근접한 간섭신호에 의한 어댑티브 어레이의 성능 열화 연구

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Approximation of the Performance Loss of an Adaptive Array due to a Neighboring Interferer

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요 약

Applebaum 타입의 beam forming 알고리듬을 사용하는 어댑티브 어레이에서 간섭신호가 원하는 신호와 매우 근접해 있을 때의 출력 신호 대 간섭 잡음비의 간단한 근사식을 도출하였다. 이 근사식은 어레이 크기, 원하는 신호와 간섭신호의 입사각 차이의 함수로 표현된다. 이 근사식에 의해 정해진 성능열화를 유발하는 간섭신호의 입사각 위치를 결정하는 식을 도출하였다. 제안된 근사식은 원하는 신호와 간섭신호가 8 도 이내의 입사각 차이를 유지할 때 컴퓨터로 계산한 정확한 신호 대 간섭 잡음비의 값과 1 dB 이내의 오차를 유지함을 보였다. 또한 어레이 엘리먼트의 숫자가 늘어남에 따라 간섭신호가 원하는 신호에 근접할 수 있는 정도도 더 늘어남을 보였다.

Key Words : Adaptive Array, Interfering Signal, Proximity, Range, Input INR

ABSTRACT

This paper derives an approximate expression for the output *SINR* (Signal to Interference plus Noise Ratio) of Applebaum type adaptive array under the scenario of the interferer's proximity to the desired signal. The approximation is made in terms of array geometry, the arrival direction of desired signal and that of an interfering signal. An interferer in the close proximity of target signal is shown to drastically impair the array performance. An approximate expression for interferer arrival direction which results in a predetermined performance loss is also obtained in terms of array configurations. Proposed approximation agrees with the computer calculated performance impairment when the two signals are apart by less than eight degrees. The allowable proximity of the interfering signal increases with the number of array elements.

I. Introduction

Adaptive array techniques offer possible solutions to the serious interference problems which may involve electronic countermeasures (ECM), RF interface, clutter scatterer returns and natural noise sources, which is performed via their flexible capabilities for automatic null steering and beam forming[1,2,3,4]. For these reasons, it has drawn a lot of attention from broad spectrum of application areas including mobile communication network. In particular, many works have been performed to enhance the capacity of CDMA system by using the beam forming capability of the adaptive array which now has been dubbed "Smart Antenna"[5,6,7].

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In CDMA cellular communication system deploying adaptive array in its basestation, in general, the number of users in a cell far exceeds the number of antenna elements. Therefore, forming nulls for all DOAs (direction of arrival) of interfering signals are impossible. Instead, the blind beam forming algorithms are chosen to track the desired signal based on the assumption of the predominance of the desired signal power. Accordingly, in this case, the proximity of an interferer to the desired signal is not seriously affecting the performance of the adaptive array.

In the mean time, Wiener solution pursued beam forming algorithm requires the exact a priori information for DOA of the desired signal and it has been shown in the literatures [1,8,9] that when the interferers are close to the desired signal source the array's performance deteriorates drastically since the array suppresses not only the interferences but also the desired signal. The interrelation between the degree of the performance degradation of an adaptive array and the DOA of the close interferer, however, has not been studied often. The objective of this paper is, therefore, to determine how the performance of an adaptive array is hampered by an interferer which is closely located to the desired signal and also determine how close the interferer can be positioned to achieve a given performance degradation.

This paper consists of five sections. In Section II, a mathematical modeling based on Applebaum type adaptive array is built in terms of various parameters and a closed expression for the array output *SINR* is derived. An analytic approximation for array output *SINR* which is a function of array geometry and signal arrival directions is obtained in Section III. In Section IV, the accuracy of the approximation in previous section is assessed and a rule of thumb determining the allowable interferer proximity is also expressed. Section V finally concludes this paper.

Ⅱ. Problem Formulation

In the steady state, the weight vector of an Applebaum adaptive array is determined by

$$\underline{w} = M^{-1}\underline{s}^{*} \tag{1}$$

where M is the input signal covariance matrix given by

$$M = E\left[\underline{v}^{*}\underline{v}^{T}\right] \tag{2}$$

and $\underline{s}$ is the steering vector which is given by

$$\underline{s} = \begin{bmatrix} \exp(jkx_1 \sin\theta) \\ \exp(jkx_2 \sin\theta) \\ \vdots \\ \exp(jkx_N \sin\theta) \end{bmatrix}$$

