Exact Error and Outage Probability Formulas for Alamouti Space Time Code $2 \times J$

Hyung Yun Kong, Ho Van Khuong, and Doo Hee Nam

Abstract: **Alamouti space-time code (STC) is a part of the UMTS-WCDMA standard. However, up to the best of our knowledge no exact closed-form outage and bit error probability (BEP) formulas for this famous code exists. Evaluating its performance through simulations is time-consuming and therefore, there should be analytical performance graphs to serve as a reference which are derived in this paper for coherently MPSK-modulated data. Additionally, analytical results take into account different channel fading levels from transmit antennas to receive ones.**

Index Terms: **AWGN, Outage probability, Rayleigh fading, Space-Time Code (STC).**

I. INTRODUCTION

In wireless networks, signal fading arising from multi-path propagation is a particularly severe channel impairment that can be mitigated through the use of diversity [1] which is provided using temporal, frequency, polarization, and spatial resources. Among them, spatial diversity which relies on the principle that signals transmitted from geographically separated transmitters, and/or to geographically separated receivers, experience fading that is independent has been intensively researched recently. This kind of diversity technique usually uses a class of special codes called space-time codes (STCs) [2] whose performance was proved to be very good in flat Rayleigh fading channel by simulation programs. The first STC was suggested by Alamouti [3] with simple structure and high code rate (equal to 1), that is, there is no bandwidth sacrifice or equivalently, the application of STC remains bandwidth as that of uncoded data. As a result, it is selected as a part of the UMTS-WCDMA standard [4].

As usual, the bit error probability (BEP) or symbol error rate (SER) is used to evaluate the performance of a communications system. Additionally, an alternative measure, called outage probability which is defined as the probability of failing to achieve simultaneously a signal-to-noise ratio sufficient to give satisfactory reception, is very appropriate to participate in this work because in wireless systems a receiver is likely to successfully decode transmitted data only if received signal-to-noise ratio remains above a certain minimum required (threshold) ratio.

Even though there are some researches on performance analysis of space-time codes [5]–[8], they only focus on finding pairwise error probability (PEP) for the derivation of union bounds to the BEP. Moreover, channels' symmetry (that is, path gains from transmit antennas (Tx) to receive ones (Rx) are Rayleigh-distributed random variables with the same variance) is assumed. [5] presented the application of the PEP to approximate the bit error rate (BER) of Alamouti code for 2Tx-2Rx but still using the assumption of the channels' symmetry. On the contrary, we will derive the exact outage probability and BER expressions for this code in a communications system with two transmit antennas and J receive antennas which account for the different conditions of propagation paths to the receiver. We consider such possible scenarios because in order for path gains to be independent, antenna elements should be spaced enough apart [9]. Then the signals arriving at each receive antenna may experience the fading with different average power. In addition, these formulas are generalized for MPSK modulation under the flat Rayleigh fading channel plus AWGN (Additive White Gaussian Noise).

The rest of this paper is organized as follows. The analytical performance expressions are derived in Section II. Section III presents simulation and numerical results as well as elaborative discussions of diversity gain, the symmetry and asymmetry of channels. Finally, the paper is concluded in Section IV.

II. FORMULA ESTABLISHMENT

Alamouti STC for two transmit antennas is represented by a transmission matrix

$$STC = \begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix} \quad (1)$$

where x_1 and x_2 are two consecutive MPSK-modulated symbols having the same amplitude at the input of the STC encoder; $(.)^*$ denotes complex conjugate operator.

The signal transmission on two transmit antennas is processed as follows. At the first time slot, x_1 and x_2 are simultaneously sent on antenna 1 and 2, respectively. Then, $-x_2^*$ and x_1^* continue to be transmitted on antenna 1 and 2 at the second time slot.

Channel model

The flat fading channel is usually assumed for most spatial diversity systems in which path gains $\alpha_{t,j}$ from transmit antenna t to receive antenna j are modeled as samples of independent zero-mean complex Gaussian random variables (r.v.s) with variances $2\sigma_{tj}^2$, and are constant during two-symbol durations but independently change over longer intervals.

Receiver

Consider the case of J receive antennas. The received signal is a superposition of signals from two transmit antennas attenuated by fading and corrupted by noise given by

$$r_{1,j} = x_1\alpha_{1,j} + x_2\alpha_{2,j} + n_{1,j} \quad (2)$$

$$r_{2,j} = -x_2^*\alpha_{1,j} + x_1^*\alpha_{2,j} + n_{2,j} \quad (3)$$

Manuscript received April 15, 2005; approved for publication by Xiang-Gen Xia, Division I Editor, April 10, 2007.

The authors are with the Department of Electrical Engineering, University of Ulsan, Korea, email: {hkong, duheeya}@mail.ulsan.ac.kr, khuongho2001@yahoo.com.

1229-2370/07/$10.00 © 2007 KICS

where $r_{1,j}$ and $r_{2,j}$ are the received signals at the $1st$ and $2nd$ time-durations of receive antenna j ($j = 1, 2, \cdots, J$), respectively; $n_{1,j}$ and $n_{2,j}$ are independent independent zero-mean complex Gaussian r.v.s with variance σ^2.

Assuming coherent detection, maximum likelihood (ML) decoding can be achieved based only on linear processing at the receiver [2]. As a result, the symbols x_1 and x_2 are estimated by

$$x_1' = \sum_{j=1}^{J} \left(r_{1,j}\alpha_{1,j}^* + r_{2,j}^*\alpha_{2,j}\right) \quad (4)$$

$$x_2' = \sum_{j=1}^{J} \left(r_{1,j}\alpha_{2,j}^* - r_{2,j}^*\alpha_{1,j}\right). \quad (5)$$

Substituting $r_{1,j}$ and $r_{2,j}$ in (2)–(3) into (4)–(5), we obtain

$$x_1' = \sum_{j=1}^{J} \left(|\alpha_{1,j}|^2 + |\alpha_{2,j}|^2\right) x_1 + n_1 \quad (6)$$

$$x_2' = \sum_{j=1}^{J} \left(|\alpha_{1,j}|^2 + |\alpha_{2,j}|^2\right) x_2 + n_2 \quad (7)$$

where

$$n_1 = \sum_{j=1}^{J} n_{1,j}\alpha_{1,j}^* + n_{2,j}^*\alpha_{2,j} \quad (8)$$

$$n_2 = \sum_{j=1}^{J} n_{1,j}\alpha_{2,j}^* - n_{2,j}^*\alpha_{1,j}\,. \quad (9)$$

(6)–(7) shows that STC provides exactly performance as the $2J$-level receive maximum ratio combining.

