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A Study on the Uncertainty of Structural Cross-Sectional Area Estimate by using Interval Method for Allowable Stress Design

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Abstract

This study presents the so-called Modified Allowable Stress Design (MASD) method for structural designs. The objective of this study is to qualitatively estimate uncertainties of tensile steel member's cross-sectional structural designs and find the optimal resulting design which can resist all uncertainty cases. The design parameters are assumed to be interval associated with lower and upper bounds and consequently interval methods are implemented to non-stochastically produce design results including the structural uncertainties. By seeking optimal uncertainty combinations among interval parameters, engineers can qualitatively describe uncertain design solutions which were not considered in conventional structural designs. Under the assumption that structures have basically uncertainties like displacement responses, the safety range of resulting designs is represented by lower and upper bounds depending on given tolerance error and structural parameters. As a numerical example uncertain cross-sectional areas of members that can resist applied loads are investigated and it demonstrates that the present design method is superior to conventional allowable stress designs (ASD) with respect to a reliably structural safety as well as an economical material.

Keywords: Structural Uncertainty, Modified Allowable Stress Design, Interval Method

1. INTRODUCTION

In tradition design procedures of structures are based on a combination of safety factors for the loads and knockdown factors for the strength. Both these factors have been utilized from the past five decades for designs of building structures and their members.

There are at least two fundamental shortcomings to these traditional design procedures. First since the procedures were developed for conventional configurations, materials and familiar structural concepts, it may be difficult to apply to structures that have unconventional configurations, use new material systems and contain novel structural concepts. Consider, for example, the case of composite materials. Adaptations of traditional design procedures to account for larger scatter in composite properties and the sensitivity of composite structures to environmental effects and damages have led to a very conservative approach for designing composite structures. In this approach it is assumed in essence that a "worst case scenario" occurs simultaneously for each design condition, e.g. temperature, moisture, damage, loading and so on. These result in substantial and unnecessary weight penalties.

The second shortcoming of traditional design procedures is that measurements of the safety and reliability are not available. As a result it is not possible to determine (with any precision) the relative importance of various design options on the safety of building structures. In addition with no measurement of the safety it is unlikely that there is a consistent level of the safety and the efficiency throughout the building structure. That situation may lead to excessive weight without corresponding improvement in overall safety.

A new structural design procedure based on the design concept considering uncertainties based non-stochastic methods can help in order to overcome many of these problems. In particular since a stochastic method for exact measurements of safety and reliability is expensive during the design process and has limits of the adaptation, the approach based on non-stochastic methods, the so-called interval method, allows the designer to produce a consistent level of safety and efficiency as characteristics of the structure -no unnecessary over - designs in some areas.

Since the mid-1960s, a Moore-computability called the interval analysis has been introduced by Moore (1966) for bounding solutions of initial value problems. Koylouglu, Cakmak and Nielsen (1995) extended the original application to the treatment of uncertainty of loading conditions and structural parameters. However it is difficult to apply these results to practical structural engineering problems, because of the complexity of the algorithm.

Rao and Berke (1997) and Rao and Chen (1998) have developed different versions of interval analysis based on finite element methods. Although these works were mainly restricted to narrow intervals and approximate numerical solutions, it is very important that interval analyses were actually applied to measure practical structures, for example, beams and frames.

In this study interval methods are used in order to represent uncertainties of structural parameters. Interval solutions do not give the details of probability density description, however each uncertain parameter can be defined using two real number: the upper and lower bound.

In order to perform cross-sectional area design of steel members, we proposes the so-called "Modified Allowable Structural Design", i.e. MASD and intuitive result data by the interval methods are investigated quantitatively by using a normal probability density function as illustrated in the following Section 4.

In this study uncertainties of variables such as structural parameters, yielding stresses and loading conditions are only considered for structural designs. Error of workmanship and structural properties by environmental conditions are excluded. Uncertainties of these variables are expressed by intervals containing tolerance error x. It represents the existence or non-existence of structural uncertainties, which consist of combinations of initial data such as a tree graph as shown in Figure 1.

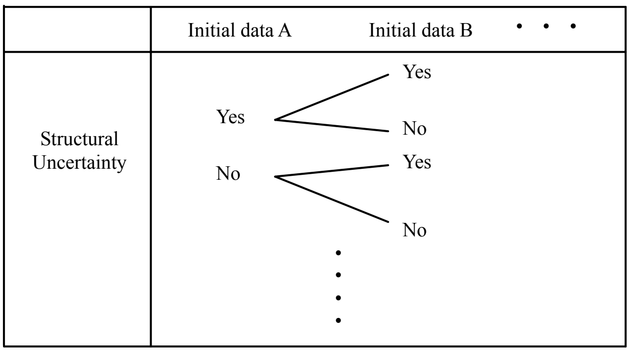


Figure 1. Tree graph representing structural uncertainties of initial data

Design conditions can be described by interval change functions which are composed of upper and lower bounds and the functions are introduced in the next Section 3.1.

