Low Density Parity Check Codes for Hybrid ARQ System

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ABSTRACT

The most appropriate low density parity check (LDPC) code for hybrid automatic repeat request (HARQ) system suitable for future multimedia communication systems is presented in this paper. HARQ system with punctured LDPC code is investigated at first. And two transmission schemes with parallel concatenated LDPC code are also presented and their performances are analyzed according to the various values of mean column weight (MCW). As a result, the parallel concatenated LDPC code with the diversity effect of information bit is considered to be more appropriate for HARQ system considering the throughput as well as error performance.

Key Words : HARQ system, punctured LDPC code, parallel concatenated LDPC code, mean column weight.

I. Introduction

The error performance and throughput can be improved simultaneously if HARQ (hybrid automatic repeat request) scheme which is a combination of ARQ and ECC (error correcting code) is used under the various channel conditions[1,2]. It is strongly required to use a powerful ECC in the future communication systems to provide high-quality and high-speed multimedia services.

LDPC (low density parity check) code[3-6] which was proposed by Gallager at 1962 shows good error performance. LDPC code is a linear block code and nearly all the elements of parity check matrix are zero. This has relatively low computational complexity compared to the turbo code. And LDPC code shows even better error performance than turbo code if the codeword length is large[7]. Also, there is no error floor at high SNR (signal-to-noise ratio) region which is the critical drawback of turbo code[5]. So LDPC code is considered to be more appropriate ECC of HARQ scheme which is suitable for high-quality multimedia services for future communication systems.

The researches on the ECC of HARQ scheme have mainly been carried on turbo code[8,9]. Therefore, LDPC code suitable for HARQ scheme is suggested and investigated in this paper. In general, the code rate of LDPC code can be varied either by puncturing parity bits or concatenating the constituent encoders in parallel. So a proper HARQ transmission scheme with punctured LDPC code is investigated at first. In addition, it is known that the performance of PC (parallel concatenated) LDPC code is largely depends on the MCW (mean column weight) of parity check matrix[10-12]. And it is considered that transmission method of PC LDPC code can affect the performance of HARQ scheme. Accordingly, the optimum MCW suitable for HARQ scheme is suggested. And two transmission types with PC LDPC code are also presented and their performances are compared and analyzed. Finally, the most appropriate LDPC code for HARQ scheme is presented by comparing and analyzing the punctured and PC LDPC codes.

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Ⅱ. Punctured LDPC code for HARQ system

HARQ scheme is generally classified into two categories such as type-1 and 2[9]. The same bit is retransmitted in type-1, but different bit is transmitted in type-2 when retransmission is requested. The second type shows better throughput than the first one. So the second type is considered for the HARQ scheme with punctured LDPC code in this paper. The system model of HARQ scheme with punctured LDPC code is shown in Fig. 1.

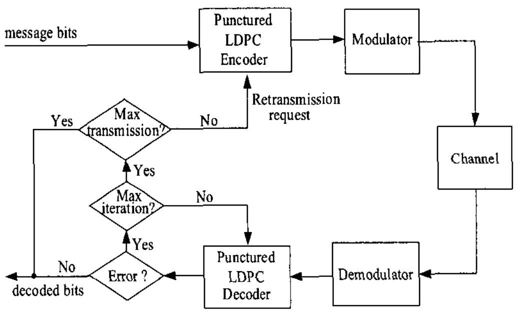


Fig. 1. System model of HARQ scheme with punctured LDPC code.

The information and parity bits are produced from the message bit at punctured LDPC encoder. And these are classified into two groups by puncturing. One is transmitted at the first transmission and the other is stored for retransmission. The information and parity bits which are selected according to the code rate are sent at the first transmission. Decoding is done by the received bits and zero values which are inserted in the punctured positions at the receiver. The decoding is completed if there is no error after each iteration. Otherwise, retransmission is requested. Then the parity bits which were stored in the encoding process are transmitted. In this case, the iterative decoding is continued with the information and parity bits which were received before and stored in the buffer at the receiver as well as the transmitted bits received presently.

