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AN EXTENDED NON-ASSOCIATIVE ALGEBRA

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Abstract. A Weyl type algebra is defined in the paper (see [2],[4], [6], [7]). A Weyl type non-associative algebra $\overline{WN_{m,n,s}}$ and its restricted subalgebra $\overline{WN_{m,n,s_r}}$ are defined in the papers (see [1], [14], [16]). Several authors find all the derivations of an associative (Lie or non-associative) algebra (see [3], [1], [5], [7], [10], [16]). We find $Der(\overline{WN_{0,0,1_n}})$ of the algebra $\overline{WN_{0,0,1_n}}$ and show that the algebras $\overline{WN_{0,0,1_n}}$ and $\overline{WN_{0,0,s_1}}$ are not isomorphic in this work. We show that the associator of $\overline{WN_{0,0,1_n}}$ is zero.

1. Preliminaries

Let $\mathbb{F}$ be a field of characteristic zero (not necessarily algebraically closed). Throughout this paper, $\mathbb{N}$ and $\mathbb{Z}$ will denote the non-negative integers and the integers, respectively. Let $\mathbb{F}[x_1, \cdots, x_{m+s}]$ be the polynomial ring with the variables $x_1, \cdots, x_{m+s}$. Let $g_1, \cdots, g_n$ be given polynomials in $\mathbb{F}[x_1, \cdots, x_{m+s}]$. For $n, m, s \in \mathbb{N}$, let us define the commutative, associative $\mathbb{F}$-algebra

$$\mathbb{F}_{g_n,m,s} = \mathbb{F}[e^{\pm g_1}, \cdots, e^{\pm g_n}, x_1^{\pm 1}, \cdots, x_m^{\pm 1}, x_{m+1}, \cdots, x_{m+s}]$$

in the formal power series ring $\mathbb{F}[[x_1, \cdots, x_{m+s}]]$ which is called a stable algebra in the paper (see [9]) with the standard basis

$$\{e^{a_1 g_1} \cdots e^{a_n g_n} x_1^{i_1} \cdots x_{m+s}^{i_{m+s}} | a_1, \cdots, a_n, i_1, \cdots, i_m \in \mathbb{Z}, i_{m+1}, \cdots, i_{m+s} \in \mathbb{N}\}$$

and with the obvious addition and the multiplication (see [9] and [14]). ∂_w, $1 \le w \le m+s$, denotes the usual partial derivative with respect to

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x_w on $\mathbb{F}_{g_n,m,s}$. For partial derivatives $\partial_u, \cdots, \partial_v$ of $\mathbb{F}_{g_n,m,s}$, the composition $\partial_u^{j_u} \circ \cdots \circ \partial_v^{j_v}$ of them is denoted $\partial_u^{j_u} \cdots \partial_v^{j_v}$ where $j_u, \cdots, j_v \in \mathbb{N}$. Let A be the set $\{\partial_u^{j_u} \circ \cdots \circ \partial_v^{j_v} | j_u, \cdots, j_v \in \mathbb{N}, \partial_w$ is the partial derivation of $\mathbb{F}_{g_n,m,s}$ with respect to x_w, $1 \leq w \leq m+s\}$.
Let us define the vector space $WN(g_n, m, s) = WN(g_n, m, s)_A$ over $\mathbb{F}$ which is spanned by the standard basis

$$\begin{aligned} &\{e^{a_1 g_1} \cdots e^{a_n g_n} x_1^{i_1} \cdots x_{m+s}^{i_{m+s}} \partial_u^{j_u} \cdots \partial_v^{j_v} | a_1, \cdots, a_n, i_1, \cdots, i_m \in \mathbb{Z}, \\ (1) \quad & i_{m+1}, \cdots, i_{m+s} \in \mathbb{N}, j_u, \cdots, j_v \in \mathbb{N}, 1 \leq u, \cdots, v \leq m+s\} \end{aligned}$$

