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(Max, +)-대수를 이용한 3-노드 유한 버퍼 일렬대기행렬 망에서 최적 버퍼 크기 결정*

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Determining the Optimal Buffer Sizes in Poisson Driven 3-node Tandem Queues using (Max, +)-algebra*

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■ Abstract ■

In this study, we consider stationary waiting times in finite-buffer 3-node single-server queues in series with a Poisson arrival process and with either constant or non-overlapping service times. We assume that each node has a finite buffer except for the first node. The explicit expressions of waiting times in all areas of the stochastic system were driven as functions of finite buffer capacities. These explicit forms show that a system sojourn time does not depend on the finite buffer sizes, and also allow one to compute and compare characteristics of stationary waiting times at all areas under two blocking rules : communication and manufacturing blocking. The goal of this study is to apply these results to an optimization problem which determines the smallest buffer capacities satisfying predetermined probabilistic constraints on stationary waiting times at all nodes. Numerical examples are also provided.

Keywords : (Max, +)-linear Systems, (Max, +)-algebra, Tandem Queues, Finite Buffers, Waiting Times

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1. Introduction

As a common model of telecommunication networks and manufacturing systems, tandem queues with infinite or finite buffers have been widely studied. Many researchers have interests in characteristics of stochastic networks such as mean waiting times, system sojourn times, invariant probabilities, blocking probabilities, etc. Because of the computational complexities and difficulties in performance evaluations for stochastic behaviors in networks, most studies over the past decades are focused on very restrictive and/or small size of stochastic networks.

In Poisson driven 2-node tandem queues with exponential service times, Grassmann and Drekic [7] studied the joint distribution of both queues by using generalized eigenvalues. For infinite buffered queues in series with non-overlapping service times and an arbitrary arrival process, Whitt [14] studied the optimal order of nodes which minimizes the mean value of sojourn times. Nakade [9] derived bounds for the mean value of cycle times in tandem queues with general service times under two blocking policies: communication and manufacturing.

Under the assumption of the capacities of buffers including the space for a customer in service and an arbitrary arrival process, Wan and Wolff [12] showed that the departure processes in tandem queues with finite buffers except for the first node and non-overlapping service times are independent of the sizes of finite buffers when they are greater than 2 under communication blocking or when they are greater than 1 under manufacturing blocking. Labetoulle and Pujolle [8] gave the same results for mean response time, but they derived mean waiting times at each queue under the assumption of infinite buffer capacities for all nodes. However, in our best knowledge, there is no result on waiting times in all sub-areas of finite buffered queues in series.

To say nothing of finite-buffer systems, at a node of a Poisson driven infinite-buffer stochastic network with non-exponentially distributed service times its departure processes and input processes are no longer Markovian. In addition, blocking phenomena at each node are caused by the finiteness of buffer capacities of all downstream nodes. In other words, overcrowding at one node can make all the upstream nodes blocked one after the other. These blocking phenomena do not guarantee the Markovian properties even in stochastic networks with Markovian service times. So, the system performances can not be treated by the well-known analytical methods.

More generous system, so called a (max, +)-linear system, has been studied. Various types of stochastic networks which are prevalent in telecommunication, manufacturing systems belong to the (max, +)-linear system. Many instances of (max, +)-linear systems can be represented by stochastic event graphs, a special type of stochastic Petri net, which allow one to analyze them. To be short, (max, +)- linear system is a choice-free net and consists of single-server queues under FIFO (First-In First-Out) service discipline. Discrete event systems (DESs) can be properly modeled by (max, +) -algebra, involving only two operators : 'max' and '+'.

Baccelli and Schmidt [6] derived a Taylor series expansion for mean stationary waiting time with respect to the arrival rate in a Poisson driven (max, +)-linear system. Their approach was generalized to other characteristics of stationary and transient waiting times by Baccelli et al. [4, 5], Ayhan and Seo [1, 2]. Recently, Seo [10, 11], by the same way, derived explicit expressions for characteristics of stationary waiting times in all areas of deterministic 2-node (3-node) tandem queues with finite buffers under two blocking policies : communication and manufacturing. He also disclosed a relationship of stationary waiting times in all areas of the systems between two blocking policies.

