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Study on MCC and Hvorslev-MCC Models

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MCC 모델 및 Hvorslev-MCC 모델의 비교 연구

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Abstract In this study, the MCC(modified Cam-Clay) model and the Hvorslev-MCC model, recently developed based on the critical state theory and with relatively few model parameters, were investigated by comparing the model predictions with the result of the conventional triaxial compression test strictly performed in laboratory. The discrepancy of the prediction capacities of the models exists on the heavily over-consolidated specimen. The Hvorslev-MCC model accurately predicts the peak strength envelope for heavily over-consolidated clayey specimens on the dry side of the critical state since it adopts the Hvorslev surface in the supercritical region other than the ellipse of the MCC model.

Key Words : Critical State, MCC, Hvorslev-MCC, Constitutive Model

요 약 본 연구에서는 한계상태이론에 기초하며 비교적 적은 매개변수를 활용하도록 최근에 개발된 Hvorslev-MCC 모델 및 전통적 MCC 모델의 거동예측 능력을 엄격하게 수행된 삼축압축시험 결과와 비교하여 고찰하였다. 모델예측 결과의 차이는 과압밀점토 시료에서 주로 발생하였다. Hvorslev-MCC 모델은 과압밀점토 시료의 첨두강도를 매우 정확히 예측하였으며, 이는 한계상태 건조 측의 항복면에 단순 타원이 아닌 Hvorslev 면을 활용하였기 때문이다.

1. Introduction

Roscoe et al.(1958) postulated a behavioral framework based on the concepts of "critical state," which has been the most widely used for the soil constitutive modeling[1]. The Cam-Clay models based on critical state concept have proven to be useful in the numerical analysis of boundary value problems requiring realistic soil models. They have been recognized for the simplicity requiring limited numbers of material parameters. The modified Cam-Clay(MCC) appears most popular by far and yet sufficiently accurate for most applications(Roscoe and Burland, 1968; Wood, 1990)[1, 2].

The MCC model has been proven to satisfactorily predict the behavior of normally to lightly over-consolidated(OC) clays that lie in the subcritical region. But, its prediction for the stress-strain behavior of heavily OC clays in the supercritical region is not satisfactory as the adopted yield curves tend to overestimate peak strength in this region. A failure surface in "cap models," assumed to be fixed without expansion or contraction, is used to replace the yield curve on the dry side of the critical state[2, 3]. Although the cap models are known to predict failure stress fairly adequately for soils on the supercritical region, their inability to describe the softening behavior and volume expansion of OC clays remains a major deficiency. After Hvorslev's experimental finding that a straight line, Hvorslev surface, would approximate the peak strength envelope of an OC soil fairly accurately, the introduction of a Hvorslev surface as a means of representing the supercritical yield surface has been made by a few authors(Mita et al., 2004)[4].

In this study, the applications of the MCC model and the Hvorslev-MCC model by Mita et al.(2004), recently developed based on the critical state theory, were investigated by comparing the model predictions with the

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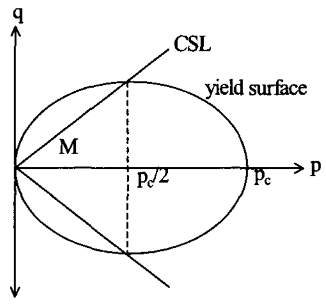
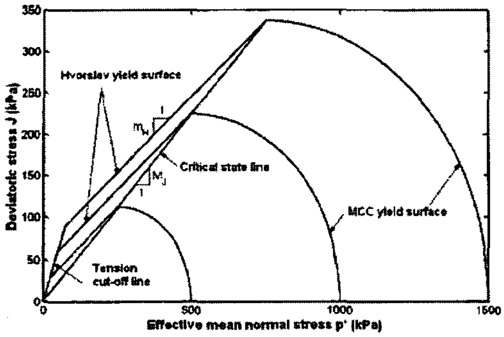
results of the experiment strictly performed in laboratory.

2. Hvorslev-MCC Model

The popular modified Cam-Clay model(MCC) predicts soil behavior in the subcritical region fairly well; however, the predictions for heavily over-consolidated soils, in the supercritical region, are not so satisfactory. Mita et al.(2004) developed the Hvorslev-Modified Cam clay model(Hvorslev-MCC) which has MCC features in the subcritical region and Hvorslev surface in the supercritical region(Fig. 1)[5].