Ⅲ. PC LDPC code for HARQ system

HARQ scheme with PC LDPC code is same as that with punctured LDPC code except the fact that the code rate is changed by adding the constituent encoders not by puncturing parity bits. The constituent encoders are characterized by the MCW of parity check matrix. If the parity check matrix is $M X N$ and the number of ones at the nth column is C_n, the MCW is represented as follows[11].

$$\overline{w_c} = \frac{1}{N}\sum_{n=1}^{N} C_n \tag{1}$$

The probability of receiving erroneous information can be increased at low SNR region compared to high SNR region if the number of edges which is connected at bit node is large[14]. This may result in the performance degradation. On the other hand, correct information is received from many check nodes if the number of edges is large at relatively high SNR region, so the erroneous bits can be corrected rapidly[11]. Accordingly, it is known that the parity check matrix with low-valued MCW is appropriate at low SNR region. On the other hand, high-valued MCW is suitable at high SNR region[10]. The second decoder of PC LDPC code shows better error performance than the first decoder because the second one can utilize the extrinsic information of the first decoder. So it is reasonable that parity check matrix with low-valued MCW is used for the first decoder and that with high-valued MCW for the second one[10-12].

The encoder of PC LDPC code is shown in Fig. 2. Here, x is information and p_i is parity bit of the ith constituent LDPC encoder. There is no interleaver in PC LDPC encoder unlike the turbo encoder. This is due to the fact that the input of the second constituent LDPC encoder is encoded different encoding rule from the first one[10].

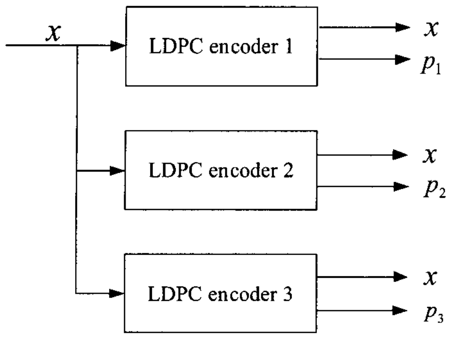


Fig. 2. Encoder of PC LDPC code.

There can be several HARQ transmission schemes with PC LDPC code according to the concatenation method of constituent encoders and transmission types of information and parity bits. Two transmission types of HARQ scheme with PC LDPC code are suggested in this paper. Information bit is transmitted at the first transmission only and parity bit of constituent encoder is sent at the following transmissions in the first type. In other words, x and p_1 are transmitted at the first transmission. So the code rate is 1/2. And p_2 and p_3 are transmitted at the second and third transmission. Accordingly, the overall code rate is changed to 1/3 and 1/4 gradually. The redundancy effect is expected in this type. On the other hand, the information bit is transmitted once again at the second transmission in the second type. That is, x and p_1 are transmitted at the first transmission. And x and p_2 are sent at the second and third transmission. So the diversity effect of the information bit is expected in this scheme. In addition, the number of required LDPC encoders and their corresponding decoders is decreased as compared to the first type.

The iteration of PC LDPC code is classified into two types such as local and global iteration in general. The local iteration is performed within each constituent decoder. On the other hand, the global iteration is done between or among the constituent decoders. The decoder of PC LDPC code of the two types according to the number of transmissions is shown in the Fig. 3 and 4. Here, y and q_i are the input bits of LDPC decoder which are corresponding to the information and ith parity bits of encoder.

y and q_1 are decoded at the LDPC decoder at the first transmission as shown in the figures. There is only one constituent decoder in this case. The retransmission is requested if there is an error after the predetermined maximum number of local iterations. Then, q_2 and y are decoded according to transmission types, respectively, as shown in Fig. 3(b) and 4(b). In this case, the decoding is performed at the first LDPC decoder and the extrinsic information is passed to the second decoder. The decoding is continued to the next global iteration if there are errors after the maximum number of local iterations of the second decoder. That is, the outputs of the second decoder are passed to the first decoder again. If there are still errors after the maximum number of global iterations, the retransmission is requested. In that case, if the first type is used, three LDPC decoders are used for decoding q_3 at the third transmission by performing the global as well as local iterations as shown in Fig. 3(c). On the other hand, if the second type is used, only two decoders are used for decoding q_2 as shown in Fig. 4(c).

