Survivable Traffic Grooming in WDM Ring Networks

Srivatsan Sankaranarayanan, Suresh Subramaniam, Hongsik Choi, and Hyeong-Ah Choi

Abstract: **Traffic grooming, in which low-rate circuits are multiplexed onto wavelengths, with the goal of minimizing the number of add-drop multiplexers (ADMs) and wavelengths has received much research attention from the optical networking community in recent years. While previous work has considered various traffic models and network architectures, protection requirements of the circuits have not been considered. In this paper, we consider survivable traffic grooming, or grooming traffic which contains a mix of circuits that need protection and that do not need protection. We assume a unidirectional ring network with all-to-all symmetric traffic with $t \geq 1$ circuits between each node pair, of which s require protection. As it turns out, survivable traffic grooming presents a significant tradeoff between the number of wavelengths and the number of ADMs, which is almost non-existent in non-survivable traffic grooming for this type of traffic. We explore this tradeoff for some specific cases in this paper. We also present some new results and solution methods for solving certain non-survivable traffic grooming problems.**

Index Terms: **Add-drop multiplexers, all-to-all traffic, circuits, lightpaths, optical networks, ring networks, survivability, traffic grooming, wavelength-division-multiplexing (WDM).**

I. INTRODUCTION

In the last few years, optical networks using wavelength division multiplexing (WDM) and wavelength routing have been considered as promising architectures for next-generation backbone networks. WDM has reduced the mismatch between electronic source rates, which are of the order of several Mbps, and optical transmission rates, which approach Terabits per second, by partitioning the fiber bandwidth into a few tens of disjoint wavelength bands, each capable of operating at a more manageable rate of a few Gbps. At the same time, technology has enabled the optical routing of wavelengths or wavelength routing, which provides the benefits of reduced switching costs (compared to high-speed electronic switches) and more transparency than electronics could provide.

Wavelength-routing optical networks provide circuit-switched optical connections called as *lightpaths*. There have been several papers on issues related to this wavelength-routing network architecture in the literature. More recently, the realization of two facts have opened up a new set of related problems. One is that there is a limit to the granularity of the fiber bandwidth partitioning process, i.e., there is a limit to the number of wavelengths that can be used in a wavelength-routing network. The second is that, even with WDM there is a mismatch between electronic source rates and the capacity of a wavelength. As remarked above, electonic source rates are typically of the order of a few tens of Mbps, whereas a wavelength can accommodate rates of upto a few Gbps. This mismatch has lead to a problem called as *traffic grooming*. Traffic grooming implies the judicious multiplexing of low-rate (electronic rate) traffic streams into lightpaths such that network costs are minimized.

A possible way of implementing traffic grooming is to time-division multiplex several low-rate circuits into a high-rate (i.e., wavelength-rate) circuit, and then convert the resulting signal into optical form. This is the kind of multiplexing that is done in synchronous optical network (SONET) networks. In such networks, the optical signal is terminated (converted to electronic form) at every node, and the approriate low-rate circuits are "dropped" and "added" at the nodes by using devices called as electronic add-drop multiplexers (EADMs). An additional complication arises with traffic grooming in WDM networks. Since a wavelength needs to be terminated at a node only if it carries an electronic circuit that needs to be dropped or if an electronic circuit needs to be added, the choice of the low-rate circuits that are multiplexed together and the choice of wavelengths play important roles in determining the cost of the network.

An obvious component of the network cost is that of the fiber infrastructure. Besides the fiber, additional components include optical cross-connects (OXCs), optical ADMs (OADMs), electronic ADMs (EADMs), electronic line-terminatals (LTs), and electronic switches (called digital crossconnect systems (DCSs)) to demultiplex and switch the low-rate electronic circuits. Most of the literature on traffic grooming has focused on the number of EADMs and the number of wavelengths that the fiber has to carry as the main cost-affecting components. This is because OADMs are necessary at every node where at least one wavelength must be terminated, and the cost of DCSs is proportional to the number of EADMs. The cost of an LT can be assumed to be half that of an EADM.

