$ is usual sample mean.

Setting $\phi(X_1, X_2, X_3) = [X_3^2 - (X_2 - X_1)^2] I(X_2 > X_1)$ and defining the symmetric kernel $\Psi(X_1, X_2, X_3) = \frac{1}{3!} \sum_R \phi(X_{i_1}, X_{i_2}, X_{i_3})$, where the sum is over all arrangements of X_1, X_2 and X_3. The $\hat{\delta}_{ca}$ in (3.7) is equivalent to the U-statistic

$$U_n = \frac{1}{\binom{n}{3}} \sum_{i<j<k} \Psi(X_i, X_j, X_k). \tag{3.8}$$

The following theorem gives a summary of the large sample properties of $\hat{\Delta}_{ca}$ or U_n.

Theorem 3.1.

(i) As $n \to \infty$, $\sqrt{n}(U_n - \Delta_{ca})$ is asymptotically normal with mean 0 and variance is given by

$$\begin{aligned} \sigma^2 &= \frac{1}{\mu^4} Var \Big\{ \mu_2 \bar{F}(X_1) - \int_{X_1}^{\infty} u^2 dF(u) + 2X_1 \int_{X_1}^{\infty} u dF(u) \\ &- X_1^2 \bar{F}(X_1) + \mu_2 F(X_1) - X_1^2 F(X_1) + 2X_1 \int_0^{X_1} u dF(u) \\ &- \int_0^{X_1} u^2 dF(u) + X_1^2 \int_0^{\infty} \bar{F}(u) dF(u) - \int_0^{\infty} \int_y^{\infty} u^2 dF(u) dF(y) \\ &+ \int_0^{\infty} \int_y^{\infty} 2uy e^{-u-y} du dy - \int_0^{\infty} y^2 \bar{F}(y) dy \Big\} \end{aligned} \tag{3.9}$$

(ii) Under H_0, the variance reduces to

$$\sigma_0^2 = Var\left(2X_1 - \frac{X_1^2}{2} - 1\right) = 1, \tag{3.10}$$

(iii) If F is continuous NBUCA, then the test is consistent.

Table 3.1. Critical values of the test statistic $\hat{\Delta}_{ca}$

n	1%	5%	10%	90%	95%	99%
5	-.91344	-.51259	-.34444	.26705	.30747	.35929
6	-.91415	-.51550	-.34551	.26319	.29978	.35362
7	-.89827	-.51946	-.34208	.26059	.29903	.35113
8	-.85615	-.48882	-.31989	.25407	.29186	.34794
9	-.86729	-.46048	-.31432	.24672	.28237	.33967
10	-.85258	-.45643	-.30258	.24185	.28098	.33845
11	-.83023	-.44745	-.29622	.23619	.27291	.32889
12	-.80278	-.43260	-.28388	.23185	.26825	.32287
13	-.75698	-.42347	-.28085	.22776	.26392	.32179
14	-.72002	-.40669	-.26693	.22074	.25805	.31856
15	-.73638	-.40639	-.26478	.21852	.25521	.31284
16	-.72999	-.38879	-.26379	.21390	.24948	.30631
17	-.69020	-.37884	-.26476	.20909	.24493	.30119
18	-.67222	-.37716	-.24795	.20883	.24177	.29950
29	-.67393	-.36427	-.24508	.20242	.23699	.29512
20	-.64034	-.36440	-.24360	.20197	.23641	.28983
21	-.66261	-.35950	-.24476	.19606	.23268	.28010
22	-.62360	-.33968	-.23533	.19862	.23090	.28444
23	-.60381	-.33306	-.23037	.19146	.22627	.28131
24	-.58831	-.32873	-.22574	.19259	.22417	.27679
25	-.58896	-.33208	-.22209	.18832	.22097	.27229
26	-.56412	-.31259	-.21205	.18496	.22023	.27342
27	-.56323	-.31939	-.22076	.17986	.21124	.26715
28	-.53163	-.32466	-.21781	.17993	.21308	.26651
29	-.58554	-.30887	-.20986	.17733	.20945	.26197
30	-.52130	-.30470	-.19863	.17502	.20758	.26026
31	-.53057	-.29564	-.20544	.17325	.20436	.25453
32	-.52811	-.30175	-.20824	.17402	.20731	.25679
33	-.51909	-.29401	-.19824	.16997	.20018	.25041
34	-.49184	-.27440	-.19162	.16713	.19865	.25295
35	-.49783	-.28306	-.19697	.16522	.19692	.24365
36	-.49462	-.27886	-.19064	.16544	.19616	.24694
37	-.45789	-.26351	-.18331	.16695	.19645	.24212
38	-.47697	-.27154	-.19007	.15899	.18939	.24248
39	-.45476	-.26459	-.18574	.16065	.19421	.24015
40	-.45908	-.26750	-.18532	.15878	.19085	.23861

Proof.