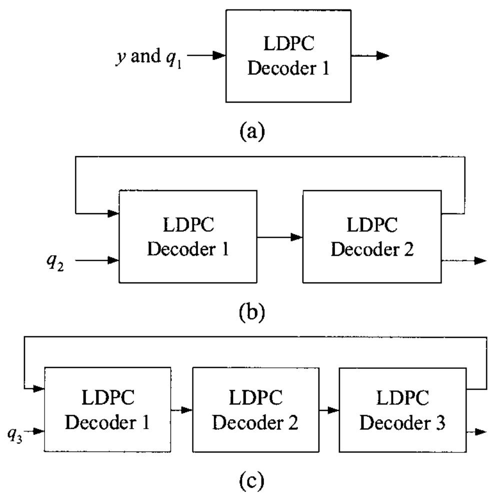


Fig. 3. Decoder of PC LDPC code of the first type according to the number of transmissions. (a) The first Tx., (b) The second Tx., (c) The third Tx.

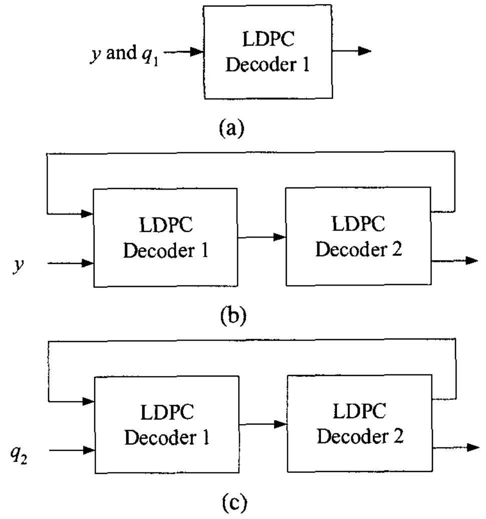


Fig. 4. Decoder of PC LDPC code of the second type according to the number of transmissions. (a) The first Tx., (b) The second Tx., (c) The third Tx.

Ⅳ. Simulation results

Simulation is carried out at AWGN (additive white Gaussian noise) channel to analyze the performance of punctured and PC LDPC codes for HARQ scheme. The length of information bit is 1024 and BPSK (binary phase shift keying) is used. And the maximum number of transmissions is set to 4, in other words, up to 3 retransmissions are possible. Accordingly, the codeword length varies from 2048 to 5120 according to the code rate which corresponds to from 1/2 to 1/5, respectively. It is known that the undetected errors are rarely found in LDPC code due to the increase in the minimum distance if the column weight is guaranteed[3]. So iteration stop and retransmission request can be performed by using this inherent error detection capability of LDPC code. This is an advantage as compared to HARQ scheme with turbo code[13] which requires the additional CRC (cyclic redundancy check) code to detect errors.

The optimum method of generating the parity check matrix has not been reported up to now if the codeword length is relatively short. So the method which was used for generating the parity check matrix by Mackay and Neal is adopted in this paper[4]. And the random puncturing method is adopted in this paper. The main purpose of this paper is finding the most appropriate LDPC code for HARQ system by analyzing and comparing the performance of the codes. So the most frequently used and well-known method of generating parity check matrix and puncturing is used in this paper for the purpose.

4.1 HARQ scheme with punctured LDPC code

About 2.6, 0.7, -0.2 and finally -0.8dB are required according to the number of transmissions to get the BER (bit error rate) of 10^{-6} if the punctured LDPC code is used as shown in Fig. 5(a). In other words, the SNR difference between the first and second transmission is about 1.9dB. And those between the last three transmissions are about 0.9 and 0.6dB. Error performance of the first transmission is relatively poor as compared to other transmissions due to many punctured bits. On the other hand, the error performance is improved rapidly from the second transmission. The throughput of punctured LDPC code is shown in Fig. 5(b). About 0.33 can be obtained between the SNR range of 0.5 and 1dB.