Numerical examples verify the superiority and the efficiency of the proposed method in comparisons to classical allowable stress designs.

2. MATHEMATICAL BACKGROUND

2.1 Computability on Continuous Domain with Real Intervals

This Section illustrates that the continuous domain of real intervals to give a computational foundation of interval arithmetic is effectively given and allows us to define a computability notion on the interval analysis, the so-called Moore-computability. So the continuous domain of real intervals is defined as $\mathbf{R} = \langle \mathbf{I(R)}, \underline{\mathbf{C}} \rangle$, where

$$\mathbf{I(R)} = \{[\mathbf{r},\mathbf{s}] : \mathbf{r},\mathbf{s} \in \mathbf{R} \text{ and } \mathbf{r} \le \mathbf{s}\} \cup \{[-\infty, +\infty]\} \quad (1)$$

$$[\mathbf{r},\mathbf{s}] \subseteq [\mathbf{t},\mathbf{u}] \text{ if, and only if, } \mathbf{r} \le \mathbf{t} \text{ and } \mathbf{u} \le \mathbf{s} \quad (2)$$

with $\le$ being the usual "less or equal" order on the extended real numbers.

The below relation associated to this continuous domain is defined by

$$[\mathbf{r},\mathbf{s}] << [\mathbf{t},\mathbf{u}] \text{ if, and only if, } \mathbf{r} < \mathbf{t} \text{ and } \mathbf{u} < \mathbf{s} \quad (3)$$

The total elements of $\mathbf{R}$ are the sets of all degenerate intervals as follows.

$$\mathbf{Total(R)} = \{[\mathbf{r},\mathbf{r}] \,/\, \mathbf{r} \in \mathbf{R}\} \quad (4)$$

Since each degenerate interval $[\mathbf{r},\mathbf{r}]$ has associated the real number $\mathbf{r}$, the total elements of the continuous domain $\mathbf{R}$ can be considered as the set of real numbers. A countable base for this continuous domain is the rational intervals, i.e. the set $\mathbf{I(S)} = \{[\mathbf{v},\mathbf{w}] : \mathbf{v},\mathbf{w} \in \mathbf{S} \text{ and } \mathbf{v} < \mathbf{w}\}$. Therefore we can have a notion of computable real interval and computability of interval function.

For many operations, including standard arithmetic operations of addition, subtraction, multiplication and division, the resulting sets are represented as intervals that can be conveniently defined in term of end-points of the argument intervals.

The following functions from $\mathbf{I(R)}^{\mathbf{n}}$ in $\mathbf{I(R)}$ are interval arithmetic of Moore-computability and can be recovered from their rational interval restrictions which are clearly computable.

Let $\mathbf{X^I} = [\underline{\mathbf{X}}, \overline{\mathbf{X}}]$ and $\mathbf{Y^I} = [\underline{\mathbf{Y}}, \overline{\mathbf{Y}}]$ be the intervals, then the operations are defined by the following formulas:

$$\mathbf{X^I} + \mathbf{Y^I} = [\underline{\mathbf{X}}, \overline{\mathbf{X}}] + [\underline{\mathbf{Y}}, \overline{\mathbf{Y}}] = [\underline{\mathbf{X}} + \underline{\mathbf{Y}}, \overline{\mathbf{X}} + \overline{\mathbf{Y}}] \quad (5)$$

$$\mathbf{X^I} - \mathbf{Y^I} = [\underline{\mathbf{X}}, \overline{\mathbf{X}}] - [\underline{\mathbf{Y}}, \overline{\mathbf{Y}}] = [\underline{\mathbf{X}} - \overline{\mathbf{Y}}, \overline{\mathbf{X}} - \underline{\mathbf{Y}}] \quad (6)$$

$$\begin{aligned}\mathbf{X^I} \times \mathbf{Y^I} &= [\underline{\mathbf{X}}, \overline{\mathbf{X}}] \times [\underline{\mathbf{Y}}, \overline{\mathbf{Y}}] \\ &= [\mathbf{min}(\underline{\mathbf{X}} \cdot \underline{\mathbf{Y}}, \underline{\mathbf{X}} \cdot \overline{\mathbf{Y}}, \overline{\mathbf{X}} \cdot \underline{\mathbf{Y}}, \overline{\mathbf{X}} \cdot \overline{\mathbf{Y}}), \\ &\quad \mathbf{max}(\underline{\mathbf{X}} \cdot \underline{\mathbf{Y}}, \underline{\mathbf{X}} \cdot \overline{\mathbf{Y}}, \overline{\mathbf{X}} \cdot \underline{\mathbf{Y}}, \overline{\mathbf{X}} \cdot \overline{\mathbf{Y}})]\end{aligned} \quad (7)$$

$$\begin{aligned}\mathbf{X^I} / \mathbf{Y^I} &= [\underline{\mathbf{X}}, \overline{\mathbf{X}}] / [\underline{\mathbf{Y}}, \overline{\mathbf{Y}}] \\ &= \begin{cases} [\underline{\mathbf{X}}, \overline{\mathbf{X}}] \times \left[\dfrac{1}{\underline{\mathbf{Y}}}, \dfrac{1}{\overline{\mathbf{Y}}}\right], & \textbf{if } \mathbf{0} \in [\underline{\mathbf{Y}}, \overline{\mathbf{Y}}] \\ [-\infty, +\infty], & \textbf{otherwise} \end{cases}\end{aligned} \quad (8)$$

