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IV ECM Threshold Cointegration Tests and Nonlinear Monetary Policy in Korea

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분계점 공적분 검정법을 사용한 한국의 비선형 테일러 통화정책 검증

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ABSTRACT

The goal of this paper is to examine the validity of nonlinear Taylor rules in Korea. To perform our tests, we utilize new IV ECM threshold cointegration tests that are invariant to nuisance parameters. The new tests have a standard chi-square distribution and the same critical values can be used throughout. This is in contrast to OLS ECM threshold cointegration tests, which depend on nuisance parameters and have nonstandard distributions. After finding significant support for nonlinear cointegration, we find that the Bank of Korea raises the call rate of interest only when inflation is above a threshold rate. We additionally find that the Bank of Korea increases the call rate of interest to possibly counter domestic currency deprecation only when the rate of currency deprecation exceeds a threshold.

본고에서는 소위 ‘테일러 룰’이라고 일컫는 통화정책론이 한국의 통화정책에도 적용될 수 있는지를 검증하고자 하였다. ‘테일러 룰’이 적용된다면 이자율, 물가상승률 및 잠재성장률 간에 공적분이 성립해야 하는데, 본고에서는 선형관계를 전제로 하는 공적분은 성립하지 않는다는 결과를 산출하였고, 더 나아가 새로운 분계점 공적분 검정법(IV ECM Threshold Cointegration Tests)을 개발하고 이를 적용하고자 하였다. 이 방법론은 기존의 공적분 검정법과 달리 성가신 파라미터(nuisance parameters)에 의존하지 않는다는 장점이 있다. 이 장점을 사용하여 적용한 결과, 본고에서는 한국에 있어서도 비선형 테일러 통화정책에 대한 분계점 공적분이 성립하고 ‘비선형 테일러 룰’이 검증되었음을 보여주었다.

Ⅰ. Introduction

Predicting the reaction of monetary authorities to changes in fundamental economic variables has long been a goal of central bank observers and monetary economists alike. In particular, observers often wish to know how the central bank responds to changing economic fundamentals when setting short-term interest rates. This research is often expressed by a simple monetary policy reaction function, or "Taylor rule." The simple policy rule was initially introduced by Taylor (1993) and can be described as follows:

$$i_t = r^* + \pi_t + \alpha_1^*(\pi_t - \pi^*) + \alpha_2 y_t + \alpha_3 i_{t-1} + \alpha_4 i_{t-2} + \varepsilon_t \quad (1)$$

where i_t is the nominal target interest rate of the central bank, r^* is the equilibrium real interest rate, and π_t is the inflation rate over the most recent four quarters. Here, π^* is the target inflation rate of the central bank, y_t is the "output gap" measured as the percentage deviation in real GDP from target or potential real GDP, and ε_t is an i.i.d. error term. Lagged values of i_t are included to allow for "interest rate smoothing," where the central bank gradually adjusts i_t to the target rate (e.g., English, Nelson, and Sack, 2003). Rearranging terms and simplifying gives the following testing equation:

$$i_t = \alpha_0 + \alpha_1\pi_t + \alpha_2 y_t + \alpha_3 i_{t-1} + \alpha_4 i_{t-2} + \varepsilon_t, \quad (2)$$

where $\alpha_0 = (r^* - \alpha_1^*\pi^*)$ and $\alpha_1 = (1 + \alpha_1^*)$. See Qin and Enders (2007) for a survey of papers that examine a linear Taylor rule.

In spite of the number of papers that test models of the Taylor rule, the validity of many of these tests has been questioned. Bunzel and Enders (2007) and Österholm (2005), for example, find that the U.S. interest rate and inflation rate each have unit roots, while the output gap is stationary. Moreover, they additionally find no evidence of a long-run linear cointegrating relationship among the variables in the model. We obtain similar results with Korean data. These findings call into question many estimates of the linear Taylor rule, since if the variables in the model are nonstationary and not cointegrated then spurious estimates can result.

Recently, a growing body of literature finds evidence of nonlinear dynamics in many economic time series. For example, given transactions costs, central banks may take action to increase the target interest rate only when the inflation rate surpasses a threshold. Similar asymmetric responses can arise if monetary policy makers care more about high inflation than low inflation. For these and other reasons, a growing number of authors argue that the Taylor rule should be modeled as a nonlinear relationship. See, for example, the papers by Clarida, Gali, and Gertler (2000), Nobay and Peel (2003), Ruge-Murcia (2003), Dolado, Maria-Dolores, and Ruge-Murcia (2004), where the authors argue that central bank preferences are likely to be asymmetric in the inflation rate and/or output gap. The presence of nonlinearities in the Taylor rule could explain the apparent lack of mean reversion in i_t and π_t, and the findings that these variables are not linearly cointegrated, since the standard unit-root and cointegration tests assume that the data-generating process is linear. As in the linear case, in order to estimate nonlinear models and avoid spurious results we must first examine the validity of the model by testing for nonlinear cointegration.

Until recently, the lack of testing for nonlinear cointegration in nonlinear models might be due to difficulties of finding practical testing methodologies. In this paper, we estimate and test the validity of nonlinear threshold models by employing new error-correction model (ECM) threshold cointegration tests. Specifically, we test for nonlinear monetary policy in Korea by estimating nonlinear Taylor rules. While previous papers have tested for nonlinear Taylor rules in the U.S., and other early industrialized countries, we contribute to the literature by testing for nonlinear Taylor rules in Korea, a newly industrializing economy. Several notable differences exist between the economies of the U.S. and Korea that might lead to different findings in a nonlinear Taylor rule. Perhaps most important, while the U.S. is a large open economy, the economy of Korea is a small open economy with proportionately greater exposure to international trade. For instance, in 2004, the shares of exports and imports of goods and services in GDP were 10% and 15%, respectively, in the U.S., while their shares were 44% and 40%, respectively, in Korea (World Bank, 2007, World Development Indicators). Second, while the goals of low inflation and full employment are likely important for monetary policy in both the U.S. and Korea, attention to the foreign exchange rate may be a relatively more important policy goal in Korea. In a small open economy, the central bank may give a lower priority to controlling inflation, relative to exchange rates, than in a large open economy like the U.S. If so, one can hypothesize that if the Korean inflation rate increases at the same time that the won appreciates relative to the U.S. dollar, the Bank of Korea might choose not to increase the call interest rate to slow inflation due to concerns that this could cause a further appreciation of the won and potentially harm or destabilize foreign trade.