Thus we may define the multiplication $*$ on $WN(g_n, m, s)$ as follows:

$$\begin{aligned} & e^{a_{11} g_1} \cdots e^{a_{1n} g_n} x_1^{i_{11}} \cdots x_{m+s}^{i_{1,m+s}} \partial_u^{j_u} \cdots \partial_v^{j_v} * \\ & \quad e^{a_{21} g_1} \cdots e^{a_{2n} g_n} x_1^{i_{21}} \cdots x_{m+s}^{i_{2,m+s}} \partial_h^{j_h} \cdots \partial_w^{j_w} \\ (2) \quad & \quad = e^{a_{11} g_1} \cdots e^{a_{1n} g_n} x_1^{i_{11}} \cdots x_{m+s}^{i_{1,m+s}} \\ & \quad \quad \partial_u^{j_u} \cdots \partial_v^{j_v} (e^{a_{21} g_1} \cdots e^{a_{2n} g_n} x_1^{i_{21}} \cdots x_{m+s}^{i_{2,m+s}}) \partial_h^{j_h} \cdots \partial_w^{j_w} \end{aligned}$$

for any basis elements $e^{a_{11} g_1} \cdots e^{a_{1n} g_n} x_1^{i_{11}} \cdots x_{m+s}^{i_{1,m+s}} \partial_u^{j_u} \cdots \partial_v^{j_v}$ and $e^{a_{21} g_1} \cdots e^{a_{2n} g_n} x_1^{i_{21}} \cdots x_{m+s}^{i_{2,m+s}} \partial_h^{j_h} \cdots \partial_w^{j_w} \in WN(g_n, m, s)$. Thus we can define the Weyl-type non-associative algebra $\overline{WN_{g_n,m,s}}$ with the multiplication $*$ in (2) and with the set $WN(g_n, m, s)$ (see [15] and [16]). For $B \subset A$, let us define the the non-associative subalgebra $\overline{WN_{g_n,m,s_B}}$ of the non-associative algebra $\overline{WN_{g_n,m,s}}$ spanned by

$$\begin{aligned} &\{e^{a_1 g_1} \cdots e^{a_n g_n} x_1^{i_1} \cdots x_s^{i_s} \partial_u^{j_u} \cdots \partial_v^{j_v} | a_1, \cdots, a_n, i_1, \cdots, i_m \in \mathbb{Z}, \\ (3) \quad & i_{m+1}, \cdots, i_s, j_u, \cdots, j_v \in \mathbb{N}, \partial_u^{j_u} \cdots \partial_v^{j_v} \in B, 1 \leq u, \cdots, v \leq m+s\} \end{aligned}$$

If we take $B = \{\partial_u^0 \cdots \partial_v^0\}$, then the algebra $\overline{WN_{g_n,m,s_B}}$ is the $\mathbb{F}$-algebra $F_{g_n,m,s}$. This implies that the algebra $\overline{WN_{g_n,m,s_B}}$ contains the polynomial ring, naturally. The simplicity of $\overline{WN_{g_n,m,s_B}}$ is depending on the set B. It is well known that the non-associative algebra $WN_{g_n,m,s}$ is simple, even though it has the right annihilator (see [7] and [8]). A non-associative algebra $\overline{WN_{g_n,m,s_B}}$ is symmetric if there is a isomorphism induced by change of variables (see [5]). Throughout the paper we put $\partial^1 = \partial$ and $\partial^k = \partial(\partial^{k-1})$.

2. Derivations of $\overline{WN_{0,0,1_3}}$

Lemma 1. *For any derivation D of the non-associative algebra $\overline{WN_{0,0,1_3}}$ $=< \{x^i\partial,\ x^i\partial^2, x^i\partial^3 | i \in \mathbb{N}\} >$, and for any $x^i\partial^j$, $1 \leq j \leq 3$, we have the followings*

$$D(\partial) = D(\partial^2) = D(\partial^3) = 0$$

$$D(x^i\partial^j) = is_1 x^{i-1}\partial^j$$

where $s_1 \in \mathbb{F}$.

