

A Modification of the Approach to the Evaluation of Collision Risk Using Sech Function

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Abstract : Evaluation of collision risk plays a key role in developing the expert system of navigation and collision avoidance. This paper presents a new collision risk model formula that is one modification model on the basis of one approach to the evaluation of collision risk using sech function produced in earlier studies. And as a tool of the evaluation field of ship collision, this paper applied the new model in appraising the collision risk and represented how to decide the safe range of own ship's action. Moreover this paper also analyzed theoretically how to determine the coefficients as described in the new modification model, and suggested the appropriate values as applicable.

Key words : Collision risk, Sech function, Distance of closest point of approach, Approach time, Before-action, After-action

1. Introduction

Evaluating the risk of collision quantitatively plays a key role in developing the expert system of navigation and collision avoidance. A new approach to collision risk using sech function (Jeong, 2003a, 2003b and 2003c) was introduced to solve the inherent problems other relevant researches have. However some problems in this new method remained unsolved.

Therefore, this paper aims at presenting a new modified model on the basis of the approach to the evaluation of collision risk using sech function produced in Jeong (2003a)'s study. And as a tool in the risk evaluation field of ship collision, this paper applied the new model in appraising the collision risk and represented how to decide the safe range of own ship's action. Moreover this paper also analyzed theoretically how to determine the coefficients as described in the new modification model formula, and suggested the appropriate values as applicable.

2. Modification Approach

2.1 Original Approach

The original approach to evaluating collision risk is given by equation (1) (Jeong, 2003a).

$$CR = p \cdot \operatorname{sech}(a \cdot dcpa) + q \cdot \operatorname{sech}(b \cdot ta) + r \cdot \Phi(\theta, \alpha) \quad (1)$$

Meanwhile the modified formula in this paper is given by equation (2).

$$CR = \frac{p \cdot \operatorname{sech}(a \cdot dcpa)}{ta} + r \cdot \Phi(\theta, \alpha) \quad (2)$$

In equation (1) and (2), CR is the collision risk; $dcpa$ is the distance of closest point of approach; ta is the approach time¹⁾; p , q and r are the amplitude coefficients; a and b are the coefficients of sech function; $\Phi(\theta, \alpha)$ is the own ship's state function. It has the relationship with the targets' position θ and aspect α , the magnitude of which is 0 if own ship is in the stand-on situation and 1 if she is in the give-way situation.

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1) An approach time is completely different from TCPA(Time to CPA) and is given by the following equation.

$$\begin{aligned} t_a &= \frac{2 \cdot dcpa}{v_r} && \text{if } 045^\circ < \zeta \leq 90^\circ \\ &= -\frac{2 \cdot dcpa}{v_r} && \text{if } 090^\circ < \zeta < 135^\circ \\ &= \frac{R}{v_r \cos \zeta} && \text{otherwise} \end{aligned}$$

Here R and v_r are denoted by the range and the relative speed of a target respectively. ζ is the absolute value of the difference of a target's bearing added to 180° and the relative course of a target.

2.2 Some Points from Modification Approach

1) When *dcpa* decreases *CR* will increase. Generally, the decrease of *dcpa* means that the risk of collision increases. In addition if *dcpa* is equal or nearly equal to zero, we regard it as equally dangerous. For example, if *dcpa* is 0 and 0.1 mile respectively, collision risk will be almost the same intuitively. By using sech function, *dcpas* of 0 and 0.1 miles will be 1 or 0.9950 respectively. In this regard Equation (2) can be considered to represent collision risk well by using sech function.

2) Because in Equation (1) *ta* values of different signs are the same ones, it means that even a target, which passed through its CPA, has still the same collision risk and it may be absurd. Equation (2) is to depict (-) value just after the passing of CPA and represents to get out of risk. The only problem in Equation (2) is that when *ta* is 0, *CR* will be discontinuous. That is to say, *CR* will change from maximum to minimum. So when target is very close to own ship, for example, the distance is within 0.1 mile, the *CR* will be so low. Of course, this situation is not fit for intrinsic observation. Considering the situation that *ta*'s value 0 seldom happens and *CR* will change from '+' to '-' when the targets pass CPAs, the formula to calculate *ta* should be modified as one in the above-mentioned footnote.

