Performance Analysis of Synchronous Downlink MC-CDMA with Precoding and Frequency Offset

Won Mee Jang and Moon Woo Lee

Abstract: **We analyze the performance of code division multiple access (CDMA) system with multicarrier (MC) that employs precoding in synchronous downlink channels. Even though considerable efforts are undergoing for frequency offset estimation and correction, it is inevitable for the system to bear the remaining frequency offset. Therefore it is important to predict accurately the system performance in the presence of the residual frequency offset. We obtain the bit error rate (BER) performance in terms of the number of users, the spreading factor, the number of sub-carriers, and frequency offset. We assume that the spreading factor is equal to the number of sub-carriers, although we can generalize the case. The simulation results show that the BER of MC-CDMA with precoding shows a performance that varies with frequency offset as well as system loading.**

Index Terms: **Frequency offset, Multi Carrier-Code Division Multiple Access (MC-CDMA), multiuser detection, precoding.**

I. INTRODUCTION

Normally, in synchronous channels that are frequency nonselective in nature, orthogonal signals can be employed. However, the orthogonality of spreading sequences in Multi Carrier-Code Division Multiple Access (MC-CDMA) can be destroyed due to frequency selective fading channels that affect the signal amplitude in each carrier differently. Also to improve the spectrum efficiency of MC-CDMA, cyclic prefix (CP) insertion is often eliminated and the result is the loss of orthogonality among user signals. Another important application is in military spread spectrum multiple access systems where orthogonal spreading is not acceptable. To combat the multiple access interference (MAI) due to the loss of orthogonality, various multiuser detection techniques also have been proposed. Linear minimum mean square error (MMSE) multiuser detection was applied to MC-CDMA to reduce the MAI introduced by eliminating cyclic prefix in the system [1]. A maximum likelihood (ML) multiuser detection and a blind adaptive decorrelating detection were employed for synchronous MC-CDMA systems in [2] and asynchronous MC-CDMA systems in [3], respectively. Space-time multiuser detection for coded MC-CDMA developed in [4] can enhance the system capacity and performance by jointly employing the maximum a posteriori probability (MAP) based multiuser detection and channel decoding techniques. Transmitter precoding has similar complexity to the corresponding receiver based multiuser detection but the computational complexity is shifted to the common transmitter, where it is less critical. To avoid the MAI, precoding was considered in complex wavelet packet based MC-CDMA [9] and in random spreading codes [10]. However, the effect of frequency offset has not been considered with precoding in the context of MC-CDMA.

Inter-carrier interference (ICI) due to carrier frequency offset in MC-CDMA can degrade system performance significantly. In order to simultaneously combat both the ICI and MAI to achieve reliable performance in MC-CDMA systems, a joint frequency offset estimation and multiuser detection was proposed using maximum-likelihood nonlinear optimization [5], constrained minimum output energy [6], multicarrier interference subspace rejection [7], and successive interference cancellation [8]. The proposed joint ICI and MAI rejection methods were shown to be able to reduce the ICI and MAI by providing simulation results [5] and the combination of simulation results and performance analysis [6]–[8]. Even though considerable efforts are undergoing for frequency offset estimation and correction, it is hardly possible to completely eliminate it without the remaining frequency offset. Our contribution in this paper is to predict accurately the system performance in the presence of the residual frequency offset. We apply precoding with random spreading sequences to MC-CDMA for system performance analysis. In summary, our principal contribution is the detailed analysis of the effects of frequency offset in MC-CDMA with precoding in synchronous downlink channels.

II. SYSTEM MODEL

A. CDMA

We begin with an additive white Gaussian noise (AWGN) channel. Let us consider a multiuser system with $K+1$ users (K interferers) sharing the channel, with the transmitted signal, $x(t)$, and the received signal, $r(t)$, given by

$$x(t) = \sum_{i=-\infty}^{\infty} \sum_{k=0}^{K} A_k s_i^k(t) b_i^k$$

and

$$r(t) = x(t) + n(t) \tag{1}$$

where $n(t)$ represents AWGN process with two-sided power spectral density σ^2. The signature waveform of the kth user, $s_i^k(t)$, is a binary random sequence defined in $iT_b \leq t < (i+1)T_b$ and consists of $n = T_b/T_c$ antipodal random binary chips where T_b and T_c are the bit and chip durations, respectively. A_k is the amplitude of the kth user, b_i^k represents the data bit of the kth user during the ith bit interval, and $b_i^k \in [-1, 1]$, $\forall k, i$. In flat fading channels, A_k is replaced with αA_k where α is the fading parameter. α can be considered as a constant during the symbol duration due to the interleaving implemented at

Manuscript received May 16, 2006; approved for publication by Lie-Liang Yang, Division II Editor, February 15, 2007.

The authors are with the Peter Kiewit Institute of Information Science, Technology & Engineering, Department of Computer and Electronics Engineering, University of Nebraska, U.S.A; email: wjang@unlnotes.unl.edu, moonlee@mail.unomaha.edu.

