Evaluation on the Selective Combining for the Detection of *M*-ary DPSK Signals over Nakagami Fading Channels

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Abstract

The performances of *M*-ary DPSK(MDPSK) for diversity reception theoretically are derived, using an *L*-branch selection combining(SC) in frequency-nonselective slow Nakagami fading channels. For integer values of the Nakagami fading parameter *m*, An exact closed-form symbol error rate(SER) multichannel performance that can be easily evaluated via numerical integration is presented. Finally, we compare these analyses with numerical analyses with integral-form expressions for the performance of MDPSK signals under the effect of two-branch SC diversity over slow and nonselective Rician fading channels with additive white Gaussian noise(AWGN).

Key words : MDPSK, SC, Nakagami Fading, Rician Fading, SER.

I. Introduction

The statistical properties of mobile radio environments can be often specified by three propagation effects: 1) short-term fading, 2) long-term fading, 3) propagation path loss[1]. In short-term fading, the scattering mechanism only results in numerous reflected components[2]. The rayleigh model is used to characterize this fading in small geographical areas and sometimes does not account for large scale effects like shadowing by building and hills[3]. In long-term fading, the change of effective height for mobile communication antenna exists due to the nature of the terrain. Its statistics follow the log-normal distribution. But in the event that there are both the information signal and the multitone interfering signal, this channel is modeled to be a Rician fading model[4]. By modeling the channel as a Rician fading channel, a result is obtained that is valid in the limit of large direct-to-diffuse power ratios for channels with no fading and in the limit of small direct-to-diffuse power ratios for rayleigh environments as well as for the general case when neither then direct nor the diffuse components of the signal are negligible[5]. A more general fading distribution which has been shown to be appropriate to model the multipath fading in urban as well as indoor situations is the Nakagami *m*-distribution, of which rayleigh fading is a special case of fading index $m=1$[6]. Nakagami distribution can model fading conditions more or less severe than those in the Rayleigh case. The Rice and lognormal distribution can also be approximated by the Nakagami distribution[7].

When *M*-ary DPSK signals experience the Nakagami-*m* channel, diversity schemes can minimize the effects of this fading, since deep fades seldom occur simultaneously during the same time intervals on two or more paths[8]. In [9], bit error rate(BER) of coherent and noncoherent FSK is analyzed theoretically for diversity reception in Nakagami fading environment, using an *L*-branch maximal ratio combining(MRC). These results can be extended to include coherent PSK and DPSK. Patenaude[10] has computed the average bit error probability of DPSK and FSK signals over slow and flat Nakagami fading with postdetection diversity reception. Fedele[11] has given the integral form, not the closed form, for the performance achievable by two-branch MRC and SC predetection system in receiving MDPSK signals under a slow and nonselective Nakagami fading conditions. Aalo[12] proposed an exact closed-form expression for the error rate of coherent MPSK signals in a Nakagami fading model. The result was extended to systems that employing MRC diversity reception at the receiver. Staley[13] presents a recursive method for evaluating the error rate performance of multichannel MRC reception of coherent MPSK system operating over slowly Rician fading channel, using an expression for the received phase density function. Annamalai[14] computed the exact SER for MDPSK and MPSK signals in conjunction with MRC in Nakagami and lognormal-Rice fading channels. Also, he evaluated exact integral expressions of MRC and equal-gain combining(EGC) diversity receivers for *M*-ary QAM on Nakagami fading channels[15]. In [16], the general formula for evaluating the bit-error probability of *M*-ary DPSK with *L* diversity equal-gain combining over frequency nonselective *m*-dis-

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tributed, uncorrelated Nakagami fading channels and correlated Gaussian noise was presented. Abu-Dayya[17] finds accurate analyses for the average bit error rates of MPSK signals using both dual-branch EGC and dual-branch SC assuming that the desired signal experiences frequency-nonselective Rician or Nakagami fading and the multiple interferers experience independent frequency-nonselective rayleigh fading. Alouini[18] provided the effect of the severity of the Nakagami fading channels on the BER performance for the conventional SC and MRC detection of binary PSK(BPSK) and FSK(BFSK). Especially, due to the mathematical complexity of SC compared to other diversity techniques, an exact closed-form analysis of SC, where the convergence of the probability density function(PDF) is considerably faster than numerical integration techniques or [18, Eq. (15)], for MDPSK signals has not been published previously, despite its practical interest.

In this paper, with reference to a traditional SC, whereby the signal with the largest signal-to-noise ratio (SNR) is selected from the original diversity branches in the channel while assuming that the noise power is constant across all branches, the order statistics is applied[19],[20]. We use new PDF instead of [18, Eq. (15)] in Nakagami-distributed slow and nonselective fadings and AWGN and represent the multichannel performance, not two-branch performance[11],[17], of SC diversity reception of MDPSK by combining integral expressions and a finite-series formula. Finally, in Appendices, we express the relationship between the Nakagami-m channel and the Rician channel of MDPSK in the independent L-branch diversity system over a Rician fading model. On account of superior complexity for the mathematical manipulations of SC in a Rician fading channel, L branches are assumed for the dual diversity of the most practical cases and the integral-form expressions of these performances are derived. But it is worthwhile that SC diversity system can attain through a Rician fading model.