The explicit expressions in Seo [10, 11], functions of finite buffer capacities, are immediately applicable to compute the characteristics of stationary waiting times using Theorem 1 in [10] (see also [1]) and Theorem 2.3 in [2] such as higher moment and tail probability. On this score, the goal of this study is to apply those results in [1, 2, 10, 11] to an optimization problem for determining the smallest buffer capacities which satisfy predetermined probabilistic constraints on stationary waiting times at all nodes. These results can be applied to various type of finite buffer tandem systems which are prevalent in real world. For example, it is applicable to a buffer allocation problem in communication networks consisting of several routers and servers, and to a facility (storage) planning in manufacturing systems with AGVs carrying (raw) materials between workstations for loading and unloading.

Reader can refer on basic (max, +)-algebra and some preliminaries on waiting times in (max, +)-linear systems to Baccelli et al. [3] (see also [1, 2, 10, 11]). This paper is organized as follows. Section 2 and 3 includes some preliminary results and the optimization problem which is our main result. Numerical examples are given in section 4. Conclusion and some future research topics are mentioned in Section 5.

2. Stationary Waiting Times in 3-node Finite Buffer Tandem Queues

In this section we introduce the explicit expressions on stationary waiting times in single-server 3-node tandem queues with finite buffers and a Poisson arrival process. We assume that service times at all nodes are distributed as either constant or non-overlapped, and only a FIFO service discipline is allowed. We also assume that a stochastic network has finite buffer at each node except for the first node.

It is worth mentioning that Seo [10, 11] assumed all service times are deterministic, but all expression are still valid for non-overlapping service times. Because the only one we need is to decide the maximum value of the random vector D_n(see also [1, 2]), so non-overlapping service time distributions can be treated as the same way as constant service times. Because every service time at each node can be different in a system with non-overlapping service times, let σ^i_{-n} be an i.i.d. random variable of the nth non-overlapping service time at node i in the sense of Palm probability on the negative half line. Let K_i be a buffer capacity at node i, i=1, 2, 3, which includes a room for a customer in service. From the basic probability theory(taking a regular summation into a summation of random variables) the Propositions shown in Seo [11] for a deterministic 3-node finite-buffer tandem queue can be rewritten as follows. For stochastic networks with non-overlapping service times, however, one should use the series expansion defined in (2.4) in [10] (not a closed form) instead of Theorem 1 in [10] (a closed form) and Theorem 2.3 in [2] in order to compute characteristics of stationary waiting time.

Proposition 1 : Under communication blocking, when $K_1 = \infty$, $K_2 \geq 3$ and $K_3 \geq 3$,

$$D_n^1 = \sum_{j=1}^{n} \sigma_{-j}^1 \text{ for } 0 \leq n < K_2,$$

$$D_n^1 = max\left\{\sum_{j=1}^{n} \sigma_{-j}^1, \sigma_{-n}^1 + \sum_{j=1}^{n-K_2+1} \sigma_{-(n+1-j)}^2\right\}$$

for $K_2 \leq n < K_2 + K_3$,

$$D_n^1 = max\left\{\sum_{j=1}^{n} \sigma_{-j}^1, \sigma_{-n}^1 + \sum_{j=1}^{n-K_2+1} \sigma_{-(n+1-j)}^2, \sigma_{-n}^1 + \left(\sigma_{-n}^2 + \sigma_{-(n-1)}^2\right) + \sum_{j=1}^{n-(K_2+K_3)+1} \sigma_{-(n+1-j)}^3\right\}$$

for $n \geq K_2 + K_3$,

$$D_n^2 = \sigma_0^1 + max\left\{\sum_{j=1}^{n} \sigma_{-j}^1, \sum_{j=1}^{n} \sigma_{-j}^2\right\} \text{ for } 0 \leq n < K_3,$$

$$D_n^2 = \sigma_0^1 + max\left\{\sum_{j=1}^{n} \sigma_{-j}^1, \sum_{j=1}^{n} \sigma_{-j}^2, \sigma_{-n}^2 + \sum_{j=1}^{n-K_3+1} \sigma_{-(n+1-j)}^3\right\} \text{ for } n \geq K_3,$$

$$D_n^3 = \sigma_0^1 + \sigma_0^2 + max\left\{\sum_{j=1}^{n} \sigma_{-j}^1, \sum_{j=1}^{n} \sigma_{-j}^2, \sum_{j=1}^{n} \sigma_{-j}^3\right\}$$

for $n \geq 0$.