Using the new IV ECM threshold cointegration tests, we estimate a nonlinear Taylor rule for Korea using quarterly data from 1991-2007. Overall, we find significant evidence of a nonlinear threshold cointegrating relationship in the Korean Taylor rule. Most important, we find that the Bank of Korea increases the call rate of interest only when inflation rises above a threshold rate. In addition, we find that the Bank of Korea increases the call rate of interest to possibly counter a depreciation of the won or to increase its stability, but only when the deprecation exceeds a threshold rate. The remainder of the paper proceeds as follows. In Section 2, we provide further background discussion of the Taylor rule in Korea and discuss some methodological issues. In Section 3, we develop our new IV ECM threshold cointegration test and examine its properties. In Section 4, we present our empirical findings. In Section 5, we summarize and provide concluding remarks.

Ⅱ. Background and Testing Issues

We will examine the validity of nonlinear Taylor rules in Korea by estimating new ECM threshold cointegration models. A linear Taylor rule for Korean monetary policy has been previously examined in the paper by Hsing and Lee (2004). The authors utilize linear cointegration tests, vector autoregressions (VAR), and impulse response functions with quarterly data from 1978-2003. In addition to the usual right-hand variables of the inflation gap, output gap, and lagged interest rate (call rate), they include variables of deviation from trend in the Korean-U.S. exchange rate and the standard deviation in Korean stock prices from trend. Trend values are estimated by using Hodrick-Prescott (1997) filters. Prior to estimating the Taylor rule in Korea, the authors perform augmented Dickey-Fuller (ADF) unit root tests to determine whether the variables in the model are stationary or nonstationary. They find that the exchange rate gap and stock price gap are stationary variables, while the other variables are nonstationary. Hsing and Lee (2004) next perform cointegration tests using their nonstationary variables and reject the null of no cointegration at the 1% level of significance. The results from their impulse response functions provide evidence that a positive increase in the inflation rate leads to a short-run increase in the call rate by the central bank. They additionally find evidence of a short-run positive effect on the call rate following an increase in the exchange rate gap (i.e., when the won depreciates relative to the dollar). In the longer-run, the authors find that the call rate of interest increases following an increase in the output gap (i.e., when output rises above full employment), and when stock prices are above trend.

While our paper complements the findings of Hsing and Lee (2004), there are several important differences. Most important, we utilize a nonlinear framework in order to allow for regime-specific threshold effects that depend on the rate of inflation and other variables. In addition, potential problems noted in the literature about estimating Taylor rules remain in their study. Most notably, Hsing and Lee (2004) omit any stationary variables from their cointegration test out of necessity of employing OLS based estimation procedures. In contrast, we include any stationary variables from the model in our IV threshold cointegration tests. It is understandable why, out of necessity, the authors omitted important stationary variables from their OLS based cointegration tests. However, it seems difficult to omit important variables from the model for the sake of estimation convenience. An interesting contribution in the work of Hsing and Lee (2004) is to include the Korea/U.S. exchange rate in the Taylor rule. We follow their suggestion and build on their paper. While can hypothesize that including the exchange rate in the Taylor rule is less important when examining a large open economy like the U.S., in a small open economy with a heavy reliance on international trade it may be important to examine reactions of the central bank to exchange rate changes. To do so, we will consider three types of threshold variables in our nonlinear Taylor rule: (1) the rate of inflation; (2) the rate of depreciation in the exchange rate; and (3) the rate of economic growth. We will then utilize a nonlinear framework to allow for regime-specific threshold effects that are functions of these variables.

As noted, the literature has been silent in regards to testing the validity of nonlinear Taylor rules by using nonlinear cointegration tests. The question is why? The answer, we expect, lies in the difficulties of finding practical testing methodologies. To begin, we note that standard unit-root and cointegration tests often indicate the absence of a valid relationship in the linear Taylor rule. This might happen since assuming a linear data-generating process will lead to lower power in the presence of nonlinear dynamics. As such, it is natural to examine nonlinear Taylor rules. However, despite the growing evidence of nonlinear dynamics in many empirical papers, to the best of our knowledge, the previous papers that estimate nonlinear Taylor rules have not explicitly examined the validity of their estimates by testing for nonlinear cointegration.[1]

Regarding the methodology of testing for threshold cointegration, Enders and Siklos (2001) initially suggest valid threshold cointegration tests that permit asymmetric adjustment in the error correction term. Their paper might be the first work in the literature to provide relevant critical values to test for threshold cointegration. The authors adopt the traditional approach of Engle and Granger (1987, EG). However, their tests cannot be applied to the model that includes a stationary variable so we cannot use their test to examine the nonlinear Taylor rule. More recently, another line of threshold cointegration test has been considered by Seo (2006) and Li (2006) using the ECM. However, these methods utilize OLS type estimation procedures and depend on nuisance parameters.

Our main point of departure from the OLS based tests is to utilize stationary instrumental variables (IVs) in the ECM threshold cointegration test. The OLS based ECM threshold cointegration tests have a non-standard distribution that depends on a mixing of the Dickey-Fuller type non-standard distribution and the standard normal distribution. As a result, the usual non-standard critical values cannot be obtained a priori without knowing the weights of the two distributions. We build on the previous important works of Hansen and Seo (2002), Seo (2006), and Enders et al. (2007), among others, and provide new solutions. In particular, we include stationary IVs in the ECM threshold cointegration test. By adopting stationary IVs in our ECM threshold cointegration test, we demonstrate that the test statistics will be free of nuisance parameters and have chi-squared or standard normal asymptotic distributions in every case. As a result, the same critical values can be used throughout. Our paper complements the work of Enders et al. (2007), who suggest utilizing stationary IVs in autoregressive distributed lag (ADL) threshold cointegration tests. The authors demonstrate that by adopting stationary IVs in ADL threshold cointegration tests, the test statistics will be free of nuisance parameters. Enders et al. (2007) utilize their test to estimate nonlinear Taylor rules for the U.S. We suggest an alternative IV threshold cointegration test based on the ECM. Our IV ECM threshold cointegration test is an important contribution, since OLS based ECM tests are increasingly popular in the literature but have a disadvantage that they often require bootstrapping.