(a) MCC model



(b)Hvorslev-MCC model(Mita et al., 2004)

Fig. 1. Failure Envelopes

The standard expression for the elastoplastic constitutive matrix $[D^{ep}]$ is obtained as

$$[D^{ep}] = [D] - \frac{[D]\left\{\frac{\partial P(\{\sigma\},m)}{\partial\sigma}\right\}\left\{\frac{\partial F(\{\sigma\},k)}{\partial\sigma}\right\}^T[D]}{\left\{\frac{\partial F(\{\sigma\},k)}{\partial\sigma}\right\}^T[D]\left\{\frac{\partial P(\{\sigma\},m)}{\partial\sigma}\right\}+A} \quad (1)$$

where $\partial F(\{\sigma\},k)=0$, $\partial P(\{\sigma\},m)=0$, and $[D]$ denote the yield function, plastic potential function, and the elastic constitutive matrix, respectively. $\{\sigma\}$ represents the vector of stress(expressed in terms of either the stress components or stress invariants); $\{k\}$ represents the state parameters; and $\{m\}$ is essentially a vector of state parameters, the value of which are immaterial because only the differentials of P with respect to the stress components are needed in the flow rule. The term A is known as the hardening/softening parameter, and its form depends on the type of plasticity. For strain hardening/softening plasticity, a linear relationship is assumed between the state parameters $\{k\}$ and the accumulated plastic strains $\{\varepsilon^p\}$. In the Hvorslev-MCC model, isotropic hardening/softening is assumed and the hardening/softening rule is given in terms of a single hardening parameter p_o' which is related to the plastic volumetric strain ϵ_v^p. Consequently, the equation for A for the Hvorslev-MCC model is written as Eq.(2), where Λ is a scalar multiplier.

$$A = -\frac{1}{\Lambda}\frac{\partial F(\{\sigma\},k)}{\partial p_o'}dp_o' \quad (2)$$

The significant feature of the surface with which Hvorslev was particularly concerned is the shear strength of a heavily OC specimen at failure is a function of both the mean effective normal stress p' and the specific volume v of the specimen at failure. The specific volume appears in the plot of Hvorslev failure surface through its influence on the equivalent stress p_e' depending directly on specific volume. The value of p_e' for any specific volume is simply the stress on the normal consolidation line at that specific volume. The equation of the idealized Hvorslev straight line in $q/p_e':p'/p_e'$ plane is Eq.(3). m_H and g_H are slope and intercept of the Hvorslev line. The virgin compression(slope=λ) and swelling lines(slope=κ) are assumed straight in the v-$\ln p'$ plane(Eq. 4).

$$\frac{J}{p_e'} = m_H\frac{p'}{p_e'} + g_H \quad (3)$$

$v=N-\lambda(\ln p')$ (virgin consolidation line)
$v=vs-\kappa(\ln p')$ (swelling line) (4)

where λ, κ, and N(value of specific volume of the virgin compression at $p'=1.0$ kPa) are the traditional material properties in critical state soil mechanics. The value of v_s is different for each swelling line. Irreversible plastic volume changes take place along the virgin consolidation line while reversible elastic volume changes occur along the swelling lines. In view of Eq.(4) and substituting this into Eq.(3) yields Eq.(5).

$$p_e'=e^{N-v/\lambda} \qquad J=m_H p' + g_H e^{N-v/\lambda} \tag{5}$$

The Hvorslev surface intersects the critical state(CL) line at p_{cs}', J_{cs}, and v_{cs} in Eq.(6). M_J denotes the CS stress ratio, Γ is value of specific volume on the CSL at $p'=1.0$ kPa. Hence, the Eq.(7) for the Hvorslev surface is obtained by manipulating Eq.(6).

$$J_{cs}=M_J\, p_{cs}' \quad v_{cs}=\Gamma-\lambda\ ln\ p_{cs}' \quad g_H=(M_J-m_H)e^{\Gamma-N/\lambda}$$
$$v=N-\lambda\ ln\ p_o'+\kappa\ ln(p_o'/p') \tag{6}$$

$$\frac{J}{(M_J-m_H)}-\frac{m_H}{(M_J-m_H)}p'-p_o'e^{(\Gamma-N/\lambda)}e^{-\kappa/\lambda\ln(p_o'-p')}=0 \tag{7}$$

The derivatives of the yield function F and plastic potential function P are as Eqs.(8) to (10), respectively(Mita et al., 2004)[4].