Commonly useful notions are the mid-point of an interval $\mathbf{X^c}$

$$\mathbf{X^c} = \frac{\overline{\mathbf{X}} + \underline{\mathbf{X}}}{2} \quad (9)$$

and the uncertainty of an interval $\mathbf{\Delta X}$

$$\mathbf{\Delta X} = \frac{\overline{\mathbf{X}} - \underline{\mathbf{X}}}{2} \quad (10)$$

2.2 The Sample Statistics to the Normal Probability Density Function

Conceptually and quantitatively, in order to investigate data by interval methods, it is usually desirable to have a continuous mathematical function. A probability density function (PDF) is such a mathematical function. It is assumed in this study that the data by the interval method is identified with the random variable, which can take on any value on the real line. There are several probability density functions which are used frequently in the structural engineering, but this study introduces normal probability density function (NPDF) which can easily apply to uncertainty problems.

The normal probability density function is defined as

$$\mathbf{p(x)} = \frac{1}{\mathbf{a}\sqrt{2\pi}} \mathbf{exp}\left\{-\frac{1}{2}\left(\frac{\mathbf{x}-\mathbf{b}}{\mathbf{a}}\right)^2\right\} \quad (11)$$

where $-\infty \le \mathbf{x} \le \infty$, $\mathbf{a} > 0$, $\mathbf{b} > -\infty$.

The mean of $\mathbf{X}$ is

$$\overline{\mathbf{X}} = \mathbf{b} \quad (12)$$

And its variance is

$$\boldsymbol{\sigma}_{\mathbf{x}}^2 = \mathbf{a}^2 \quad (13)$$

The degree Θ of uncertainty of probabilistic model is written as follows.

$$\Theta = \frac{\sigma_x}{\bar{X}} \ (\%) \qquad (14)$$

3. MODIFIED ALLOWABLE STRESS DESIGN (MASD)

3.1 Interval Change Function (ICF)

In the tensile structures with the uncertainty of variables which consist of steel materials, it is reasonable to introduce an interval change function that contains the uncertainties of structural parameters, yielding stresses and loading conditions. They denote design variables in cross-sectional designs. Since the uncertain variables are changeable values between upper and lower bounds defined by intervals in structural systems, the values of analytical results vary between certain upper and lower bound.

The interval change function which is composed of the upper and lower bounds is a mathematical formulation with respect to the tolerance error $\mathbf{x}$. The basic idea behind this function is qualitatively to calculate the variations to the required results that take place when a small change (i.e. uncertainty) is made by the uncertain variables against some nominal values in the structural system.

Considering if numerical uncertainties of the structural variables exist or not, generalized scenario function for the uncertainty, i.e. interval change function $\hat{\mathbf{I}}_i$ may be defined as follows.

$$\hat{\mathbf{I}}_i = \frac{b_1^c b_2^c b_3^c \cdots b_q^c}{a_1^c a_2^c a_3^c \cdots a_p^c}\left[\mathrm{f(x)}_i,\ \mathrm{g(x)}_i\right] \qquad (15)$$

where

$\mathbf{a}^c$, $\mathbf{b}^c$: Mid-point of each uncertain parameter

$\mathbf{i}$: 2^{p+q}, Number of the uncertainty scenario

$\mathbf{p}$, $\mathbf{q}$: Number of uncertain parameters

$\mathbf{f(x)}_i = \frac{(1-\mathbf{x})^q}{(1+\mathbf{x})^p}$: ICF for lower bound

$\mathbf{g(x)}_i = \frac{(1+\mathbf{x})^q}{(1-\mathbf{x})^p}$: ICF for upper bound

$\mathbf{x}$: Tolerance error, $0 \le \mathbf{x} < 1$, $\mathbf{x} \in \mathbf{R}$

3.2 Design Formulation and Numerical Algorithm

Uncertain variables are expressed as the formulation of the upper and lower bound functions. Using interval change functions, the governing equation of the modified allowable stress design is written as follows.

$$\hat{\sigma}_t = \frac{\hat{\mathbf{N}}}{\hat{\mathbf{A}}_n} = [\mathbf{f(x)}_i, \mathbf{g(x)}_i] \le \hat{\mathbf{f}}_t = \frac{\hat{\mathbf{f}}_y}{\zeta} \qquad (16)$$

where $\mathbf{f(x)}_i$ and $\mathbf{g(x)}_i$ denote interval change functions of the lower and upper bounds, respectively. $\mathbf{x}$ is the tolerance error which is $0 \le \mathbf{x} < 1$ $\mathbf{x} \in \mathbf{R}$ about real value $\mathbf{R}$. ζ is the factor of safety. $\hat{\mathbf{f}}_y$ and $\hat{\mathbf{f}}_t$ are respectively the design yielding strength and the design allowable stress of tension about the steel material structure. $\hat{\mathbf{A}}_n$ is the effective cross-sectional area of members. $\hat{\mathbf{N}}$ is the applied internal force and assumed as response results by the finite element method.