Ⅱ. SER of Multichannel Selective Diversity Receiver

It is assumed that diversity branches are statistically independent with each other over Nakagami fading channels.

The probability density function(PDF) of the received instantaneous SNR per symbol, r at the output of L-branch SC in a Nakagami fading channel is given by [21]

$$f(\gamma)_{SC}=\frac{L}{\Gamma(m)}\left(\frac{m}{\gamma_0}\right)^{m}\gamma^{m-1}\exp\left(-\frac{m}{\gamma_0}\gamma\right)\cdot\left[1-\exp\left(-\frac{m}{\gamma_0}\gamma\right)\sum_{i=0}^{m-1}\left(\frac{m}{\gamma_0}\gamma\right)^{i}\frac{1}{i!}\right]^{L-1} \quad (1)$$

with assumption that γ_0 is the average received SNR. In (1), we use the SC technique to select the signal with the largest SNR from L diversity branches for equal fading severity.

The parameter m is defined as the fading index. By setting $m=1$, we observe that (1) reduces to a rayleigh PDF. For values of m in the range $\frac{1}{2}\le m\le 1$, we obtain PDFs that have larger tails than a rayleigh-distributed random variable. For values of $m>1$, the tail of the PDF decays faster than that of the rayleigh. As $m\to\infty$, we have a channel that becomes nonfading. At the other extremes, for $m=\frac{1}{2}$ we have a one-sided Gaussian distribution[4].

Then we find the symbol error probability for MDPSK to be

$$P_{e,\,Nakagami,\,SC}\equiv\int_0^{\infty}P_{e,\,nonfading}\,f(\gamma)_{SC}\,d\gamma. \quad (2)$$

where $P_{e,\,nonfading}$ is the conditional PDF with respect to γ when MDPSK signals experience no fading and is given by[11],[14],[22]

$$P_{e,\,nonfading}=\frac{\sin\frac{\pi}{M}}{2\pi}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\cdot\frac{\exp\left[-\gamma\left(1-\cos\frac{\pi}{M}\cos\theta\right)\right]}{1-\cos\frac{\pi}{M}\cos\theta}\,d\theta. \quad (3)$$

Now, to find new PDF, we can express the last term of (1) by using the binomial theorem as follows:

$$\left[1-\exp\left(-\frac{m}{\gamma_0}\gamma\right)\sum_{i=0}^{m-1}\left(\frac{m}{\gamma_0}\gamma\right)^{i}\frac{1}{i!}\right]^{L-1}=\sum_{n_0=0}^{L-1}(-1)^{n_0}\binom{L-1}{n_0}\left[\sum_{i=0}^{m-1}\left(\frac{m}{\gamma_0}\gamma\right)^{i}\frac{1}{i!}\right]^{n_0}\exp\left(-\frac{m}{\gamma_0}n_0\gamma\right). \quad (4)$$

Also, the square bracketed term in (4) is

$$\left[\sum_{j=0}^{m-1}\left(\frac{m}{\gamma_0}\gamma\right)^{i}\frac{1}{i!}\right]^{n_0}=\sum_{n_1=0}^{n_0}\sum_{n_2=0}^{n_1}\cdots\sum_{n_{m-1}=0}^{n_{m-2}}\binom{n_0}{n_1}\binom{n_1}{n_2}\cdots\binom{n_{m-2}}{n_{m-1}}\left(\frac{1}{1!}\right)^{n_1-n_2}\left(\frac{1}{2!}\right)^{n_2-n_3}\cdots\left(\frac{1}{(m-2)!}\right)^{n_{m-2}-n_{m-1}}\cdot\left(\frac{1}{(m-1)!}\right)^{n_{m-1}}\left(\frac{m}{\gamma_0}\gamma\right)^{n_1+n_2+\cdots+n_{m-1}}. \quad (5)$$

Finally, from (4) and (5), the new expression for (1) can be written as

$$f(\gamma)_{SC}=\frac{L}{\Gamma(m)}\left(\frac{m}{\gamma_0}\right)^{m}\sum_{n_0=0}^{L-1}(-1)^{n_0}\binom{L-1}{n_0}\cdot\sum_{n_1=0}^{n_0}\sum_{n_2=0}^{n_1}\sum_{n_{m-1}=0}^{n_{m-2}}\left[\prod_{j=1}^{m-1}\binom{n_{j-1}}{n_j}\cdot\left(\frac{1}{j!}\right)^{n_j-n_{j+1}}\left(\frac{m}{\gamma_0}\right)^{n_j}\right]\exp\left[-\frac{m}{\gamma_0}(1+n_0)\gamma\right]\gamma^{n+m-1} \quad (6)$$