[1] The one exception is the paper by Enders, Lee, and Strazicich (2007), where the authors examine nonlinear U.S. Taylor rules. We wish to contribute to the literature in this regard, by examining the validity of nonlinear Taylor rules in Korea. As we will explain in more detail, in terms of methodologies our paper complements the work of Enders et al. (2007). The underlying concept of the approach used in this paper is similar to theirs. However, our paper focuses on a different test statistic than is the focus in Enders et al. (2007). As such, our suggested tests provide solutions to the limitations found in previous tests that utilize the ECM type approach.

Ⅲ. New Testing Methodology

The question of interest is whether a stable linear or nonlinear Taylor rule does in fact exist. Specifically, given that at least some variables in the Taylor rule are non-stationary, it is important to ascertain whether there is a valid cointegration relationship among them. Initial examination of our tests reveals little or no support for linear cointegration in equation (2). Testing for cointegration in the Taylor rule, however, incurs a difficulty since there is a trivial cointegration relationship among the target interest rate and its lagged values. Given our discussion above, it is quite possible that the Taylor rule should be modeled as a nonlinear relationship. In particular, if a right-hand variable in the model exceeds a certain threshold, the central bank will react in a different manner than when the variable is below the threshold. For instance, when the inflation rate is relatively low (less than 4%, for example) the central bank may not intervene at all. In contrast, when inflation is relatively high (above 4%, for example), a standard Taylor rule response will apply. This type of nonlinear model can make significant progress towards explaining misspecifications in the standard Taylor rule, such as a finding of unreasonably high interest rate smoothing that resembles a random walk and/or lack of cointegration. However, in order to accurately test the validity of nonlinear Taylor rules requires testing procedures not subject to nuisance parameters.

Balke and Fomby (1997) initially introduced the so-called threshold cointegration test, which permits a threshold effect in the long-run adjustment process of the ECM. The authors assume that cointegration exists only within a certain range of deviations from the long-run equilibrium implied by the null, but they did not consider an explicit test for threshold cointegration. Hansen and Seo (2002) suggest a procedure to estimate and test for the existence of threshold effects in a vector ECM, but also did not provide an explicit test for threshold cointegration.[2] We consider models with nonlinearity in the short-run dynamics, as demonstrated in the literature on testing for threshold cointegration. Hansen and Seo (2002) and Seo (2006) consider different sets of parameters in two regimes as follows

$$\Delta x_t = c + \gamma_1 I_t z_{t-1} + \gamma_2 (1-I_t) z_{t-1} + (\textit{stationary dynamics}) + u_t, \tag{3}$$

and define Heaviside indicator functions as

$$I_t = 1 \text{ if } z_{t-1} > \delta \text{ and } I_t = 1 \text{ if } z_{t-1} \leq \delta \tag{4a}$$

or

$$I_t = 1 \text{ if } \Delta z_{t-1} > \delta \text{ and } I_t = 1 \text{ if } \Delta z_{t-1} \leq \delta. \tag{4b}$$

[2] Gonzalo and Pitarakis (2004) propose a procedure to test the presence of threshold effects in nonstationary ECM models with or without cointegration. However, they also do not provide explicit tests for threshold cointegration.

While these "level" and "momentum" threshold variables are common in the literature, additional threshold indicators can be adopted in the IV ECM threshold cointegration tests due to the invariance properties of the test. For instance, using our test, we can adopt additional threshold indicators for inflation rates such as

$$I_t = 1 \text{ if } \pi_{t-1} > \tau \text{ and } I_t = 0 \text{ if } \pi_{t-1} \leq \tau. \tag{4c}$$

The threshold indicator in (4c) allows us to model different regimes that depend directly on whether inflation is relatively high or low. We focus on estimating equation (3) and utilize versions of (4a) - (4c) to allow for different regimes in our short-run dynamics. Note that the number of parameters in the testing equation is smaller for the ECM based threshold cointegration tests than the ADL based tests, since the ECM based tests impose the common factor restriction (CFR). When the CFR holds, the ECM based tests are more powerful than the ADL based tests, while the reverse will be true when the CFR does not hold.

Our threshold cointegration tests differ from Enders et al. (2007) in that we consider IV ECM type tests instead of IV ADL type tests. Thus, while we follow the corresponding framework suggested in Enders et al. (2007) for the IV ADL test, we instead consider the IV ECM threshold cointegration test equation (3) with the IVs,

$$w_t = z_t - z_{t-m}, \tag{5}$$

where m is a finite number and m << T. From (5), it is clear that w_t is stationary since w_t consists of the stationary variables ($z_t - z_{t-m}$), regardless of whether the system in (3) is cointegrated or not. Specifically, it is simple to demonstrate that $w_t = (z_{t-1} - z_{t-2}) + (z_{t-2} - z_{t-3}) + \ldots + (z_{t-m+1} - z_{t-m}) = z_{t-1} + z_{t-2} + \ldots + z_{t-m+1}$, where each differenced term is stationary even if each individual z is I(1). The test for threshold cointegration in (3) is performed by testing the following hypotheses:

H_o: $\gamma_1 = 0$ and $\gamma_2 = 0$ vs. H_1: at least one of these is not zero.

Rejection of the null hypothesis (H_o) indicates that cointegration is supported in at least one regime. For simplicity, we can rewrite equation (3) as

$$\Delta x_{1t} = \gamma_1 I_t z_{t-1} + \gamma_2 (1-I_t) z_{t-1} + \phi_1' I_t q_t + \phi_2'(1-I_t) q_t + u_t, \tag{6}$$

where q_t includes the deterministic terms of the constant, lags of x_{1t} and x_{2t}, and any stationary covariates. We consider the usual t-statistic on $\gamma_i = 0$, i = 1, 2, in (6). Alternatively, we can consider the Wald test statistic for the joint hypothesis:

$$Wald = (R\hat{\theta})' [\hat{\sigma}_1^2 R(\tilde{z}'\tilde{w})^{-1}(\tilde{w}'\tilde{w})(\tilde{w}'\tilde{z})^{-1}R']^{-1} (R\hat{\theta}), \tag{7}$$

where $\theta_i = (\gamma_1, \gamma_2, \phi_1', \phi_2')'$, $\hat{\sigma}_1^2$ is the estimated error variance from (3), R is a selection matrix that selects the parameters under the null hypothesis, and r is the number of restrictions. In the expression of (6), we use simplified notations $\tilde{z}$ and $\tilde{w}$ to denote that the effect of q_t is controlled by using the residuals from the regression of z_t on q_t or the regression of w_t on q_t. Letting $z = (z_{m+1}, .., z_T)'$ and $q = (q_{m+1}, .., q_T)'$, we obtain the residuals as $\tilde{z} = M_q z$, where M_q is the projection onto the orthogonal space of q_t with $M_q = I_{T-m} - q(q'q)^{-1}q'$. Similarly, we obtain the residuals $\tilde{w} = M_q w$, where $w = (w_{m+1}, .., w_T)'$. Then, we use $\tilde{w}$ with ($\tilde{w}_t I_{1t}$, $\tilde{w}_t (1-I_{1t})$) as the instruments for $\tilde{z}$. It can be shown that the asymptotic distribution of the resulting t-statistic is standard normal and the distribution of the Wald statistic is chi-squared.