Let

$$\lambda = \sum_{j=1}^{J} \left(|\alpha_{1,j}|^2 + |\alpha_{2,j}|^2\right) = \sum_{t=1}^{2}\sum_{j=1}^{J} z_{tj} \quad (10)$$

where $z_{tj} = |\alpha_{t,j}|^2$ are exponentially distributed r.v.s with means of $2\sigma_{tj}^2 = 1/\lambda_{tj}$.

Now we consider three possible cases of the r.v.s z_{tj}.

Case 1: They have the same variance denoted as $2\sigma_\alpha^2$

This scenario happens when all signals from transmit antennas to receive antennas experience identical fading level. Since $\alpha_{t,j}$ are zero-mean complex Gaussian r.v.s with variance $2\sigma_\alpha^2$, λ is chi-square-distributed with $4J$ degrees of freedom

$$f_\lambda(\lambda) = \frac{1}{\sigma_\alpha^{4J} 2^{2J}\Gamma(2J)} \lambda^{2J-1} e^{-\lambda/2\sigma_\alpha^2} \quad (11)$$

where $\lambda \geq 0$ and

$$\Gamma(n) = \int_0^\infty t^{n-1}e^{-t}dt. \quad (12)$$

Case 2: They have the different variances represented as $2\sigma_{tj}^2 = 1/\lambda_{tj}$

This is an asymmetric case where transmitted signals are attenuated by distinct fading degrees. Since z_{tj} are statistically independent r.v.s, the characteristic function of λ denoted as $\Psi_\lambda(iw)$ is equal to the product of characteristic functions of the individual r.v.s z_{tj}, that is,

$$\Psi_\lambda(iw) = \prod_{t=1}^{2}\prod_{j=1}^{J} \Psi_{z_{tj}}(iw) \quad (13)$$

where $i^2 = -1$ and $\Psi_{z_{tj}}(iw)$ is the characteristic function of z_{tj} given by

$$\Psi_{z_{tj}}(iw) = \frac{1}{1 - iw/\lambda_{tj}}\,.$$

For the sake of exposition, we call $\Psi_{z_{tj}}(iw)$ the Fourier transform of $f_\lambda(\lambda)$ which is slightly different from the exact definition of the Fourier transform.

Taking Heaviside's expansion on the product in (13), we can rewrite $\Psi_\lambda(iw)$ in the more compact form as

$$\Psi_\lambda(iw) = \sum_{t=1}^{2}\sum_{j=1}^{J} \frac{A_{tj}}{1 - iw/\lambda_{tj}} \quad (14)$$

where

$$A_{tj} = \prod_{n=1, n\neq t}^{2}\ \prod_{m=1, m\neq j}^{J} \frac{1}{1 - \lambda_{tj}/\lambda_{nm}}\,. \quad (15)$$

From (14), it is straightforward to find the pdf of λ as follows

$$f_\lambda(\lambda) = \sum_{t=1}^{2}\sum_{j=1}^{J} A_{tj}\lambda_{tj}e^{-\lambda_{tj}\lambda} \quad (16)$$

where $\lambda \geq 0$.

Case 3: Generalization of the above two cases

Assume that there are K r.v.s of the same variance $2\sigma_\alpha^2 = 1/\lambda_e$ among $2J$ r.v.s z_{tj} and $M = 2J - K$ remaining r.v.s of different variances denoted as $2\sigma_m^2 = 1/\lambda_m$ where $m = 1, \cdots, M$. Then, the characteristic function of λ from (13) is of the following form:

$$\begin{aligned}\Psi_\lambda(iw) &= \left(\frac{1}{1 - iw/\lambda_e}\right)^K \prod_{m=1}^{M} \frac{1}{1 - iw/\lambda_m} \\ &= \sum_{m=1}^{M} \frac{E_m}{1 - iw/\lambda_m} + \sum_{k=1}^{K} \frac{A_k}{\left(1 - iw/\lambda_e\right)^k}\end{aligned}$$

where

$$E_m = \left(\frac{1}{1 - \lambda_m/\lambda_e}\right)^K \prod_{u=1, u\neq m}^{M} \frac{1}{1 - \lambda_m/\lambda_u}.$$

To find the coefficients A_k, we can solve the system of K equations which is established by randomly choosing K distinct values of iw but not equal to λ_m and λ_e [10]. Denote K values of iw as B_n with $n = 1, 2, \cdots, K$, we obtain the equation system as

$$\left(\frac{1}{1 - \frac{B_n}{\lambda_e}}\right)^K \prod_{m=1}^{M} \frac{1}{1 - \frac{B_n}{\lambda_m}} = \sum_{m=1}^{M} \frac{E_m}{1 - \frac{B_n}{\lambda_m}} + \sum_{k=1}^{K} \frac{A_k}{\left(1 - \frac{B_n}{\lambda_e}\right)^k}\,.$$

Thus, $A = [A_1\, A_2 \cdots A_K]^T$ is given by

$$A = C^{-1}D$$

where $[.]^T$ is a transpose operator and C is a KxK square matrix whose elements C_{uv} are given by

$$C_{uv} = \frac{1}{(1 - B_u/\lambda_e)^v}$$

and $D = [D_1\, D_2 \cdots D_K]^T$ with

$$D_u = \left(\frac{1}{1 - B_u/\lambda_e}\right)^K \prod_{m=1}^{M} \frac{1}{1 - B_u/\lambda_m} - \sum_{m=1}^{M} \frac{E_m}{1 - B_u/\lambda_m}$$

and $u, v = 1, 2, \cdots, K$.

Finally, the pdf of λ is determined from the inverse Fourier transform of $\Psi_\lambda(iw)$ as follows

$$f_\lambda(\lambda) = \sum_{m=1}^{M} E_m \lambda_m e^{-\lambda_m \lambda} + \sum_{k=1}^{K} A_k \frac{\lambda^{k-1}\lambda_e^k e^{-\lambda_e \lambda}}{\Gamma(k)} \quad (17)$$

where $\lambda \geq 0$.