where k is the wave number which is equal to $2\pi/\lambda$, λ is the wavelength of the narrowband signal, x_n $(n = 1,2,\cdots,N)$ is the array element position, N is the number of the array elements, θ is the array steering direction, * and T means the complex conjugate and transpose, respectively. In this paper, the matrix and vector quantities are denoted by the upper-case and underlined lower-case letters, respectively. In Eq. (2), E denotes the expectation and $\underline{v}$ is the input signal vector such that

$$\underline{v} = \alpha(t)\underline{s}_d + \beta(t)\underline{s}_i + \underline{n}(t) \tag{3}$$

where $\alpha(t)$ is the desired signal waveform, $\underline{s}_d$ is the desired signal arrival phase vector, $\beta(t)$ is the interfering signal waveform, $\underline{s}_i$ is the interfering signal arrival phase vector, $\underline{n}(t)$ is the antenna element thermal noise vector.

Each component of $\underline{n}(t)$ is derived from a Gaussian process and are mutually uncorrelated. From Eq. (3) together with the assumption that the components of $\underline{s}_d$, $\underline{s}_i$, $\underline{n}(t)$ are mutually uncorrelated, Eq. (2) becomes

$$M = E\left[|\alpha(t)|^2\right]\underline{s}_d^{*}\underline{s}_d^{T} + E\left[|\beta(t)|^2\right]\underline{s}_i^{*}\underline{s}_i^{T} + \sigma_n^{2}I \tag{4}$$

where σ_n^2 is the power of the thermal noise and

I is $N \times N$ identify matrix. In Eq. (4) M consists of two parts. One is the desired signal component and the other is the unwanted signal component, i.e., interference and thermal noise. If we define the latter as M_{nn}, Eq. (4) becomes

$$M = E\left[|\alpha(t)|^2\right] \underline{s}_d^* \underline{s}_d^T + M_{nn} \tag{5}$$

Using a matrix inversion lemma the inverse of covariance matrix is

$$M^{-1} = M_{nn}^{-1} - \frac{E\left[|\alpha(t)|^2\right] M_{nn}^{-1} \underline{s}_d^* \underline{s}_d^T M_{nn}^{-1}}{E\left[|\alpha(t)|^2\right] \underline{s}_d^T M_{nn}^{-1} \underline{s}_d^* + 1} \tag{6}$$

When the steering vector is perfect, i.e., the array steers the exact signal source, Eq. (1) becomes

$$\underline{w} = \left[M_{nn}^{-1} - \frac{E\left[|\alpha(t)|^2\right] M_{nn}^{-1} \underline{s}_d^* \underline{s}_d^T M_{nn}^{-1}}{E\left[|\alpha(t)|^2\right] \underline{s}_d^T M_{nn}^{-1} \underline{s}_d^* + 1} \right] \underline{s}_d^* \tag{7}$$

Equation (7) can be further simplified to

$$\underline{w} = \mu M_{nn}^{-1} \underline{s}_d^* \tag{8}$$

where

$$\mu = \frac{1}{E\left[|\alpha(t)|^2\right] \underline{s}_d^T M_{nn}^{-1} \underline{s}_d^* + 1}$$

We partition the array output power by the desired signal power and the unwanted signal power as we did in deriving Eq. (5). Then the desired signal power P_d is expressed as

$$P_d = E\left[|\alpha(t)|^2\right] \underline{w}^{T*} \underline{s}_d^* \underline{s}_d^T \underline{w} \tag{9}$$

and the unwanted power P_{nn} is given by

$$P_{nn} = \underline{w}^{T*} M_{nn} \underline{w} \tag{10}$$

From Eq. (9) and Eq. (10) the output $SINR$ is determined from

$$SINR = \frac{E\left[|\alpha(t)|^2\right] \underline{w}^{T*} \underline{s}_d^* \underline{s}_d^T \underline{w}}{\underline{w}^{T*} M_{nn} \underline{w}} \tag{11}$$

Substituting Eq. (8) into Eq. (11) we have

$$SINR = E\left[|\alpha(t)|^2\right] \underline{s}_d^T M_{nn}^{-1} \underline{s}_d^* \tag{12}$$