where

$$n = n_1 + n_2 + \cdots + n_{m-1} \tag{7}$$

and

$$n_m = 0. \tag{8}$$

Substituting (6) and (3) into (2), after some mathematical manipulations, we can show that the integral-form SER with SC on a Nakagami fading channel is

$$P_{e,Nakagami,SC} = \frac{L}{\Gamma(m)}\left(\frac{m}{\gamma_0}\right)^m \sum_{n_0=0}^{L-1}(-1)^{n_0}\binom{L-1}{n_0} \cdot \sum_{n_1=0}^{n_0}\sum_{n_2=0}^{n_1}\cdots\sum_{n_{m-1}=0}^{n_{m-2}}\left[\prod_{j=1}^{m-1}\binom{n_{j-1}}{n_j}\left(\frac{1}{j!}\right)^{n_j-n_{j+1}}\left(\frac{m}{\gamma_0}\right)^{n_j}\right] \cdot \frac{\sin\frac{\pi}{M}}{2\pi}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{\Gamma(n+m)}{1-\cos\frac{\pi}{M}\cos\theta} \cdot \left[\frac{\gamma_0}{m(1+n_0)+\gamma_0-\gamma_0\cos\frac{\pi}{M}\cos\theta}\right]^{n+m} d\theta \tag{9}$$

Note that SER for MDPSK without diversity(L=1) under the rayleigh fading(m=1) reduces to the familiar result[22]

$$P_{e,Rayleigh} = \frac{\sin\frac{\pi}{M}}{2\pi}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{1}{1-\cos\frac{\pi}{M}\cos\theta} \cdot \frac{1}{1+\gamma_0-\gamma_0\cos\frac{\pi}{M}\cos\theta}\,d\theta. \tag{10}$$

Also, we can find that the result of (9) for M=2 without SC diversity is equivalent to the bit error probability of DPSK signals by letting the signal alphabet size M=2 and replacing the bit energy E_b by $2E_b$ from Crepeau's result for that of noncoherent MFSK signals in a Nakagami fading channel[23].

III. Numerical Results

Fig. 1 shows the required SNR of a selection diversity system for MDPSK signals with SC diversity on a Nakagami fading channel for the number of diversity branches, respectively. These results are plotted with M=2, 4, 8, 16 and m=1, 5. The required SNR per bit over MR-combined noncoherent frequency-shift keying (NCFSK) in a slow flat rayleigh-fading environment is presented in [24]. To produce this figure, the symbol error probability of 10^{-2} is chosen. This figure illustrates that, by increasing the number of M, the SNR per symbol increases to achieve an equal SER for given values of m and L. It is noted that as the number of diversity branches decreases, the SNR per symbol required over SC is deviated and a substantial gain in the SNR per symbol required over SC for m=5 is more

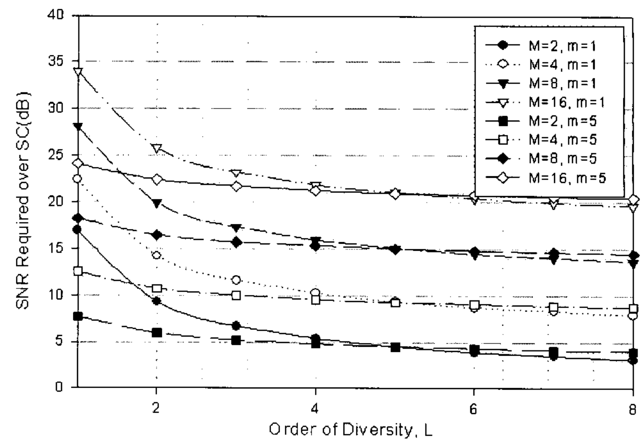

Fig. 1. Average SNR versus order of diversity for MDPSK with SC diversity receivers for M=2, 4, 8, and 16, m=1, 5. The symbol error probability is 10^{-2}.

achieved than that for m=1, for equal alphabet sizes M. But for larger the order of diversity, L, these curves become more horizontal approximately. So, the SNR fits more closely and its improvement for m=1 is slightly better than that for m=5.

Next in Figs. 2~4, the SER performances of MDPSK signals in a Nakagami fading channel is plotted against the order of diversity with M=2, 4, 8, 16, respectively. It is expected result as the diversity branches L increase, fading depth decreases. However, we can find that SER fits nearly with increasing alphabet size M under different fading severity indexes. The effective improvement in fading parameter m for a fixed alphabet size M does not improve the performance of MDPSK signals in proportion to increasing L. Also, the actual evaluation with increasing M under different SNRs does not have

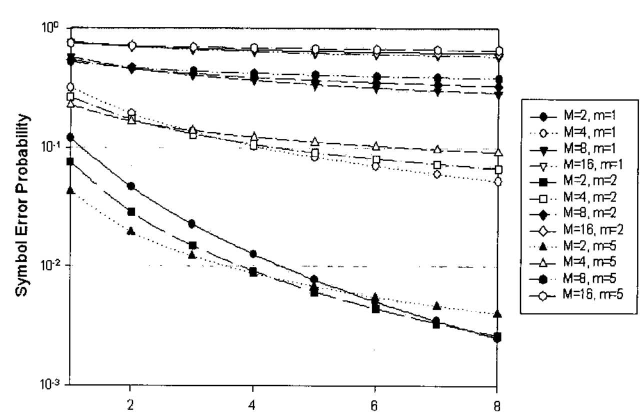

Fig. 2. Error performances of MDPSK signals adopting SC diversity technique. The parameters for this figure are: M=2, 4, 8, and 16, m=1, 2, and 5, SNR=5 dB.