4.2 HARQ scheme with PC LDPC code

Simulation according to various MCW's to check the most appropriate combination of MCW's in PC LDPC code is carried out at first in this paper. The maximum number of iterations at the first transmission is set to 300 since only the local iteration is performed at the first transmission. And those of local and global iterations at the other transmissions are set to 10. That is, the total number of iterations is 300 if decoding is performed after passing three decoders and executed till the maximum number of global iterations. It is known that the MCW of 3 shows the best error performance if the code rate is 1/2[2]. So the MCW of the first transmission is set to 3. And simulation is carried out varying

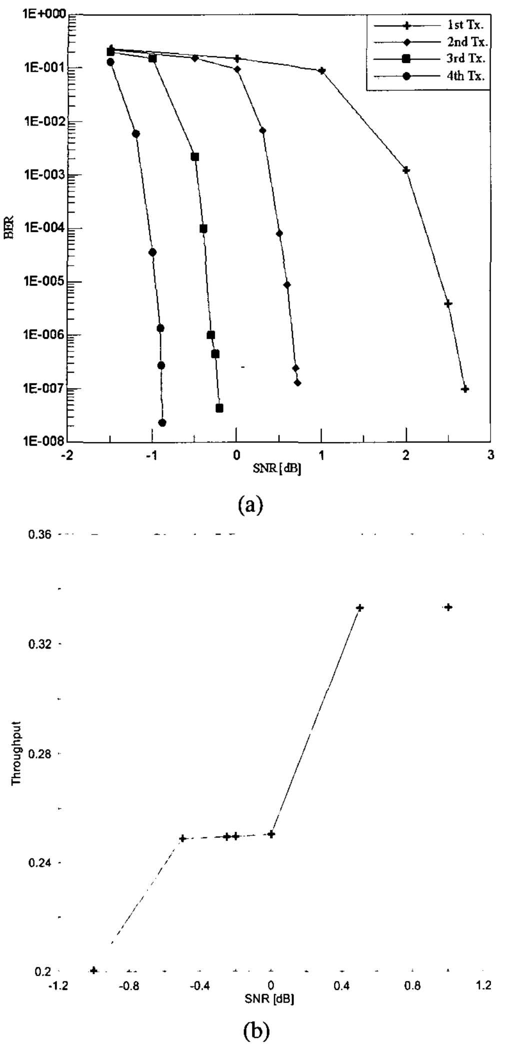


Fig. 5. Performance of HARQ scheme with punctured LDPC code. (a) BER, (b) Throughput

MCW according to code rate at the other transmissions. The error performances of PC LDPC code of the first type according to the code rates of 1/3, 1/4, 1/5 are shown in Fig. 6.

The MCW between 1.5 and 2.5 shows better error performance than other values when the code rate is 1/3 as shown in Fig. 6(a). The error performance according to various MCW's of the third transmission of the first scheme with three constituent decoders is shown in Fig. 6(b). MCW's of (1.5, 2.5, 3) show the best error performance. And those of (1.5, 2, 3) is the second. The error performance of the final transmission which is required four constituent decoders