3) When own ship state function $\Phi(\theta, a)$ is used, its appropriate value should be taken according as own ship maintains her course and speed or alters.

3. Application of Modification Approach to Verify Its Effect

Before determining the detailed coefficients of modification approach, the verification of the approach should be researched first. Meanwhile the characteristic of the coefficients also can be observed.

For simplicity, the coefficients of modification approach are supposed as $p=1$, $r=0$ and $a=1.15$. Here $r=0$ means ignoring the effect of own ship state function temporarily.

3.1 When Different Targets Approaching Own ship:(Own ship Does Not Take An Action)

In Table 1, own ship and targets' initial positions are given and in Fig. 1 the *dcpas* of the targets are 0.5 miles and 1.5 miles respectively. Fig. 2 expresses the *CRs* of the two targets. When they pass the CPAs respectively, the *CR* is 0 and *CR*'s values are minus after they pass the CPAs. At a short time before the targets pass CPAs the collision risk value reaches its maximum. The target with

the *dcpa* of 0.5 mile has *CR* maximum of 0.6065, while the one with *dcpa* of 1.5 miles has *CR* maximum of 0.0716.

But as shown in Fig. 2, the *CR* value reaches its maximum just before the targets pass CPAs and it decreases sharply to 0 after they pass CPAs. This is not fit for our aim to design the modification approach and *ta* should be modified as described in footnote 1).

Table 1 Own ship with no action and targets' situation

	Own ship	Target 1	Target 2
<i>dcpa</i> (mile)	--	0.5	1.5
Range(mile)	--	8.0	8.0
Bearing(°)	--	023.9	050
Course(°)	000	235	258
Speed (mile/min)	0.4	0.4	0.4

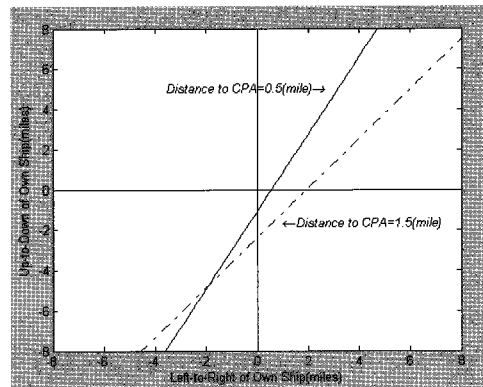


Fig. 1 Two targets with different *dcpas* approaching own ship

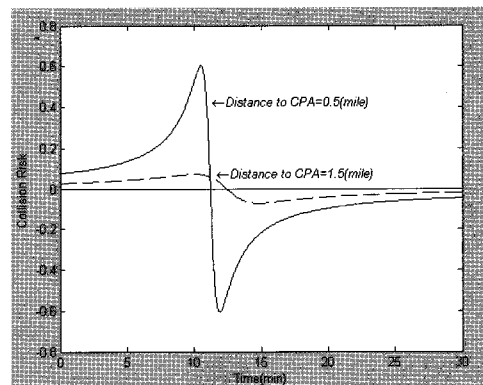


Fig. 2 Comparison of collision risk of two targets with different *dcpas*

3.2 The Collision Risk When Own ship Alters Her Course or Speed

In Table 2, the information of own ship and target is

shown, and when the target is as near as 4.2 miles, own ship makes an alteration of course or speed. Then the *dcpa* will be changed from 0.5 mile to 1.5 miles. Course alteration is from 000° to 035° and speed alteration is from 0.4 mile/min to 0.18 mile/min.

Table 2 Own ship with action and target's situation

	Own ship	Target 1	Avoidance Action
Range(mile)	--	8.0	4.2
Bearing(°)	--	023.9	050
Course(°)	000	235	035
Speed (mile/min)	0.4	0.4	0.18

In Fig. 3, the approach time *ta* of course-altered case is smaller than that of speed-altered case. The reason is that the relative speed of course-altered situation is bigger than that of speed-changed situation. And under these situations the collision risk sometimes can be bigger than *CR* threshold again. Then we should consider the re-avoidance action. However such a problem can be solved by adjusting the appropriate coefficients *p* and *a*, and sufficiently considering the own ship state function $\Phi(\theta, \alpha)$ into modification approach.