Expanding the lower or upper bound of Eq. (16), the relation between the tolerance error and the safety factor is rewritten as follows.

$$\zeta \le \frac{(\hat{\mathbf{f}}_y)_{min}}{[\mathbf{f(x)},\ \mathbf{g(x)}]_{max}} \qquad (17)$$

The MATHEMATICA Version 4.0 and the FORTRAN POWER STATION Version 4.0 are implemented to perform the structural analysis based on interval methods and calculate optimal tolerance errors and safety factors.

The numerical algorithm of MASD is shown in Figure 2.

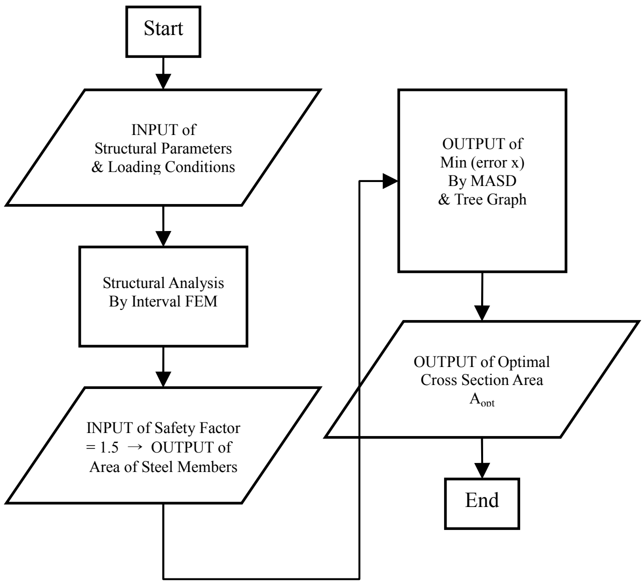


Figure 2. MASD algorithm for the cross-sectional area design based on the interval method

4. NUMERICAL EXAMPLE

A numerical example testing the present method is discussed here. The considered structure is composed of the two-dimensional 6 truss members. Its structural mechanism is shown in Figure 3.

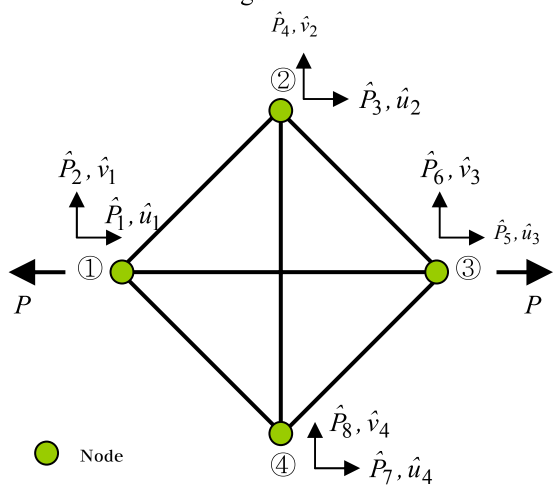


Figure 3. Loading-displacement mechanism of plane truss structure

The details of the rivet connectivity in the tensile design of the steel material and required structural parameters are shown in Figure 4. Standard cross-section properties of the rivet and steel members appear in Table 1 and 2. It is assumed that horizontal axial forces apply only in the structural connectivity, for example in node 1, and design variables with properties of the uncertainty are defined as yielding stress $\hat{\mathbf{f}}_y$, internal force $\hat{\mathbf{N}}$ and effective cross-sectional area $\hat{\mathbf{A}}_n$. Here $\hat{\mathbf{A}}_n$ is the formulation with uncertainty terms of the thickness of steel $\hat{\mathbf{t}}$, the width of steel $\hat{\mathbf{b}}$ and the radius of rivet hole $\hat{\mathbf{d}}$.

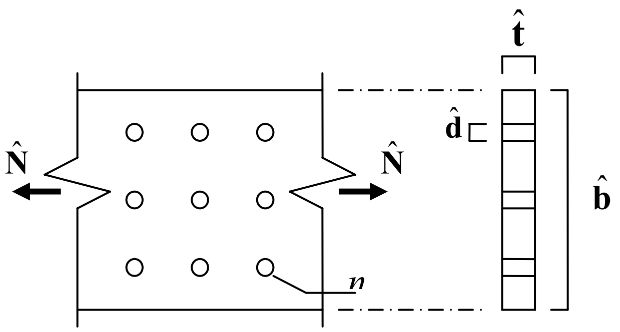


Figure 4. Section shape of connectivity in steel members

Table 1. Steel properties of the standard section

Steel properties of standard section			Section Area (cm^2)
TYPE	t (mm)	b (mm)	
A	16	150	24.0