Due to the assumption of the mutual independence of $n_{t,j}$, n_1, and n_2 are zero-mean Gaussian r.v.s with the same variance ς^2, given the channel realizations

$$\varsigma^2 = \sum_{j=1}^{J} \left(|\alpha_{1,j}|^2 + |\alpha_{2,j}|^2\right)\sigma^2 = \lambda\sigma^2.$$

The signal-to-noise ratio at the receiver can be calculated from (6)–(7) as $\gamma_r = \lambda^2/\varsigma^2 = \lambda/\sigma^2$ with the assumption that the amplitudes of modulated symbols are normalized to be 1. Therefore, the pdf of γ_r is easily derived as

$$f_{\gamma_r}(\gamma_r) = \sigma^2 f_\lambda(\sigma^2\gamma_r). \quad (18)$$

A. Outage Probability Analysis

The outage probability is defined as the probability that γ_r is less than or equal to an threshold SNR γ

$$P_{outage}(\gamma) = P(\gamma_r \leq \gamma) = \int_0^{\gamma} f_{\gamma_r}(\gamma_r)d\gamma_r. \quad (19)$$

Case 1: z_{tj} have the same variance denoted as $2\sigma_\alpha^2$

Substituting $f_\lambda(\lambda)$ from (11) into (18), we obtain the pdf of γ_r as

$$\begin{aligned} f_{\gamma_r}(\gamma_r) &= \sigma^2 \frac{1}{\sigma_\alpha^{4J} 2^{2J}\Gamma(2J)} (\sigma^2\gamma_r)^{2J-1} e^{-\sigma^2\gamma_r/2\sigma_\alpha^2} \\ &= \frac{1}{\left(\frac{2\sigma_\alpha^2}{\sigma^2}\right)^{2J}\Gamma(2J)} \gamma_r^{2J-1} e^{-\gamma_r/(2\sigma_\alpha^2/\sigma^2)}. \quad (20) \end{aligned}$$

Then, replacing $f_{\gamma_r}(\gamma_r)$ from (20) into the outage probability expression and performing integration by parts, we have

$$P_{outage}(\gamma) = 1 - e^{-\sigma^2\gamma/2\sigma_\alpha^2} \sum_{k=0}^{2J-1} \frac{1}{k!}\left(\frac{\sigma^2\gamma}{2\sigma_\alpha^2}\right)^k. \quad (21)$$

Case 2: z_{tj} have the different variances

From (16), (18), and (19), we have

$$\begin{aligned} P_{outage}(\gamma) &= \int_0^{\gamma} \sigma^2 \sum_{t=1}^{2}\sum_{j=1}^{J} A_{tj}\lambda_{tj} e^{-\lambda_{tj}\sigma^2\gamma_r} d\gamma_r \\ &= \sum_{t=1}^{2}\sum_{j=1}^{J} A_{tj}\left(1 - e^{-\lambda_{tj}\sigma^2\gamma}\right). \quad (22) \end{aligned}$$

Case 3: Generalization of the above two cases

The outage probability in this case can be found by combining the results derived from (21)–(22):

$$\begin{aligned} P_{outage}(\gamma) = & \sum_{m=1}^{M} E_m\left(1 - e^{-\lambda_m\sigma^2\gamma}\right) + \\ & \sum_{k=1}^{K} A_k\left(1 - e^{-\sigma^2\gamma/2\sigma_\alpha^2} \sum_{u=0}^{k-1}\frac{1}{u!}\left(\frac{\sigma^2\gamma}{2\sigma_\alpha^2}\right)^u\right). \end{aligned} \quad (23)$$

B. Error Probability Expressions

Rewrite (6)–(7) in the following form

$$\begin{aligned} x_1' &= \lambda x_1 + n_1 \quad (24) \\ x_2' &= \lambda x_2 + n_2. \quad (25) \end{aligned}$$

Because x_1 and x_2 are attenuated and corrupted by the same fading and noisy level, their error probability is equal. As a result, SER of x_1 is sufficient to evaluate the performance of the system.

The SER of MPSK modulation conditioned on the instantaneous received SNR γ_r is given by [p. 268, 1]

$$P_{eM} = 1 - \int_{-\pi/M}^{\pi/M} p_{\theta_r}(\theta_r)d\theta_r \quad (26)$$

where

$$p_{\theta_r}(\theta_r) = \frac{1}{2\pi} e^{-\gamma_r \sin^2\theta_r} \int_0^{\infty} V e^{-\left(V - \sqrt{2\gamma_r}\cos\theta_r\right)^2/2} dV \quad (27)$$

and $M = 2^k$ is the number of possible phases of the carrier in MPSK modulation.

Now, the average SER is found by averaging the P_{eM} over γ_r

$$P_{eAVG} = \int_0^{\infty} P_{eM} f_{\gamma_r}(\gamma_r)d\gamma_r. \quad (28)$$

It is impossible to deduce an explicit expression for P_{eAVG} except the case of $M = 2$. However, (28) can be calculated by approximating the integrals as sums [11].

The equivalent bit error probability for MPSK modulation can be asymptotic as follows when a Gray-code is used in the mapping process [1]:

$$BER \approx \frac{P_{eAVG}}{k}. \quad (29)$$

A special case: $M = 2$

For coherent BPSK modulation ($M = 2$), the symbol x_1 is detected by

$$\overline{x}_1 = \text{sign}(\text{Re}\{\lambda x_1 + n_1\}) = \text{sign}(\lambda x_1 + N_1) \tag{30}$$

where sign($\cdot$) is a signum function; Re$\{\cdot\}$ denotes the real part of a complex number; $N_1 = \text{Re}\{n_1\}$ represents a zero-mean Gaussian r.v. with variance $\varsigma^2/2 = \lambda\sigma^2/2$.

Therefore, it is straightforward to derive P_{e2} directly as

$$\begin{aligned} P_{e2} &= P(N_1 > \lambda|\lambda) \\ &= \int_\lambda^\infty \frac{1}{\sqrt{2\pi\varsigma^2/2}} \exp\left(-\frac{N_1^2}{2\pi\varsigma^2/2}\right) dN_1 \\ &= \text{Q}\left(\sqrt{\frac{\lambda^2}{\varsigma^2/2}}\right) \\ &= \text{Q}\left(\sqrt{2\gamma_r}\right) \end{aligned} \tag{31}$$

where

$$\text{Q}(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right) dy.$$

Finally, by averaging P_{e2} over the parameter γ_r, we obtain the average BER

$$P_{e2AVG} = \int_0^\infty P_{e2} f_{\gamma_r}(\gamma_r) d\gamma_r. \tag{32}$$

Now, we consider three possible cases of the pdf of γ_r given by (18) with the individual functions of $f_\lambda(\lambda)$ in (11), (16), and (17).