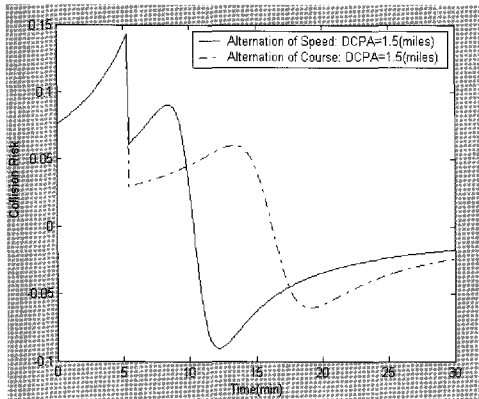


Fig. 3 After avoiding action with alternation of course and speed respectively

3.3 Own ship's Safety Action Range

1) Single Target Situation

Using the modification approach, own ship's safety action range can be taken. In Table 3 the information of own ship and target is shown. But safety action range only can be calculated by considering course altered only or speed altered only respectively. Assume that own ship's initial

course is 000° and speed is 0.4 mile/min. The expected *dcpa* after avoidance action should be at least 1.5 miles.

Table 3 Own ship with full range of action and target's situation

	Own ship	Target	Range of Action
Range(mile)	--	4.2	Course : 0~360° Speed : -0.4~0.4 mile/min
Bearing(°)	--	040	
Course(°)	000	230	
Speed (mile/min)	0.4	0.3	

When one appropriate threshold is given, the safety course range without speed change or safety speed range without course change can be taken easily if own ship takes corresponding avoidance actions. In Table 3, the target's initial course is 000° and speed is 0.3 mile/min, and the distance to own ship is 4.2 miles. When own ship takes avoidance actions by taking course changed from 000° to 360° without speed change, the collision risk is shown in Fig. 4. And when own ship takes avoidance actions with speed change from -0.4~0.4 mile/min(-24~24 knot), the collision risk is shown in Fig. 5. In the two figures, 0.04 is used as the threshold of collision risk. Then own ship's safety course range and speed range can be taken respectively. According to Fig. 4, the safety course range is the course value under *CR* threshold, that is 101.5° ~ 338.5°; and according to Fig. 5, the safety speed range is -0.4~0.249 (mile/min), i.e. -24.00~14.94 knots. Here the safety threshold is just an example. An appropriate threshold should be taken by analyzing from practical data and be verified by many experiments at sea.

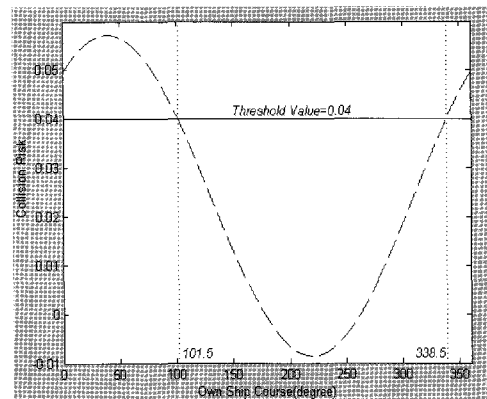


Fig. 4 Collision risk of a target against alternation of own ship's course

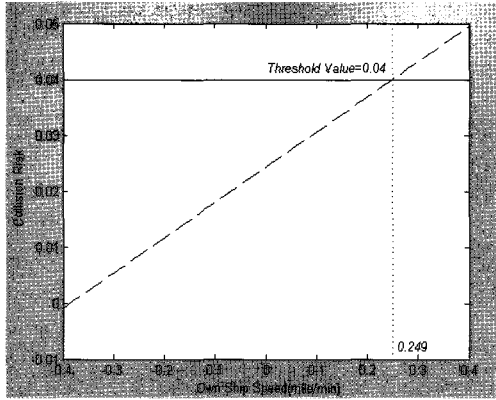


Fig. 5 Collision risk of a target against alternation of own ship's speed

2) Multi-Targets Situation

Sometimes the targets are more than one and at that time this method should also can be used to get the kind of safety range. Using the data in the Table 4, safety course range without speed change can be taken. Own ship's initial course is 000° and speed is 0.4 mile/min. And the expected *dcpa* after avoidance action should be at least 1.5 miles.