In deriving Eq. (12) we use the fact that M_{nn}, therefore M_{nn}^{-1} is Hermitian. M_{nn}^{-1} is expressed as

$$M_{nn}^{-1} = \frac{1}{\sigma_n^2} \left(I - \frac{\gamma_i \underline{s}_i^* \underline{s}_i^T}{\gamma_i N + 1} \right) \tag{13a}$$

where γ_i is the input interference to noise ratio (INR) and is given by

$$\gamma_i = \frac{E\left[|\beta(t)|^2\right]}{\sigma_n^2} \tag{13b}$$

From Eq. (12) and Eq. (13) we have

$$SINR = \gamma_d \left(N - \frac{\gamma_i \left|\underline{s}_d^T \underline{s}_i^*\right|^2}{\gamma_i N + 1} \right) \tag{14a}$$

where γ_d is the input signal to noise ratio (SNR) and is given by

$$\gamma_d = \frac{E\left[|\alpha(t)|^2\right]}{\sigma_n^2} \tag{14b}$$

To evaluate the term $\left|\underline{s}_d^T \underline{s}_i^*\right|^2$, we express $\underline{s}_d$ as

$$\underline{s}_d = \begin{bmatrix} \exp(jkx_1 \sin\theta_d) \\ \exp(jkx_2 \sin\theta_d) \\ \vdots \\ \exp(jkx_N \sin\theta_d) \end{bmatrix} \tag{15}$$

where θ_d is the desired signal direction.

Similarly we express $\underline{s}_i^*$ as

$$\underline{s}_i^* = \begin{bmatrix} \exp(-jkx_1 \sin\theta_i) \\ \exp(-jkx_2 \sin\theta_i) \\ \vdots \\ \exp(-jkx_N \sin\theta_i) \end{bmatrix} \tag{16}$$

where θ_i is the interference direction.

From Eq. (15) and Eq. (16), $\underline{s}_d^T \underline{s}_i^*$ is computed from

$$\underline{s}_d^T \underline{s}_i^* = \sum_{n=1}^{N} \exp[jkx_n(\sin\theta_d - \sin\theta_i)] \quad (17)$$

Using Eq. (17) we have

$$\left|\underline{s}_d^T \underline{s}_i^*\right|^2 = \sum_{n=1}^{N}\sum_{m=1}^{N} \exp[jk[(x_n - x_m)(\sin\theta_d - \sin\theta_i)]] \quad (18)$$

Ⅲ. Performance Measure Approximation

For the case of an interferer far removed from the desired signal source, it is well known that an Applebaum type adaptive array is able to suppress the interference using the adaptive nulling method [1]. Therefore, we restrict our analysis to the case wherein the DOA of the interfering signal is near the DOA of the desired signal source. In accordance with the above argument, i.e., for a small deviation of θ_i from θ_d, Eq. (18) can be approximated as

$$\left|\underline{s}_d^T \underline{s}_i^*\right|^2 \approx N^2 - G(\theta_i) \quad (19)$$

In Eq. (19), $G(\theta_i)$ is given by

$$G(\theta_i) = \varepsilon u_i^2 \quad (20)$$

where

$$\varepsilon = k^2\left[N\sum_{n=1}^{N} x_n^2 - \left(\sum_{n=1}^{N} x_n\right)^2\right] \qquad u_i = \sin\theta_d - \sin\theta_i$$

$G(\theta_i)$ is an effective measure of how far the interferer is located from the desired signal source in angular direction. In its extremity, for example, if the DOA of the interferer is the same as that of the desired signal, $G(\theta_i)$ is equal to zero. In this case, from Eq. (14) and Eq. (19) we see that $\left|\underline{s}_d^T \underline{s}_i^*\right|^2 = N^2$ and the output $SINR$ reaches its minimum value which is expressed as

$$SINR_{\min} = \frac{N\gamma_d}{N\gamma_i + 1} \quad (21)$$

In general $N\gamma_i >> 1$ and Eq. (21) is then simplified to

$$SINR_{\min} = \frac{\gamma_d}{\gamma_i} \quad (22)$$

Eq. (21) and Eq. (22) indicates that when the DOA of the interferer and desired signal are same as each other the adaptive array can not suppress the interferer. The array input signal power and the array input interference power remain unchanged at the array output. In other words, our adaptive array does nothing better than a single antenna. Using Eq. (14) and Eq. (19) the output $SINR$ can be written as

$$SINR = N\gamma_d \frac{1 + \frac{\gamma_i}{N} G(\theta_i)}{N\gamma_i + 1} \quad (23)$$

Noting that $N\gamma_d$ is the array output SNR without interference, we define the ratio R as follows.

$$R = \frac{SINR}{N\gamma_d} = \frac{1 + \frac{\gamma_i}{N} G(\theta_i)}{N\gamma_i + 1} \quad (24)$$

