# Static Output Feedback Control Synthesis for Discrete-time T-S Fuzzy Systems

**Jiuxiang Dong and Guang-Hong Yang***

**Abstract:** This paper considers the problem of designing static output feedback controllers for nonlinear systems represented by Takagi-Sugeno (T-S) fuzzy models. Based on linear matrix inequality technique, a new method is developed for designing fuzzy stabilizing controllers via static output feedback. Furthermore, the result is also extended to $H_\infty$ control. Examples are given to illustrate the effectiveness of the proposed methods.

**Keywords:** $H_\infty$ control, linear matrix inequalities, static output feedback, T-S fuzzy systems.

## 1. INTRODUCTION

In the past two decades, Takagi-Sugeno (T-S) fuzzy model has received much attention [1,2]. The main reason is that it can represent nonlinear systems with a set of linear subsystems, which are connected by IF-THEN rules, then the technique in the conventional linear system theory can be applied to T-S fuzzy systems. In general, the fuzzy control system design for T-S fuzzy models is based on the so-called parallel distributed compensation (PDC) scheme which establishes that a linear control is designed for each local linear system. The overall controller is a fuzzy blending of all local linear controllers, which is usually nonlinear [3]. Based on linear matrix inequality (LMI) technique, state feedback controller designs for T-S fuzzy systems have been studied in [4-10] and many important progresses have been achieved. However, the above controller design methods of fuzzy control systems are based on the assumption of the states are available for controller implementation, which is not true in many practical cases. Therefore, output feedback of fuzzy control systems is very important and some results based on output feedback have been obtained [11-17].

Recently, there have appeared a number of approaches for designing dynamic output feedback controller for fuzzy control systems, see [11-14] and the references therein. Since dynamic output feed back problems can be transformed into static output feedback problems, the static output feedback formulation is more general than the dynamic output feedback formulation and it can be easily implemented with low cost [17]. In recent years, static output feedback designs for TS fuzzy systems have received much attention. In [18], by iterative linear matrix inequality (ILMI) approach and using structural information of membership functions, a technique for $H_\infty$ static output feedback controller design is given. In [19], by using parameterized linear matrix inequality (PLMI) approach and diagonal structured Lyapunov matrix, a sufficient condition for $H_\infty$ static output feedback control design is obtained. Based on a new quadratic stabilization condition for T-S fuzzy control systems, [17] gives an LMI-based method for fuzzy static output feedback controller design. Among these results, [18] and [19] are only applicable for the systems in which the measured output is linearly dependent on the states. [17] presents a method for designing static output feedback controllers for the nonlinear systems in which the measured output may be nonlinear function of states, but it involves a strict technical condition, which might be difficult to be implemented and satisfied. For overcoming the difficulty, the paper will continue to study the problem of designing fuzzy static output feedback controllers for T-S fuzzy systems in which the measured output may be nonlinear function of

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states. A new design method is given in terms of a set of LMIs.

The paper is organized as follows. In the nextsection, system description and some preliminaries are given. In Section 3, a new sufficient condition for fuzzy static output feedback control design is proposed, and the result is also extended to $H_\infty$ guaranteed cost control. Section 4 presents two examples to illustrate the effectiveness of the proposed design methods. Finally, Section 5 concludes this paper.

## 2. PROBLEM STATEMENT AND PRELIMINARIES

Takagi-Sugeno (T-S) fuzzy modes can be written as the following form:

$$\begin{aligned} x(k+1) &= A(\alpha)x(k) + B_1(\alpha)w(k) + B_2(\alpha)u(k), \\ z(k) &= C_1(\alpha)x(k) + D_1(\alpha)w(k) + D_2(\alpha)u(k), \\ y(k) &= C_2(\alpha)x(k), \end{aligned} \tag{1}$$

where $x(k)$ is the state, $u(k)$ is the controlled input, $y(k)$ is the measured output, $z(k)$ is the controlled output, $w(k)$ is unknown but energy-bounded disturbance input, and

$$\begin{aligned} A(\alpha) &= \sum_{i=1}^{r} \alpha_i(k)A_i, \quad B_2(\alpha) = \sum_{i=1}^{r} \alpha_i(k)B_{2i}, \\ B_1(\alpha) &= \sum_{i=1}^{r} \alpha_i(k)B_{1i}, \quad C_1(\alpha) = \sum_{i=1}^{r} \alpha_i(k)C_{1i}, \\ D_1(\alpha) &= \sum_{i=1}^{r} \alpha_i(k)D_{1i}, \quad K(\alpha) = \sum_{i=1}^{r} \alpha_i(k)K_i, \end{aligned} \tag{2}$$