1229-2370/07/$10.00 © 2007 KICS

the transmitter [11]–[13]. We use the binary phase-shift keying (BPSK) modulation to simplify the presentation, although we can extend the analysis to other modulation schemes. Throughout this paper, we assume equal power for all users; therefore $A_k = A$ for all k for simplicity. It is also assumed that the signature waveforms have unit energy, $\int_{iT_b}^{(i+1)T_b} |s_i^k(t)|^2 dt = 1, \forall k, i$. The output of the matched filters (MF) matched to the kth user signature waveform at the kth mobile station, during the ith bit interval, is

$$y_i^k = \int_{iT_b}^{(i+1)T_b} r(t) s_i^k(t) dt = A_k b_i^k + \sum_{\substack{j=0 \\ j \neq k}}^{K} A_j R_i^{k,j} b_i^j + \eta \quad (2)$$

where the crosscorrelation of user k and user j during the ith bit interval is

$$R_i^{k,j} = \int_{iT_b}^{(i+1)T_b} s_i^k(t) s_i^j(t) dt \quad (3)$$

and η is an independent Gaussian random variable with variance equal to σ^2. By combining the MF outputs from $K+1$ receiving sites into a single vector,

$$\hat{\mathbf{y}} = [y_i^0 \; \ldots \; y_i^K]^\tau = \mathbf{RAb} + \mathbf{n} \quad (4)$$

where τ denotes the transpose of a matrix and $\mathbf{R}$ is a positive semidefinite[1] crosscorrelation matrix with the kth row and jth column equal to $R_i^{k,j}$. $\mathbf{n}$ is a zero-mean Gaussian noise vector with the covariance matrix equal to $\text{diag}\{\sigma_i^2\}$. Without loss of generality, we delete the suffix i in the matrices, but notice that $\mathbf{R}$ is a time varying matrix.

B. Precoding

With precoding, the vector of MF outputs is $\mathbf{y} = \mathbf{RTAb} + \mathbf{n}$. To find $\mathbf{T}$ we employ the MMSE criterion. The optimum precoding transformation $\mathbf{T}$ that minimizes the mean square error is $\mathbf{T} = \mathbf{R}^{-1}$ [14]. That is, with optimum precoding we have: $\mathbf{y} = \mathbf{Ab} + \mathbf{n}$. Thus, the multiuser detection problem is decoupled into $K+1$ separate single-user detection problems without the noise enhancement at the receiver. However, precoding results in a transmit power increase by the factor, $C = (\sum_{i=0}^{K} A_i^2 R_{ii}^{-1})/(\sum_{i=0}^{K} A_i^2)$ [14]. To maintain the average transmit power with precoding the same as without precoding, we modify the precoding transformation as $\mathbf{T} = \sqrt{C^{-1}}\mathbf{R}^{-1}$. The performance of precoding and that of receiver-based decorrelator are asymptotically equivalent in AWGN channels as [15]

$$P_e = Q\left\{\sqrt{\frac{A^2}{\sigma^2} C^{-1}}\right\} = Q\left\{\sqrt{2\left(\frac{E_b}{N_o}\right) C^{-1}}\right\}$$
$$= Q\left\{\sqrt{2\left(\frac{E_b}{N_o}\right)(1-\beta)}\right\} \quad (5)$$

where

$$\beta = \lim_{K,n \to \infty} \frac{K+1}{n} \quad (6)$$

[1]Throughout this paper we will assume that $\mathbf{R}$ is positive definite so that $\mathbf{R}^{-1}$ exists.

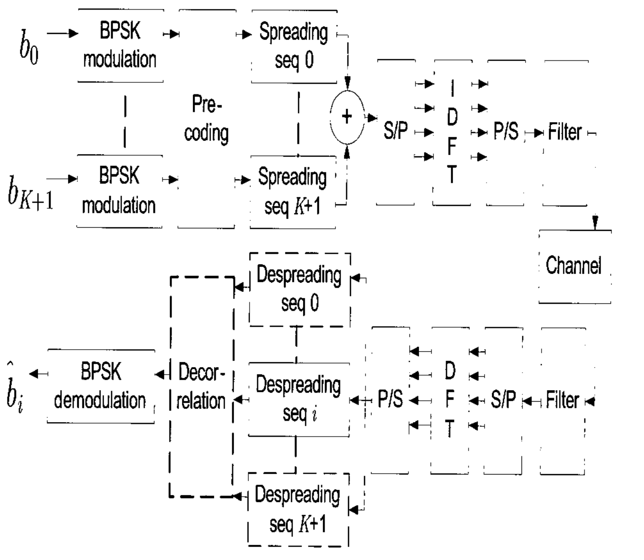


Fig. 1. Downlink MC-CDMA transmitter and receiver (solid line: MC-CDMA with precoding).