A brief review of the published literature on grooming is in order now. Henceforth, we use ADM to denote an EADM. An enumeration of various architectural options provided by unidirectional and bidirectional SONET rings to optimize the network cost which is dominated by the SONET transmission equipment is given in [1]. The work in [2] quantifies the maximum possible ADM savings using a "super-node" approximation technique for uniform and distance-dependent traffic in bidirectional rings. The merit of cross-connecting the traffic streams in SONET uindirectional and bidirectional rings in the context of reducing the SONET ADM costs is evaluated in [3]. It also gives three network architectures that have low ADM costs. Several

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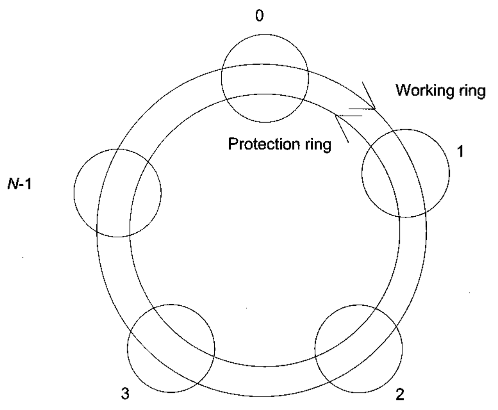

Fig. 1. A UPSR-like ring network.

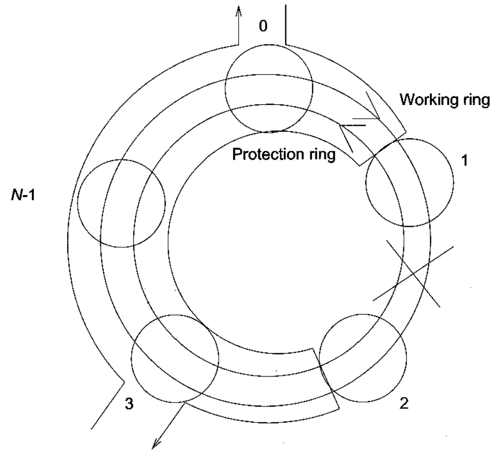

Fig. 2. A bidirectional connection between nodes 0 and 3 after link (1,2) has failed.

network architectures that have low ADM costs for static as well as dynamic traffic are given in [4]. A heuristic based on circle construction is proposed in [5] for grooming arbitrary traffic in SONET/WDM unidirectional and bidirectional ring networks. A heuristic based on circle construction is also proposed in [6] but it incorporates fully matrix-based operation into traffic grooming and wavelength assignment. A unidirectional ring network with a special case of dynamically changing traffic called t-allowable traffic in which there is an upper bound on the number of circuits a node can source or sink is considered in [7]. It proposes a heuristic to reduce the number of ADMs. Several heuristics for reducing the number of ADMs in BLSRs with arbitrary traffic along with their performance ratios are proposed in [8]. A new capacity correlation model which takes into account the capacity distribution on a wavelength, the arrival rates of calls of varying capacity and the load correlation on neighboring links to compute the call blocking performance on a multihop single wavelength path is the topic of [9]. The logical topology design problem in the context of minimizing the amount of electronic routing in unidirectional rings with arbitrary traffic is dealt with in [10]. Several grooming solutions for all-to-all traffic in unidirectional rings are presented in [11]. These solutions are tailored for specific values of the grooming factor, which is defined as the maximum number of low-rate circuits that can be multiplexed onto a wavelength. Those solutions are the best-performing ones to date for the model considered. The problem of traffic grooming in a WDM-based optical mesh network with the objective of improving the network throughput is investigated in [12].