$r$ is the number of fuzzy rules; $\alpha_i(k)$ is membership function and satisfying $0 \le \alpha_i(k) \le 1$, and $\sum_{i=1}^{r}\alpha_i(k) = 1$, $A_i$, $B_{1i}$, $B_{2i}$, $C_{1i}$, $D_{1i}$, $D_{2i}$, $C_{2i}$ are of appropriate dimensions. Assume $C_{2i}$, $1 \le i \le r$, are of full row rank, and let invertible matrices $T_i$, $1 \le i \le r$, such that

$$C_{2i}T_i = \begin{bmatrix} I & 0 \end{bmatrix}, \quad \text{for } 1 \le i \le r. \tag{3}$$

**Remark 1:** For each $C_{2i}$, the corresponding $T_i$ generally is not unique. A special $T_i$ can be obtained by the following formula,

$$T_i = \begin{bmatrix} C_{2i}^T(C_{2i}C_{2i}^T)^{-1} & C_{2i}^{\perp} \end{bmatrix}, \tag{4}$$

where $C_{2i}^{\perp}$ denotes an orthogonal basis for the null space of $C_{2i}$.

In this paper, the concept of parallel distributed compensation (PDC) is used to design fuzzy controller, i.e., the designed fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts, more details can found in [3]. For the fuzzy model (1), the following static output feedback controller is exploited:

$$u(k) = \sum_{i=1}^{r} \alpha_i(k)K_i y(k). \tag{5}$$

In this sequel, $H_\infty$ control will be considered, and the following preliminary lemma will be used in this sequel.

**Lemma 1** [20]: Given a prescribed $H_\infty$ performance index $\gamma > 0$, if there exists a symmetric matrix $Q$, satisfying

$$\begin{bmatrix} -Q & * & * & * \\ 0 & -\gamma I & * & * \\ (A(\alpha)+B_2(\alpha)K(\alpha)C_2(\alpha))Q & B_1(\alpha) & -Q & * \\ (C_1(\alpha)+D_2(\alpha)K(\alpha)C_2(\alpha))Q & D_1(\alpha) & 0 & -\gamma I \end{bmatrix} < 0$$

then the system (1) with the fuzzy static output feedback controller (5) is stable while satisfying $H_\infty$ performance bound $\gamma$.

## 3. MAIN RESULTS

In this section, a new LMI-based method for fuzzy static output feedback control design of discrete-time fuzzy systems will be given, and it is also extended to $H_\infty$ control. Now a new sufficient condition for fuzzy static output feedback control design is presented by the following theorem.

**Theorem 1:** If there exist symmetric matrices $Q$, $J_{ijl}$, $R_{ijl}$, $1 \le i,j,l \le r$, and matrices $S_{ijl}$, $L_i$, $1 \le i, j, l \le r$, with

$$S_{ijl} = \begin{bmatrix} S_{11} & 0 \\ S_{21}^{ijl} & S_{22}^{ijl} \end{bmatrix}, \quad L_i = \begin{bmatrix} L_{ai} & 0 \end{bmatrix} \tag{6}$$

satisfying the following LMIs,

$$\begin{bmatrix} Q - T_l S_{ijl} - S_{ijl}^T T_l^T & * \\ A_i T_l S_{ijl} + B_{2i} L_j & -Q + J_{ijl} + R_{ijl} \end{bmatrix} < 0, \quad 1 \le i,j,l \le r \tag{7}$$

$$[J_{ijl}]_{r\times r} + [J_{ijl}]_{r\times r}^T > 0, \quad 1 \le l \le r, \tag{8}$$

$$R_{iii} > 0, \quad 1 \le i \le r, \tag{9}$$

$$R_{iij} + R_{iji} + R_{jii} > 0, \quad 1 \le i \ne j \le r, \tag{10}$$

$$R_{ijl}+R_{ilj}+R_{jil}+R_{jli}+R_{lij}+R_{lji}>0,\quad 1\le i<j<l\le r \tag{11}$$