Table 4 Own ship with full range of action and targets' situation

	Own ship	Target 1	Target 2	Target 3	Range of Action
Range (mile)	--	5.0	8.0	5.0	Course: 0~300°
Bearing (°)	--	040	170	300	
Course (°)	000	260	012	120	
Speed (mile/min)	0.4	0.3	0.6	0.4	

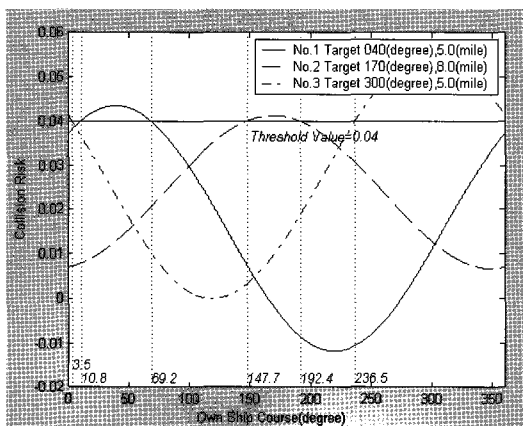


Fig. 6 Collision risk of multi-targets with own ship's course alternation

Here are 3 targets, and own ship changes course from

000° ~ 360°. The 3 targets' collision risk by course change is shown in Fig. 6. Also, 0.04 is as the course threshold. Then the safety course range can be taken. Just as shown in Fig. 6, the safety course range with speed maintaining is 3.5° ~ 10.8°, 69.2° ~ 147.7° and 192.4° ~ 236.5°. Moreover the threshold is also effected by the own ship state function $\Phi(\theta, a)$. So all possible factors should be considered analytically in deciding the threshold value.

4. Coefficient Decision of Modification Approach

4.1 Determining Coefficient a

For determining coefficient 'a' simply, the amplitude coefficients of modification approach are supposed as *p* is 1 and *r* is 0.

Commonly, when target approaches own ship, target's *CR* value's difference between *CR* values before and after actions is a criterion to judge whether the action is effective or not. If the difference is minus, that is to say after-action *CR* is bigger than before-action, it is so say the action is not effective and should be adjusted. Obviously, if the difference can get to its maximum, the action taken is the most effective one. Here, the difference can be defined as follows:

$$F = CR_1 - CR_2 = \frac{\text{sech}(a \cdot dcpa_1)}{ta_1} - \frac{\text{sech}(a \cdot dcpa_2)}{ta_2} \quad (3)$$

Here, *dcpa*₁ and *ta*₁ mean target's values of approaching time before action taken, and *dcpa*₂ and *ta*₂ are the values when the approach time is maximum after action taken. The result of *F* in (3) should bigger than 0.

When *dcpa*₁, *dcpa*₂, *ta*₁, *ta*₂ are all known, *F* is a function of 'a' as equation (4).

$$F(a) = \frac{\text{sech}(a \cdot dcpa_1)}{ta_1} - \frac{\text{sech}(a \cdot dcpa_2)}{ta_2} \quad (4)$$

When the derivative of *F(a)* is 0, the value of 'a' can be obtain and the maximum of *F(a)* can be given.

$$\begin{aligned} \frac{dF}{da} &= -\frac{\text{sech}(a \cdot dcpa_1) \tanh(a \cdot dcpa_1)}{ta_1} dcpa_1 \\ &+ \frac{\text{sech}(a \cdot dcpa_2) \tanh(a \cdot dcpa_2)}{ta_2} dcpa_2 \\ &= 0 \end{aligned} \quad (5)$$

The value of 'a' obtained from (5) is that when the value of dF/da is 0, and $dcpa_1 < dcpa_2$, then $F(a)$ can reach its maximum. The bigger $F(a)$'s value is, the better the effect of the avoidance action is.