Proof. Let D be the derivation in the lemma. Since ∂_x is the annihilator of itself, we have that $\partial * D(\partial) = 0$. This implies that

$$D(\partial) = \sum_{1 \leq p \leq 3} a_p \partial^p \tag{4}$$

where $a_p \in \mathbb{F}$, $1 \leq p \leq 3$. Similarly, we can prove the followings: $D(\partial^2) = \sum_{1 \leq p \leq 3} b_p \partial^p$ and $D(\partial^3) = \sum_{1 \leq p \leq 3} c_p \partial^p$ with appropriate coefficients. Since $x\partial$ is a right identity of ∂, we have that $\partial * D(x\partial) = a_2\partial^2 + a_3\partial^3$. This implies that

$$D(x\partial) = a_2 x\partial^2 + a_3 x\partial^3 + s_1\partial + s_2\partial^2 + s_3\partial^3 \tag{5}$$

where $s_m \in \mathbb{F}$, $1 \leq m \leq 3$. Since $x\partial$ is an idempotent, we have that

$$D(x\partial) = a_2 x\partial^2 + a_3 x\partial^3 + s_1\partial$$

Since $D(\partial * x^2\partial) = 2D(x\partial)$, we can prove that

$$\partial * D(x^2\partial) \quad = \quad 2a_2 x\partial^2 + 2a_3 x\partial^3 + 2s_1\partial - 2a_2\partial - 2a_1 x\partial$$

Similarly, we can prove that

$$D(x^2\partial) = -a_1x^2\partial + a_2x^2\partial^2 + a_3x^2\partial^3 + 2s_1x\partial - 2a_2x\partial + t_1\partial + t_2\partial^2 + t_3\partial^3$$

where $t_1, t_2, t_3 \in \mathbb{F}$. Since $D(x\partial * x^2\partial) = 2D(x^2\partial)$, we can also prove that

$$D(x^2\partial) = -a_1x^2\partial + a_2x^2\partial^2 + a_3x^2\partial^3 + 2s_1x\partial$$

By $D(\partial * x^3\partial) = 3D(x^2\partial)$, we have that

$$\begin{aligned} D(x^3\partial) &= -2a_1x^3\partial + a_2x^3\partial^2 + a_3x^3\partial^3 + 3s_1x^2\partial - 3a_2x^2\partial \\ &\quad -6a_3x\partial + r_1\partial + r_2\partial^2 + r_3\partial^3 \end{aligned}$$

where $r_1, r_2, r_3 \in \mathbb{F}$. Since $D(x\partial * x^3\partial) = 3D(x^3\partial)$, we also have that

$$D(x^3\partial) = -2a_1x^3\partial + a_2x^3\partial^2 + a_3x^3\partial^3 + 3s_1x^2\partial$$

By $D(x^2\partial * x^2\partial) = 2D(x^3\partial)$, we have that $-2a_1x^3\partial + a_2x^3\partial^2 + a_2x^2\partial + a_3x^3\partial^3 + 3s_1x^2\partial = -2a_1x^3\partial + a_2x^3\partial^2 + a_3x^3\partial^3 + 3s_1x^2\partial$. This implies that $a_2 = 0$ and we have the followings:

$$\begin{aligned} &D(x\partial) = a_3x\partial^3 + s_1\partial \\ &D(x^2\partial) = -a_1x^2\partial + a_3x^2\partial^3 + 2s_1x\partial \\ (6) \qquad &D(x^3\partial) = -2a_1x^3\partial + a_3x^3\partial^3 + 3s_1x^2\partial \end{aligned}$$