Case 1: z_{tj} have the same variance denoted as $2\sigma_\alpha^2$

Combining (20) and (32), we obtain

$$\begin{aligned} P_{e2AVG} &= \int_0^\infty \text{Q}\left(\sqrt{2\gamma_r}\right) \frac{\gamma_r^{2J-1} e^{-\gamma_r/(2\sigma_\alpha^2/\sigma^2)}}{\left(\frac{2\sigma_\alpha^2}{\sigma^2}\right)^{2J} \Gamma(2J)} d\gamma_r \\ &= \frac{(2J-1)!}{\Gamma(2J)} \cdot \\ & \int_0^\infty \text{Q}\left(\sqrt{2\gamma_r}\right) \frac{\gamma_r^{2J-1} e^{-\gamma_r/(2\sigma_\alpha^2/\sigma^2)}}{(2J-1)! \left(\frac{2\sigma_\alpha^2}{\sigma^2}\right)^{2J}} d\gamma_r . \end{aligned}$$

The above integral can be computed based on the results in [p. 825, 1] as

$$\begin{aligned} P_{e2AVG} &= \frac{(2J-1)!}{\Gamma(2J)} \left[\frac{1-\beta}{2}\right]^{2J} \cdot \\ & \sum_{k=0}^{2J-1} \binom{2J-1+k}{k} \left[\frac{1+\beta}{2}\right]^k \\ &= \frac{1}{\Gamma(2J)} \left[\frac{1-\beta}{2}\right]^{2J} \cdot \\ & \sum_{k=0}^{2J-1} \frac{(2J-1+k)!}{k!} \left[\frac{1+\beta}{2}\right]^k \end{aligned} \tag{33}$$

where

$$\beta = \sqrt{\frac{2\sigma_\alpha^2/\sigma^2}{1 + 2\sigma_\alpha^2/\sigma^2}} . \tag{34}$$

Case 2: z_{tj} have the different variances

The average error probability is achieved by using (16), (18), and (32) as follows

$$\begin{aligned} P_{e2AVG} &= \int_0^\infty \text{Q}\left(\sqrt{2\gamma_r}\right) \sigma^2 \sum_{t=1}^{2} \sum_{j=1}^{J} A_{tj} \lambda_{tj} e^{-\lambda_{tj}\sigma^2\gamma_r} d\gamma_r \\ &= \sum_{t=1}^{2} \sum_{j=1}^{J} A_{tj} \int_0^\infty \text{Q}\left(\sqrt{2\gamma_r}\right) \lambda_{tj}\sigma^2 e^{-\lambda_{tj}\sigma^2\gamma_r} d\gamma_r \\ &= \sum_{t=1}^{2} \sum_{j=1}^{J} \frac{A_{tj}}{2} \left(1 - \frac{1}{\sqrt{1+\lambda_{tj}\sigma^2}}\right). \end{aligned} \tag{35}$$

Case 3: Generalization of the above two cases

Substituting (17)–(18) into (32), we have

$$\begin{aligned} P_{e2AVG} &= \sum_{m=1}^{M} \frac{E_m}{2} \left(1 - \frac{1}{\sqrt{1+\lambda_m\sigma^2}}\right) + \\ & \sum_{k=1}^{K} \frac{A_k}{\Gamma(k)} \left[\frac{1-\beta}{2}\right]^k \sum_{m=0}^{k-1} \frac{(k-1+m)!}{m!} \left[\frac{1+\beta}{2}\right]^m \end{aligned} \tag{36}$$

where β is given in (34).

(33), (35), and (36) are closed-form BER expressions for the STC with 2-transmit antennas and J-receive antennas associated with coherent BPSK modulation.

III. NUMERICAL RESULTS

In the following presented simulation results, the average signal-to-noise ratio SNR is given by

$$\begin{aligned} SNR &= \mathbf{E}\{\gamma_r\} \\ &= \frac{\sum_{t=1}^{2} \sum_{j=1}^{J} 2\sigma_{tj}^2}{\sigma^2} \end{aligned} \tag{37}$$

and σ^2 and $|x_i|^2$ are chosen to be 1. Additionally, the channel state information is assumed to be perfectly known.

It is noted that the definition of the average SNR in (37) derived from (6)–(7) is slightly different from that of the average SNR in [2] which is essentially inferred from (2)–(3). For example in the case 1 where z_{tj} have the same variance, their relationship is

$$(SNR \text{ in our paper}) = (SNR \text{ in [2]}) + 10\log_{10}(J)$$

where all quantities in the above expression have the unit of dB.

The purpose of introducing a new definition of SNR is only to facilitate in discussing all possible cases of channels' fading degree. In what follows, we shall evaluate the performance of the Alamouti STC with different number of receive antennas in term of the outage probability and the error probability

Table 1. Variance assignment for channel gains in case 2.

t	Variances of channel gains			
1	$2\sigma_{11}^2 = a$	$2\sigma_{12}^2 = 2a$	$\cdots$	$2\sigma_{1J}^2 = Ja$
2	$2\sigma_{21}^2 = (J+1)a$	$2\sigma_{22}^2 = (J+2)a$	$\cdots$	$2\sigma_{2J}^2 = 2Ja$

Table 2. Variance assignment for channel gains in case 3.

t	Variances of channel gains			
1	$2\sigma_{11}^2 = b$	$2\sigma_{12}^2 = b$	$\cdots$	$2\sigma_{1J}^2 = b$
2	$2\sigma_{21}^2 = 2b$	$2\sigma_{22}^2 = 3b$	$\cdots$	$2\sigma_{2J}^2 = (J+1)b$