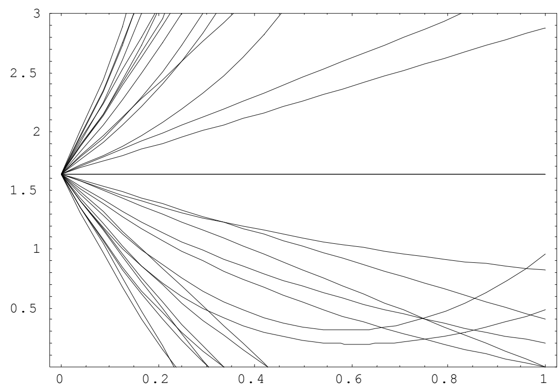
Table 2. Rivet properties of the standard section

TYPE	River radius (mm)	Rivet hole radius (mm)	Yielding stress (t/cm^2)
a	20	21.5	2.4
b	20	21.5	3.3

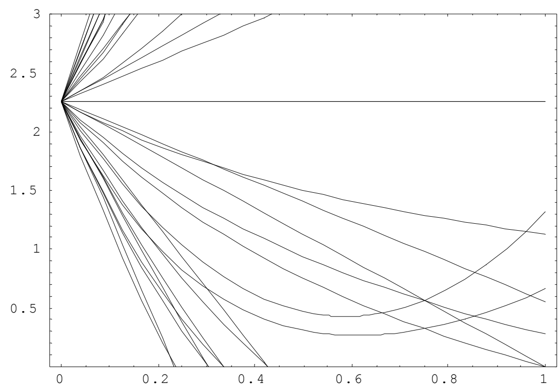
The relationship between the safety factor and tolerance error with the upper and lower bounds of the interval change function for each yielding stress are shown in Table 3 and Figure 5, where the horizontal and vertical axes denote the tolerance error and the safety factor, respectively. Practically the value of tolerance errors can not be large values for structural design. Thus tolerance errors must be decided by being compared with values of the safety factor. Through using limit values of the tolerance error and the safety factor, we can process the flexible design. This is the goal of the MASD algorithm.

In case of the safety factor = 1.5 (i.e. in conventional Allowable Stress Design or ASD), the limit of the tolerance error $\mathbf{x}$ (1.0E-02%) is shown in Figure 6 (b). The limited tolerance errors of certain or uncertain properties of yielding stress are changed by their scenario combinations of the uncertainty and we must determine the minimum of them, since the minimum can only cover all combinations with non-stochastic methods to save the structural safety. Thus in yielding stresses 2.4 t/cm^2 and 3.3 t/cm^2, the limited values are respectively 1.41 % and 5.91 % in case 3rd-combination.

Maximums of the safety factor can be produced through minimums of the tolerance error and are shown in Figure 6(a). In yielding stresses 2.4 t/cm^2 and 3.3 t/cm^2, maximal safety factor are respectively 1.64 and 2.26 in case 16th-combination. Therefore, in comparisons to the conventional value (i.e. 1.5) for the structural design, we can modify this value as flexible values including characteristics of uncertainties without the fracture of the structural system. Figure 7 (a) and (b) show uncertain cross-sectional areas described as upper and lower bounds in yielding stresses 2.4 t/cm^2 and 3.3 t/cm^2, respectively. From Figure 8, it can be seen that results of MASD are effective in the cross-sectional area design. In the yielding stress = 2.4 t/cm^2 and 3.3 t/cm^2, error values in compared to final design results by the conventional ASD are respectively 1.56 % and 3.89 %.