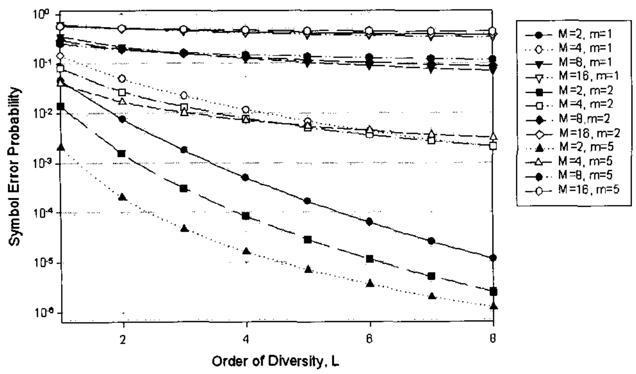

Fig. 3. Error performances of MDPSK signals adopting SC diversity technique. The parameters for this figure are: M=2, 4, 8, and 16, m=1, 2, and 5, SNR=10 dB.

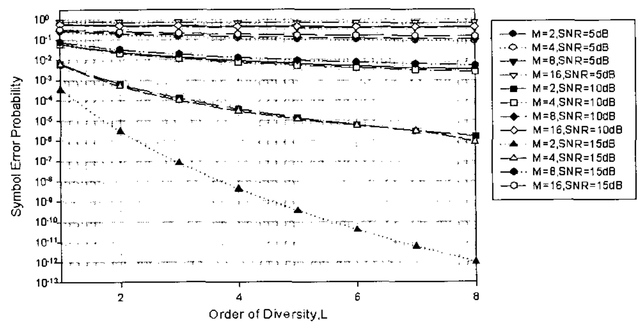

Fig. 4. Error performances of MDPSK signals with SC diversity receiver structures for M=2, 4, 8, and 16, m=3, SNR=5, 10, 15 dB.

an important effect on the performance of MDPSK signals in the SC diversity system in proportion to increasing L for a given value of m. But the results indicate that the discrepancy between the error performance for alphabet size M of MDPSK signals becomes more apparent as the order of diversity grows for a given value of SNR.

Figs. 5~7 show numerical results to evaluate the performance gain of MDPSK signals achievable by SC on Nakagami fading channels with respect to the triple diversity branches, which is considered the most practical case. It is clear from these results that a practical improvement in performance is achieved by increasing the diversity branch L. The most remarkable gain occurs in going from a single-branch receiver to a two-branch receiver.

Figs. 8~11 depict the comparisons of coherent MDPSK performance on Nakagami and Rician fading channels. Under the assumption that all SC diversity branches experience independent Nakagami fading with m=2, 3, and 5, these fading severity indexes correspond to a Rician fading channel with Rician factor K=2.414,

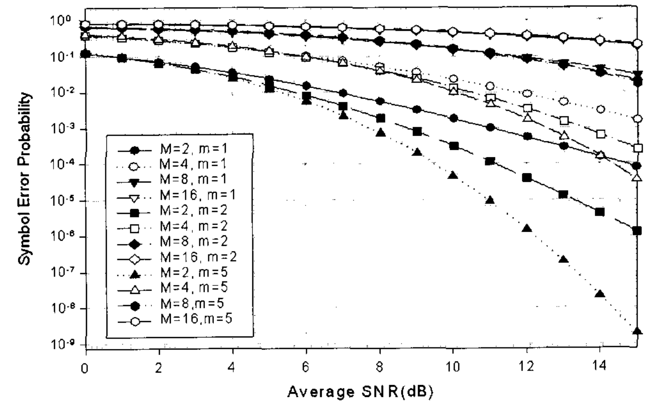

Fig. 5. Error performances of MDPSK signals adopting SC diversity technique. The parameters for this figure are: M=2, 4, 8, and 16, L=3, m=1, 2, and 5.

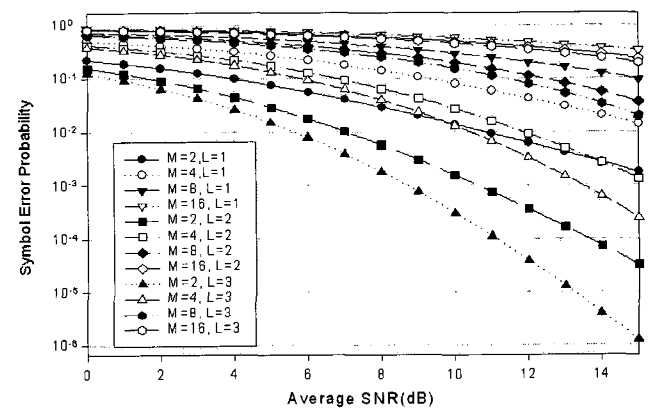

Fig. 6. Error performances of MDPSK signals adopting SC diversity technique. The parameters for this figure are: M=2, 4, 8, and 16, L=1, 2, and 3, m=2.