Proposition 2 : Under manufacturing blocking, when $K_1 = \infty$, $K_2 \geq 2$ and $K_3 \geq 2$,

$$D_n^1 = \sum_{j=1}^{n} \sigma_{-j}^1 \text{ for } 0 \leq n \leq K_2,$$

$$D_n^1 = max\left\{\sum_{j=1}^{n} \sigma_{-j}^1, \sigma_{-n}^1 + \sum_{j=1}^{n-K_2} \sigma_{-(n+1-j)}^2\right\}$$

for $K_2 < n \leq K_2 + K_3$,

$$D_n^1 = max\left\{\sum_{j=1}^{n} \sigma_{-j}^1, \sigma_{-n}^1 + \sum_{j=1}^{n-K_2} \sigma_{-(n+1-j)}^2, \sigma_{-n}^1 + \sigma_{-n}^2 + \sum_{j=1}^{n-(K_2+K_3)} \sigma_{-(n+1-j)}^3\right\} \text{ for } n > K_2 + K_3,$$

$$D_n^2 = \sigma_0^1 + max\left\{\sum_{j=1}^{n} \sigma_{-j}^1, \sum_{j=1}^{n} \sigma_{-j}^2\right\} \text{ for } 0 \leq n \leq K_3,$$

$$D_n^2 = \sigma_0^1 + max\left\{\sum_{j=1}^{n} \sigma_{-j}^1, \sum_{j=1}^{n} \sigma_{-j}^2, \sigma_{-n}^2 + \sum_{j=1}^{n-K_3} \sigma_{-(n+1-j)}^3\right\} \text{ for } n > K_3,$$

$$D_n^3 = \sigma_0^1 + \sigma_0^2 + max\left\{\sum_{j=1}^{n} \sigma_{-j}^1, \sum_{j=1}^{n} \sigma_{-j}^2, \sum_{j=1}^{n} \sigma_{-j}^3\right\}$$

for $n \geq 0$.

3. Optimal Buffer Capacities

These results are applicable to an optimization problem which determines the minimal finite buffers subject to probabilistic constraints on stationary waiting times in (max,+)- linear systems, where D_n^i sequence has the structure given in (2.5) of [10]. This probabilistic constraint, keeping waiting times within an acceptable range with a pre-specified probability, will ensure predictable completion times.

With the fact that the stationary waiting times at node 3 are independent of the capacities of finite buffers (see [10~12]), an optimization problem determining the smallest finite buffer capacities can consider only sub-areas of systems. For node $i(i=1, 2)$, let $\tau_i \geq 0$ be a pre-specified bound on stationary waiting time W^i and let $0 < \beta_i < 1$ be a pre-specified probability value, like a QoS (Quality of Service). Since W^i is stochastically non-increasing in K_2 and K_3, one is able to numerically determine the smallest (optimal) values of finite buffers K_2 and K_3 by using the explicit expressions of D_n^i, introduced in the previous section and in [11], together with Theorem 2.3 in [2]. Optimal sizes of finite buffers can be computed as a solution of the following optimization problem, for a given arrival rate $\lambda \in [0, a_i^{-1})$,

$$\begin{aligned} & min\ K_2 + K_3 \\ & s.t.\ \Pr(W^i > \tau_i) \le \beta_i,\ for\ i = 1,2 \\ & \quad K_2, K_3 \in \mathrm{N} \end{aligned}$$

where a_i is given in the i-th component of D_n vector defined in (2.5) of [1, 2, 10, 11].