where n is the spreading factor and $Q(\xi) = \frac{1}{\sqrt{2\pi}} \int_{\xi}^{\infty} e^{-\frac{x^2}{2}} dx$. P_e denotes the probability of bit error. E_b is the bit energy, and N_o is the one-side noise power spectral density. The decorrelation detection that decouples the signal at the receiver is to multiply $\hat{\mathbf{y}}$ by the inverse crosscorrelation matrix, $\mathbf{R}^{-1}$. Thus, $\tilde{\mathbf{y}} = \mathbf{Ab} + \mathbf{R}^{-1}\mathbf{n}$. Apart from the attractive asymptotic efficiency properties, further justification for the decorrelation detection is provided by the fact that it is known to be the solution to the maximum likelihood detector when the energies are not known by the receiver [16]. We add the decorrelation detection simulations results to some figures just for verification since the BER of precoding and decorrelation detection with random spreading sequences displays the same performance in AWGN channels [15].

C. MC-CDMA

Fig. 1 shows the block diagram of MC-CDMA. The $K+1$ sequences of symbols, obtained by BPSK modulation, are spread and added together. We have used random spreading sequences under the assumption of perfect synchronization. We add a cyclic prefix at the output of the inverse discrete Fourier transform (IDFT) to protect the MC-CDMA from inter-symbol interference (ISI) due to multipath spreads and against timing-offset errors at the receiver. At the receiver side, after removing the cyclic prefix, the discrete Fourier transform (DFT) is applied to the M useful samples of each multicarrier symbol.

Let v_m be the summation of $K+1$ users' mth chip during the ith bit interval with precoding, then $v_m = \sum_{k=0}^{K} \sum_{l=0}^{K} (\mathbf{R}^{-1})_i^{k,l} b_i^l c_{i,m}^k$, $m = 0, \cdots, M-1$, where $c_{i,m}^k$ is the kth user's mth chip in the ith bit interval. For simple presentation, we assume that the spreading factor (n) is equal to the number of sub-carriers (M). However, we can extend the result to arbitary spreading factors with a little additional complexity. The sequence v_m is serial-to-parallel (S/P) converted and its IDFT is taken. The IDFT of v_m corresponding to a block of M symbols is

$$w_h = \frac{1}{M}\sum_{m=0}^{M-1} v_m \exp\left(j2\pi m \frac{h}{M}\right), \quad h = 0,\cdots, M-1. \quad (7)$$

The P/S conversion and zero-th order interpolation give the continuous-time signal, $w(t) = \sum_{h=0}^{M-1} w_h q(t - hT_c)$, where $q(t)$ is the unit rectangular pulse over a chip interval. Let the normalized frequency offset, ϵ, be $f_o/\Delta f$, where f_o is a frequency offset and $\Delta f = 1/MT_c$. At the receiver input, the noiseless component of the received signal during the bit interval, impaired by the frequency offset, is

$$\hat{r}(t) = \frac{1}{M}\sum_{h=0}^{M-1}\sum_{m=0}^{M-1} v_m q(t - hT_c)\exp\left\{j2\pi(m+\epsilon)\frac{h}{M}\right\}. \quad (8)$$

The sampled values of the noiseless component of the received signal at the output of the receiver filter are

$$\begin{aligned} y_h &= \frac{1}{T_c}\int_{hT_c}^{(h+1)T_c} \hat{r}(t) q(t - hT_c) dt \\ &= \frac{1}{M}\sum_{m=0}^{M-1} v_m \exp\left(j2\pi h \frac{m+\epsilon}{M}\right), \\ & h = 0,\cdots, M-1. \end{aligned} \quad (9)$$

The frequency offset included in Eq. (8) produces ICI, and y_h in (9) is the hth noiseless DFT input. Now, the M values of y_h corrupted by AWGN samples are fed to the DFT. Thus the DFT output z_g is

$$\begin{aligned} z_g &= \sum_{h=0}^{M-1} y_h \exp\left(-j2\pi h\frac{g}{M}\right) + \eta_g \\ &= \sum_{h=0}^{M-1}\sum_{m=0}^{M-1} v_m \frac{1}{M}\exp\left\{j\frac{2\pi}{M}(m+\epsilon-g)h\right\} + \eta_g, \\ g &= 0,\cdots, M-1 \end{aligned} \quad (10)$$

where η_g is the noise variable. The output of the DFT is then fed into the despreading block after the P/S conversion. Precoding can be applied before spreading followed by IDFT at the transmitter. Decorrelation detection can be employed after DFT and despreading at the receiver as shown in Fig. 1.