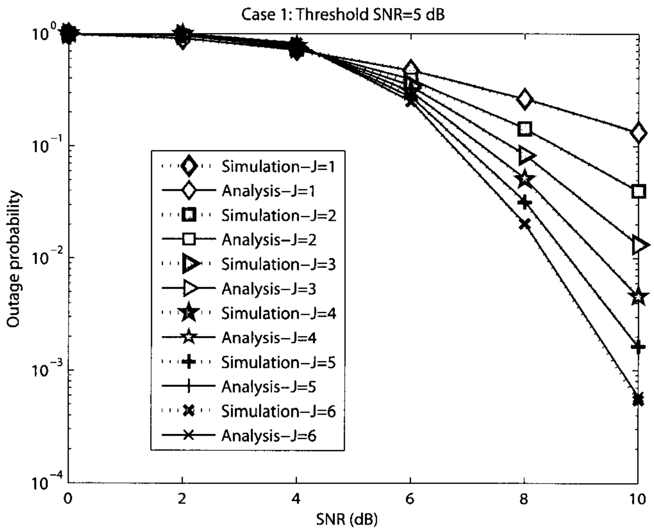


Fig. 1. Outage probability of Alamouti STC for case 1 with $\gamma = 5$ dB.

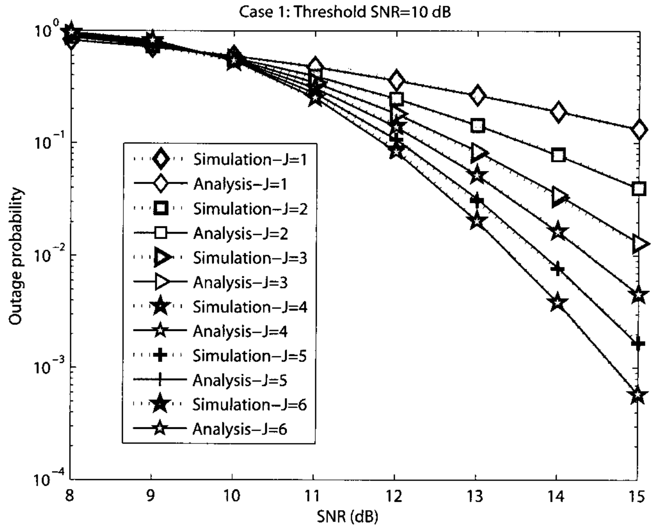


Fig. 2. Outage probability of Alamouti STC for case 1 with $\gamma = 10$ dB.

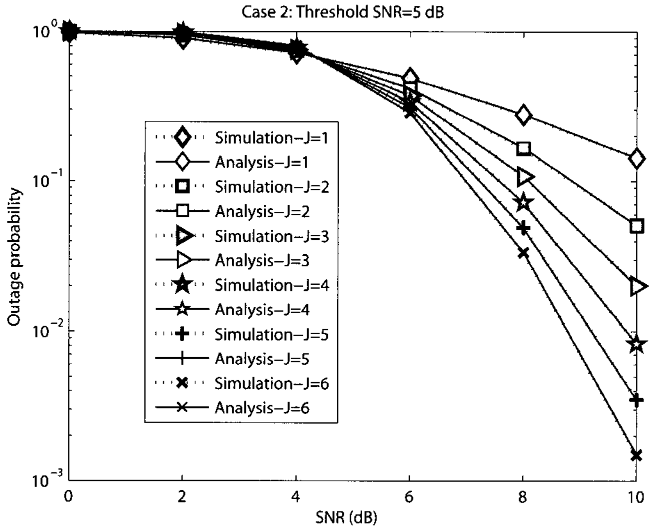


Fig. 3. Outage probability of Alamouti STC for case 2 with $\gamma = 5$ dB.

for three scenarios of channels as mentioned above. For the first case, all $2\sigma_{tj}^2 = 1/\lambda_{tj}$ are chosen as $\sigma^2(SNR)/(2J)$. To investigate the second situation, we assign the variances of channel gains in the ascending order given in Table 1 where $a = 2\sigma^2(SNR)/(2J)/(2J+1)$.

In the case 3, the channels from the transmit antenna 1 to all receive antennas are assumed to have the same quality while those from the transmit antenna 2 experience the distinct fading levels in the ascending order. Specifically, their variances are illustrated in Table 2 where $b = \sigma^2(SNR)/(J - 1 + (J+1)(J+2)/2)$.

Figs. 1–6 plot the outage probability curves of the Alamouti STC for distinct threshold SNRs ($\gamma = 5$ dB and $\gamma = 10$ dB) for three interested cases. They illustrate that the analysis exactly agrees with the simulation. Moreover, the more the diversity gain $2J$ increases, the more the outage probability reduces.

Performance of the Alamouti STC in terms of the outage probability is significantly degraded with respect to the asymmetry of channels from the transmit antennas to the receive antennas. This can be deduced from Fig. 7 which compares the performances of Alamouti STC in three cases. The symmetric degree of channels in descending order follows case 1, case 2, and case 3. As seen in Fig. 7, the best performance is achieved for case 1 and the worst for case 3. Therefore, the more symmetric the propagation paths, the better performance the Alamouti STC exhibits. This is because if one of the links to the receiver is severely faded, the diversity gain is reduced and thus, leading to the performance degradation.

Figs. 8–10 depict the error probability of the Alamouti STC for distinct conditions of the paths from the transmitter to the receiver. It is also recognized the theoretical formulas are consistent with simulation results. Moreover if we pay a little attention to the definition of SNR, the BER curves corresponding to $J = 1$ and $J = 2$ in Fig. 8 coincide with the simulation results in Fig. 6 for 2Tx-1Rx and Fig. 8 for 2Tx-2Rx of [2], respectively. This comes to a conclusion that the mathematical analysis is completely exact. Furthermore, in order to examine the effect of channel asymmetry on the performance of the encoding, we plot all BER curves of three cases in the same figure (see Fig. 11) and the similar comment to Fig. 7 can be drawn,

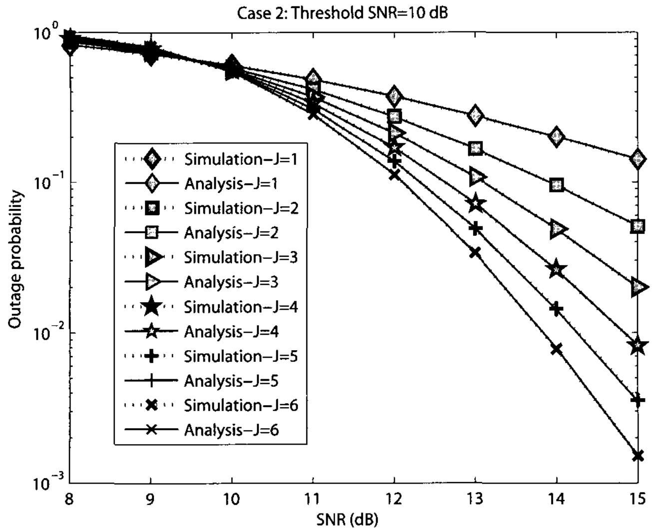


Fig. 4. Outage probability of Alamouti STC for case 2 with $\gamma = 10$ dB.