It is very hard to show the convexities of polynomial $p_k(\cdots)$ and $q_k(\cdots)$ in K_2 and K_3. Instead, because for a fixed n, each argument of D_n^i is monotonously decreasing in K_2 and K_3, and 'max' function is a convex so that D_n^i for all $n \ge 0$ is a decreasing convex. Consequently, W^i, the elapsed time from the arrival until the beginning of service at node i, is a decreasing convex (composition of convex functions, see equations (2.2) and (2.3) in [1, 10]) in K_2 and K_3, thus the smallest finite buffer capacities can be chosen numerically.

On the other hand, when the value of $K_2 + K_3$ is increased by one, the explicit expressions of D_n^1 (see Proposition 1 in [11]) are obtained as followings.

From this fact we can immediately conclude the following Proposition which shows that the stationary waiting time in system with blocking is more affected by the buffer capacity of the closest node among the downstream nodes as we may expect.

Proposition 3: D_n^1 is more decreasing in K_2 than in K_3 because the arguments of D_n^1 with K_2+1 and K_3 are less than or equal to those with K_2 and K_3+1 for all $n \ge 0$. Moreover, for a fixed value of K_2+K_3 the larger value of K_2 results in less stationary waiting time W^1.

K_2+1, K_3	$D_n^1 = n\sigma^1$ for $0 \le n < K_2$, $D_{K_2}^1 = K_2\sigma^1$, $D_n^1 = max\ \{n\sigma^1, \sigma^1 + (n-K_2)\sigma^2\}$ for $K_2+1 \le n < K_2+K_3+1$, $D_n^1 = max\ \{n\sigma^1, \sigma^1 + (n-K_2)\sigma^2, \sigma^1 + 2\sigma^2 + [n-(K_2+K_3)]\sigma^3\}$ for $n \ge K_2+K_3+1$
K_2, K_3+1	$D_n^1 = n\sigma^1$ for $0 \le n < K_2$, $D_{K_2}^1 = max\ \{K_2\sigma^1, \sigma^1 + \sigma^2\}$, $D_n^1 = max\ \{n\sigma^1, \sigma^1 + (n-K_2+1)\sigma^2\}$ for $K_2+1 \le n < K_2+K_3+1$, $D_n^1 = max\ \{n\sigma^1, \sigma^1 + (n-K_2+1)\sigma^2, \sigma^1 + 2\sigma^2 + [n-(K_2+K_3)]\sigma^3\}$ for $n \ge K_2+K_3+1$

Note that the explicit expressions of D_n^2 and W^2 are functions of K_3 whereas those of D_n^1 and W^1 are functions of K_2 and K_3. Thus, since the convexity of W^i together with Proposition 3, an optimization problem can be solved in two steps: one for K_3 and then one for K_2 under the optimal value of K_3^*, which derives the global optimum. That is,

step 1	step 2
$min\ K_3$ $s.t.\ \Pr(W^2 > \tau_2) \le \beta_2$ $K_3 \in \mathrm{N}$	$min\ K_2 + K_3^*$ $s.t.\ \Pr(W^1 > \tau_1) \le \beta_1$ $K_2 \in \mathrm{N}, K_3^*$

4. Numerical Examples

Even though our methods are valid for both deterministic and non-overlapping service times, in order to avoid computational complexity and for the sake of understanding our results more easily we consider a 3-node finite-buffer queues in series with deterministic service times under two blocking policies in this section.

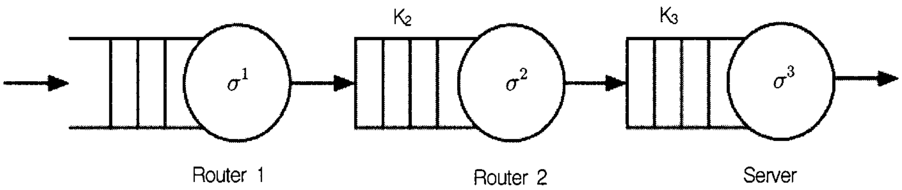
The following <Figure 3> shows an example for 3-node finite-buffer tandem queues with communication blocking in which all incoming calls can feed forward to a main server according to a predetermined sequence of routers whenever there is a room in the next one.