By $D(\partial * x^4\partial) = 4D(x^3\partial)$, we have that

$$\begin{aligned} D(x^4\partial) &= -3a_1x^4\partial - 12a_3x^2\partial + a_3x^4\partial^3 + 4s_1x^3\partial \\ &\quad +u_1\partial + u_2\partial^2 + u_3\partial^3 \end{aligned}$$

where $u_1, u_2, u_3 \in \mathbb{F}$. By $D(x\partial * x^4\partial) = 4D(x^4\partial)$, we can also prove that

$$(7) \qquad D(x^4\partial) = -3a_1x^4\partial + a_3x^4\partial^3 + 4s_1x^3\partial$$

By $D(x^2\partial * x^3\partial) = 3D(x^4\partial)$, we have that

$$(8) \qquad D(x^4\partial) = -3a_1x^4\partial + 2a_3x^2\partial + a_3x^4\partial^3 + 4s_1x^3\partial$$

By comparing (7) and (8), we have that $a_3 = 0$. The formula (6) and (7) become

$$\begin{aligned} D(x\partial) &= s_1\partial \\ D(x^2\partial) &= -a_1x^2\partial + 2s_1x\partial \\ D(x^3\partial) &= -2a_1x^3\partial + 3s_1x^2\partial \\ D(x^4\partial) &= -3a_1x^4\partial + 4s_1x^3\partial \end{aligned} \tag{9}$$

By $D(\partial * x\partial^2) = D(\partial^2)$ and $D(x\partial * x\partial^2) = D(x\partial^2)$, we can also prove that

$$D(x\partial^2) = -a_1x\partial^2 + b_1x\partial + b_2x\partial^2 + b_3x\partial^3 + s_1\partial^2$$

Similarly, we can prove the followings:

$$\begin{aligned} D(x^2\partial^2) &= -2a_1x^2\partial^2 + b_1x^2\partial + b_2x^2\partial^2 + b_3x^2\partial^3 + 2s_1x\partial^2 \\ D(x^3\partial^2) &= -3a_1x^3\partial^2 + b_1x^3\partial + b_2x^3\partial^2 + b_3x^3\partial^3 + 3s_1x^2\partial^2 \end{aligned} \tag{10}$$

Since $x\partial^2$ annihilates itself, we have that b_1 is zero. By $D(\partial^2 * x^2\partial^2) = 2D(\partial^2)$ and $D(x\partial^2 * x^2\partial^2) = 2D(x\partial^2)$, we can prove that a_1 and b_2 are zeroes. This implies that

$$\begin{aligned} D(x\partial^2) &= b_3x\partial^3 + s_1\partial^2 \\ D(x^2\partial^2) &= b_3x^2\partial^3 + 2s_1x\partial^2 \\ D(x^3\partial^2) &= b_3x^3\partial^3 + 3s_1x^2\partial^2 \end{aligned} \tag{11}$$

By $D(x^2\partial^2 * x^3\partial^2) = 6D(x^3\partial^2)$, we have that $b_3 = 0$. This implies that $D(\partial) = D(\partial^2) = 0$ and

$$\begin{aligned} D(x\partial) &= s_1\partial \\ D(x^2\partial) &= 2s_1x\partial \\ D(x^3\partial) &= 3s_1x^2\partial \end{aligned} \tag{12}$$

We also have that $D(x\partial^2) = s_1\partial^2$, $D(x^2\partial^2) = 2s_1x\partial^2$, and $D(x^3\partial^2) = 3s_1x^2\partial^2$. Similarly, we are able to prove that $D(\partial^3) = 0$, $D(x\partial^3) = s_1\partial^3$, $D(x^2\partial^3) = 2s_1x\partial^3$, and $D(x^3\partial^3) = 3s_1x^2\partial^3$. By induction on i of $x^i\partial$, we can prove that

$$D(x^i\partial) = is_1x^{i-1}\partial$$

Similarly, we are able to prove that

$$\begin{aligned} D(x^i\partial^2) &= is_1x^{i-1}\partial^2 \\ D(x^i\partial^3) &= is_1x^{i-1}\partial^3 \end{aligned}$$