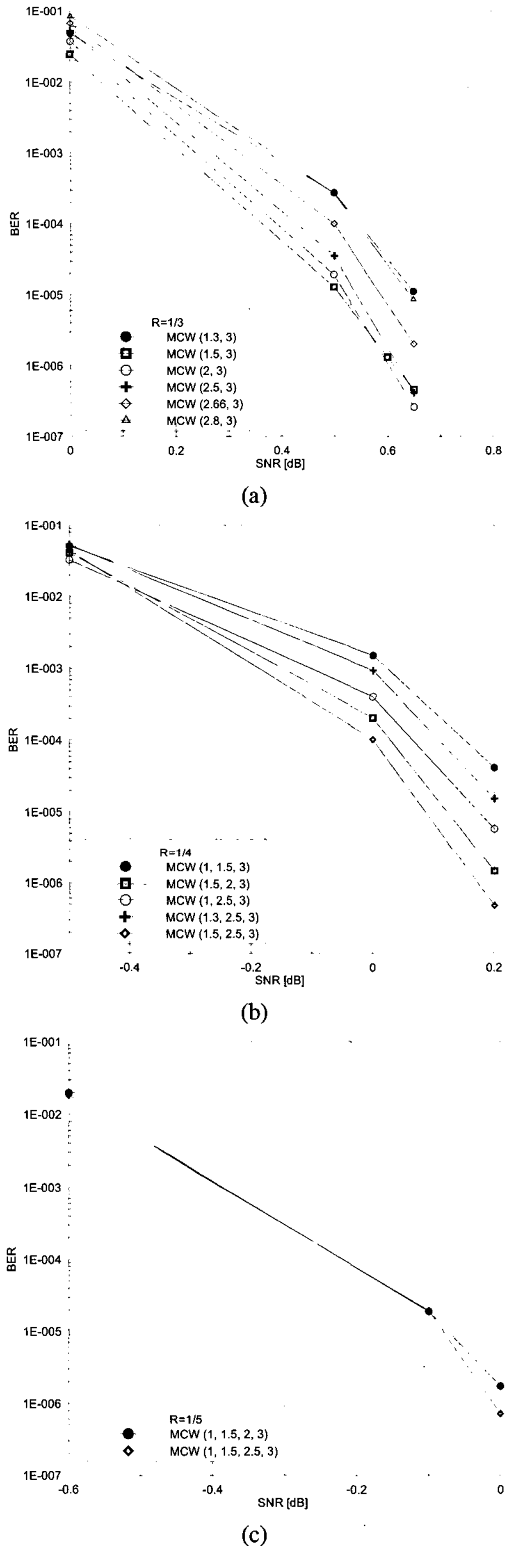


Fig. 6. Performance of PC LDPC code of the first scheme according to the various MCW's. (a) Code rate of 1/3, (b) Code rate of 1/4, (c) Code rate of 1/5

is shown in Fig. 6(c). MCW's of (1, 1.5, 2.5, 3) shows the best performance considering the overall error performance.

The performance of the second type according to various MCW's is shown in Fig. 7. The performance is nearly same regardless of MCW if the code rate is 1/3 as shown in Fig. 7(a). And the MCW between 2.66 and 2 shows better error performance than other values when the code rate is 1/4 which corresponds to the third transmission. Finally, MCW's of (1.5, 2.5, 3) show the best error performance in case of code rate of 1/5. So the MCW's of (1.5, 2.5, 3) are the most appropriate choice for the second type of HARQ scheme with PC LDPC code.

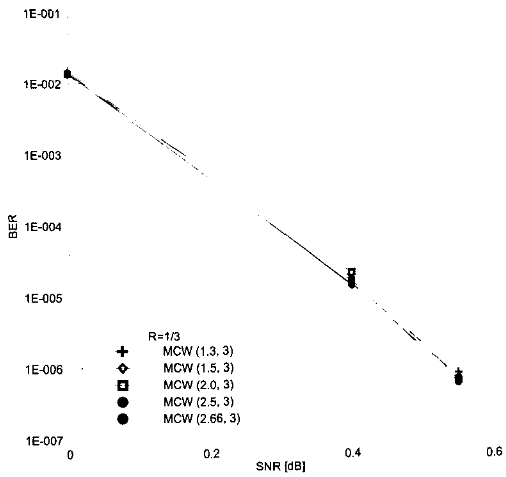
The performance of the two types of HARQ schemes with PC LDPC code using the MCW of the previous simulation results is shown in Fig. 8. The BER at the first transmission of two schemes is same because the transmitted bits are same. But the second scheme outperforms the first one about 0.1, 0.3 and 0.5dB to get the BER of 10^{-6} after the second transmission. So the performance difference between two schemes is increased further as the number of transmissions is increased.

Thus the diversity effect of retransmitting the information bit is observed to be more influential to the performance than the redundancy effect of transmitting the parity bit. The throughput of two schemes is shown in Fig. 8(b). The second scheme shows better throughput than the first one at low SNR region. Also the second scheme with less encoders and decoders is considered to be a better one as compared to the first scheme in terms of complexity.