$$\frac{\partial F(\{\sigma\},k)}{\partial p'}=-\frac{m_H}{(g(\theta)-m_H)}-\frac{\kappa}{\lambda}\frac{1}{p'}e^{(\Gamma-N/\lambda)}e^{-\kappa/\lambda\ln(p_o'/p')}$$
$$\frac{\partial F(\{\sigma\},k)}{\partial J}=-\frac{1}{(g(\theta)-m_H)} \tag{8}$$
$$\frac{\partial F(\{\sigma\},k)}{\partial\theta}=-\frac{(J-m_H)}{(g(\theta)-m_H)^2}g(\theta)^2\frac{\frac{1}{\sqrt{3}}cos\theta\sin\Phi_{cs}'-\sin\theta}{\sin\Phi_{cs}'}$$

$$\frac{\partial P(\{\sigma\},m)}{\partial p'}=-\frac{1}{p'}\left[1-\left(\frac{J}{p'g(\theta(\sigma'))}\right)^2\right] \tag{9}$$

$$\frac{\partial P(\{\sigma\},m)}{\partial J}=\frac{2J}{(p'g(\theta(\sigma')))^2} \qquad \frac{\partial P(\{\sigma\},m)}{\partial\theta}=0 \tag{10}$$

Manipulating the above equations, the hardening/softening parameter A is expressed as

$$A=\frac{\nu}{\lambda}\frac{p_o'}{p'}e^{\Gamma-N^{-\kappa\ln(p_o'-p')/\lambda}}\left[1-\left(\frac{J}{p'g(\theta(\sigma'))}\right)^2\right] \tag{11}$$

An additional parameter m_H is required to define the slope of the Hvorslev surface in the Hvorslev-MCC model along with the five material parameters used in the MCC model(N, λ, κ, M_J, and G or ν).

3. Model Predictions and Discussion

The conventional triaxial compression and oedometer tests were conducted. Kaolin clay(liquid limit=54%, plastic limit=28%, specific gravity=2.66, coefficient of permeability=2.41x10^{-8}cm/sec) was used to prepare test specimens. The dry soil sample was mixed with deaired water at a water content of twice liquid limit. The slurry was carefully placed into the small slurry consolidometer, then consolidation was performed using dead weights(to 206.85 kPa). Through this slurry consolidometer technique, very homogeneous, undisturbed, and reproducible specimens were prepared with known stress history. The cell pressure of 262.01 kPa was applied during the consolidation phase in the triaxial test then the cell pressure was kept or reduced to make the specimens normally consolidated or lightly over-consolidated(OCR=2) and heavily over-consolidated(OCR=10) states. After that, the undrained strain-controlled shearing(0.01%/min) was conducted. The model parameters, whose identification and determination were specified in section 2, are given in Table 1.

Table 1. Model Parameters

Parameter	identification	value	usage
N	specific volume of virgin compression at p'=1.0 kPa	3.80	both models
λ	slope of virgin compression line in v-ln p' space	0.268	both models
κ	slope of swelling line in v-ln p' space	0.058	both models
$M_{(J)}$	slope of critical state line in q-p' space	0.95	both models
m_H	slope of Hvorslev line in q-p' space	0.34	Hvorslev-MCC model
v	Poisson's ratio	0.3	both models

The values of parameters λ, κ, $M_{(\theta)}$, and m_H in Table 1 have a significant effect on the predictions. However, these parameters could be accurately obtained, through clear procedures stated earlier, from the experiments conducted on the test clay specimens. The parameter N has very little influence on the predictions. Another parameter that could influence the predicted stress-strain response in the Poisson's ratio ν. For most geotechnical applications, the range of ν falls between 0.15 and 0.35(Mita et al., 2004)[4]. Within this range, a maximum variation of the predicted peak strength was observed to be 10%. The usual value of 0.3 was used for the Poisson's ratio in this study. Figs. 2 to 4 are plots of the principal stress difference versus the axial strain of the triaxial tests.