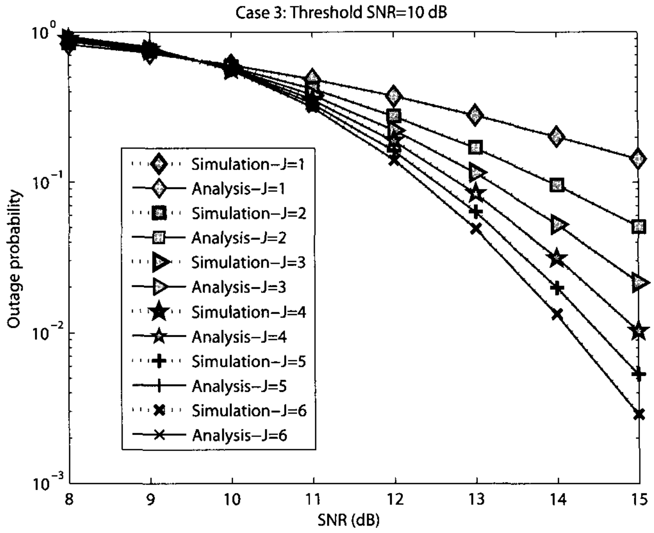


Fig. 6. Outage probability of Alamouti STC for case 3 with $\gamma = 10$ dB.

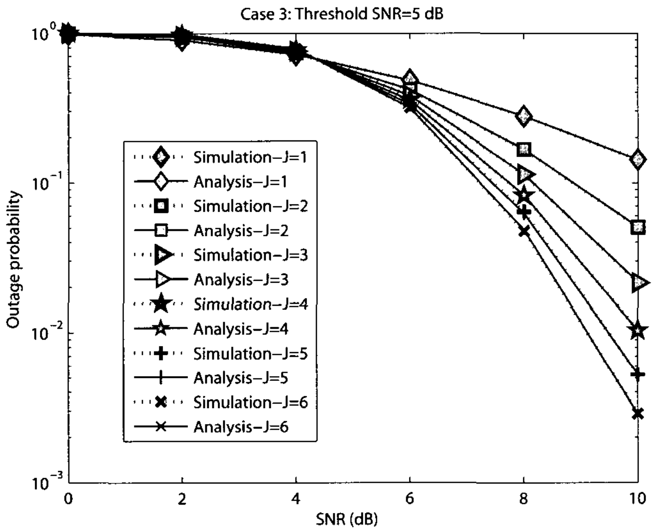


Fig. 5. Outage probability of Alamouti STC for case 3 with $\gamma = 5$ dB.

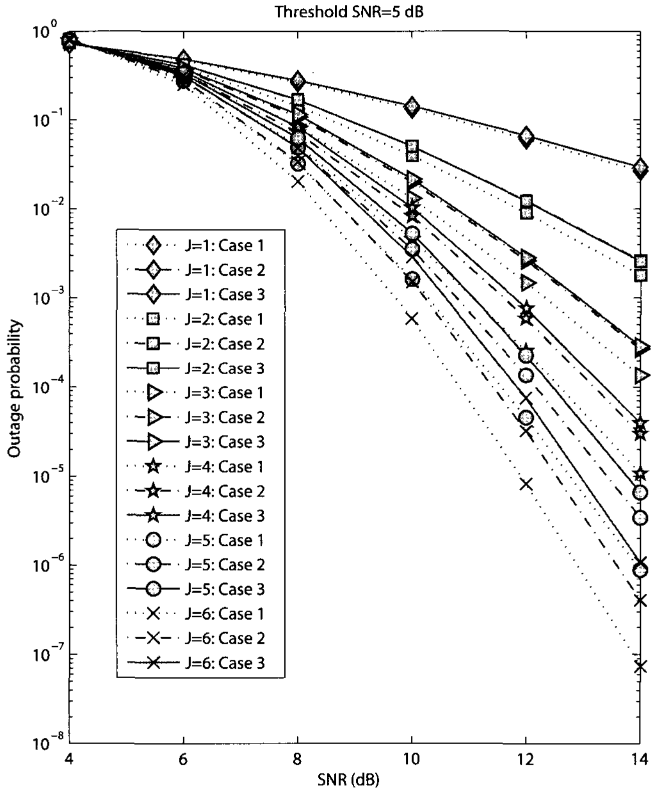


Fig. 7. Comparison on outage probability of Alamouti STC among three cases.

that is, STC yields the best performance when the symmetry of channels to the destination is reached.

The error probability of the Alamouti STC for case 1 corresponding to different modulation levels M is demonstrated in Figs. 12 and 13. It is found that for any MPSK modulation level, the performance of the system is improved proportionally to the number of receive antennas. In other words, it reveals the agreement between the theoretical formulas and simulation results.

From investigated figures, we realize that both outage probability and symbol error rate are approximate measures to assess the performance of the system because they express the performance improvement when the quality of the channel becomes better (SNR is larger). However, to have a quick evaluation of a certain system, it is obvious that adopting the outage probability measure is more reasonable because its closed-form expression is much easier to be established and observed than that of SER.

IV. CONCLUSION

The exact outage and error probability formulas for analyzing the performance of one of the most important STCs, Alamouti STC, are derived. All possible cases of the asymmetric and symmetric property of channels to the destination are also examined. The simulation results verified the validity of those expressions.