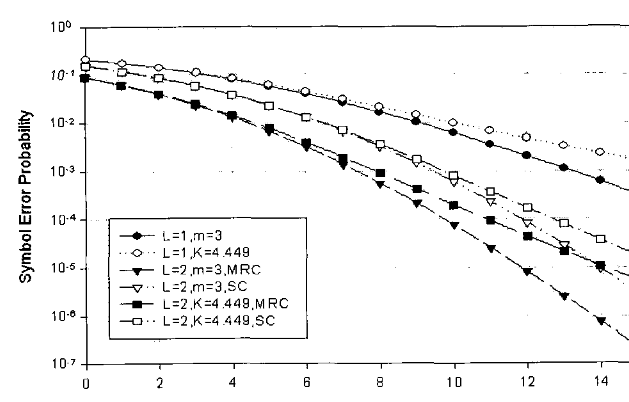

Fig. 7. Error performance comparisons of MDPSK signals with MRC and SC diversity receiver structures in Nakagami and Rician fading channels. These parameters for this figure are: M=2, L=1, 2, m=3 and K=4.449.

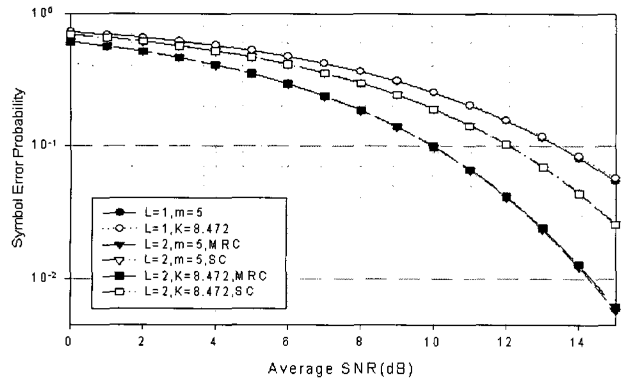

Fig. 8. Error performance comparisons of MDPSK signals with MRC and SC diversity receiver structures in Nakagami and Rician fading channels. These parameters for this figure are: M=8, L=1, 2, m=5 and K=8.472.

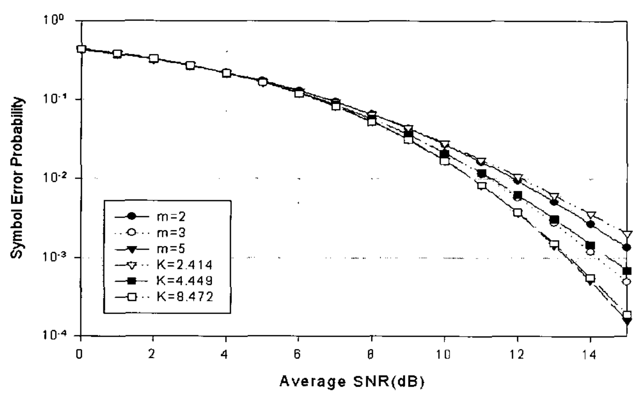

Fig. 9. Error performance comparisons of MDPSK signals with SC diversity receiver structures in Nakagami and Rician fading channels. These parameters for this figure are: M=4, L=2, m=2, 3, 5 and K=2.414, 4.449, 8.472.

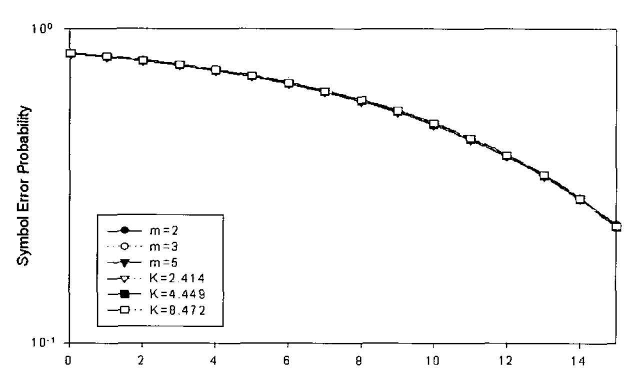

Fig. 10. Error performance comparisons of MDPSK signals with SC diversity receiver structures in Nakagami and Rician fading channels. These parameters for this figure are: M=16, L=2, m=2, 3, 5 and K=2.414, 4.449, 8.472.