III. MULTIPLE ACCESS AND INTER-CARRIER INTERFERENCE

The variable of the MAI, ICI and noise for the desired user, during the ith bit duration without precoding, can be written as [18]

$$f_i^d = \sum_{m=0}^{M-1} u_m S_m \quad (11)$$

where [17]

$$\begin{aligned} S_m &= \frac{1}{M}\sum_{h=0}^{M-1}\exp\left\{j\frac{2\pi}{M}(m+\epsilon)h\right\} \\ &= \frac{\sin(\pi(m+\epsilon))}{M\sin(\frac{\pi}{M}(m+\epsilon))}\exp\left(j\{\pi(1-\frac{1}{M})(m+\epsilon)\}\right) \end{aligned} \quad (12)$$

and u_m is the aggregated MAI and noise in all the sub-carriers defined as

$$u_m = \sum_{l=0}^{M-1} c_{i,l}^d \mu_m + n_m$$

and

$$\mu_m = \sum_{\substack{k=0 \\ k\neq d}}^{K} b_i^k c_{i,m}^k \quad (13)$$

where $c_{i,m}^d$ is the desired user's mth chip in the ith bit interval. The conditional pdf of the MAI, ICI and noise with random spreading sequences (without precoding) in frequency selective fading channels is given as [18]

$$\begin{aligned} f_{MC-CDMA}(x|\vec{\alpha}) &= \frac{1}{2^{nKM}}\sum_{i_0=0}^{nK}\cdots\sum_{i_{M-1}=0}^{nK} \\ & \binom{nK}{i_0}\cdots\binom{nK}{i_{M-1}}\frac{1}{\sqrt{2\pi\sigma^2\sum_{k=0}^{M-1}S_k^2}} \\ & \exp\Big\{-\{x - A[K\sum_{k=0}^{M-1}\alpha_k S_k \\ & -2(\sum_{k=0}^{M-1} i_k\alpha_k S_k)/n]\}^2/(2\sigma^2\sum_{k=0}^{M-1}S_k^2)\Big\} \end{aligned} \quad (14)$$

where the fading vector $\vec{\alpha} = [\alpha_0 \cdots \alpha_{M-1}]$ and α_i is the fading parameter in the ith sub-carrier. The corresponding variance is [18]

$$\sigma_{MC-CDMA}^2 = \sigma^2\sum_{k=0}^{M-1}S_k^2 + \frac{K}{n}A^2\sum_{k=0}^{M-1}\alpha_k^2 S_k^2. \quad (15)$$

With decorrelation detection, multiple access interference in S_0 can be eliminated, but ICI is retained. Furthermore, noise and ICI are enhanced by $1/(1-\beta)$, thus [15], [18]

$$\begin{aligned} \sigma_{MC-CDMA,decor}^2 &= \frac{\sigma^2}{(1-\beta)}\sum_{k=0}^{M-1}S_k^2 \\ &+ \frac{K}{n}\frac{A^2}{(1-\beta)}\sum_{k=1}^{M-1}\alpha_k^2 S_k^2. \end{aligned} \quad (16)$$

Precoding reduces the received signal power by a factor of $(1-\beta)$ and eliminates multiple access interference in S_0 but retains the same ICI. Thus [15], [18]

$$\sigma_{MC-CDMA,pre}^2 = \sigma^2\sum_{k=0}^{M-1}S_k^2 + \frac{K}{n}A^2\sum_{k=1}^{M-1}\alpha_k^2 S_k^2. \quad (17)$$

IV. PERFORMANCE ANALYSIS

From Eqs. (5) and (17), we can apply the Gaussian approximation of the MAI and ICI (since the noise is already Gaussian) to AWGN channels [18]. Then the Gaussian approximation BER for precoding or decorrelation detection can be written as [15], [18]

$$P_b = Q\left(\sqrt{\frac{S_0^2A^2(1-\beta)}{\sigma^2\sum_{k=0}^{M-1}S_k^2+\frac{K}{n}A^2(\sum_{j=1}^{M-1}S_j^2)}}\right) = Q\left(\sqrt{\frac{2S_0^2(E_b/N_o)(1-\beta)}{\sum_{k=0}^{M-1}S_k^2+2\frac{K}{n}(E_b/N_o)(\sum_{j=1}^{M-1}S_j^2)}}\right) \quad (18)$$

which reduces to CDMA without ICI. For flat fading channels, the fading parameter $\gamma = \alpha^2$ where $\alpha_0 = \alpha_1 = \cdots = \alpha_{M-1} = \alpha$, and the corresponding BER performance can be shown as [15], [18]

$$P_b = \int_0^\infty Q\left(\sqrt{\frac{2S_0^2\gamma(E_b/N_o)(1-\beta)}{\sum_{k=0}^{M-1}S_k^2+2\frac{K}{n}\gamma(E_b/N_o)(\sum_{j=1}^{M-1}S_j^2)}}\right)e^{-\gamma}d\gamma \quad (19)$$

which reduces to AWGN channel with $\gamma = 1$. In frequency selective fading channels the BER performance can be written as follows where $\vec{\gamma} = [\gamma_0 \cdots \gamma_{M-1}]$, $\gamma_i = \alpha_i^2$, and $f_{\vec{\gamma}}(\vec{\gamma})$ is the joint pdf of $\gamma_0, \cdots, \gamma_{M-1}$. Then the Gaussian approximation BER can be obtained as follows as shown in Appendix

$$P_b \approx \int_0^\infty Q\left(\sqrt{\sum_{i=0}^{M-1}\frac{2S_0^2\gamma_i(E_c/N_o)(1-\beta)}{\sum_{k=0}^{M-1}S_k^2+2K\left(E_c/N_o\right)\left(\sum_{j=1}^{M-1}\gamma_jS_j^2\right)}}\right)f_{\vec{\gamma}}(\vec{\gamma})d\vec{\gamma} \quad (20)$$

which reduces to flat fading channel with $\gamma_0 = \cdots = \gamma_{M-1} = \gamma$. E_c/N_o is the chip energy-to-noise ratio and equal to $(E_b/N_o)/n$. Eq. (20) results from (18) and (24).