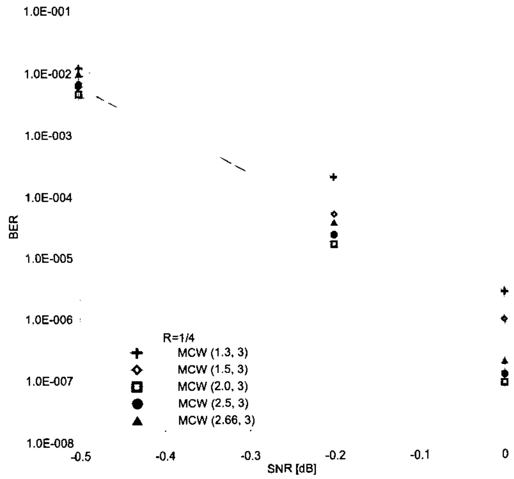
4.3 Comparisoins

The performance comparisons between punctured and PC LDPC codes are shown in Fig. 9.

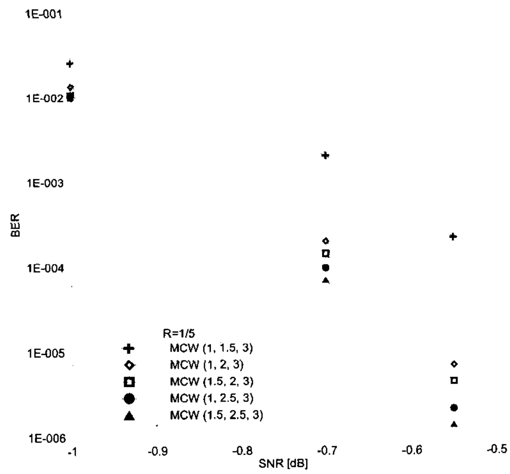
The HARQ scheme with PC LDPC code shows better error performance than punctured LDPC code at the first and second transmission. That is, the performance difference is about 1.5



(a)



(b)



(c)

Fig. 7. Performance of PC LDPC code of the second scheme according to various MCW's. (a) Code rate of 1/3, (b) Code rate of 1/4, (c) Code rate of 1/5