Theorem 1. *Suppose that Assumption* 1 *in Enders, Lee, and Strazicich* (2007) *holds. Also, suppose that* $\gamma_1 = \gamma_2 = 0$ *in the data generating process* (3), *and the threshold parameter* τ *is consistently estimated or is known a priori. Then, as* $T \rightarrow \infty$, *the Wald statistic in* (6) *follows*

$$Wald \rightarrow \chi_r^2 ,$$

and each t-statistic t_i *on* $\gamma_1 = 0$ *or* $\gamma_2 = 0$ *has the standard normal distribution*

$$t_i \rightarrow Z.$$

Proof: See the Appendix.

In order to conduct the threshold cointegration test, we need to estimate the threshold parameter τ. For this, we adopt a grid search method. First, we sort the threshold variable from the lowest to the highest value and determine the threshold estimate within the range of 10 to 90 percentiles of the threshold variable at the value where the sum of squared residuals from regression (6) is minimized. The idea is that the threshold value cannot be smaller (greater) than the lowest (highest) value of the threshold variable, and we eliminate both end points, which is standard procedure in the literature. Minimizing the sum of squared residuals yields the same consistent estimates of the threshold parameters as maximizing the F-statistic on the coefficients that separate two regimes.

The threshold parameter is consistently estimated when the coefficients in two regimes are different. The consistency of the threshold parameter is warranted under three different cases. First, different regimes will occur when there is a structural change in the level term ($c_1 \neq c_2$), or in any deterministic terms including trend functions. Second, a regime change is evident when the short-run dynamics of Δx_{1t} and Δx_{2t} are different in each regime. Third, the threshold parameter is consistently estimated when the persistent parameters are different, such that $\gamma_1 \neq \gamma_2$. It is important to note that our consistency results do not hinge solely on the third case of $\gamma_1 \neq \gamma_2$. Instead, we allow for the first two cases in addition to the third case. Thus, our consistency result of the threshold parameter can be stronger than a supreme type test statistic that relies solely on the third case. Although consistency of the threshold parameter estimate is also maintained in a supreme type test as given in the literature, it often requires the assumption that $c_1 = c_2$ (the coefficients of the level and trend terms) and $\phi_1 = \phi_2$ (the coefficients of short-run dynamics) and examines whether $\gamma_1 = \gamma_2$ or $\gamma_1 \neq \gamma_2$. When the required assumption ($c_1 = c_2$ and $\phi_1 = \phi_2$) does not hold, the supreme type test involves nuisance parameters and diverges whenever $c_1 \neq c_2$ or $\phi_1 \neq \phi_2$. In our case, we do not need to employ such a supreme type test, and each of the separate restrictions of $\phi_1 = \phi_2$ or $\gamma_1 = \gamma_2$ is decisively rejected in our analysis of the Korean Taylor rule.

Clearly, our IV based ECM testing strategy differs from the existing OLS based

ECM threshold tests. For instance, the OLS based tests do not permit us to examine the testing hypothesis described in (3) with (4c), since no asymptotic result is readily available when the indicator function is defined differently from the equilibrium error term. In contrast, our threshold classification rules are well tailored to testing the Taylor rule. This outcome is due to the fact that in the OLS based ECM threshold cointegration tests the distribution of the test statistic depends on the particular indicator function that is adopted. As a result, new critical values must be simulated, if the relevant asymptotic distributions can be possibly developed. Rather than model the threshold variables solely by the magnitude or change of their deviations from the long-run equilibrium, we can adopt other threshold variables that may be better suited to the Taylor rule. As a result, in examining the long-run relationship, we hypothesize that the central bank follows the standard Taylor rule when inflation is higher than the threshold rate.

We will consider four different threshold variables as follows:

$$I_t = 1 \text{ if } \pi_{t-1} > \tau \text{ and } I_{1t} = 0 \text{ otherwise,} \tag{8a}$$

$$I_t = 1 \text{ if } (\ln e_{t-1} - \ln e_{t-2}) > \tau \quad \text{and } I_t = 0 \text{ otherwise,} \tag{8b}$$

$$I_t = 1 \text{ if } |\ln e_{t-1} - \ln e_{t-2}| > \tau \quad \text{and } I_t = 0 \text{ otherwise,} \tag{8c}$$

$$I_t = 1 \text{ if } \ln realGDP_{t-1} - \ln realGDP_{t-2} > \tau \quad \text{and } I_t = 0 \text{ otherwise.} \tag{8d}$$

The threshold function described in (8a) is the focus of our paper, and allows for different reactions by the monetary authorities depending on whether inflation is above or below a threshold rate. We hypothesize that the Bank of Korea will take stronger action to increase the call rate of interest when inflation is above the threshold rate than when inflation is below the threshold rate. Threshold function (8b) allows for a different interest rate policy response when the rate of depreciation of the won relative to the U.S. dollar exceeds a threshold rate. Thus, we hypothesize that the Bank of Korea will take action to counteract and/or stabilize the value of the won only when the rate of depreciation exceeds a threshold rate. The threshold function in (8c) is similar to (8b), but removes the sign on the rate of change in e. This threshold variable allows for the possibility that the Bank of Korea is more concerned with preventing general fluctuations in the exchange rate rather than taking a particular action to prevent the won from depreciating per se. We hypothesize that the Bank of Korea will take action to counteract a change in the exchange by changing the call rate, regardless of whether the change is a depreciation or appreciation, only when the rate of change exceeds a threshold rate. The threshold function in (8d) is described by the rate of growth of real GDP. This allows for the possibility that the Bank of Korea will respond differently to changing i depending on whether the rate of growth in output is above or below the threshold rate. Thus, we hypothesize that the Bank of Korea will increase the call rate of interest only when the rate of growth in real GDP is above a threshold rate. It is important to note that the four threshold functions defined in (8a) – (8d) could not be considered in the OLS based ECM threshold cointegration tests without adopting a bootstrap procedure. As previously noted, if some of the variables in the Taylor rule are I(0) while others are I(1), then a bootstrap procedure may be problematic.