4.449, and 8.472. The performance of SC (See Appendix B) with respect to MRC (See Appendix A) is improved very restrictedly in Nakagami and Rician fading conditions with increasing diversity branches for a single value of M, m and K. (See Fig. 8 and 9) It is noted that for a given number of the diversity branches L the SER fits more closedly with increasing the signal alphabet size M under different fading severity indexes (See Fig. 10 and 11)

Ⅳ. Conclusion

We have analyzed the exact performances for MDPSK systems employing multichannel SC diversity, which are based on the order statistics, in the presence of Nakagami-distributed slow and nonselective fading. It is desirable to obtain a simpler expression without integration for the integral-form performance by use of integration tables. Average SER formula of MDPSK signals achievable by SC systems in a Rician fading channel has also been derived in terms of integral expressions that can be computed via numerical manipulations. It is valuable to investigate the effects of multichannel diversity reception for MDPSK signals over Rician fading channels with accurate closed-form analyses. These results show that the obvious performances gain is achievable with increasing the diversity branches, L, however the performance improvement is more limited. The results suggested are sufficiently general to offer a convenient method to evaluate the performance of wireless personal communications.

Appendix A

Comparisons of Coherent MDPSK Performance with a MRC Diversity Reception in Rician and Nakagami Fading Channels

In this Appendix, an exact integral expressions are derived for calculating SER of MDPSK in conjunction with L-order MRC diversity on Rician and Nakagami fading channels, respectively.

The PDF at the output of an L-branch MRC on a Nakagami fading channel is [12]

$$f(\gamma)_{MRC} = \frac{\gamma^{Lm-1}}{\Gamma(Lm)}\left(\frac{m}{\gamma_0}\right)^{Lm} e^{-\frac{m}{\gamma_0}\gamma} \quad \text{(A1)}$$

Then we find SER for MDPSK to be

$$P_{e,\,Nakagami,\,MRC} \equiv \int_0^{\infty} P_{e,\,nonfading}\, f(\gamma)_{MRC}\, d\gamma. \quad \text{(A2)}$$

Substituting (3) and (A1) into (A2) for a Nakagami fading, we get

$$P_{e,Nakagami,MRC} = \frac{\sin\frac{\pi}{M}}{2\pi}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{1}{1-\cos\frac{\pi}{M}\cos\theta}\cdot \left(\frac{m}{m+\gamma_0-\gamma_0\cos\frac{\pi}{M}\cos\theta}\right)^{Lm}d\theta. \tag{A3}$$

The PDF at the output of an L-branch MRC on a Rician fading channel is [13]

$$f(\gamma)_{Rician.MRC} = \left(\frac{K+1}{\gamma_0}\right)^{\frac{L+1}{2}}\left(\frac{\gamma}{KL}\right)^{\frac{L-1}{2}}\cdot \exp\left[-KL-\frac{(K+1)\gamma}{\gamma_0}\right]I_{L-1}\left(2\sqrt{\frac{K(K+1)L\gamma}{\gamma_0}}\right) \tag{A4}$$

where $I_{L-1}(\cdot)$ is $(L-1)$th-order modified Bessel function of the first kind for identical diversity branches.

The Rician factor K is defined as the ratio of the mean direct power to the mean diffused power. When Rician factor K is 0, the error performances lead to those of rayleigh fading model.

We get SER with MRC on a Rician fading by averaging (B1) over underlying fading SNR. Then we find symbol error probability under the Rician fading model to be

$$P_{e,Rician,MRC} \equiv \int_0^\infty P_{e,nonfading}\, f(\gamma)_{Rician,MRC}\, d\gamma. \tag{A5}$$

Substituting (3) and (A4) into (A5) for a Rician fading, we get

$$P_{e,Rician,MRC} = \frac{\sin\frac{\pi}{M}}{2\pi}\left(\frac{K+1}{\gamma_0}\right)^L\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\cdot \frac{1}{\left(1+\frac{1+K}{\gamma_0}-\cos\frac{\pi}{M}\cos\theta\right)^L}\cdot\frac{1}{1-\cos\frac{\pi}{M}\cos\theta}\cdot \exp\left[-KL+\frac{\frac{K(K+1)L}{\gamma_0}}{1+\frac{1+K}{\gamma_0}-\cos\frac{\pi}{M}\cos\theta}\right]d\theta. \tag{A6}$$

Then we can find that the result of (A6) for L=1 corresponds to [22, Eq. (5)].

There is a close fit between the Nakagami m-distribution and the Rician distribution when the following parameter relationship holds[7]:

$$K=\frac{\sqrt{m^2-m}}{m-\sqrt{m^2-m}},\quad m\geq 1. \tag{A7}$$

The fit is exact in the extremes

$m=1$, $K=0$ (rayleigh fading)

$m\to\infty$, $K\to\infty$ (nonfading)

Appendix B

Comparisons of Coherent MDPSK Performance with a Two-branch Selective Diversity Reception in Rician and Nakagami Fading Channels

It is assumed that statistical characteristics of diversity branches for SC of MDPSK signals in a Rician fading channel are equivalent to those of MDPSK signals in a Nakagami fading channel.