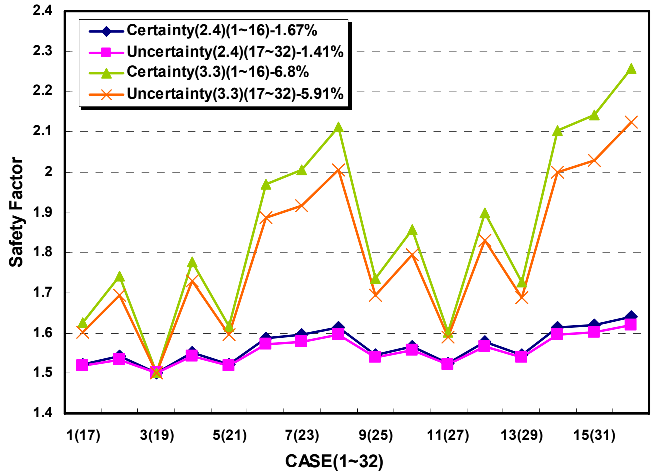
(a) f_y=2.4 t/cm^2



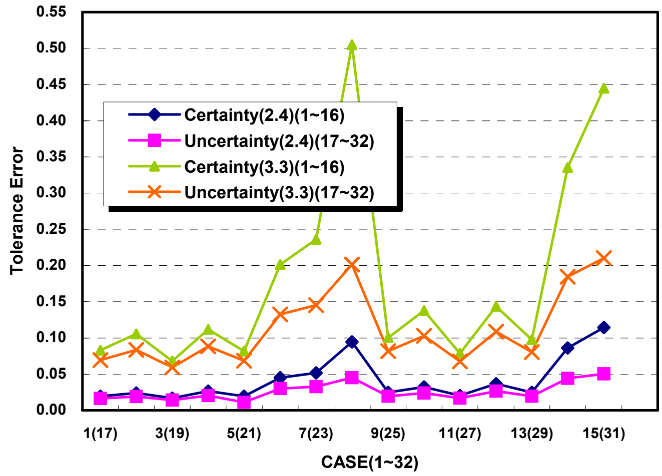
(b) f_y=3.3 t/cm^2

Figure 5. Interval change function: relationship among tolerance error and safety factor for 16 cases of scenarios and Exact value of safety error=1.5

Uncertainty degrees of the yielding stresses 2.4 t/cm^2 and 3.3 t/cm^2 are visualized as follows through a mathematical function which denotes a normal probability density function conceptually and quantitatively. The value of the upper bound is overestimated for the degree of the safety of the structural design both the yielding stresses 2.4 t/cm^2 and 3.3 t/cm^2 and is shown in Figures 9 and 10.



(a) Change and flexibility of safety factor with fixed tolerance error



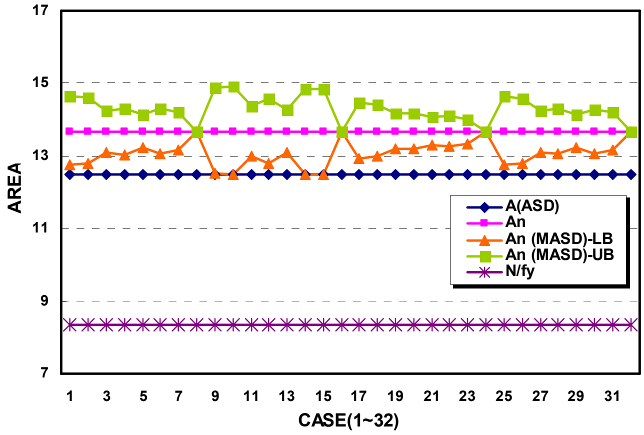
(b) Change and flexibility of tolerance error with fixed safety factor

Figure 6. The flexible values of tolerance error and safety factor as each case combination of uncertain parameters

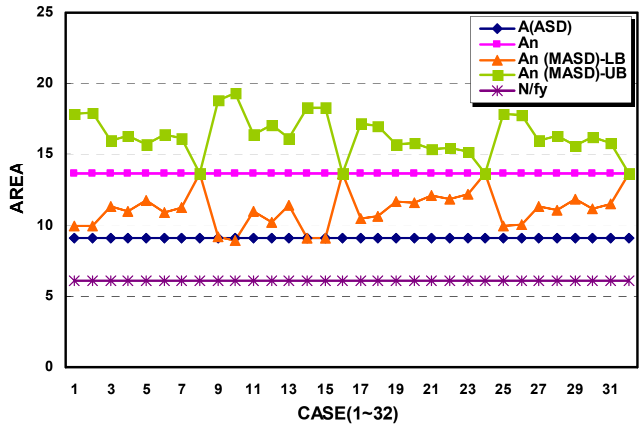
Table 3. Interval change function with lower and upper bounds for combinations of uncertainties of parameters, expression of uncertainty scenarios, $(*)^C$=exact value, $(*)^X$=non-uncertainty, $(*)^O$=uncertainty