then the system (1) with $w(k)=0$ is asymptotically stable via static output feedback gain

$$K_i=L_{ai}S_{11}^{-1}. \tag{12}$$

**Proof:** Form (6) and (12), we have

$$L_j=\begin{bmatrix}L_{aj} & 0\end{bmatrix}=\begin{bmatrix}K_jS_{11} & 0\end{bmatrix}=\begin{bmatrix}K_j & 0\end{bmatrix}\begin{bmatrix}S_{11} & 0\\ S_{21}^{ijl} & S_{22}^{ijl}\end{bmatrix}$$
$$=K_j\begin{bmatrix}I & 0\end{bmatrix}S_{ijl}=K_j(C_{2l}T_l)S_{ijl}=K_jC_{2l}T_lS_{ijl}.$$

Combining it with (7), then we can obtain that

$$\begin{bmatrix}Q-T_lS_{ijl}-S_{ijl}^TT_l^T & *\\ A_iT_lS_{ijl}+B_{2i}K_jC_{2l}T_lS_{ijl} & -Q+J_{ijl}+R_{ijl}\end{bmatrix}<0,\quad 1\le i,j,l\le r.$$

From (7), (8), (9), we have Q>0 and $T_lS_{ijl}+S_{ijl}^TT_l^T>0$. Then we have $T_lS_{ijl}$ is invertible. Let $V_{ijl}=(T_lS_{ijl})^{-1}$ and pre- and post-multiplying (7) by $\begin{bmatrix}V_{ijl}^T & 0\\ 0 & I\end{bmatrix}$ and its transpose, then we have that

$$\begin{bmatrix}V_{ijl}^TQV_{ijl}-V_{ijl}-V_{ijl}^T & *\\ A_i+B_{2i}K_jC_{2l} & -Q+J_{ijl}+R_{ijl}\end{bmatrix}<0,\quad 1\le i,j,l\le r. \tag{13}$$

From Q>0, it follows

$$-Q^{-1}\le V_{ijl}^TQV_{ijl}-V_{ijl}-V_{ijl}^T. \tag{14}$$

From (13) and (14), it follows that

$$\begin{bmatrix}-Q^{-1} & *\\ A_i+B_{2i}K_jC_{2l} & -Q+J_{ijl}+R_{ijl}\end{bmatrix}<0,\quad 1\le i,j,l\le r. \tag{15}$$

Multiplying (15) by $\alpha_i\alpha_j\alpha_l$, $1\le i,j,l\le r$ and suming them, then it follows that

$$\begin{bmatrix}Q & *\\ A(\alpha)+B_2(\alpha)K(\alpha)C_2(\alpha) & -Q+J(\alpha)+R(\alpha)\end{bmatrix}<0, \tag{16}$$

where $A(\alpha)$, $B_2(\alpha)$, $C_2(\alpha)$, $K(\alpha)$ are same as in (2), and $J(\alpha)=\sum_{i=1}^r\sum_{j=1}^r\sum_{l=1}^r\alpha_i\alpha_j\alpha_lJ_{ijl}$, $R(\alpha)=\sum_{i=1}^r\sum_{j=1}^r\sum_{l=1}^r\alpha_i\alpha_j\alpha_lR_{ijl}$.

Multiplying (8) by $[\alpha_1I\cdots\alpha_rI]$ and its transpose and summing them, we obtain

$$\sum_{i=1}^r\sum_{j=1}^r\alpha_i\alpha_jJ_{ijl}+\sum_{i=1}^r\sum_{j=1}^r\alpha_i\alpha_jJ_{jil}=2\sum_{i=1}^r\sum_{j=1}^r\alpha_i\alpha_jJ_{ijl}>0,\quad 1\le l\le r,$$

which implies that

$$J(\alpha)=\sum_{i=1}^r\sum_{j=1}^r\sum_{l=1}^r\alpha_i\alpha_j\alpha_lJ_{ijl}>0. \tag{17}$$