On the other hand, <Figure 4> describes another example for one with manufacturing blocking. AGV 1 transports a raw material to a machine, then after staying at the machine during some amount of time (its processing time plus delay time) it advances to the next process (es) by AGV 2. In this example we assumed that AGVs can handle a material at a time and a material processed at the machine can only moves into a storage for waiting the AGV 2 if the storage is not full. Otherwise, the material holds the machine until a room in the storage is available.

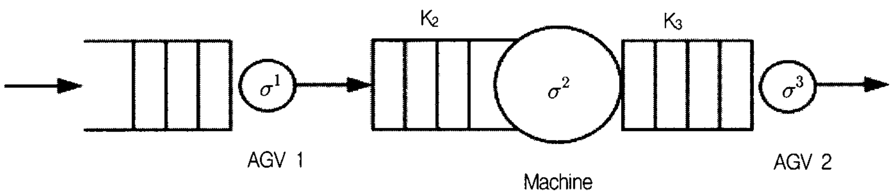
For a constant or non-overlapping system no delay is occurring at nodes after a node with the biggest service time, so we assume that the last node has the largest service time. Let $\sigma^1 = 1$, $\sigma^2 = 3$ and $\sigma^3 = 5$ be the constant service times at each node, and let $K_2 = 5$ and $K_3 = 5$ be the capacities of finite buffers at node 2 and 3. In this particular example, the Lyapunov maximum (maximum of service times, see details in [3]) a is 5. Note that for an infinite-buffer system the Lyapunov exponent is depend on the node i, whereas it is not for a finite-buffer one. We assume that we are only interested in first two moments of stationary waiting time W_i.

From the explicit expressions for D_n^i together with Theorem 1 in [10] one is able to compute the exact values of first two moments for stationary waiting times. Explicit expressions for each blocking policy are given as follows:

Communication Blocking Policy
$D_n^1 = n$ for $0 \le n \le 5$,
$D_n^1 = 7+3(n-6)$ for $6 \le n < 14$,
$D_n^1 = 32+5(n-14)$ for $n \ge 14$,
$D_n^2 = 1+3n$ for $0 \le n < 8$,
$D_n^2 = 29+5(7-9)$ for $n \ge 9$



〈Figure 3〉 3-node tandem queue with finite buffers under communication blocking



〈Figure 4〉 3-node tandem queue with finite buffers under manufacturing blocking

Manufacturing Blocking Policy
$D_n^1 = n$ for $0 \le n \le 6$,
$D_n^1 = 7+3(n-7)$ for $7 \le n < 16$,
$D_n^1 = 34+5(n-16)$ for $n \ge 16$,
$D_n^2 = 1+3n$ for $0 \le n < 10$,
$D_n^2 = 34+5(n-11)$ for $n \ge 11$

<Tables 1~2> and <Tables 3~4> show exact values and simulation values for the first and second moments of stationary waiting times at node 1 and 2 for various values of traffic intensity. From the numerical results, we can see that the expressions of D_n^i are accurate and that the first and second moments of waiting times under manufacturing blocking rule are smaller than or equal to those under communication blocking rule (see [11]). In addition, from comparing <Tables 3~4> to <Tables 5~6> we can obtain the exactly same values of first and second moments of waiting times at node 2 for the systems with one difference buffer capacity at node 3 under two blocking policies which is the fact addressed in Theorem 1 of [11]. In <Table 4> and <Table 6>* indicates the computational error.

〈Table 1〉 First Moment of Stationary Waiting Times Under Communication Blocking with $K_1 = \infty, K_2 = 5, K_3 = 5$

Traffic Intensity (ρ)	$E(W^1)$		$E(W^2)$	
	Solution	Simulation	Solution	Simulation
0.1	0.01020	0.00991 ∓ 0.00083	1.09574	1.0925 ∓ 0.00522
0.2	0.02083	0.02024 ∓ 0.00096	1.20455	1.1997 ∓ 0.00606
0.5	0.05594	0.05501 ∓ 0.00082	1.65546	1.6488 ∓ 0.00824
0.8	0.47410	0.4740 ∓ 0.06407	4.20172	4.1567 ∓ 0.13331
0.9	4.75394	5.0469 ∓ 0.63141	12.79257	13.0683 ∓ 0.77876