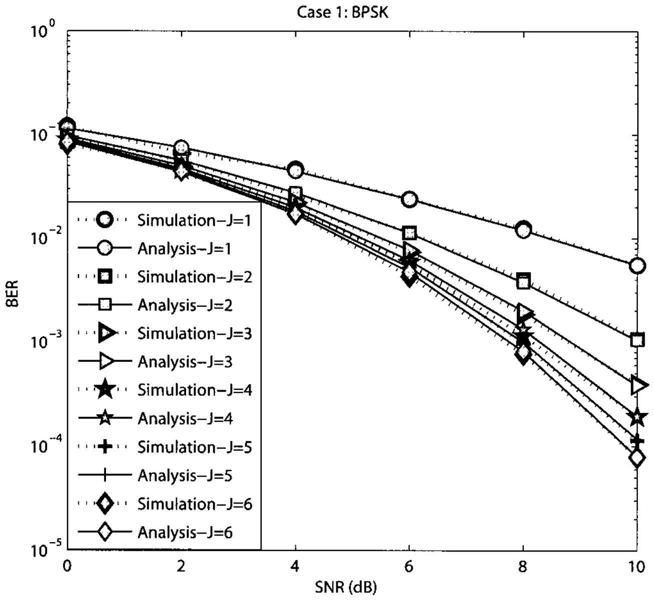


Fig. 8. BER performance of Alumouti STC for BPSK modulation in the case 1.

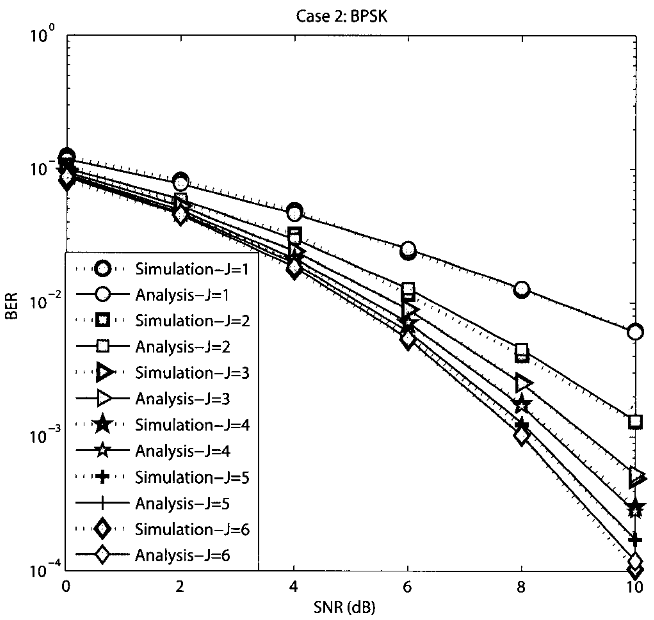


Fig. 9. BER performance of Alumouti STC for BPSK modulation in the case 2.

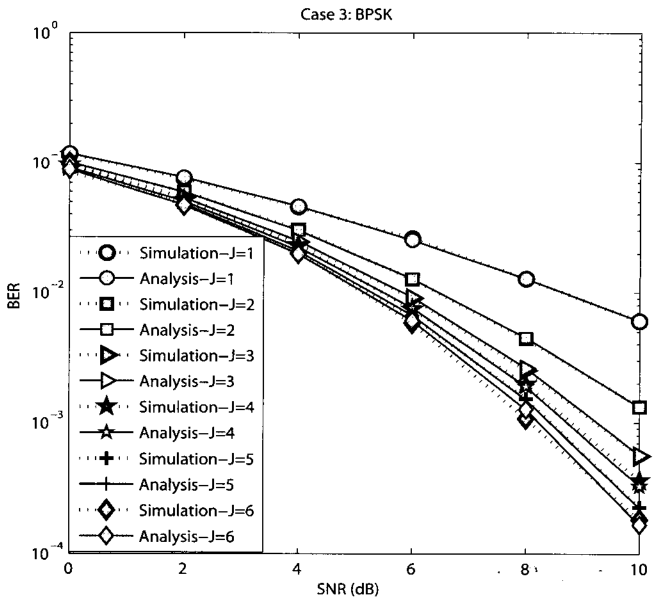


Fig. 10. BER performance of Alumouti STC for BPSK modulation in the case 3.

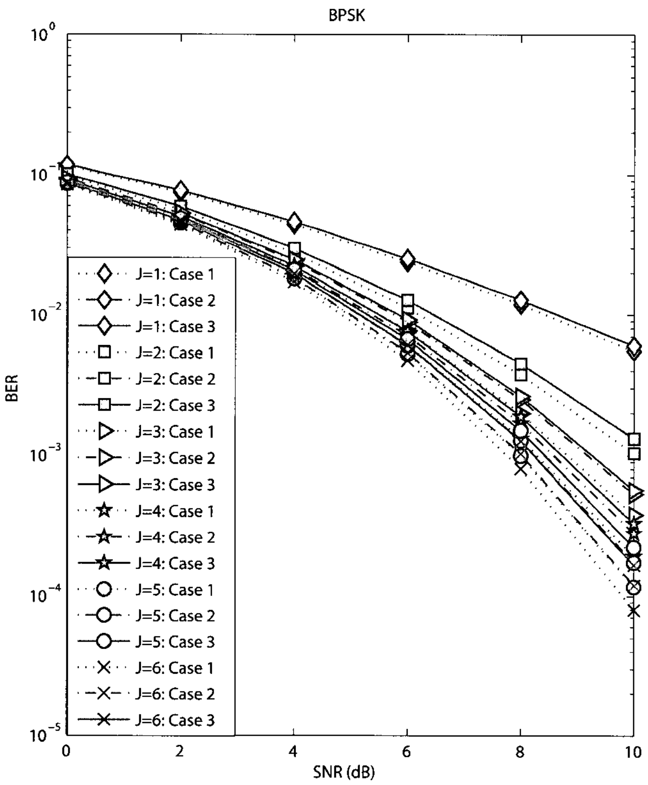


Fig. 11. Comparison on error probability of Alamouti STC among three cases.

Therefore, they serve well as a reference for quickly estimating the system performance without time-consuming simulations.

ACKNOWLEDGMENTS

This research was supported by the MIC (Ministry of Information and Communication), Korea, under the ITRC (Information technology Research Center) support program supervised by the IITA (Institute of Information Technology Assessment).

This work was supported by 2007 Research Fund of University of Ulsan.

REFERENCES

[1] J. G. Proakis, *Digital communications*, 4th Ed. McGraw-Hill, 2001.