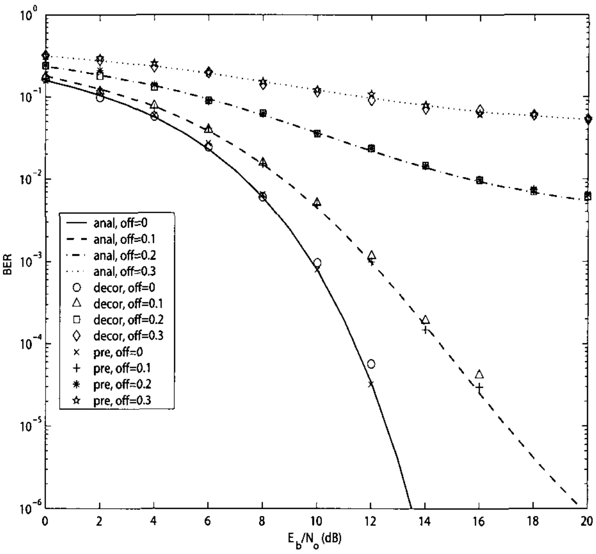


Fig. 2. Analytical and simulation results, precoding and decorrelation, $K = 63$, $n = 128$, $\beta = 0.5$, frequency offset=0, 0.1, 0.2, 0.3, AWGN.

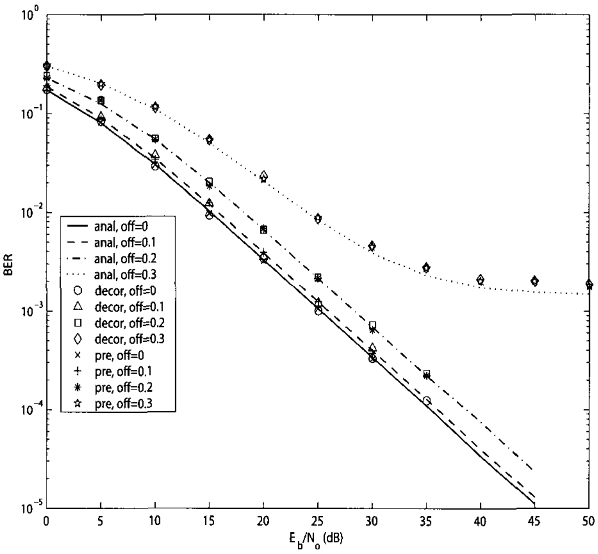


Fig. 3. Analytical and simulation results, precoding and decorrelation, $K = 31$, $n = 128$, $\beta = 0.25$, frequency offset=0, 0.1, 0.2, 0.3, Rayleigh fading.

V. NUMERICAL RESULTS

In Fig. 2 the BER of the system loading, $\beta = 0.5$ (64 users, 63 interferers and 128 chips/bit), and frequency offset of 0, 0.1, 0.2, and 0.3 is shown for AWGN channels. We observe that the BER saturates for the frequency offset of 0.2 and 0.3. In Fig. 3, we show the BER in flat fading channels for the system loading, $\beta = 0.25$ (32 users and 128 chips/bit). The BER is linear with the signal-to-noise ratio (SNR) for frequency offset of 0, 0.1, and 0.2. However, with 0.3 frequency offset, the BER begins to saturate at the bit energy-to-noise ratio (E_b/N_o) of 25 dB.

As system loading increases to $\beta = 0.5$ (64 users and 128 chips/bit) in Fig. 4, the effect of frequency offset is more distinct. The BER is linear with SNR for 0 and 0.1 frequency offsets but begins to saturate at $E_b/N_o = 20$ dB and 10 dB for 0.2 and 0.3 frequency offsets, respectively. With $\beta = 0.75$ (96 users and 128 chips/bit) in Fig. 5, we see that the BER already begins to saturate for 0.1 frequency offset. All figures show that precoding and decorrelation demonstrate the same performance and agree well with the analytical result. The critical frequency offsets in flat fading channels are 0.2, 0.1, and 0 for the system loading of 0.25, 0.5, and 0.75, respectively. Beyond the critical point, the BER saturates and the system performance becomes catastrophic. Therefore any further carrier frequency mismatch should be avoided.