〈Table 2〉 Second Moment of Stationary Waiting Times Under Communication Blocking with $K_1 = \infty, K_2 = 5, K_3 = 5$

Traffic Intensity (ρ)	$E[(W^1)^2]$		$E[(W^2)^2]$	
	Solution	Simulation	Solution	Simulation
0.1	0.00701	0.00682 ∓ 0.00072	1.40131	1.3854 ∓ 0.02463
0.2	0.01476	0.01425 ∓ 0.00084	1.90189	1.8716 ∓ 0.03132
0.5	0.04547	0.04313 ∓ 0.00108	4.58941	4.5312 ∓ 0.08059
0.8	8.92257	8.8183 ∓ 2.31177	64.86605	63.9672 ∓ 6.25132
0.9	225.3716	248.77 ∓ 54.8541	565.0797	601.10 ∓ 84.6312

〈Table 3〉 First Moment of Stationary Waiting Times Under Manufacturing Blocking with $K_1 = \infty, K_2 = 5, K_3 = 5$

Traffic Intensity (ρ)	$E(W^1)$		$E(W^2)$	
	Solution	Simulation	Solution	Simulation
0.1	0.01020	0.01034 ∓ 0.00132	1.09574	1.0919 ∓ 0.00789
0.2	0.02083	0.02005 ∓ 0.00140	1.20455	1.1980 ∓ 0.00806
0.5	0.05560	0.05465 ∓ 0.00073	1.64603	1.6372 ∓ 0.00854
0.8	0.28457	0.3039 ∓ 0.05227	3.55408	3.5639 ∓ 0.13519
0.9	3.44201	3.6750 ∓ 0.46991	10.89937	11.1558 ∓ 0.66526

〈Table 4〉 Second Moment of Stationary Waiting Times Under Manufacturing Blocking with $K_1=\infty, K_2=5, K_3=5$

Traffic Intensity (ρ)	$E[(W^1)^2]$		$E[(W^2)^2]$	
	Solution	Simulation	Solution	Simulation
0.1	0.00701	0.00694 ∓ 0.00099	1.40131	1.3839 ∓ 0.03786
0.2	0.01476	0.01428 ∓ 0.00129	1.90186	1.8636 ∓ 0.04455
0.5	0.04351	0.04226 ∓ 0.00091	4.44712*	4.3773 ∓ 0.07241
0.8	4.51553	4.8656 ∓ 1.88514	45.61556	47.0966 ∓ 6.04797
0.9	161.8125	171.33 ∓ 33.3185	461.7548	484.00 ∓ 60.0964

〈Table 5〉 First Moment of Stationary Waiting Times Under Communication Blocking with $K_1=\infty, K_2=5, K_3=6$

Traffic Intensity (ρ)	$E(W^1)$		$E(W^2)$	
	Solution	Simulation	Solution	Simulation
0.1	0.01020	0.00998 ∓ 0.00039	1.09574	1.0943 ∓ 0.00238
0.2	0.02083	0.02082 ∓ 0.00040	1.20455	1.2039 ∓ 0.00265
0.5	0.05583	0.05564 ∓ 0.00063	1.64603	1.6448 ∓ 0.00471
0.8	0.34415	0.3549 ∓ 0.02974	3.55408	3.6005 ∓ 0.05790
0.9	3.88841	4.3043 ∓ 0.34109	10.89937	11.4326 ∓ 0.44745

〈Table 6〉 Second Moment of Stationary Waiting Times Under Communication Blocking with $K_1=\infty, K_2=5, K_3=6$

Traffic Intensity (ρ)	$E[(W^1)^2]$		$E[(W^2)^2]$	
	Solution	Simulation	Solution	Simulation
0.1	0.00701	0.00682 ∓ 0.00033	1.40131	1.3953 ∓ 0.01164
0.2	0.01476 .	0.01477 ∓ 0.00045	1.90186	1.8983 ∓ 0.01479
0.5	0.04450	0.04423 ∓ 0.00126	4.44713*	4.4339 ∓ 0.04058
0.8	5.84079	6.2967 ∓ 1.65551	45.61556	47.328 ∓ 3.30847
0.9	183.240	220.23 ∓ 37.0036	461.7548	519.99 ∓ 52.395