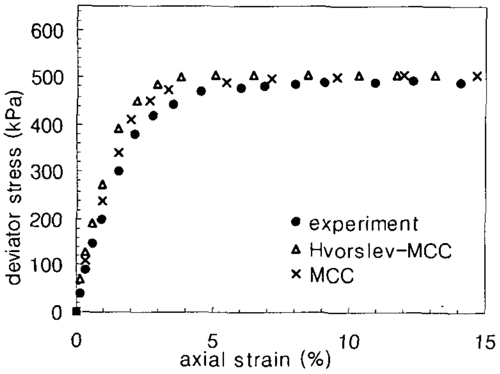


Fig. 2. Model Prediction and Test Result(OCR=1)

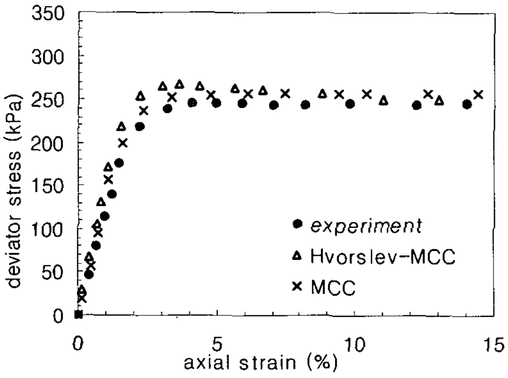


Fig. 3. Model Prediction and Test Result(OCR=2)

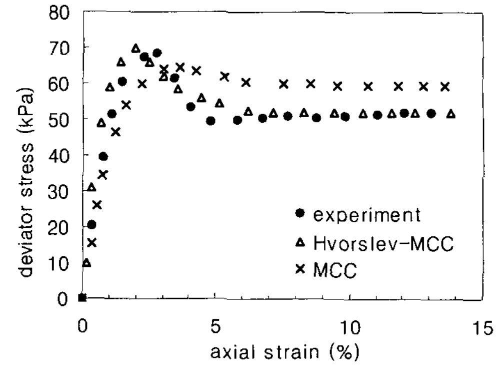


Fig. 4. Model Prediction and Test Result(OCR=10)

It can be seen from Figs. 2 through 4 that the predictions using the MCC model generally agree with the measurement but not so well as those using the Hvorslev-MCC model do. This is because the MCC model principally does not simulate the yielding behavior of the over-consolidated clays. The Hvorslev-MCC model is able to capture the essential features of the behavior of the soils. It is quite evident from the results in Fig. 4 that the model accurately predicts the peak as well as residual strength of heavily OC soil. Although the actual kinematics of the postpeak stress pattern cannot be captured by it, the trend of the soil behavior was predicted fairly closely by the Hvorslev-MCC model. The instability in geological materials is due to postpeak material strain softening and localization of plastic strains and subsequent development of shear bands. The Hvorslev-MCC model is capable of simulating the postpeak material strain softening through the reduced size of the yield surface. However, the actual kinematics of shear bands cannot be reflected by the model. From the comparison of theoretical predictions and test results presented herein, it may be concluded that, for monotonic loading, the simple general model formulated in the Hvorslev-MCC model can produces realistic results up to the peak stress for heavily OC clays, beyond which results might be improved using a regularization scheme.

The theoretical model for predicting the general response of soils, either on the dry or wet side of critical state, is based on the assumption of a continuum. The states of heavily OC clays fall on the supercritical region, and tend to develop intense shear bands upon reaching peak strength. The material becomes discontinuous and hence the continuum models, in principle, cease to apply. The occurrence of such discontinuous zones affects the numerical implementation of the constitutive equations for soils, as well as the experimental technique for determining the corresponding material parameters. The formation of discontinuities makes the single element problem a boundary value problem. During the strain softening in the Hvorslev-MCC model, the value of hardening/softening parameter A, given in Eq.(2) may cause tangent modulus to be negative and yield mesh sensitive results. To overcome this problem, the nonlocal regularization methods, verified by Tanaka et al.(2004)[5], was used in this study. The nonlocal method assumes that local state of the material at a given point may not be sufficient to evaluate the stresses at that point. The application of the nonlocal method to Hvorslev-MCC type models involves substitution of local softening variable A with its nonlocal equivalent. While computing a nonlocal softening variable in a representative volume, a mesh-size dependent softening modulus is introduced. The mesh-size dependent softening modulus will yield a mesh-size independent shear band width. Incorporation of regularization schemes will predict realistic shear band width.

4. Conclusions

In this study, the MCC and the Hvorslev-MCC models, recently developed based on the critical state theory, were investigated by comparing the model predictions with experimental results. The Hvorslev-MCC model accurately predicts the peak strength envelope for heavily OC clays on the dry side of the critical state since it adopts the Hvorslev surface, instead of ellipse in the MCC model, as the yield surface in the supercritical region. However, the MCC model showed also good predictions for the normally and slightly consolidated clays. This reflects the good capacity of the critical state theory in simulation of yielding behavior of clayey soils.

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