Fig. 6 displays the BER in frequency selective fading channels. For numerical results, we assume three independent coherent frequency bands and carrier fadings with unit correlation coefficient within each coherent frequency band. The simulation of precoding shows a similar performance of the analytical

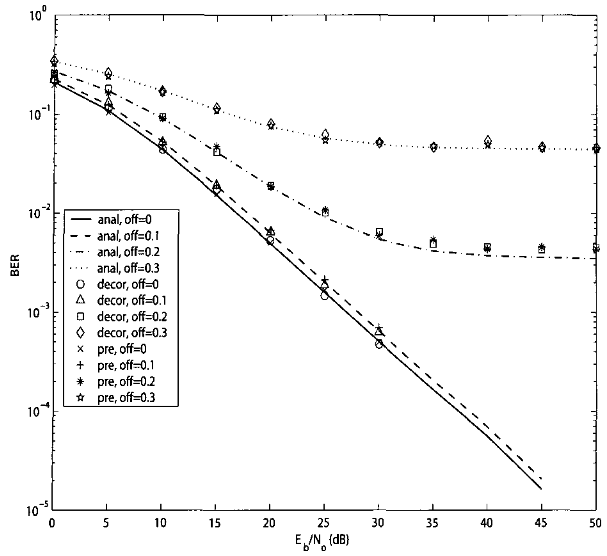


Fig. 4. Analytical and simulation results, precoding and decorrelation, $K = 63$, $n = 128$, β=0.5, frequency offset=0, 0.1, 0.2, 0.3, Rayleigh fading.

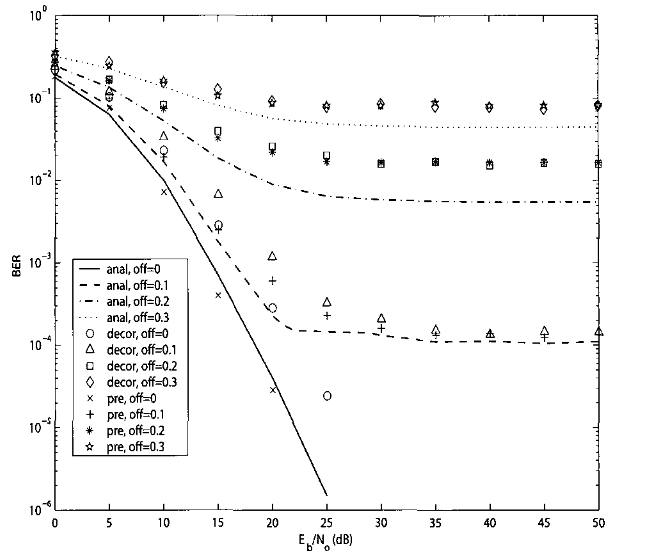


Fig. 6. Analytical and simulation results, precoding and decorrelation, $K = 63$, $n = 128$, β=0.5, frequency offset=0, 0.1, 0.2, 0.3, frequency selective fading.

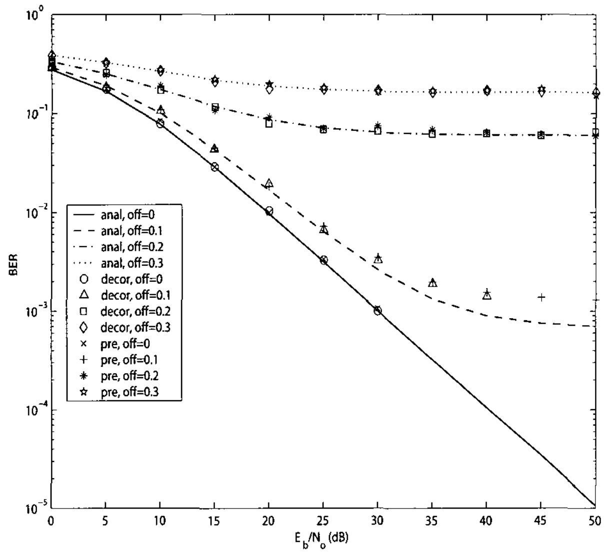


Fig. 5. Analytical and simulation results, precoding and decorrelation, $K = 95$, $n = 128$, β=0.75, frequency offset=0, 0.1, 0.2, 0.3, Rayleigh fading.

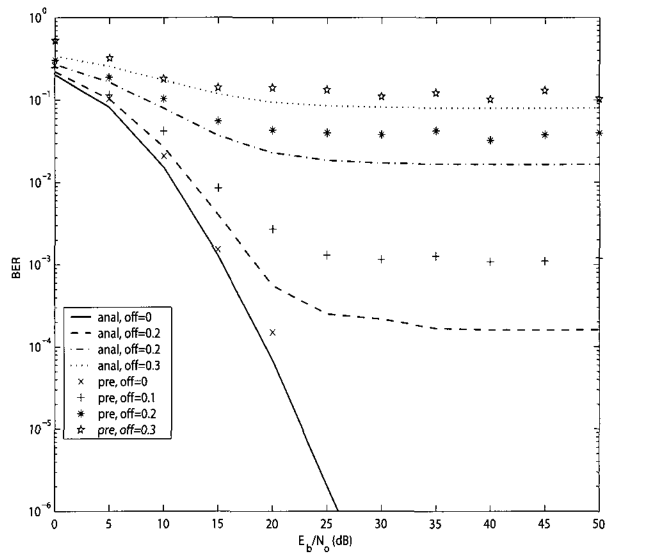


Fig. 7. Analytical and simulation results, precoding, $K = 77$, $n = 128$, β=0.6, frequency offset=0, 0.1, 0.2, 0.3, frequency selective fading.