4.2 Flow Chart of obtaining Coefficient a

The below method to get the value of coefficient a is shown in Fig. 7. There are two points should be explained:

1) The value of coefficient a should be such that the value of F is bigger than 0. Otherwise, ta_2 should be calculated and adjusted again. If the value of a is such that F is not bigger than 0, target's CR, will not be decreased only by the action taken. So some other actions should be taken. This process will continue until F is bigger than 0.

2) Usually, $ta_2 < ta_1$. Only when the vessel's speed is very low, $ta_2 > ta_1$ could happen. But this situation almost has no effects to $F > 0$.

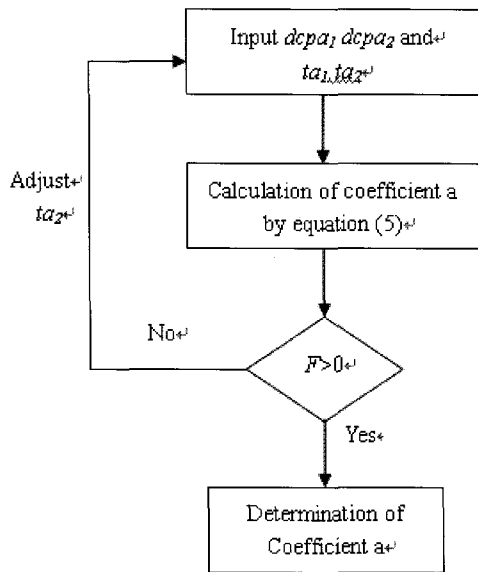


Fig. 7 Procedure of getting coefficient a

4.3 The Meaning of Getting Value of Coefficient a

The meaning of the formula (5) is shown in Fig. 8 as follows. In Fig. 8, for obtaining coefficient a, we assume that $dcpa_1$ is 1.5 mile and $dcpa_2$ is 2.3 mile, ta_1 is 6.998 minutes and ta_2 is 4.635 minutes. Substituting these into formula (4) and formula(5), the graphics of F and dF/da can be taken in Fig. 8. In Fig. 8, at the point where dF/da is 0, the value of a is 1.1491. And at this point, $F(a) = 0.018869$ is the maximum. So the value of coefficient a is determined to be 1.1491.

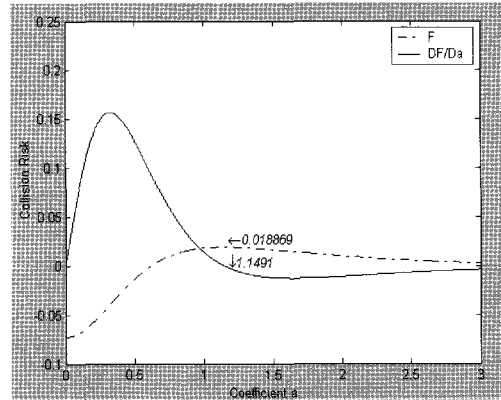


Fig. 8 Collision risk difference F by coefficient a

4.4 Validating Coefficient a

To verify the coefficient a, some examples are given. Here before-action $dcpa$ is 1.5 miles and the $dcpa$ after the avoidance action is expected to be at least 2.3 miles. And the CR maximum after the avoidance action will be considered as collision risk.

Own ship's initial course is 000° and the target's initial course is 180° . Assume that the relative speed is 1.0(mile/min) the speeds of own ship and the target are 0.5(mile/min) respectively. And the target's initial position is 9.0 mile and its bearing is 000° . When the approach time is 7.0 minutes, the avoidance action will be taken. The result will be reasonable.

The relative speed is 0.1(mile/min) the speed of own ship and the target is 0.05(mile/min) respectively. The target's initial position is 5.0 mile and bearing is 000° . If the avoidance action is taken when the approach time is 7.0 minutes, the target is too close to own ship, it is not appropriate. So the avoidance action should be taken when the range is 3.25 mile.