Therefore we have proven the lemma. □

Note 1. For any basis element $x^i\partial^k$, $1 \leq k \leq 3$, of the algebra $\overline{WN_{0,0,1_3}}$, if we define $\mathbb{F}$-linear map D_1 of the algebra $\overline{WN_{0,0,1_3}}$ as follows:

$$D_1(x^i\partial^k) = ix^{i-1}\partial^k$$

then the map D_1 of the algebra $\overline{WN_{0,0,1_3}}$ can be linearly extended to a derivation of the algebra $\overline{WN_{0,0,1_3}}$. □

Theorem 1. *For any derivation D of the algebra $\overline{WN_{0,0,1_3}}$, $D = cD_1$ such that D_1 is the derivation in Note 1 where $c \in \mathbb{F}$.*

Proof. The proof of the theorem is straightforward by Lemma 1 and Note 1. □

Corollary 1. *The dimension $Dim(Der(\overline{WN_{0,0,1_3}}))$ of the algebra $\overline{WN_{0,0,1_3}}$ is one and every derivation of the algebra $\overline{WN_{0,0,1_3}}$ is the inner derivation $ad_{c\partial}$ where $c \in \mathbb{F}$.*

Proof. The proof of the corollary is straightforward by Note 1 and Theorem 1. □

Lemma 2. *For any derivation D of the algebra $\overline{WN_{0,0,1_n}} =< \{x^i\partial^r | i \in \mathbb{N}, 1 \leq r \leq n, n > 1\} >$, for any basis element $x^i\partial^j$, $1 \leq j \leq n$, of the algebra $\overline{WN_{0,0,1_n}}$, we have the followings*

$$D(\partial) = D(\partial^2) = \cdots = D(\partial^n) = 0 \text{ and } D(x^i\partial^j) = is_1x^{i-1}\partial^j$$

where $s_1 \in \mathbb{F}$.

Proof. Let D be the derivation in the lemma. Let D be any derivation of $D \in Der_{non}(\overline{W(0,0,1)_n})$. Since ∂^j annihilates ∂^i, $1 \le i, j \le n$, we have that $D(\partial) = \sum_{j=1}^{n} r_j \partial^j$ with $r_j \in \mathbb{F}$, $1 \le i \le n$. By $D(\partial^j * x^n\partial) = n(n-1)\cdots(n-j+1)D(x^{n-j}\partial)$, we can prove that

$$\begin{aligned} &nr_1x^{n-1}\partial + n(n-1)r_2x^{n-2}\partial + \cdots + n!r_n\partial + \partial^j * ns_1x^{n-1}\partial \\ &= n(n-1)\cdots(n-j+1)(n-j)s_1x^{n-j-1}\partial \end{aligned}$$

Since $\partial^j * ns_1x^{n-1}\partial = n(n-1)\cdots(n-j+1)(n-j)s_1x^{n-j-1}\partial$, we have that $r_1 = \cdots = r_n = 0$. This implies that $D(\partial^j)$ is zero. Since $x\partial$ is a left identity of $x\partial^j$, we can prove that

$$s_1\partial^j + x\partial * D(x\partial^j) = D(x\partial^j) \tag{13}$$

Let us put $D(x\partial^j) = \sum_{i=1}^{n} b_{0,i}\partial^i + \sum_{i=1}^{n} b_{1,i}x\partial^i + \sum_{i=1}^{n} x^2 b_{2,i}\partial^i + \#_1$ where $\#_1$ the sums of its remaining terms $\sum_{i=1}^{n} x^k b_{k,i}\partial^i$, $k \ge 3$, with non-zero scalars. By (13), we prove that