result for 0 and 0.1 frequency offsets but some discrepancies for 0.2 and 0.3 offsets. The difference is originated from the approximations we introduced in the performance analysis in frequency selective fading channels. The simulation result of decorrelation detection is a little worse than precoding due to noise enhancement that is larger than power increase in precoding. The performance of an MC-CDMA system with the same parameters in different channels can be compared in Figs. 2, 4, and 6. The performance in frequency selective fading channels is better than flat fading channels due to frequency diversity. In Fig. 7 we increase the system loading by 10 percents from Fig. 6, and we can observe a gradual performance degradation for all frequency offsets both simulation and analysis.

VI. CONCLUSIONS

The analytical BER performance in terms of frequency offset and system loading was obtained and verified with the simulation result. Our contribution is to predict accurately the system performance in the presence of the residual frequency offset. MC-CDMA with precoding agrees well with the analytical result and demonstrates a similar performance with decorrelation detection. There is a critical frequency offset that varies with system loading. Beyond the critical frequency offset, the BER saturates and the system performance becomes catastrophic.

APPENDIX

FINDING C^{-1} FOR $K, n \to \infty$ IN FREQUENCY SELECTIVE FADING CHANNELS.

Let ζ be the eigen value of the crosscorrelation matrix of random spreading sequences, then $E[\zeta^{-1}] = 1/(1-\beta)$ [15]. We may rewrite Eq. (3) as

$$R_i^{k,j} = \int_{iT_b}^{(i+1)T_b} s_i^k(t) s_i^j(t) dt = \mathbf{s}^k(\mathbf{s}^j)^\tau$$

or

$$\mathbf{R} = \mathbf{S}\mathbf{S}^\tau \tag{21}$$

where $\mathbf{s}^k$ is the vector of the kth user's spreading sequence of $\pm 1/\sqrt{n}$ during the ith bit interval, and $\mathbf{S}$ is the matrix where the kth row is $\mathbf{s}^k$. Let λ be the eigen value of the crosscorrelation matrix incorporated with fading factors, $\hat{\mathbf{R}} = \mathbf{S}\Gamma^2\mathbf{S}^\tau$ where Γ is the diagonal matrix of fading factors, $\alpha_0, \cdots, \alpha_{M-1}$. Since $\hat{\mathbf{R}}$ is a Hermitian matrix,

$$\text{trace}(\hat{\mathbf{R}}) = \sum_{i=0}^{M-1} \alpha_i^2 = \sum_{k=0}^{K} \lambda_k = \frac{1}{n} \sum_{i=0}^{M-1} \alpha_i^2 \sum_{k=0}^{K} \zeta_k$$

Hence,

$$E[\lambda] = \frac{1}{n} \sum_{i=0}^{M-1} \alpha_i^2 E[\zeta] \tag{22}$$

where $\sum_{k=0}^{K} \zeta_k = n$ since $\mathbf{R} = \mathbf{S}\mathbf{S}^\tau$ is a normalized crosscorrelation matrix. Then, the power control factor C^{-1} in frequency selective fading channels can be written as

$$\begin{aligned} C = E[\lambda^{-1}] \geq \frac{1}{E[\lambda]} &= \frac{1}{\sum_{i=0}^{M-1} \alpha_i^2 E[\zeta]/n} \\ &\leq \frac{1}{\sum_{i=0}^{M-1} \alpha_i^2 \{1/(E[\zeta^{-1}]n)\}} \\ &= \frac{1}{\sum_{i=0}^{M-1} \alpha_i^2 (1-\beta)/n}. \end{aligned} \tag{23}$$

Since the two inequalities in Eq. (23) are introduced due to the convexity property of the same inverse function, we will use an approximation notation,

$$C^{-1} \approx \sum_{i=0}^{M-1} \alpha_i^2 (1-\beta)/n. \tag{24}$$

REFERENCES

[1] X. Wu, Q. Yin, J. Zhang, and K. Deng, "Time-domain multiuser detection for MC-CDMA systems without cyclic prefix," in *Proc. IEEE ICC*, vol. 2, Apr. 28 - May 2, 2002, pp. 921–925.

[2] L. Brunel, "Multiuser detection techniques using maximum likelihood sphere decoding in multicarrier CDMA systems," *IEEE Trans. Wireless Commun.*, vol. 3, no. 3, pp. 949–957, May 2004.