The avoidance action of own ship is supposed to change course from 000° to 035° in the two situations. And the value of coefficient a is 1.1491.

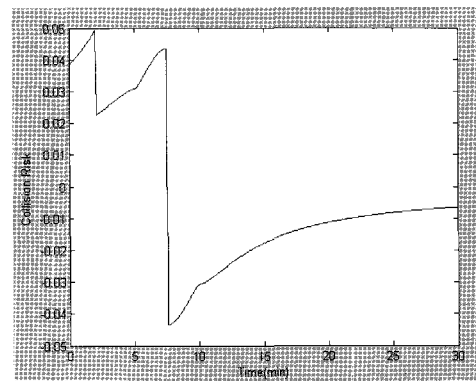


Fig. 9 Collision risk in case of relative speed 1.0 (mile/min)

Fig. 9 shows the CR of relative speed of 1.0(mile/min) with, the before-action $dcpa$ of 1.5 mile. When the approach time is 7.0 minutes(i.e. approach distance is 7.0 miles) and own ship changes its course, the $dcpa$ will be 2.3 miles. And the before-action CR is 0.0494, and it is bigger than 0.0435 which is the maximum value of collision risk after the action.

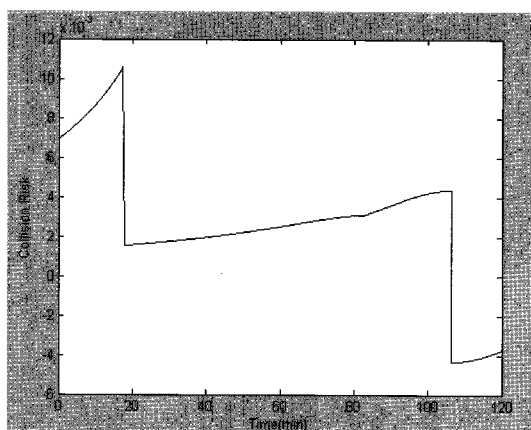


Fig. 10 Collision risk incase of relative speed 0.1 (mile/min)

Fig. 10 also shows the CR of relative speed of 0.1(mile/min) with the before-action $dcpa$ of 1.5 mile. When the approach distance is 3.25 miles and own ship changes its course, the $dcpa$ will be 2.3 miles. And the before-action CR is 0.010608, and it is bigger than 0.004353 which is the maximum value of collision risk after action.

Another two situations of relative speeds of 0.7 and 0.5 mile/min are also applied to validate coefficient a . It appears the same result that the before-action CR is bigger than CR maximum after action taken.

As a result the value of coefficient a is 1.1491 is fit when the CR values are smaller than before-action. If the relative speed is high, the appropriate time to take the avoidance action should be selected when the approach time is 7.0 minutes. And when the relative speed is low the time to take the avoidance action should be selected when the approach distance is 3.25 miles.

The value of coefficient a should be determined carefully, because the value has close relationship with the threshold of collision risk.

5. Conclusion

This paper introduced a modification approach using sech function to evaluate collision risk. The modification approach introduced in this paper is to the evaluation collision risk using sech function and is to resolve the

problems the other existing approaches have.

As a result, some conclusions can be conclude as follows:

1) $\text{sech}(dcpa)$ and ta are both used into the modification approach, which is more scientific and effective.

2) When a target approaches own ship, modified ta makes CR value increase until just before it will pass CPA . This is fit for our intuition..

3) By using the method in the flow chart, the value of coefficient a can be taken. Coefficient a of 1.1491 is taken and used in modification approach.

4) When the $dcpa$ is 1.5 miles and the target's relative speed is around 1.0 mile/min, the time to take avoidance action is when the approach time is 7 minutes; and when the target's relative speed is around and below 0.5 mile/min, the time to take avoidance action is when the approach distance is 3.25 miles.

The modification approach can solve some problems and suggest good results. However, such results should be checked by many experiments at sea. And own ship state function $\Phi(\theta, a)$ should also be considered and researched. Also, the threshold should be decided carefully because it decides the safety range of own ship. All of these will be dealt with in the future study.

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