We now determine the bounds for the DOA of the interferer with a given performance degradation. Suppose the given ratio in Eq. (24) is C, then the possible range of θ_i is determined by

$$\frac{1 + \frac{\gamma_i}{N} G(\theta_i)}{N\gamma_i + 1} \geq C \quad (25a)$$

or

$$G(\theta_i) \geq C' \quad (25b)$$

where

$$C' = \frac{N}{\gamma_i}[C(N\gamma_i + 1) - 1] \quad (25c)$$

Using Eq. (20) and Eq. (25), the allowable location of the interferer is expressed as follows.

$$\theta_i \geq \sin^{-1}\left(\sin\theta_d + \sqrt{C'/\varepsilon}\right) \qquad (26a)$$

or

$$\theta_i \leq \sin^{-1}\left(\sin\theta_d - \sqrt{C'/\varepsilon}\right) \qquad (26b)$$

Eq. (26) is a governing equation from which we can calculate how close the interferer can be located from the desired signal and the square root term $\sqrt{C'/\varepsilon}$ in Eq. (26) determines the proximity. ε can be approximated as

$$\varepsilon \approx \frac{3N^2L^2}{\lambda^2} \qquad (27)$$

where L is the array length.

Note that to obtain the approximation in Eq. (27) we assumed: the array elements are uniformly spaced; $N \geq 10$; $\sum_{n=1}^{N} x_n^2 \approx \frac{NL^2}{12}$.

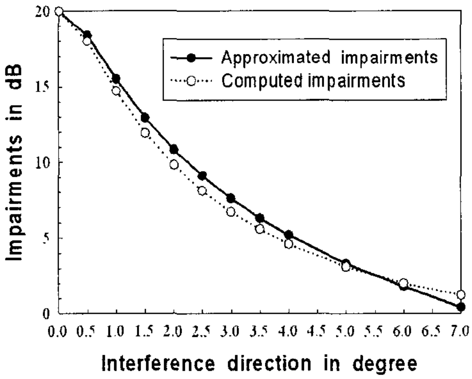

Fig. 1 Approximated and computed impairments as a function of direction, γ_d = 20 dB, $\gamma_i = 10$ dB, $N = 10$, $\theta_d = 0°$, $d = \lambda/2$

Assuming $N\gamma_i >> 1$, C' in Eq. (25c) also can be approximated as

$$C' \approx CN^2 \qquad (28)$$

Using Eq. (27) and Eq. (28), Eq. (26), without loss of generality, becomes

$$\theta_i \geq \sin^{-1}\left(\sin\theta_d + \sqrt{C/3}\,\frac{\lambda}{L}\right) \qquad (29a)$$

or

$$\theta_i \leq \sin^{-1}\left(\sin\theta_d - \sqrt{C/3}\,\frac{\lambda}{L}\right) \qquad (29b)$$

From Eq. (29) we observe that to maintain a specific output $SINR$, the angular difference between θ_d and θ_i should become larger as the center frequency of signal spectrum decreases. If we put $L = \varsigma\lambda(N-1)$ where $\varsigma\lambda$ is inter-element spacing, equation (29a) becomes

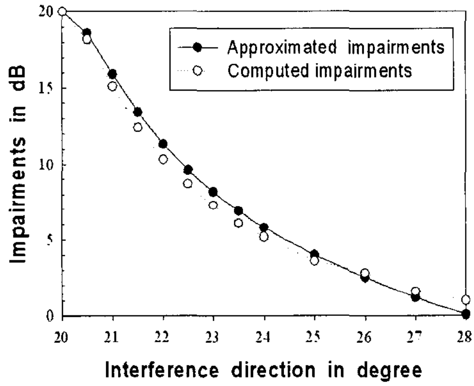

Fig. 2. Approximated and computed impairments as a function of direction, γ_d = 20 dB, $\gamma_i = 10$ dB, $N = 10$, $\theta_d = 20°$, $d = \lambda/2$

$$\theta_i \geq \sin^{-1}\left(\sin\theta_d + \sqrt{C/3}\,\frac{1}{\varsigma(N-1)}\right) \qquad (30)$$

Eq. (29) is a rule of thumb determining the required angular distance of the interferer. From Eq. (30), we note that the allowable proximity of the interferer is increased as the number of array elements increases.

Ⅳ. Computation Results

To get the computational results we assume for all cases the following: input $SNR(\gamma_d)$ is 20 dB; input $INR(\gamma_i)$ is 20 dB or 10 dB; number of array elements (N) is 10; desired signal direction (θ_d) is 0° or 20°; array element spacing is $\lambda/2$.