A brief explanation is necessary to explain how the lag order *m* is determined. The procedure to estimate the threshold parameter was discussed in the above. For the time period considered, we obtain the estimated threshold value of τ by minimizing the sum of squared residuals in the OLS estimation of the testing regression. Then, given this threshold value, we perform IV estimation using values of $m = 2, \ldots, 10$. We select the value of *m* that results in the smallest residual variance. Using this value of *m*, we re-estimate the threshold value τ. In this manner *m* and τ are jointly determined.

Ⅳ. Empirical Results

Our quarterly data on the nominal target call interest rate, inflation rate, output gap, and nominal exchange rate (the won price of one U.S. dollar) for 1991-2007 was obtained from the web site of the Bank of Korea. We begin our investigation by testing the linear Taylor rule described in (2), including the exchange rate (e_t) and lagged values of the call rate (i_{t-1} and i_{t-2}). The results of estimation are displayed in Table 1. While the sign on the coefficient of the inflation rate ($_t$) is positive in both Model 1 and 2, the coefficient is only marginally significant at the 10% level. The sign on the output gap variable is positive and significant at the usual levels. There is no evidence that the Bank of Korea responds to a depreciation of the won by changing the target rate of interest. We give more credit to Model 2, given that the second lag interest rate variable is statistically significant. Most noteworthy, in both Model 1 and 2, is the finding that the coefficient on i_{t-1} is highly significant and approximately equal to one. This is especially true in the more significant Model 2. This finding casts doubt on the validity of the linear Taylor rule, and suggests that i_t

<Table 1> Estimates of the Linear Taylor Rule in Korea

Dependent variable: i_t	Model 1	Model 2
Constant	-0.399 (-0.260)	0.573 (0.371)
π_t	0.392 (1.687)	0.373 (1.662)
y_t	0.233 (3.359)	0.185 (2.636)
e_t	0.130 (0.102)	-0.511 (-0.407)
i_{t-1}	0.831 (9.894)	1.070 (8.017)
i_{t-2}		-0.260 (-2.252)

Notes: *t*-statistics are in parentheses.

<Table 2> Unit Root Tests of the Taylor Rule Variables

Variable	Rho	t-value	lags
i_t	-0.082	-1.888	2
π_t	-0.088	-0.790	12
y_t	-0.496	-4.561	4
e_t	-0.080	-1.748	1

Note: In each case we estimated a model of the general form $\Delta x_t = a_0 + \rho x_{t-1} + \Sigma a_i \Delta x_{t-i} + \varepsilon_t$. Lag lengths were chosen using a maximum lag length (i_{max}) of 12. If the *t*-statistic for the last lag was not significant at the 5% level, i_{max} was reduced by one and the equation was re-estimated. τ_μ is the sample value of the *t*-statistic for the null hypothesis $\rho = 0$. With 50 observations, the critical values at the 10% and 5% significance levels are –2.60 and –2.93, respectively.

behaves as a random walk. This outcome is further supported when we test for a unit root in each of the variables. The results of performing augmented Dickey-Fuller unit root tests are displayed in Table 2. The results indicate that the call rate of interest (i_t), inflation rate ($_t$), and exchange rate (e_t) variables are each nonstationary, while the output gap (y_t) is stationary.

The above results suggest a need to perform tests to determine if the variables in the linear Taylor rule are cointegrated. Both EG and Johansen linear cointegration tests were performed with results displayed in Tables 3 and 4, respectively. Results for the EG cointegration test are reported with and without the stationary variable y_t. In each case, the EG tests cannot reject the null of no cointegration at the usual significance levels. Results using the Johansen linear cointegration test omit y_t and reject the null of no cointegration. However, as noted, the results from estimating the linear Taylor rule in Table 1 indicate little reaction of the target call rate to an increase in the inflation rate, and the coefficient on the lagged interest rate variable suggests that movements in the call rate resemble a random walk. We conclude that there is little support for the linear Taylor rule in Korea.

We next examine results using the IV ECM threshold cointegration test as reported in Table 5. The Q-statistic is provided as a test for serial correlations. We utilize the four different threshold indicators described in (8a) – (8d) noted as Case 1 to 4, respectively. We begin by examining our most important model in Case 1, where the threshold variable is the rate of inflation. The results show no support for cointegration in the nonlinear Taylor rule for Korea. However, the results for the other three threshold functions, Case 2 to 4, reject the null of no cointegration and support a valid threshold cointegration model at the 10%, 10%, and 1% levels of significance respectively.

When using the IV ECM threshold cointegration test, stationary covariates can be included in the testing equation to increase power. While y_t is included in the test results in Table 5, we wish to consider a further increase in power by including the

<Table 3> Engle-Granger Linear Cointegration Tests

Dependent variable: i_t	Model 1	Model 2
Constant	5.028 (2.33)	4.715 (1.99)
π_t	2.242 (10.4)	2.258 (10.14)
y_t		0.036 (0.331)
e_t	-5.277 (-3.05)	-5.030 (-2.65)
EG cointegration test statistics	-0.210 (-1.574)	-0.300 (-1.963)
# of lags	4	12

Notes: *t*-statistics are in parentheses. For each sample period, we estimated a potential long-run equilibrium relationship of the form $i_t = \beta_0 + \beta_1\pi_t + \beta_2 e_t + u_t$. The second step was to use the estimated residuals to estimate an equation of the form $\Delta e_t = \rho e_{t-1} + \Sigma\alpha_i\Delta e_{t-1} + v_t$. With 50 (100) observations, the critical value at the 10% and 5% significance levels are –3.31 (–3.09) and –3.46 (–3.40), respectively.