Moreover, from (9)-(11), we have

$$\begin{aligned}R(\alpha)&=\sum_{i=1}^r\sum_{j=1}^r\sum_{l=1}^r\alpha_i\alpha_j\alpha_lR_{ijl}z\\&=\sum_{i=1}^r\alpha_i^3R_{iii}+\sum_{1\le i<j\le r}\alpha_i^2\alpha_j(R_{iij}+R_{iji}+R_{jii})\\&+\sum_{1\le i<j\le r}\alpha_i\alpha_j^2(R_{jji}+R_{jij}+R_{ijj})\\&+\sum_{1\le i<j<l\le r}\alpha_i\alpha_j\alpha_l(R_{ijl}+R_{ilj}+R_{jil}\\&+R_{jli}+R_{lij}+R_{lji})>0.\end{aligned}$$

Combining it and (16), (17), it follows that

$$\begin{bmatrix}-Q^{-1} & *\\ A(\alpha)+B_2(\alpha)K(\alpha)C_2(\alpha) & -Q\end{bmatrix}<0. \tag{18}$$

Let $P=Q^{-1}$ and applying the Schur complement to (18), it follows that

$$(A(\alpha)+B_2(\alpha)K(\alpha)C(\alpha))^TP\,(A(\alpha)+B_2(\alpha)K(\alpha)C(\alpha))-P<0. \tag{19}$$

If we choose Lyapunov function candidate $V(k)=x^T(k)Px(k)$, then from (19), we have $V(k+1)-V(k)<0$ for $x(k)\neq 0$, which implies that the system (1) is asymptotically stable. □

**Remark 2:** By introducing slack variables $S_{ijl}$ to separating the system matrix and Lyapunvo matrix, Theorem 1 presents an LMI-based sufficient condition for designing fuzzy static output feedback controllers for discrete-time T-S fuzzy systems, which can be effectively solved via LMI Control Toolbox [21]. Compared with the result in [17], Theorem 1 does not involve the technical condition $PB_{2i} = B_{2i}M$, $1 \le i \le r$. Moreover, the design condition given by Theorem 1 may be less conservative than that given by the results of [17] (see Example 1).

It should be noted that for each $C_{2i}$, there may exist different choices of $T_i$ satisfying (3). The following theorem shows that the feasibility of the condition of Theorem 1 is independent of the choices of $T_i$.

**Theorem 2:** If the condition of Theorem 1 is feasible for some $T_i$ satisfying (3), then it is feasible for any $\tilde{T}_i$ satisfying (3), i.e., $C_{2i}\tilde{T}_i = \begin{bmatrix} I & 0 \end{bmatrix}$.

**Proof:** Since $T_i$ and $\tilde{T}_i$ satisfy (3), $C_iT_i = \begin{bmatrix} I & 0 \end{bmatrix}$. Then

$$\begin{bmatrix} I & 0 \end{bmatrix} = \begin{bmatrix} I & 0 \end{bmatrix}\tilde{T}_i^{-1}T_i. \tag{20}$$

Denote $H_i = \tilde{T}_i^{-1}T_i = \begin{bmatrix} H_{11}^i & H_{12}^i \\ H_{21}^i & H_{22}^i \end{bmatrix}$, then from (20), it follows that $H_{11}^i = I, H_{12}^i = 0$. Consider

$$\begin{aligned} T_iS_i &= \tilde{T}_i\tilde{T}_i^{-1}T_iS_i = \tilde{T}_iH_iS_i \\ &= \tilde{T}_i\begin{bmatrix} I & 0 \\ H_{21}^i & H_{22}^i \end{bmatrix}\begin{bmatrix} S_{11} & 0 \\ S_{21}^i & S_{22}^i \end{bmatrix} = \tilde{T}_i\begin{bmatrix} S_{11} & 0 \\ \tilde{S}_{21}^i & \tilde{S}_{22}^i \end{bmatrix}, \end{aligned} \tag{21}$$

where $\tilde{S}_{21}^i = H_{21}^iS_{11} + H_{22}^iS_{21}^i$, $\tilde{S}_{22}^i = H_{22}^iS_{22}^i$. Let $\tilde{S}_i = \begin{bmatrix} S_{11} & 0 \\ \tilde{S}_{21}^i & \tilde{S}_{22}^i \end{bmatrix}$, then (21) can be rewritten as follows: $T_iS_i = \tilde{T}_i\tilde{S}_i$. Therefore, if (7) holds for $T_i$, then (7) holds for $\tilde{T}_i$, which implies that the conditions of Theorem 1 is feasible for $\tilde{T}_i$. Thus, the proof is complete. □

What it follows, the new technique is extended to $H_\infty$ control.