Almost every previous effort on this topic tries to reduce the terminating equipment (ADM) costs while using the minimum number of wavelengths that are needed to support a given traffic. In this paper, we add two new dimensions to the traditional grooming problem as follows. First, due to the domination of the electronic equipment cost, it is very important to try to reduce the electronic costs further by using more wavelengths if need be. Thus, the tradeoff between the number of wavelengths and the electronic terminating equipment cost is of fundamental importance to quantify. Secondly, since an optical network is a very attractive and promising backbone network architecture for the next generation Internet (NGI) due to its enormous bandwidth and transparency, and because of the rapidly growing popularity of Integrated Services for the NGI, a diverse array of network services with different quality of service (QoS) requirements needs to be supported in future networks. This means that some of the connections (called best-effort) may not need to be protected while others with high priority (called guaranteed service connections) may impose stringent protection requirements on the network. The earlier traffic models in which all the circuits either need to be protected (or not) is insufficient for the above scenario. Our work is an attempt to address this deficiency.

The rest of the paper is organized as follows. In the next section, the network model is defined and the problem we address is formulated. The main results of the paper are given in Section III. Conclusions and directions for future work in Section IV complete the paper.

II. NETWORK MODEL AND PROBLEM DEFINITION

We consider a ring network with nodes numbered from 0 to $N-1$ in the clockwise direction as shown in Fig. 1 where N is the number of nodes in the network. There are two fibers running around the ring in opposite directions as in the SONET unidirectional path switched ring (UPSR) network. The working traffic is routed along one direction of the ring, say the clockwise direction. The fibers in the counter-clockwise direction are used for protection. While the SONET UPSR uses path protection, our network architecture assumes a loopback link-protection mechanism as in a bidirectional line switched ring (BLSR). We also assume that the restoration is done at the wavelength level. In other words, working traffic is always routed on the clockwise ring, and if a wavelength λ is used on a working link, we assume that λ is provisioned for protection on the counter-clockwise ring, if it does need to be protected. Thus, our network architecture is different from both the UPSR and the BLSR and is actually a hybrid of the two. An example of how recovery is done in the assumed architecture is shown in Fig. 2.

We assume symmetric traffic wherein if there exists a circuit from i to j, there is also a circuit from j to i. Thus, every connection is bidirectional. We also assume that the same two ADMs which are used for establishing a bidirectional connection in a given direction are also used for establishing the circuit in the other direction. A circuit occupies only a fraction of a wavelength capacity. This fraction is denoted by g, called the *grooming factor*. Thus, at most g circuits can be assigned the same wavelength without contention. These circuits are said to be groomed onto the given wavelength. We assume static, all-to-all, uniform traffic with arbitrary t, where t is the number of connections between any given pair of nodes (i,j), $0 \leq i, j \leq N-1$. Of these t connections, s $(0 \leq s \leq t)$ are survivable connections. A survivable connection (or circuit) is defined to be one that must be protected and a non-survivable circuit is one that need not be. If a wavelength λ carries even a single survivable circuit, λ must be provisioned on the protection ring. A survivable circuit is called as an s-circuit and a non-survivable (or a normal) circuit is called as an n-circuit.

We will find it useful in the next section to define a *connection graph.* Given a ring network of N nodes, the connection graph $G = (V, E)$ is an undirected graph which has the same nodes as the ring network, and an undirected edge exists between two nodes i and j if there is a bidirectional circuit to be served between the ring nodes i and j. Thus, the connection graph for all-to-all traffic is simply K_N, the complete graph of N nodes. Let C denote the number of edges in this graph. Thus, $C = \frac{N(N-1)}{2}$. C is also the exact number of connections for uniform all-to-all traffic with $t = 1$. Grooming the C connections corresponds to forming sets of edges in the connection graph, each set being no more than size g. Each set of edges corresponds to the set of connections that are groomed together.

Let W denote the total number of wavelengths and A the total number of ADMs used by a given grooming solution. Here, W stands for the total number of wavelengths on the working and protection rings combined. (Note that, in general, the number of working and protection wavelengths are not identical in our model). $W_{\min}$ is the minimum number of wavelengths required and $A_{\min}$ is the minimum number of ADMs required for the given traffic. $W(A_{\min})$ is the number of wavelengths needed by the grooming solution which achieves $A_{\min}$. $A(W_{\min})$ is the number of ADMs needed by the grooming solution which achieves $W_{\min}$.