<Table 4> Johansen Linear Cointegration Tests

Hypothesized No. of CE(s)	Eigen-value	Trace Statistic	5 Percent Critical Value	1 Percent Critical Value
None **	0.5263	87.068	47.21	54.46
At most 1 **	0.3401	40.738	29.68	35.65
At most 2	0.1822	14.969	15.41	20.04
At most 3	0.0394	2.4926	3.76	6.65

Notes: *(**) denotes rejection of the null hypothesis of no cointegration at the 5% (1%) level. Trace test indicates 2 cointegrating equation(s) at both 5% and 1% levels. One lag was used in the above result.

first differenced right-hand variables in the testing equation (i.e., the first-difference inflation rate, output gap, and nominal exchange rate). These conditional IV ECM threshold cointegration tests can be undertaken given the invariance to nuisance parameters and the assumption of weakly exogenous variables; see Li (2006) for justification on the use of the conditional ECM test. Note that a similar conditional OLS ECM threshold cointegration test could not be practically undertaken due to the

<Table 5> ECM Threshold Cointegration Estimates of the Nonlinear Taylor Rule in Korea

Dependent variable: i_t	Case 1 π_t	(t-stat)	Case 2 % Growth Rates of e_t	(t-stat)	Case 3 | % Growth Rates of e_t |	(t-stat)	Case 4 Growth Rates of GDP	(t-stat)
$I_t z_{t-1}$	0.148	1.17	-0.416	-1.46	-0.986	-2.21	-0.720	-3.29
$(1-I_t)z_{t-1}$	-0.160	-0.52	0.087	1.73	0.043	0.23	0.103	1.46
Plus	0.199	0.83	-0.658	-1.83	-2.377	-2.18	-3.636	-4.71
Minus	-1.875	-1.83	0.509	0.97	-0.006	-0.02	0.214	1.03
*Q-stat**	30.48	(0.000)	21.488	(0.044)	17.506	(0.177)	23.352	(0.025)
m	10		10		9		10	
Threshold value	5.084		1.234		0.464		-0.054	
F-statistic for cointegration*	1.635	(0.441)	5.116	(0.077)	2.619	(0.083)	12.95	(0.002)

Notes: * p-values in parentheses. Here, $I_t = 1$ if the threshold variable > threshold, and 0 otherwise.

nuisance parameter problem when including stationary variables. The test results are displayed in Table 6. In contrast to Table 5, the results in Table 6 strongly reject the null of no cointegration in most cases and demonstrate stronger support for the validity of a nonlinear Taylor rule in Korea. Note that the first differenced variables act like stationary covariates and increase power in IV based tests.

Given that the IV ECM threshold cointegration tests find significant support for the nonlinear Taylor rule in Korea, we wish to examine the individual coefficients of different variables in the model. To estimate the individual coefficients and to compare results, we repeat our tests using the IV ADL threshold cointegration test of Enders *et al.* (2007). The results are displayed in Table 7. We focus our discussion on Case 1 and 2, since the tests in Case 3 and 4 cannot reject the null of no cointegration at the usual significance levels. The results of our most important Case 1 support our earlier expectations regarding the inflation rate and monetary policy. The coefficient on the inflation rate is positive and significant only when inflation is above the threshold rate. These findings support our conjecture of nonlinear monetary policy and suggest that the Bank of Korea will increase the call rate only when inflation rises above a threshold rate (approximately 4% rate of inflation). In contrast, when inflation is below the threshold rate, the Bank of Korea will reduce the call rate of interest. However, it should be noted that while the negative coefficient on the inflation rate variable when inflation is below the threshold is statistically significant, its absolute size is much smaller than when inflation is above the threshold rate. The coefficient on the output gap variable (y_t) is positive and statistically significant regardless of the rate of inflation. While the results for y_t when inflation is below the threshold rate were not expected, these findings suggest that the Bank of Korea places a relatively strong weight on a rising output gap as a signal to increase the call rate regardless of the current rate of inflation.

While the sign on the exchange rate variable is negative when inflation is below the threshold rate and positive when inflation is above the threshold rate, the coefficient on the exchange rate variable is not significant in either case. We next examine the results for Case 2, where the threshold variable is determined by whether the rate of currency depreciation is above or below the threshold rate (approximately 3% rate of depreciation in the won relative to the U.S. dollar). The results in Case 2 suggest that the Bank of Korea will increase interest rates to possibly counter currency depreciation only when the rate of depreciation rises above the threshold rate. In contrast, when the rate of depreciation is below the threshold rate, the Bank of Korea lowers the call rate. While the sign on the coefficient of the exchange rate variable is negative and statistically significant when the rate of deprecation is below the threshold rate, a possible explanation can be noted here. Perhaps the Bank of Korea is concerned about stopping currency deprecation by raising the call rate only when the rate of depreciation is relatively large. However, in more normal times the Bank of Korea prefers to keep to the call rate relatively low to keep the value of the won lower and encourage exports. The coefficients on the other variables of the inflation rate and exchange rate are not significant in any case at the usual levels. The results in Table 7 for Case 3 and 4 are less interesting, since we cannot reject the null of no cointegration in these models and nearly all of the

<Table 6> Conditional ECM Threshold Cointegration Estimates of the Nonlinear Taylor Rule in Korea

Dependent variable: i_t	Case 1 π_t	(t-stat)	Case 2 % Growth Rates of e_t	(t-stat)	Case 3 | % Growth Rates of e_t |	(t-stat)	Case 4 Growth Rates of GDP	(t-stat)
$I_t z_{t-1}$	-1.684	-7.55	0.066	0.72	-0.220	-1.14	-0.017	-0.12
$(1-I_t) z_{t-1}$	-0.080	-0.46	-0.571	-2.64	-0.877	-1.89	-2.350	-5.38
$I_t \quad \pi_t$	-1.931	-2.03	-0.005	-0.01	0.746	2.00	0.359	1.03
$(1-I_t) \quad \pi_t$	0.539	2.38	0.973	3.89	2.057	2.14	-1.895	-2.90
$I_t \quad y_t$	-2.217	-5.69	-0.056	-0.91	-0.038	-0.58	-0.026	-0.50
$(1-I_t) \quad y_t$	0.018	0.21	0.149	1.38	-0.088	-0.43	-2.997	-3.57
$I_t \quad e_t$	63.44	3.81	13.94	10.6	10.93	3.78	10.68	5.70
$(1-I_t) \quad e_t$	10.89	5.47	5.134	1.16	-0.264	-0.03	93.56	3.96
Plus	-3.582	-3.36	0.274	1.35	0.058	0.33	0.101	0.66
Minus	-0.007	-0.045	-0.505	-1.84	-1.732	-2.29	-1.364	-1.49
*Q-stat**	10.69	(0.556)	13.30	(0.425)	17.07	(0.196)	15.68	(0.206)
m	10		9		8		10	
Threshold value	5.730		1.233		0.6458		0.0219	
F-statistic for cointegration*	28.61	(0.000)	7.472	(0.024)	4.874	(0.087)	14.494	(0.000)

Notes: * p-values in parentheses. Here, $I_t = 1$ if the threshold variable > threshold, and 0 otherwise.