**Theorem 3:** Given a prescribed $H_\infty$ performance index $\gamma > 0$, if there exist symmetric matrices $Q$, $J_{ijl}$, $R_{ijl}$, $1 \le i,j,l \le r$ and matrices $S_{ijl}$, $L_i$, $1 \le i$, $j,l \le r$, with $S_{ijl} = \begin{bmatrix} S_{11} & 0 \\ S_{21}^{ijl} & S_{22}^{ijl} \end{bmatrix}$ $L_i = \begin{bmatrix} L_{ai} & 0 \end{bmatrix}$ satisfying the following LMIs,

$$\begin{bmatrix} Q - T_lS_{ijl} - S_{ijl}^TT_l^T & * & * & * \\ 0 & -\gamma I & * & * \\ A_iT_lS_{ijl} + B_{2i}L_j & B_{1i} & -Q & * \\ C_{1i}T_lS_{ijl} + D_{2i}L_j & D_{1i} & 0 & -\gamma I \end{bmatrix} + \begin{bmatrix} 0 & * & * & * \\ 0 & & & \\ 0 & & J_{ijl} + R_{ijl} & \\ 0 & & & \end{bmatrix}$$

$$< 0, 1 \le i,j,l \le r,$$

$$\begin{bmatrix} J_{ijl} \end{bmatrix}_{r\times r} + \begin{bmatrix} J_{ijl} \end{bmatrix}_{r\times r}^T > 0, \quad 1 \le l \le r,$$

$$R_{iii} > 0, \quad 1 \le i \le r,$$

$$R_{iij} + R_{iji} + R_{jii} > 0, \quad 1 \le i \ne j \le r,$$

$$R_{ijl} + R_{ilj} + R_{jil} + R_{jli} + R_{lij} + R_{lji} > 0, \quad 1 \le i < j < l \le r,$$

then the closed-loop system (1) via the fuzzy static output feedback controller (5) with

$$K_i = L_{ai}S_{11}^{-1}, \quad 1 \le i \le r$$

is stable while satisfying an $H_\infty$ performance bound $\gamma$.

**Proof:** By the technique similar to the proof of Theorem 1 and using Lemma 1, the proof is easily obtained and omitted. □

**Remark 2:** Similar to Theorem 2, the feasibility of the condition of Theorem 3 also is independent of the choices of $T_i$.

## 4. EXAMPLE

In this section, two examples are given to illustrate the validity of the presented conditions for fuzzy static output feedback control design.

**Example 1:** Consider a discrete-time nonlinear system with two states represented by the following T-S fuzzy system:

$R_1$ : If $x_1(k)$ is $\Lambda_1$
then $x(k+1)=A_1x(k)+B_1u(k)$
$y(k) = C_{21}x(k)$

$R_2$ : If $x_1(k)$ is $\Lambda_2$
then $x(k+1)=A_2x(k)+B_2u(k)$
$y(k) = C_{22}x(k)$,

where $x^T(k) = [x_1(k)\, x_2(k)]$,

$$A_1 = \begin{bmatrix} 1.1 & -0.5 \\ 0.4 & 0.6 \end{bmatrix}, B_{21} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C_{21} = \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 1.2 & 0 \\ 0.1 & 0.9 \end{bmatrix}, B_{22} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, C_{22} = \begin{bmatrix} 2 \\ 1 \end{bmatrix},$$

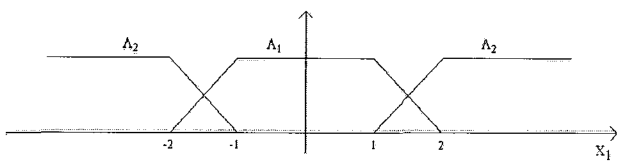


Fig. 1. Membership function of the IF parts in Example 1.

and the memberships $\Lambda_1$ and $\Lambda_2$ are depicted in Fig. 1.