We first compare the approximation of the output $SINR$ expressed in Eq. (24) to the closed expression in Eq. (14). Fig. 1 and Fig. 2 illustrate the values obtained from the approximation and the values computed from the closed expression under the parameters specified before. Fig. 1 shows the difference in performance impairments between the approximation and the closed expression when the interferer arrival direction varies from 0^{o} to 7^{o}. Desired signal is assumed to arrive from broadside. From Fig. 1 we note that our approximation in Eq. (24) shows close agreement to the closed expression in Eq. (14) with no more than 1 dB deviation. We also note from Fig. 1 that Eq. (24) holds effective when the two signals of interest are apart by up to seven degrees. As a matter of fact, when we put 8^{o} in Eq. (24) under the parameters given in Fig. 1, the degradation becomes negative, i.e., array performance exceeds the ideal output $SINR$. This happens because, as we mentioned in previous section, the approximation in Eq. (24) only works under a certain extent of the signals' proximities. Another important observation made from Fig. 1 is that the performance improvement due to the deployment of an adaptive array is completely vanished when the interferer is collocated with the desired signal. This phenomenon was illustrated in [9]. Comparing Fig. 1 and Fig. 2, it has been shown that the array performance becomes better as the desired signal approaches broadside [2, 8]. The parameters given in Fig. 2 are same as ones in Fig. 1 except the desired signal arrival direction. In Fig. 2, θ_d is set to 20^{o} instead of broadside. It is noted that for a given proximity the impairment in Fig. 2 is greater than that in Fig. 1 by up to 0.7 dB.

In Table 1 and Table 2, we examine the accuracy of the approximation in Eq. (24) by alternative way. In this examination, the allowable interferer direction is obtained first using Eq. (30) under the parameters given including predetermined impairment. Approximated value in degree is then applied to Eq. (14) to get the computer calculated value. By comparing the two numbers of impairments, the accuracy of the approximation can be measured. From Tables 1 and 2, we reassure that the output $SINR$ degradation based on our approximation differ by no more than 1 dB from those based on the closed expression.

Table 1. Accuracy of the approximation, $\gamma_d = \gamma_i = 20$ dB, $N = 10$, $\theta_d = 0^{o}$, $d = \lambda/2$

Allowable impairments (in dB)	0.5	1	2	3	4	5
Interferer proximity from Eq.(30) (in degree)	7.0	6.6	5.9	5.2	4.6	4.1
Computed impairments from Eq.(14) (in dB)	1.2	1.5	2.2	2.9	3.6	4.4

Table 2. Accuracy of the approximation, $\gamma_d = 20$ dB, $\gamma_i = 10$ dB, $N = 10$, $\theta_d = 20^{o}$, $d = \lambda/2$

Allowable impairments (in dB)	0.5	1	2	3	4	5
Interferer proximity from Eq.(30) (in degree)	27.6	27.2	26.4	25.7	25.0	24.5
Computed impairments from Eq.(14) (in dB)	1.2	1.5	2.2	2.8	3.6	4.4

We finally explore the relation between the degree of proximity and number of array elements in Fig. 3. Setting the impairment to 2 dB in Eq. (30), we vary the number of array elements to obtain the allowable degree of proximities. In Fig. 3, the allowable interference proximity to the desired

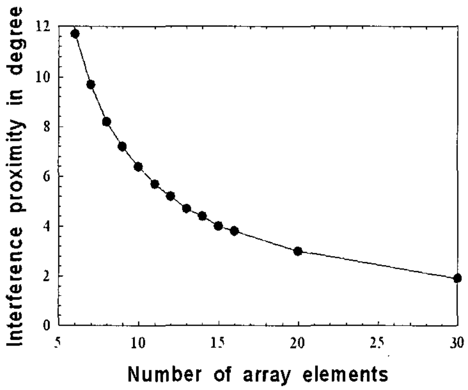

Fig. 3 Allowable interference direction to maintain the impairment within 2 dB, $\gamma_d = 20$ dB, $\gamma_i = 10$ dB, $\theta_d = 20^{o}$, $d = \lambda/2$

signal direction has been drawn against the number of array elements. It is evident from Fig. 3 that as the number of array elements increases, freedom of the interferer proximity is also increased

V. Conclusions

For the case of perfect steering Applebaum adaptive array, the expression for the output SINR due to an interferer present was derived. It is a function of the interferer direction, array input SNR, INR and array configuration. An approximation of the allowable interferer direction for a given degradation has been determined and it also has been found that the validity of the approximation holds up when the angular distance between the desired signal and the interferer is within 7 or 8 degrees. We have also shown that as the number of array elements increases, the tolerable proximity of the interferer also increases to maintain a given performance.

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