〈Table 7〉 $\Pr(W^1>0.1)$ for various K_2 and K_3

K_2 \ K_3	3	4	5	6	7	8
3	0.295798	0.253218	0.226571	0.209611	0.198776	0.191807
4	0.239165	0.208751	0.189434	0.177056	0.169109	0.163983
5	0.206094	0.185646	0.172519	0.164060	0.158606	0.155079
6	0.185113	0.171662	0.162971	0.157349	0.153712	0.151356
7	0.171567	0.162791	0.157101	0.153412	0.151020	0.149469
8	0.162774	0.157063	0.153355	0.150947	0.149385	0.148370

By using the explicit expressions of D_n^i together with Theorem 2.3 in [2], <Table 7> shows the values of $\Pr(W^1 > 0.1)$ for various K_is when the arrival rate $\lambda = 0.16$(traffic intensity $\rho = 0.8$). It also show the properties of monotonously decreasing of tail probabilities in K_is and Proposition 3. Therefore, one can numerically determine the smallest values of K_is that satisfy $\Pr(W^i > \tau_i) \le \beta_i$ with various but fixed values of τ_i and β_i, for $i = 1, 2$.

〈Table 8〉 The Smallest Buffer Capacities when $\rho = 0.8$ under Communication Blocking

	$\beta_1 = \beta_2 = 0.03$		$\beta_1 = \beta_2 = 0.01$	
	K_2^*	K_3^*	K_2^*	K_3^*
$\tau_1 = 4.0$ and $\tau_2 = 25.0$	5	5	5	8
$\tau_1 = 4.0$ and $\tau_2 = 15.0$	4	7	4	10
$\tau_1 = 4.0$ and $\tau_2 = 12.0$	3	8	4	12
$\tau_1 = 6.0$ and $\tau_2 = 25.0$	5	5	4	8
$\tau_1 = 6.0$ and $\tau_2 = 15.0$	3	7	3	10
$\tau_1 = 6.0$ and $\tau_2 = 12.0$	3	8	3	12
$\tau_1 = 8.0$ and $\tau_2 = 25.0$	4	5	4	8
$\tau_1 = 8.0$ and $\tau_2 = 15.0$	3	7	3	10
$\tau_1 = 8.0$ and $\tau_2 = 12.0$	3	8	3	12

<Table 8> shows the numerical solutions for the smallest values of buffer capacities K_2^* and K_3^* for given values of τ_i and β_i under communication blocking. From the table, when a prespecified values of τ_i or β_i is decreasing (increasing), the optimal buffer capacities are increasing (decreasing) as we expect.

By the relationship between two blocking policies addressed in [11] or the same way as done in communication blocking, one is also able to determine the optimal buffer capacities for systems with manufacturing blocking policy. We omitted them here.

5. Concluding Remark

In this study we introduced the explicit expressions for stationary waiting times in a Poisson driven finite-buffer 3-node tandem queue with non-overlapping service times under two blocking policies: communication blocking and manufacturing blocking. Also, we consider an optimization problem as an application of previous results in [11], which determines the smallest buffer capacities under probabilistic constraints on the waiting times at all nodes. From the fact that the characteristics of stationary waiting times are more affected by the finiteness of the closest node among the downstream nodes the optimization problem can be solved stepwise in two steps: one for K_3, the other for $K_2 + K_3^*$.

These results can be extended to more general complex (max,+)-linear systems with finite buffers such as m-node tandem queues, fork-and-join type queues (a special case of tandem queues), (maybe) mixture of them, and so forth. Moreover, they are also applicable to systems with more various blocking policies. Computational complexity and difficulty are growing fast as the number of nodes and the size of finite buffers are increasing, which also depend on the service times. So, developing more efficient computational algorithms will be a interesting topic in the future in order to apply to real world systems with large scale.

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