By using the standard theory of U-statistic [Lee (1990)], one can easily prove parts (i) and (ii). To prove part (iii), let $D(x,t) = \bar{F}(t)\mu_2 - \int_t^\infty (x-t)^2 dF$ in (3.3) and since F is NBUCA and continuous, $D(x,t) > 0$ for at least one value of (x,t), call it (x_0, t_0). Now set

$$(x_1, t_1) = inf\{(x,t)|(x < x_0 \text{ and } t \leq t_0)\}, \bar{F}(x) = \bar{F}(x_0) \text{ and } \bar{F}(t) = \bar{F}(t_0)\}.$$

Thus,

$$\begin{aligned} D(x_1, t_1) &= \bar{F}(t_1)\mu_2 - \int_{t_1}^\infty (x-t_1)^2 dF) \geq \bar{F}(t_1)\mu_2 - \int_{t_0}^\infty (x-t_0)^2 dF \\ &= \bar{F}(t_0)\mu_2 - \int_{t_0}^\infty (x-t_0)^2 dF = D(x_0, t_0) > 0, \end{aligned}$$

and $F(x_1 + \delta_1) - F(x_1) > 0$ and $F(t_1 + \delta_2) - F(t_1) > 0$ and since x_1 and t_1 are points of increases of F, thus $\Delta_{ca} > 0$, then the test is consistent .

To use the above test, we calculate $\sqrt{n}\hat{\Delta}_{ca}/\sigma_0$ and reject H_0 if it exceeds the normal variate $Z_{1-\alpha}$.

By using Monte Carlo methods, we calculate the empirical critical points of $\hat{\Delta}_{ca}$ in (3.7), for different samples. Table 3.1 represents the lower and the upper percentile points for 1%, 5%, 10%, 90%, 95% and 99% and calculations are based on 1000 simulated samples $n = 5(1)40$.

4. THE POWER ESTIMATES

The power of the test statistic $\hat{\Delta}_{ca}$ in (3.5) is considered for the significant level at 95% upper percentile and for most commonly used alternatives. These are:

(i) Linear failure rate:

$$\bar{F}_1(x) = \exp(-x - \frac{\theta}{2}x^2), \theta > 0, \ x \geq 0,$$

(ii)Gamma:

$$\bar{F}_2(x) = \int_x^\infty u^{(\theta-1)} \exp(-u) du/\Gamma(\theta), \theta > 0, \ x \geq 0,$$

(iii) Weibull:

$$\bar{F}_3(x) = \exp(-x^\theta), \ \theta > 0, \ x \geq 0.$$

These distributions are reduced to exponential distribution for appropriate values of θ. The power of the test presented in Table 4.1 shows the power estimate of $\hat{\Delta}_{ca}$-statistic The power estimates in Table(4.1) shows clearly the departure from exponentiality towards (NBUCA) properties as θ increases and also as n increases.

Table 4.1. Power Estimate of $\hat{\Delta}_{ca}$-Statistic

Distribution	θ	Sample size $n=10$	$n=20$	$n=30$
F_1 : Linear failure rate	2	0.243	0.433	0.626
	3	0.284	0.508	0.721
	4	0.314	0.597	0.791
F_2 : Gamma	2	0.318	0.583	0.704
	3	0.628	0.905	0.973
	4	0.822	0.984	0.998
F_3 : Weibull	2	0.729	0.972	1.000
	3	1.000	1.000	1.000
	4	1.000	1.000	1.000