	Interval change function with lower (F(x)) and upper (G(x)) bounds		Case
Combination	F(x)	G(x)	
N^X, B^X, t^X, d^X	$\frac{N^C(1-x)}{(B^C t^C - nd^C t^C)x^2 + (2B^C t^C)x + B^C t^C - nd^C t^C}$	$\frac{N^C(1+x)}{(B^C t^C - nd^C t^C)x^2 - (2B^C t^C)x + B^C t^C - nd^C t^C}$	1
N^X, B^X, t^X, d^O	$\frac{N^C(1-x)}{(B^C t^C)x^2 + (2B^C t^C - nd^C t^C)x + B^C t^C - nd^C t^C}$	$\frac{N^C(1+x)}{(B^C t^C)x^2 - (2B^C t^C - nd^C t^C)x + B^C t^C - nd^C t^C}$	2
N^X, B^X, t^O, d^X	$\frac{N^C(1-x)}{(B^C t^C + nd^C t^C)x + B^C t^C - nd^C t^C}$	$\frac{N^C(1+x)}{-(B^C t^C + nd^C t^C)x + B^C t^C - nd^C t^C}$	3
N^X, B^X, t^O, d^O	$\frac{N^C(1-x)}{(B^C t^C)x + B^C t^C - nd^C t^C}$	$\frac{N^C(1+x)}{-(B^C t^C)x + B^C t^C - nd^C t^C}$	4
N^X, B^O, t^X, d^X	$\frac{N^C(1-x)}{(nd^C t^C)x^2 + (B^C t^C)x + B^C t^C - nd^C t^C}$	$\frac{N^C(1+x)}{(nd^C t^C)x^2 - (B^C t^C)x + B^C t^C - nd^C t^C}$	5
N^X, B^O, t^X, d^O	$\frac{N^C(1-x)}{(B^C t^C - nd^C t^C)x + B^C t^C - nd^C t^C}$	$\frac{N^C(1+x)}{-(B^C t^C - nd^C t^C)x + B^C t^C - nd^C t^C}$	6
N^X, B^O, t^O, d^X	$\frac{N^C(1-x)}{(nd^C t^C)x + B^C t^C - nd^C t^C}$	$\frac{N^C(1+x)}{-(nd^C t^C)x + B^C t^C - nd^C t^C}$	7
N^X, B^O, t^O, d^O	$\frac{N^C(1-x)}{B^C t^C - nd^C t^C}$	$\frac{N^C(1+x)}{B^C t^C - nd^C t^C}$	8
N^O, B^X, t^X, d^X	$\frac{N^C}{(B^C t^C - nd^C t^C)x^2 + (2B^C t^C)x + B^C t^C - nd^C t^C}$	$\frac{N^C}{(B^C t^C - nd^C t^C)x^2 - (2B^C t^C)x + B^C t^C - nd^C t^C}$	9
N^O, B^X, t^X, d^O	$\frac{N^C}{(B^C t^C)x^2 + (2B^C t^C - nd^C t^C)x + B^C t^C - nd^C t^C}$	$\frac{N^C}{(B^C t^C)x^2 - (2B^C t^C - nd^C t^C)x + B^C t^C - nd^C t^C}$	10
N^O, B^X, t^O, d^X	$\frac{N^C}{(B^C t^C + nd^C t^C)x + B^C t^C - nd^C t^C}$	$\frac{N^C}{-(B^C t^C + nd^C t^C)x + B^C t^C - nd^C t^C}$	11
N^O, B^X, t^O, d^O	$\frac{N^C}{(B^C t^C)x + B^C t^C - nd^C t^C}$	$\frac{N^C}{-(B^C t^C)x + B^C t^C - nd^C t^C}$	12
N^O, B^O, t^X, d^X	$\frac{N^C}{(nd^C t^C)x^2 + (B^C t^C)x + B^C t^C - nd^C t^C}$	$\frac{N^C}{(nd^C t^C)x^2 - (B^C t^C)x + B^C t^C - nd^C t^C}$	13
N^O, B^O, t^X, d^O	$\frac{N^C}{(B^C t^C - nd^C t^C)x + B^C t^C - nd^C t^C}$	$\frac{N^C}{-(B^C t^C - nd^C t^C)x + B^C t^C - nd^C t^C}$	14
N^O, B^O, t^O, d^X	$\frac{N^C}{(nd^C t^C)x + B^C t^C - nd^C t^C}$	$\frac{N^C}{-(nd^C t^C)x + B^C t^C - nd^C t^C}$	15
N^O, B^O, t^O, d^O	$\frac{N^C}{B^C t^C - nd^C t^C}$	$\frac{N^C}{-B^C t^C - nd^C t^C}$	16

The degree of uncertainty of models of the yielding stress 3.3 t/cm^2 (i.e. the degree of uncertainties is the lower bound=0.09 and the upper bound=0.117) is greater than that of the yielding stress 2.4 t/cm^2 (i.e. the degree of uncertainties is the lower bound 0.024 and the upper bound=0.025).



(a) Uncertain area, in yielding stress = 2.4t/cm^2



(b) Uncertain area, in yielding stress = 3.3t/cm^2

Figure 7. Uncertainties of cross-section area in truss through the MASD and ASD by each scenario