The Rician PDF of the instantaneous SNR at the output of L-branch SC is [25]

$$f(\gamma)_{Rician.SC} = L\left(\frac{K+1}{\gamma_0}\right)\left[1-e^{-K}\sum_{n=0}^{\infty}\frac{K^n}{n!}\cdot\frac{\Gamma\left(n+1,\frac{(K+1)\gamma}{\gamma_0}\right)}{\Gamma(n+1)}\right]^{L-1}\cdot\exp\left[-K-\frac{(K+1)\gamma}{\gamma_0}\right]\cdot I_0\left(2\sqrt{\frac{K(K+1)\gamma}{\gamma_0}}\right) \tag{B1}$$

where

$$\Gamma(1+n,x)=n!\,e^{-x}\sum_{s=0}^{n}\frac{x^s}{s!},\quad n=0,1,\cdots \tag{B2}$$

and $I_0(\cdot)$ is zeroth-order modified Bessel function of the first kind.

For the practical cases of $L\leq 2$, we find that

$$f(\gamma)_{Rician.SC}=f_1(\gamma)_{Rician.SC}-f_2(\gamma)_{Rician.SC},\quad L=1,2. \tag{B3}$$

From (B3), $f_1(\gamma)_{Rician.SC}$ and $f_2(\gamma)_{Rician.SC}$ can be expressed as

$$f_1(\gamma)_{Rician.SC} = L\left(\frac{K+1}{\gamma_0}\right)\exp\left[-K-\frac{(K+1)\gamma}{\gamma_0}\right]I_0\left(2\sqrt{\frac{K(K+1)\gamma}{\gamma_0}}\right) \tag{B4}$$

and

$$f_2(\gamma)_{Rician.SC} = L\left(\frac{K+1}{\gamma_0}\right)(L-1)e^{-K}\sum_{n=0}^{\infty}\frac{K^n}{n!}\exp\left[-\frac{(K+1)\gamma}{\gamma_0}\right]\cdot \sum_{s=0}^{n}\frac{1}{s!}\left[\frac{(K+1)\gamma}{\gamma_0}\right]^s\exp\left[-K-\frac{(K+1)\gamma}{\gamma_0}\right]\cdot I_0\left(2\sqrt{\frac{K(K+1)\gamma}{\gamma_0}}\right). \tag{B5}$$

Then we find the symbol error probability for MDPSK in a Rician fading channel to be

$$P_{e,Rician,SC} = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\int_0^\infty P_{e,nonfading}\cdot [f_1(\gamma)_{Rician,SC}-f_2(\gamma)_{Rician,SC}]d\gamma d\theta \equiv P_{e1,Rician,SC}-P_{e2,Rician,SC}. \tag{B6}$$

Then we represent $P_{e1,Rician,SC}$ and $P_{e2,Rician,SC}$ as

$$P_{e1,Rician,SC}=\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\int_0^\infty P_{e,nonfading}\, f_1(\gamma)_{Rician,SC}\, d\gamma d\theta \tag{B7}$$

and

$$P_{e2,\,Rician,\,SC}=\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\int_0^{\infty}P_{e,\,nonfading}\,f_2(\gamma)_{Rician,\,SC}\,d\gamma\,d\theta. \tag{B8}$$

Substituting (B4) and (3) into (B7), we change the dummy variable to $x=\sqrt{\gamma}$ and use identity[26]

$$\int_0^{\infty}x^{\mu}e^{-\alpha x^2}J_{\nu}(\beta x)dx=\frac{\beta^{\nu}\Gamma\left(\frac{\nu}{2}+\frac{\mu}{2}+\frac{1}{2}\right)}{2^{\nu+1}\alpha^{\frac{1}{2}(\mu+\nu+1)}\Gamma(\nu+1)}\cdot {}_1F_1\left(\frac{\nu+\mu+1}{2};\nu+1;-\frac{\beta^2}{4\alpha}\right) \tag{B9}$$

where $J_{\nu}(\cdot)$ is νth-order Bessel function of the first kind.

So, $P_{e1,\,Rician,\,SC}$ may be evaluated as

$$P_{e1,\,Rician,\,SC}=L\frac{\sin\frac{\pi}{M}}{2\pi}\left(\frac{1+K}{\gamma_0}\right)\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\cdot\frac{1}{1-\cos\frac{\pi}{M}\cos\theta}\frac{\exp\left[-\frac{K\left(1-\cos\frac{\pi}{M}\cos\theta\right)}{1+\frac{1+K}{\gamma_0}-\cos\frac{\pi}{M}\cos\theta}\right]}{1+\frac{1+K}{\gamma_0}-\cos\frac{\pi}{M}\cos\theta}d\theta\cdot \tag{B10}$$