5. PITMAN'S ASYMPTOTIC RELATIVE EFFICACY

Finally, to assess how good this procedure is relative to others in the literature, we employ the concept of "Pitman's asymptotic relative efficiency" (PARE), which is defined as follows: Let T_{1n} and T_{2n} be two test statistics for testing $H_0 : F_\theta \in \{F_{\theta_n}\}, \theta_n = \theta + cn^{-1/2}$ with c an arbitrary constant, then the asymptotic relative efficiency of T_{1n} relative to T_n is defined by

$$e(T_{1n}, T_{2n}) = \{\mu_1'(\theta_0)/\sigma_1(\theta_0)\}/\{\mu_2'(\theta_0)/\sigma_2(\theta_0)\} \quad (5.1)$$

where

$$\mu_i'(\theta_0) = lim_{n\to\infty}\{\frac{\partial}{\partial\theta}E(T_{in})\}_{\theta\to\theta_0} \text{ and} \quad (5.2)$$

$$\sigma_i^2(\theta_0) = lim_{n\to\infty}var_0(T_{in}), \text{ i=1,2 is the null variance} \quad (5.3)$$

We choose the following three alternatives:

(i) Linear failure rate:

$$\bar{F}_1(x) = \exp(-x - \frac{\theta}{2}x^2), \theta > 0, \ x \geq 0,$$

(ii)Makeham:

$$\bar{F}_2(x) = \exp[-x - \theta(x + e^{-x} - 1)], \theta > 0, \ x \geq 0,$$

(iii) Weibull:

$$\bar{F}_3(x) = \exp(-x^\theta), \ \theta > 0, \ x \geq 0.$$

[see, Hollander and Proschan(1975)].

Note that H_0 (the exponential) is attained at $\theta = 0$ in (i) and (ii) and attained at $\theta = 1$ in (iii) of above alternatives. Now to evaluate the "Pitman's asymptotic efficiency" (PAE) for our test $\hat{\Delta}_{ca}$ and compare it (by taking ratios) to PAE of other tests to get PARE. For the suggested above in (3.7), the PAE is given by

$$\frac{\mu'(\theta_0)}{\sigma_0} = \frac{\partial}{\partial\theta}\Delta_{ca}\,|_{\theta\to\theta_0} = \frac{1}{\sigma_{\theta_0}}\frac{\partial}{\partial\theta}\left[\frac{1}{\mu_\theta^2}\left\{\int_0^\infty \mu_\theta(2)\bar{F}_\theta(x)dF_\theta(x) - \int_0^\infty\int_x^\infty (u-x)^2 dF_\theta(u)dF_\theta(x)\right\}\right]_{\theta\to\theta_0}. \tag{5.4}$$

Since the above test $\hat{\Delta}_{ca}$ is new and no other tests are known for this class NBUCA, we compare it with some other class tests such as U_n is presented by Kango(1993) and the results are summarized in Table 5.1 and Table 5.2.

Table 5.1. PAE of $\hat{\Delta}_{ca}$ and U_n

	$PAE = \mu_i'(\theta_0) = \mu'(\theta)/\sigma_i(\theta_0), i = 1,2,3$	
Distribution	$\hat{\Delta}_{ca}$	U_n
F_1 : Linear failure rate	1.00	0.433
F_2 : Makeham	0.25	0.144
F_3 : Weibull	1.00	0.132

Table 5.2. PARE of $\hat{\Delta}_{ca}$, with respect to U_n

Distribution	$e_{F_i}(\hat{\Delta}_{ca}\ ,\ U_n)$
F_1 : linear failure rate	1.0/0.433=1.736
F_2 : Makeham	0.25/0.144=2.31
F_3 : Weibull	1.0/0.132=7.576

It is clear from Table 5.1 and 5.2 that our new test statistic $\hat{\Delta}_c a$ performs well for $\bar{F}_1$, $\bar{F}_2$ and $\bar{F}_3$ and it is more efficient than the statistic U_n which is proposed by Kango(1993).

6. APPLICATIONS

In this section, we calculate the $\hat{\Delta}_{ca}$ test statistic for the data set of 40 patients suffering from blood cancer (Leukemia) from one of Ministry of Health Hospitals in Saudi Arabia [see Abouammah et al. (1994)]. The ordered life times (in days) are: 115, 181, 255, 418, 441, 461, 516, 739, 743, 789, 807, 865, 924, 983, 1024, 1062, 1063, 1165, 1191, 1222, 1222, 1251, 1290, 1357, 1369, 1408, 1455, 1478, 1549, 1578, 1578, 1599, 1603, 1605, 1696, 1735, 1799, 1815, 1852.

It was found that the test statistic $\hat{\Delta}_{ca}$ in (3.7) has the following result $\hat{\Delta}_{ca} =$ 0.390331 and this value is greater than the critical value in Table 3.1 at 95% upper percentile. We therefore accept H_1, which states that the data has NBUCA property.

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