<Table 7> ADL Threshold Cointegration Estimates of the Nonlinear Taylor Rule in Korea

Dependent variable: i_t	Case 1 π_t	(t-stat)	Case 2 % Growth Rates of e_t	(t-stat)	Case 3 | % Growth Rates of e_t |	(t-stat)	Case 4 Growth Rates of GDP	(t-stat)
$I_t i_{t-1}$	-0.371	-2.25	0.042	0.25	0.194	1.56	0.070	0.08
$(1-I_t) i_{t-1}$	-0.018	-0.10	-0.577	-2.49	-0.184	-0.27	-1.124	-2.94
$I_t \pi_{t-1}$	0.968	1.97	-0.074	-0.15	-0.410	-1.33	-0.131	-0.09
$(1-I_t) \pi_{t-1}$	-0.191	-2.51	0.742	1.54	0.098	0.06	5.703	2.17
$I_t y_{t-1}$	0.395	3.08	0.157	0.40	0.064	0.56	0.151	0.77
$(1-I_t) y_{t-1}$	0.148	3.41	0.079	1.17	0.296	2.32	0.295	1.22
$I_t e_{t-1}$	-3.185	-1.61	2.979	1.99	-2.336	-0.60	0.164	0.03
$(1-I_t) e_{t-1}$	1.522	0.87	-7.259	-3.59	-2.405	-0.51	82.110	-2.07
Plus	1.847	0.77	-3.692	-1.76	2.660	0.62	-0.102	-0.02
Minus	-1.113	-0.51	8.878	4.01	3.408	0.90	85.695	2.08
*Q-stat**	23.751	(0.069)	50.18	(0.001)	19.450	(0.194)	23.509	(0.052)
m	4		3		2		9	
Threshold value	4.0877		3.0064		0.6458		0.0219	
F-statistic for cointegration*	3.301	(0.045)	3.109	(0.053)	1.825	(0.161)	4.336	(0.298)

Notes: * p-values in parentheses. Here, $I_t = 1$ if the threshold variable > threshold, and 0 otherwise.

estimated coefficients are insignificantly different from zero.

For the sake of robustness, we examine two additional cases. We note that the Bank of Korea began to adopt a policy of inflation targeting in late 1998. In this regard, it will be interesting to analyze the Taylor rule in Korea using only data from the sub-sample of 1999 to 2007. We also utilize data on the target rate of inflation from 1998:3 to 2007:1, which are obtained from the web site of the Bank of Korea, and use the target inflation rate as a time varying threshold level.[3] Since only the time-varying inflation target rates are used, threshold parameters are undefined in this analysis.

An important caveat from using this smaller sub-sample should be noted. There is a significant loss in degrees of freedom. However, in spite of this, overall, we obtain results that confirm our previous findings of valid nonlinear Taylor rule. In Table 8, we report our main results from adopting the inflation targeting sample period of 1999-2007. The results for the target rate of inflation as the threshold variable are similar to those in Table 6. Using the sub-sample of 1999-2007, the results for Case 1A reject the null of no cointegration at the 5% level of significance and support the validity of a nonlinear Taylor rule in Korea. The estimated threshold rate of inflation is 2.731% from the ECM model and 3.566% from the ADL model, respectively, when compared with inflation rates over the most recent four quarters. Note that these threshold rates of inflation are lower than the rate of 5.730% found for the whole sample period (Table 6). This outcome is expected given that inflation rates were falling in Korea by the late 1990s. The results for other threshold variables reported in Case 2 to 4 in Table 8 cannot reject the null of no cointegration at the usual significance levels, although the rate of depreciation of the won nearly rejects the null at the 10% level. We next consider using the Bank of Korea's target rate of inflation as a time-varying threshold variable. The results are displayed in Case 1B in Table 8. The results using the target rate of inflation (Case 1B) again reject the null of no cointegration in the ADL IV test, and nearly reject the null (p-value = 10.3%) in the ECM IV test. Overall, the results for using the inflation rate threshold variable in the inflation targeting sub-sample support those of the whole sample period and provide additional evidence of a nonlinear Taylor rule in Korea.

Ⅴ. Conclusion

In this paper, we develop new ECM threshold cointegration tests that include stationary IVs. The tests are invariant to nuisance parameters found in the OLS based ECM threshold cointegration tests. As a result, bootstrapping is unnecessary and the same critical values can be used throughout. This is the case regardless of the threshold variables adopted, deterministic terms, or inclusion of stationary covariates. In contrast to the OLS based ECM threshold cointegration tests, including stationary covariates in the IV ECM threshold cointegration test increases

[3] We are grateful to an anonymous referee who suggested using the target rate of inflation as a threshold variable.

<Table 8> Additional Test Results on the Nonlinear Taylor Rule in Korea

	Case 1 π_t	Case 2 % Growth Rates of e_t	Case 3 |% Growth Rates of e_t|	Case 4 Growth Rates of GDP
A. Sub-sample (1998:3 – 2007:1) is used				
1. ECM Test				
Threshold value	2.731	-1.572	1.328	0.046
F-statistic for cointegration*	3.214 (0.042)	1.930 (0.239)	0.954 (0.437)	1.575 (0.207)
2. ADL Test				
Threshold value	3.566	1.327	3.690	0.034
F-statistic for cointegration*	6.486 (0.009)	1.215 (0.324)	3.690 (0.100)	0.375 (0.693)
B.Inflation Target Rates are used (1998:3 – 2007:1)				
1. ECM Test				
F-statistic for cointegration*	2.558 (0.103)	n.a.	n.a.	n.a.
2. ADL Test				
F-statistic for cointegration*	36.43 (0.000)	n.a.	n.a.	n.a.