Also, substituting (B5) and (3) into (B8), after some mathematical manipulations, we can express (B8) as

$$P_{e2,\,Rician,\,SC}=L(L-1)\frac{\sin\frac{\pi}{M}}{2\pi}e^{-2K}\sum_{n=0}^{\infty}\sum_{s=0}^{n}\frac{K^n}{n!}\cdot\left(\frac{K+1}{\gamma_0}\right)^{s+1}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{1}{1-\cos\frac{\pi}{M}\cos\theta}\cdot\frac{1}{\left[1+\frac{2(1+K)}{\gamma_0}-\cos\frac{\pi}{M}\cos\theta\right]^{s+1}}\cdot {}_1F_1\left(s+1;\ 1;\ \frac{\frac{K(K+1)}{\gamma_0}}{1+\frac{2(1+K)}{\gamma_0}-\cos\frac{\pi}{M}\cos\theta}\right)d\theta \tag{B11}$$

where

$${}_1F_1(\alpha\,;\beta\,;x)=\sum_{n=0}^{\infty}\frac{\Gamma(\beta)\Gamma(\alpha+n)x^n}{\Gamma(\alpha)\Gamma(\beta+n)n!},\quad \beta\neq 0,-1,-2,\cdots \tag{B12}$$

is the confluent hypergeometric function[27].

Invoking the identity[28]

$${}_1F_1\left(s+1;\ 1;\ \frac{\frac{K(K+1)}{\gamma_0}}{1+\frac{2(1+K)}{\gamma_0}-\cos\frac{\pi}{M}\cos\theta}\right)=\sum_{k=0}^{\infty}\frac{\Gamma(1)\Gamma(s+1+k)}{\Gamma(s+1)\Gamma(1+k)}\frac{1}{k!}\cdot\left[\frac{\frac{K(K+1)}{\gamma_0}}{1+\frac{2(1+K)}{\gamma_0}-\cos\frac{\pi}{M}\cos\theta}\right]^k$$
$$=\exp\left[\frac{\frac{K(K+1)}{\gamma_0}}{1+\frac{2(1+K)}{\gamma_0}-\cos\frac{\pi}{M}\cos\theta}\right]\cdot\sum_{j=0}^{s}\frac{s!}{(s-j)!j!j!}\left[\frac{\frac{K(K+1)}{\gamma_0}}{1+\frac{2(1+K)}{\gamma_0}-\cos\frac{\pi}{M}\cos\theta}\right]^j, \tag{B13}$$

we can reduce the number of infinite-series on the right-hand side of (B11) as follows:

$$P_{e2,\,Rician,\,SC}=L(L-1)\frac{\sin\frac{\pi}{M}}{2\pi}\sum_{n=0}^{\infty}\sum_{s=0}^{n}\sum_{j=0}^{s}\cdot\frac{K^{n+j}}{n!}\frac{s!}{(s-j)!(j!)^2}\cdot\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{1}{1-\cos\frac{\pi}{M}\cos\theta}\cdot\exp\left[-2K+\frac{\frac{K(K+1)}{\gamma_0}}{1+\frac{2(1+K)}{\gamma_0}-\cos\frac{\pi}{M}\cos\theta}\right]\cdot\left[\frac{\frac{K+1}{\gamma_0}}{1+\frac{2(1+K)}{\gamma_0}-\cos\frac{\pi}{M}\cos\theta}\right]^{j+s+1}d\theta. \tag{B14}$$

Therefore, the performance of MDPSK signals with two-branch SC diversity on a Rician fading channel is

$$P_{e,\,Rician,\,SC}=L\frac{\sin\frac{\pi}{M}}{2\pi}\left(\frac{1+K}{\gamma_0}\right)\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\cdot\frac{1}{1-\cos\frac{\pi}{M}\cos\theta}\cdot\frac{\exp\left[\frac{-K\left(1-\cos\frac{\pi}{M}\cos\theta\right)}{1+\frac{1+K}{\gamma_0}-\cos\frac{\pi}{M}\cos\theta}\right]}{1+\frac{1+K}{\gamma_0}-\cos\frac{\pi}{M}\cos\theta}d\theta$$
$$-L(L-1)\frac{\sin\frac{\pi}{M}}{2\pi}\sum_{n=0}^{\infty}\sum_{s=0}^{n}\sum_{j=0}^{s}\frac{K^{n+j}}{n!}\cdot\frac{s!}{(s-j)!(j!)^2}\cdot\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{1}{1-\cos\frac{\pi}{M}\cos\theta}\exp\left[-2K+\frac{\frac{K(K+1)}{\gamma_0}}{1+\frac{2(1+K)}{\gamma_0}-\cos\frac{\pi}{M}\cos\theta}\right]\cdot\left[\frac{\frac{K+1}{\gamma_0}}{1+\frac{2(1+K)}{\gamma_0}-\cos\frac{\pi}{M}\cos\theta}\right]^{j+s+1}d\theta,\quad L=1,2. \tag{B15}$$

For the particular case when L is 1, we find this result is perfectly equivalent to [22, Eq. (5)]. Also, when K goes to zero, which corresponds to a rayleigh fading channel, we can observe that a substitution of K=0 in (B15) when there is no diversity channel is equal to the result of [29].

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