$$\begin{aligned} &s_1\partial^j + \sum_{i=1}^{n} b_{1,i}x\partial^i + 2\sum_{i=1}^{n} x^2 b_{2,i}\partial^i + \#_2 \\ &= \sum_{i=1}^{n} b_{0,i}\partial^i + \sum_{i=1}^{n} b_{1,i}x\partial^i + \sum_{i=1}^{n} x^2 b_{2,i}\partial^i + \#_1 \end{aligned}$$

where $\#_2$ the sums of its remaining terms with appropriate coefficients. We can prove that $b_{0,j} = s_1$, $b_{0,1} = \cdots = b_{0,j-1} = b_{0,j+1} = \cdots = b_{0,n} = 0$ and $b_{2,i} = \cdots = 0$. This implies that $D(x\partial^j) = s_1\partial^j + \sum_{i=1}^{n} b_{1,i}x\partial^i$. Since $D(\partial * x\partial^j) = 0$, we can prove that $b_{1,i} = 0$, $1 \le i \le n$ and

$$D(x\partial^j) = s_1\partial^j \tag{14}$$

Since $D(x^i\partial * x\partial^j) = D(x^i\partial^j)$, we have that $is_1x^{i-1}\partial * x\partial^j + x^i\partial * s_1\partial^j = D(x^i\partial^j)$. This implies that

$$D(x^i\partial^j) = is_1x^{i-1}\partial^j$$

Therefore we have proven the lemma. □

Note 2. For any basis element $x^i\partial^k$, $1 \leq k \leq n$, $n > 1$, of the algebra $\overline{WN_{0,0,1_n}}$ if we define $\mathbb{F}$-linear map D_1 of $\overline{WN_{0,0,1_n}}$ as follows:

$$D_1(x^i\partial^k) = ix^{i-1}\partial^k$$

then the map D_1 of $\overline{WN_{0,0,1_n}}$ can be linearly extended to a derivation of $\overline{WN_{0,0,1_n}}$. We remark that if $n = 1$, then $D_0(x^i\partial) = c_1(1-i)x^i\partial + c_2ix^{i-1}\partial$, $c_1, c_2 \in \mathbb{F}$ can be linearly extended to a derivation of $\overline{WN_{0,0,1_1}}$ [1] □

Theorem 2. *For any derivation D of the algebra $\overline{WN_{0,0,1_n}}$, $D = cD_1$ such that D_1 is the derivation in Note 2 where $c \in \mathbb{F}$.*

Proof. The proof of the theorem is straightforward by Lemma 1 and Note 2. □

Corollary 2. *The dimension $Dim(Der(\overline{WN_{0,0,1_n}}))$ of the algebra $\overline{WN_{0,0,1_n}}$ is one and every derivation of $\overline{WN_{0,0,1_n}}$ is the inner derivation $ad_{c\partial}$ where $ad_{c\partial}$ is induced by the element $c\partial$ for $c \in \mathbb{F}$.*

Proof. The proof of the corollary is straightforward by Note 2 and Theorem 2. □

Corollary 3. *The associator of the algebra $\overline{WN_{0,0,1_n}}$ is zero (see [17]).*

Proof. The proof of the corollary is straightforward by Note 1 and Theorem 1. □

Corollary 4. *For any $s \in \mathbb{N}$, the algebras $\overline{WN_{0,0,1_n}}$ and $\overline{WN_{0,0,s_1}}$ are not isomorphic.*

Proof. The proof of the corollary is straightforward by Corollary 2 and Theorem 1 in the paper (see [5]). □

Proposition 1. *The matrix ring $M_m(\mathbb{F})$ is not a subalgebra of the algebra $\overline{WN_{0,0,1_n}}$.*

Proof. Let A be a finite dimensional subalgebra of the algebra $\overline{WN_{0,0,1_n}}$. If $Dim(A) \geq 2$, then it is easy to prove that A has no identity. This implies that the matrix ring $M_m(\mathbb{F})$ is not a subalgebra of the algebra $\overline{WN_{0,0,1_n}}$. Therefore we have proven the proposition. □

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