[3] G. Zhang, G. Bi, and L. Zhang, "Blind multiuser detection for asynchronous MC-CDMA systems without channel estimation," *IEEE Trans. Veh. Technol.*, vol. 53, no. 4, pp. 1001–1013, Jul. 2004.

[4] J. Li, K. B. Letaief, and Z. Cao, "Space-time Turbo multiuser detection for coded MC-CDMA," *IEEE Trans. Wireless Commun.*, vol. 4, no. 2, pp. 538–549, Mar. 2005.

[5] H. Y. Lu and W. H. Fang, "Joint frequency offset estimation and multiuser detection using genetic algorithm in MC-CDMA," in *Proc. IEEE ISCAS*, vo. 2, 23–26 May 2006, pp. 1726–1729.

[6] B. Seo and H. M. Kim, "Frequency offset estimation and multiuser detection for MC-CDMA systems," in *Proc.*, MILCOM, vol. 2, 7–10 Oct. 2002, pp. 804–807.

[7] B. Smida, S. Affes, K. Jamaoui, and P. Mermelstein, "A multicarrier-CDMA receiver with full interference suppression and carrier frequency offset recovery," in *Proc. IEEE 6th Workshop on Signal Processing Advances in Wireless Communications*, 5–8 June 2005, pp. 435–439.

[8] R. M. A. Tannious and M. A. Mokhtar, "Analysis of uplink successive interference cancellation CDMA system with carrier frequency offsets," in *Proc. the Twenty-First National Radio Science Conference*, 16–18 Mar. 2004, pp. C22–1–10.

[9] X. Yu, X. Zhang, and G. Bi, "Performance of complex wavelet packet based multicarrier CDMA system with precoding technique," in *Proc. Joint Conference of the 10th Asia-Pacific Conference on Communications and the 5th International Symposium on Multi-Dimensional Mobile Communications*, vol. 2, Aug. 29–Sept. 1, 2004, pp. 923–926.

[10] M. Debbah, P. Loubaton, and M. de Courville, "Asymptotic performance of successive interference cancellation in the context of linear precoded OFDM systems," *IEEE Trans. Commun.*, vol. 52, no. 9, pp. 1444–1448, Sept. 2004.

[11] J. G. Proakis, *Digital Communications*, 4th Ed. New York: McGraw-Hill, 2001.

[12] S. Haykin, *Communication Systems*, 4th Ed. John Wiley & Sons, 2001.

[13] S. L. Miller, M. Honig, and L. B. Milstein, "Performance analysis of MMSE receivers for DS-CDMA in frequency selective fading channels," *IEEE Trans. Commun.*, vol. 48, no. 11, pp. 1919–1929, Nov. 2000.

[14] B. Vojcic and W. M. Jang, "Transmitter precoding in synchronous multiuser communications," *IEEE Trans. Commun.*, vol. 46, no. 10, pp. 1346–1355, Oct. 1998.

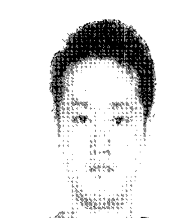
[15] W. M. Jang, L. Nguyen, and M. Hempel, "Precoded random spreading multiple access system in AWGN channels," *IEEE Trans. Wireless Commun.*, vol. 3, no. 5, pp. 1477–1480, Sept. 2004.

[16] S. Verdu, *Multiuser Detection*, p. 234. New York: Cambridge University Press, 1998.

[17] J. Armstrong, "Analysis of new and existing methods of reducing intercarrier interference due to carrier frequency offset in OFDM," *IEEE Trans. Commun.*, vol. 47, no. 3, pp. 365–369, Mar. 1999.

[18] W. M. Jang, L. Nguyen, and P. Bidarkar, "MAI and ICI of synchronous downlink MC-CDMA with frequency offset," *IEEE Trans. Wireless Commun.*, vol. 5, no. 3, pp. 693–703, Mar. 2006.

Won Mee Jang received the B.A. degree in Computer Science from University of Minnesota in 1984, the M.S. degree in Computer Engineering from George Mason University in 1987, and the D.Sc. in Electrical Engineering from the George Washington University in 1996. She was with Information and Electronics Division at the Research Institute of Science and Technology (RIST), Korea, from 1988 to 1991. From 1995 to 1998, she was a wireless engineer at Comsearch, VA, U.S.A. She has been working in the Department of Computer and Electronics Engineering at the University of Nebraska since 1998. Her research interests include spread spectrum, satellite communications, CDMA, OFDM, signal modulation/demodulation, coding, information theory, and communication theory.



Moon Woo Lee received the B.S. degree in Electronics Engineering from the University of Nebraska-Lincoln, in 2003 and the M.S. degree in Telecommunications Engineering from the University of Nebraska-Lincoln, in 2005. His research interests include multicarrier CDMA, multiuser detection and precoding system. Currently, he is working as a product engineer at LG Electornics, Huntsville, Alabama, U.S.A.