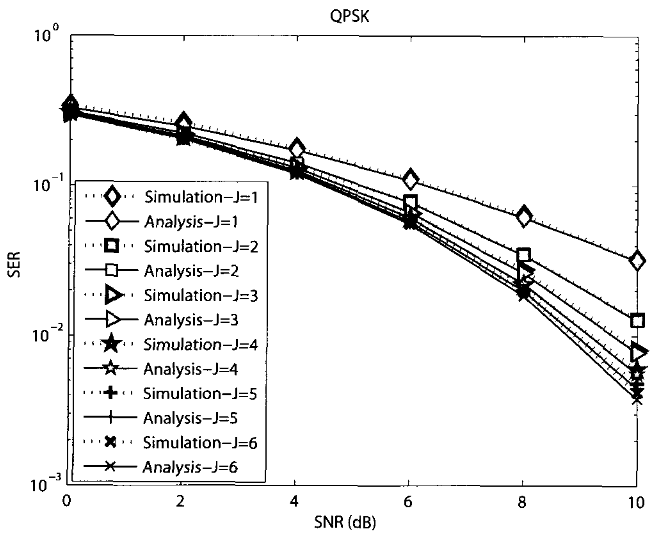


Fig. 12. SER performance of Alumouti STC for QPSK modulation.

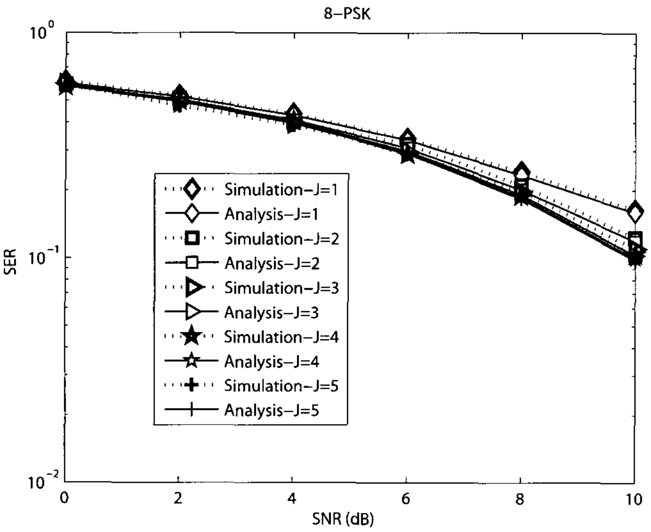


Fig. 13. SER performance of Alumouti STC for 8-PSK modulation.

[2] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block coding for wireless communications: Performance results," *IEEE Trans. Commun.*, vol. 17, pp. 451–460, Mar. 1999.

[3] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 16, pp. 1451–1458, Oct. 1998.

[4] K. Fazel and S. Kaiser, *Multi-Carrier and Spread Spectrum Systems*. John Wiley & Sons Ltd., 2003.

[5] M. Uysal and C. N. Georghiades, "Error performance analysis of space-time codes over Rayleigh fading channels," *J. Commun. Networks*, vol. 2, pp. 351–355, Dec. 2000.

[6] M. K. Simon, "Evaluation of average bit error probability for space-time coding based on a simpler exact evaluation of pairwise error probability," *J. Commun. Networks*, vol. 3, pp. 257–264, Sept. 2001.

[7] G. Taricco and E. Biglieri, "Exact pairwise error probability of space-time codes," *IEEE Trans. Inf. Theory*, vol. 48, pp. 510–513, Feb. 2002.

[8] M. K. Simon and M. S. Alouini, *Digital communication over fading channels*. John Wiley & Sons Inc., 2005.

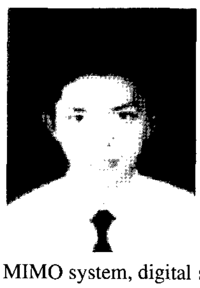
[9] B. Vucetic and J. Yuan, *Space-Time Coding*. John Wiley & Sons Ltd., 2003.

[10] N. K. Dinh, "Complex Functions and Operators," *Lecture Note*, HoChiMinh City University of Technology, 1999.

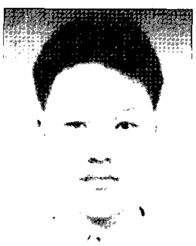
[11] A. Papoulis and S. U. Pillai, *Probability, Random Variables and Stochastic Process*, 4th Ed. McGraw Hill, 2002.

[12] M. R. Spiegel, *Mathematical Handbook of Formulas and Tables*. McGraw-Hill Inc., 1968.

Hyung Yun Kong received the Ph.D. and M.E. degrees in Electrical Engineering from Polytechnic University, Brooklyn, New York, USA, in 1996 and 1991, respectively. And he received B.E. in Electrical Engineering from New York Institute of Technology, New York in 1989. Since 1996, he was with LG electronics Co., Ltd. in multimedia research lab developing PCS mobile phone systems and LG chair man's office planning future satellite communication systems from 1997. Currently he is a professor in Electrical Engineering at University of Ulsan, Korea. He performs several government projects supported by ITRC (Information Technology Research Center), KOSEF (Korean Science and Engineering Foundation), etc. His research area includes high data rate modulation, channel coding, detection and estimation, cooperative communications, and sensor network. He is member of IEEK, KICS, KIPS, and IEICE.



Ho Van Khuong received the B.E. and the M.S. degrees in Electronics and Telecommunications Engineering from HoChiMinh City University of Technology, Vietnam, in 2001 and 2003, respectively. From April 2001 to September 2004, he was a lecturer at Telecommunications Department, HoChiMinh City University of Technology. He received the Ph.D degree in Electrical Engineering from University of Ulsan, Korea in 2006. Currently, he is a PostDoctoral Fellow at McGill University, Canada. His major research interests are modulation and coding techniques, MIMO system, digital signal processing, and cooperative communications.



Doo Hee Nam received the B.E. and M.E. degrees in Electrical Engineering from University of Ulsan, Korea in 2004 and 2006, respectively. Currently he is with TTA., GSM/WCDMA Team, Mobile communications testing center. He also participates a project supported by KOSEF (Korean Science and Engineering Foundation) and ITRC (Information Technology Research Center). His research area includes multicode modulation, LDPC, and sensor network.