Notes: * p-values in parentheses.

power when the alternative is true while leaving asymptotic properties under the null unchanged. Our testing methodology builds on the work of Enders *et al*. (2007), who find similar invariance properties in IV ADL threshold cointegration tests. We apply the IV ECM threshold cointegration methodology to test for threshold cointegration in the nonlinear Taylor rule of Korea. While previous works find evidence of nonlinear monetary policies in different countries, these papers seldom test for nonlinear cointegration. However, if the variables in a nonlinear model are nonstationary and not cointegrated, then estimation results can be spurious. Following Enders *et al*. (2007), we seek to contribute to the literature by providing new procedures to test for threshold cointegration in nonlinear models. Our new methodologies will also prove useful in other applications in macroeconomics and related areas. In addition, we note that little work has been undertaken to test for nonlinear monetary policies in newly industrializing countries.

We utilize our new testing procedures to examine and test nonlinear Taylor rules in Korea. In addition to the usual variables of the interest rate, inflation rate, and output gap, we follow the suggestion of Hsing and Lee (2004) in the linear case and include the nominal Korea/U.S. exchange rate in our nonlinear Taylor rules. Overall, we find little evidence to support the linear Taylor rule in Korea. In contrast, we find significant support for nonlinear Taylor rules with threshold effects. Four different threshold functions are examined. Most important among our results, we find that the Bank of Korea will increase the call rate of interest in response to an increase in inflation only when inflation rises above a threshold rate. In addition, we find that the Bank of Korea increases the call rate of interest to possibly counteract depreciation of the won only when the rate of depreciation is above a threshold rate.

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Appendix. Proof of Theorem 1.

We wish to prove Theorem 1 and show that the asymptotic distribution of the IV ECM test for threshold cointegration is chi-square. Our proof is an extension of the proofs in Enders, Im, and Lee (2005) and Enders, Lee, and Strazicich (2007). The difference is that in the present paper the error correction term (ECM) is instrumented by stationary instrumental variables rather than lagged nonstationary variables in a nonlinear model setting. First, we consider a sample splitting regression

$$x_t = \theta_1 f_t + v_{1t}, \qquad n_t \geq \tau \tag{A.1}$$

$$x_t = \theta_2 f_t + v_{2t}, \qquad n_t < \tau.$$

We define an indicator function $d_t(\tau) = \{ n_t \geq \tau \}$, where $d_t(\tau) = 1$ if $n_t \geq \tau$ and $d_t(\tau) = 0$ otherwise. We define $f_{1t}^* = f_t\, d_t(\tau)$ and $f_{2t}^* = f_t\,(1-d_t(\tau))$. Then, we can rewrite (A.1) as

$$x_t = \theta_1 f_{1t}^* + \theta_2 f_{2t}^* + v_t. \tag{A.2}$$

We let $\theta = (\theta_1', \theta_2')'$ and $f_t^* = (f_{1t}^*, f_{2t}^*)$. For instance, for the regression of the conditional ECM, we have $x_t = \Delta r_t$, $f_{1t}^* = d_t(\gamma)\ (z_{t-1}, \Delta f_t$, lags of $\Delta f_t)$, and $f_{2t}^* = (1-d_t(\gamma))(z_{t-1}, \Delta f_t$, lags of $\Delta f_t)$, where z_{t-1} is the error correction term. Therefore, we have $\Delta f_t = (\Delta\pi_t, \Delta y_t, \Delta e_t)$ in our application to the Korean nonlinear Taylor rule. Further, we can include I(0) regressors s_t in f_{1t}^* and f_{2t}^* as stationary covariates.

We assume that ε_t, $t = 1,..,\infty$, is an *iid* process with mean zero, variance σ^2, and finite fourth moment. Define a partial sum process $S_{[rT]} = \Sigma_{j=1}^{rT}\varepsilon_j$ with $r\in[0,1]$ and $\xi_t = \varepsilon_{t-1} + .. + \varepsilon_{t-m}$, where m is a finite positive integer. Then, following Enders, Im, and Lee (2005) we show that

$$T^{-1}\,\Sigma_{t=1}^{T} S_{t-1}\varepsilon_t \rightarrow 0.5\sigma^2[\mathrm{W}(1)^2 - 1] \tag{A.3}$$

$$T^{-1/2}\,\Sigma_{t=1}^{T}\xi_t\varepsilon_t \rightarrow \sqrt{m}\sigma^2\mathrm{W}(1) \tag{A.4}$$

$$T^{-1}\,\Sigma_{t=1}^{T}\xi_t^{\,2} \rightarrow m\sigma^2. \tag{A.5}$$

The proof is found in the above reference. Letting F = $\{ f_1^*, f_2^*, \ldots, f_T^*\}$ with $f_t^* = (f_{1t}^*, f_{2t}^*)$, we can easily expect that the moment matrix F F is a diagonal matrix, since $\mathrm{E}(f_{1t}^* f_{2t}^*) = 0$. Also, we define

$$\mathrm{B_T} = \sum_{t=1}^{T} w_t^*\, a_t - \sum_{t=1}^{T} w_t^*\, f_t^{(0)\prime}[\sum_{t=1}^{T} f_t^{(0)\prime}\, f_t^{(0)}]^{-1} \sum_{t=1}^{T} f_t^{(0)}\, a_t \tag{A.6}$$

$$C_T = \sum_{t=1}^{T} w_t^{*2} - \sum_{t=1}^{T} w_t f_t^{(0)\prime}[\sum_{t=1}^{T} f_t^{(0)\prime} f_t^{(0)}]^{-1} \sum_{t=1}^{T} f_t^{(0)} w_t^{*}. \tag{A.7}$$

Then, as shown in the above reference, it is straightforward to obtain the following results

$$\frac{1}{\sqrt{T}} B_T \rightarrow \sqrt{m}\, \sigma^2 W(1) \tag{A.8}$$

$$\frac{1}{T} C_T \rightarrow m\, \sigma^2. \tag{A.9}$$

Then, by collecting the results in (A.8) and (A.9), we can show that

$$t_{\gamma 1} = \frac{\hat{\gamma}_{1iv}}{s(\hat{\gamma}_{1iv})} = \frac{\frac{1}{\sqrt{T}} B_T}{\hat{\sigma}\sqrt{\frac{1}{T} C_T}} = W(1) \sim N(0,1).$$

$t_{\gamma 2}$ can be obtained in a similar manner. The distribution of the Wald statistic on the joint hypothesis is given as the sum of the square of the above *t*-statistics. Then, the distribution of the Wald statistic is chi-square, since the sum of standard normal random variables has the chi-square distribution with degree of freedom equal to the number of restrictions. This completes the proof.