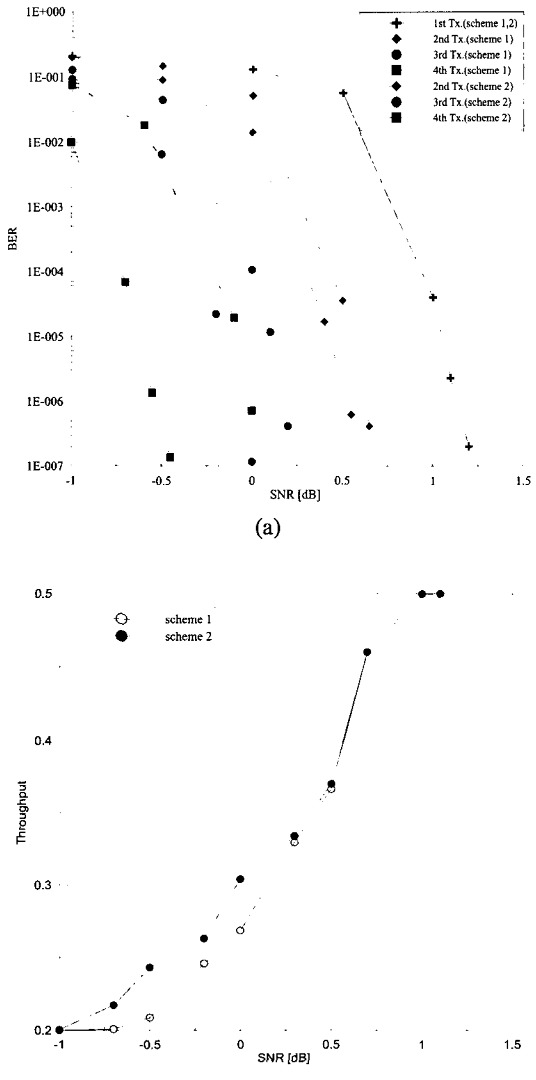


Fig. 8. Performance of HARQ schemes with PC LDPC code. (a) BER, (b) Throughput

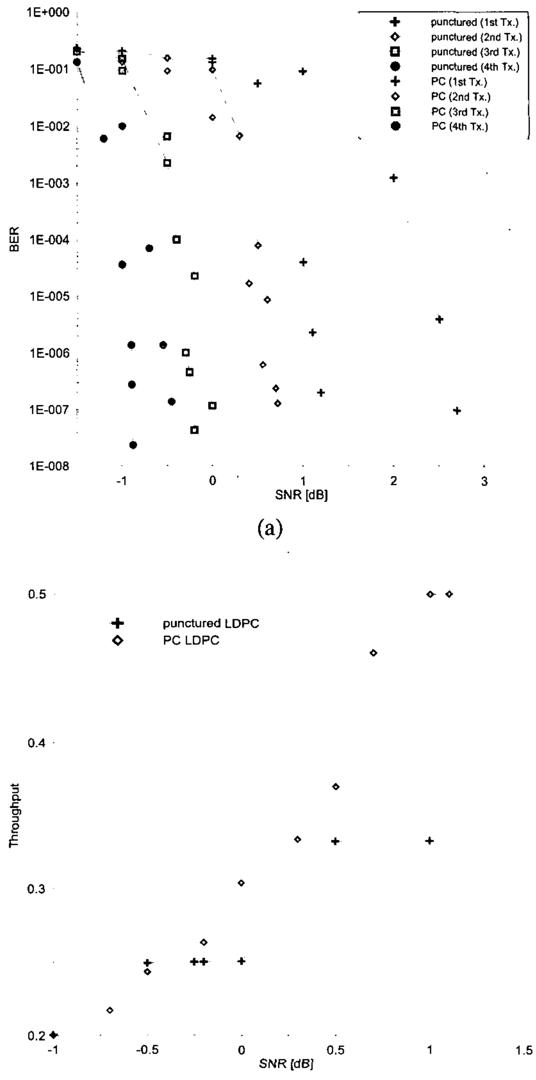


Fig. 9. Performance of HARQ scheme with punctured and PC LDPC code. (a) BER, (b) Throughput

and 0.2dB at the first two transmissions to get the BER of 10^{-6}. This is due to the fact that this scheme uses the non-punctured LDPC code. On the other hand, the punctured LDPC code shows better error performance about 0.2 and 0.4dB at the final two transmissions to get the same BER. And throughput of PC LDPC code is better than that of punctured LDPC code above -0.4dB from Fig. 9(b).

Code rates from 1/2 to 1/5 are used according to the number of retransmissions in this paper. So it is considered that maximum throughput of 0.5 can be obtained if the SNR is sufficiently large. It is desirable that almost all the errors are corrected at the first and second transmission when the HARQ scheme is used because the throughput as well as error performance should be guaranteed in this scheme. So the performance of the first two transmissions of HARQ scheme is more influential to the whole performance of a system. Therefore, PC LDPC code is considered to be a better choice for HARQ scheme on account of its good performance at the first and second transmission.

Ⅳ. Conclusions

Punctured and PC LDPC codes for HARQ scheme are presented and their performances are analyzed in this paper. HARQ scheme with punctured LDPC code shows considerably poor error performance and throughput at the first and second transmission due to many punctured bits. But the performance is rapidly improved after the second transmission.

Two HARQ schemes with PC LDPC code are presented and their performances are analyzed according to the various values of the MCW. The second scheme in which the information bits are transmitted once again when retransmission is requested shows better error performance than the first one in which the information bit is transmitted at the first transmission only. Therefore, the second scheme with the diversity effect of information bit is considered to be an appropriate transmission scheme with less encoder and decoder complexity as well as better error and throughput performance.

Finally, HARQ scheme with PC LDPC code shows good error performance and throughput due to use of non-punctured LDPC code at the first and second transmission as compared to the punctured LDPC code. As a result, the PC LDPC code seems to be more appropriate for HARQ scheme considering the throughput as well as error performance at all transmission.

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