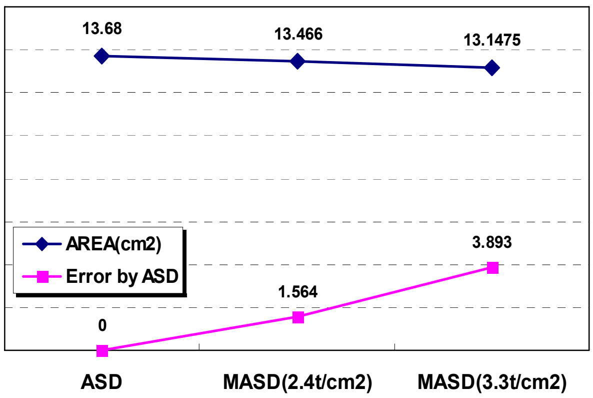
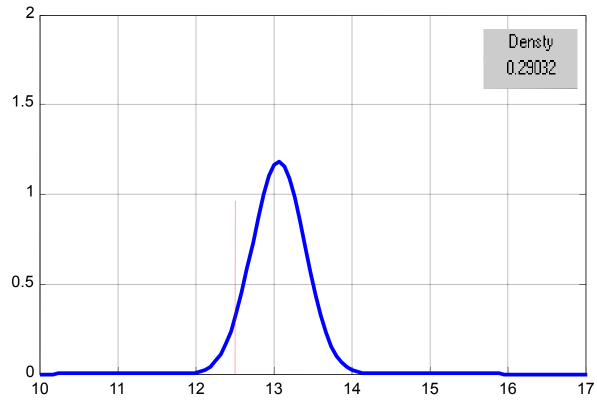
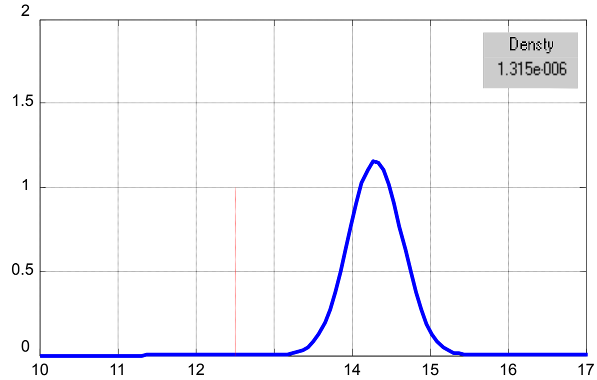


Figure 8. Error of results comparison with the MASD and conventioanl ASD, yielding stress of steel with 2.4 t/cm^2 and 3.3 t/cm^2

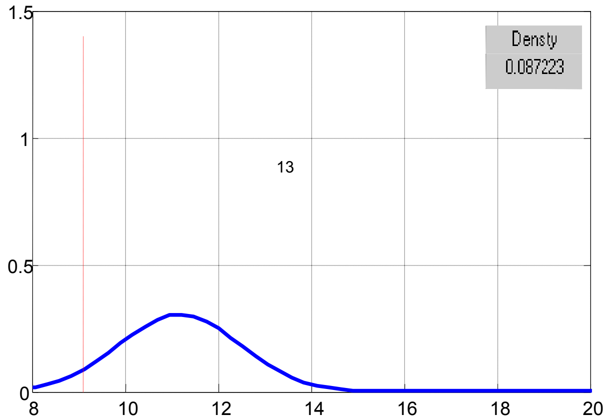


(a) Normal PDF of the lower bound of the cross-sectional area, mean=14.3078, standard deviation =0.344754, value of ASD=12.5, selected design value=13.68



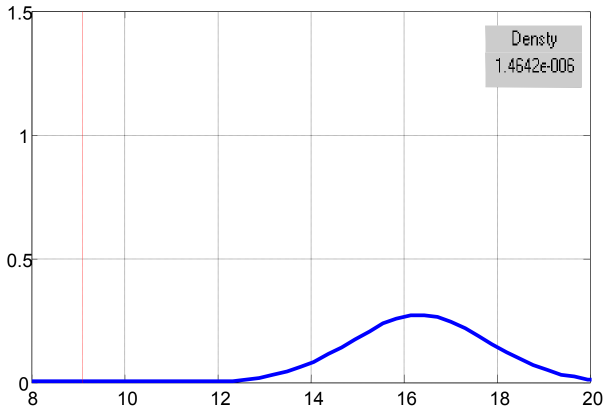
(b) Normal PDF of the upper bound of the cross-sectional area, mean=13.06457, standard deviation =0.336585, value of ASD=12.5, selected design value=13.68

Figure 9. Normal probability density function of cross-sectional area values of (a) lower bound and (b) upper bound by MASD in yielding stress=2.4 t/cm^2 and 32 cases of uncertainty scenarios



(a) Normal PDF of the lower bound of the cross-sectional area, mean=16.3417, standard deviation =1.472173, value of ASD=9.09091, selected design value=13.68

(continuous in the next page)



(b) Normal PDF of the upper bound of the cross-sectional area, mean=11.16869, standard deviation =1.316579, value of ASD=9.09091, selected design value=13.68

Figure 10. Normal probability density function of cross-sectional area values of (a) lower bound and (b) upper bound by MASD in yielding stress=3.3 t/cm^2 and 32 cases of uncertainty scenarios

5. CONCLUSIONS

This study addresses a non-stochastic innovative method in order to determine the favorable cross-sectional area of steel truss members with respect to structural uncertainties, which can appropriately resist in applied stresses. This method is a modified version of the allowable stress designs and yields approximate solutions with the lower and upper bounds. The present method allows the engineering practice to intuitively account for uncertain conditions in structural designs of members and to calculate very sharp bounds on the system response for all possible scenarios of uncertainty, existing in a single analysis.

The extended form of interval methods is appropriate for practically applying structural uncertainties, when the parameter uncertainties are reasonably small. In practice, in most cases the structural parameter errors or uncertainties are small.

The numerical example demonstrates that the present method provides the safety as well as the effective usage of materials for designing cross-sectional areas of steel truss members with response uncertainties which result from uncertainties of structural parameters.

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