Both the approaches in [17] and Theorem 1 are applicable to design static output feedback controller for the example. The LMIs of the approaches in [17] are infeasible. However, Theorem 1 gives $K_1 = -0.6112$, $K_2 = -0.2722$ which illustrates the effectiveness of the new approach.

**Example 2:** The following truck-trailer model taken from [19], is given as follows:

$$x(k+1) = \sum_{i=1}^{2} \alpha_i (A_i x(k) + B_{1i} w(k) + B_{2i} u(k)),$$
$$z(k) = \sum_{i=1}^{2} \alpha_i (C_{1i} x(k) + D_{2i} u(k)),$$
$$y(k) = \sum_{i=1}^{2} \alpha_i C_{2i} x(k),$$

where

$$B_{11} = B_{12} = \begin{bmatrix} 0 \\ 0.2000 \\ 0.1000 \end{bmatrix}, \quad B_{21} = B_{22} = \begin{bmatrix} -0.7143 \\ 0 \\ 0 \end{bmatrix}.$$

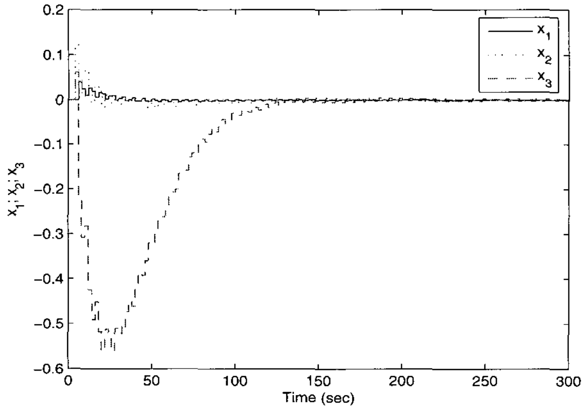


Fig. 2. State responses with initial condition $x(0) = [0\ \ 0\ \ 0]^T$ in Example 2.

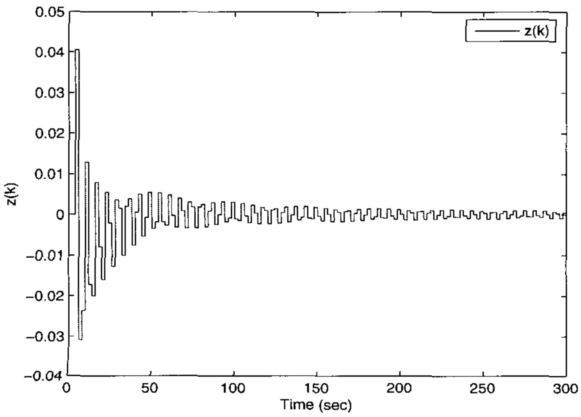


Fig. 3. The controlled output $z(k)$ with initial condition $x(0) = [0\ \ 0\ \ 0]^T$ in Example 2.

Using Theorem 3, we can design a fuzzy static output feedback controller that guarantees $H_\infty$ performance. The obtained results are given as follows:

$$K_1 = 0.2372, \quad K_2 = 0.2266, \quad \gamma_{opt} = 0.2957.$$

In the following simulations, we assume that $w(k) = 4\sin(k)/(k+4)$ and the initial condition to be $x(0) = [0\ \ 0\ \ 0]^T$. By using the obtained controller gains, the responses of state $x(k)$ and the controlled output $z(k)$ are respectively given in Figs. 2 and 3.

From the simulation results, it can be seen that the designed controller can guarantee system asymptotically stable with $H_\infty$ performance bound $\gamma = 0.2957$.

## 5. CONCLUSIONS

In this paper, the problem of designing robust static output feedback controllers for discrete-time T-S fuzzy systems has been investigated. A new sufficient condition for static output feedback stabilizing controller design is given in terms of solutions to a set of linear matrix inequalities, and the result is also extended to $H_\infty$ static output feedback controller design. The numerical examples have shown the effectiveness of the proposed design methods.

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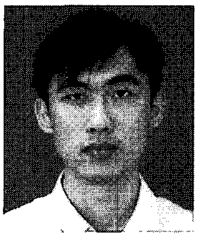

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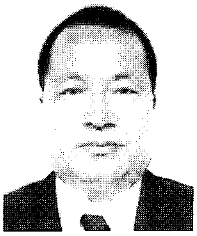

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