To motivate the tradeoff between W and A, consider the following example of a unidirectional ring with 6 nodes numbered 0 to 5 clockwise. Traffic is non-uniform with $t = 3$, $g = 3$ and $s = 1$. Three copies of the following bidirectional circuits exist between the diametrically opposite pairs of nodes, viz., (0,3), (1,4) and (2,5). To achieve $A_{\min}$, it is obvious that the three connections with the same source-destination pair need to be groomed together. Thus, $A_{\min}$ = 6 and $W(A_{\min})$ = 6. To achieve $W_{\min}$, we have to groom all the s-circuits together. This reduces the overhead in the number of wavelengths that need to be set aside for protection and thus achieves the minimum possible number of wavelengths. In this way, $W_{\min}$ = 4 and $A(W_{\min})$ = 14. Thus, we see the significance of the tradeoff that is possible. Even though we have considered non-uniform traffic in this example, it can be expected that similar tradeoffs exist for uniform traffic as well.

We now formally define the survivable traffic grooming problem we address in this paper. Given a ring network with all-to-all uniform traffic and t, s, and g, and given W wavelengths where $W \geq W_{\min}$, determine the minimum number and placement of ADMs that are needed for a grooming solution that uses no more than W wavelengths.

The above formulation gives the tradeoff between W and A. Depending on the relative costs of wavelengths and ADMs, the network can be optimized for the total cost by using a grooming solution at the appropriate tradeoff level.

The complexity of the problem is actually dependent on the parameters t, s, and g. In general, solutions to the problem with one set of parameters do not necessarily translate into solutions for another set of parameters, and results from the published literature suggest that a "one-size-fits-all" approach is unlikely to yield good solutions, even if such an approach does exist. Since minimizing ADM and wavelength costs is a problem of great practical importance in today's optical networks, we focus on obtaining the best possible solution for a specific set of parameters of interest, rather than getting a general solution methodology that works for a large range of parameters, but rather poorly. We restrict g to be no more than 16 in the results below. $g = 4$ and $g = 16$ are of particular importance in SONET networks because they correspond to the case of grooming optical carrier-3 (OC-3) rate circuits into an OC-12-rate and OC-48-rate wavelength respectively.

In the next section, we first give some new results for non-survivable traffic grooming. We then present the first results for the survivable traffic-grooming problem.

III. MAIN RESULTS

Throughout this section, we speak of grooming circuits or edges interchangeably. By this, we mean that circuits in the ring, or equivalently, edges in the connection graph are groomed together.

A. New Results on Traffic Grooming in Rings

In this section, we give some new results for non-survivable traffic grooming for the cases of $g = 2$ and $g = 3$ for arbitrary t, and $g = 16$ for $t = 1$. Optimal results for the case of $g = 4$ for $t = 1$ were presented in [11], and hence we do not consider this particular case in this section. We attempt to minimize the number of ADMs by finding the grooming pattern with the best possible *ADM efficiency*, defined as the largest ratio of the number of circuits served to the number of ADMs required, for the given value of g.

Before we present the grooming solutions, we give the following general lower bound for A that holds for $t = 1$ and arbitrary g. Let k be the largest integer such that $\frac{k(k-1)}{2} \leq g$. Then the best possible ADM efficiency is achieved when either $\frac{k(k-1)}{2}$ or g circuits are served by either k or $k+1$ ADMs, respectively. This means that to groom C circuits, we will need no less than $\lceil \frac{C}{\max(\frac{g}{k+1}, \frac{k-1}{2})} \rceil$ ADMs.

We now present our grooming algorithms by starting with the case of $g = 2$.

A.1 $g = 2$

As far as we are aware, solutions to this case have not been provided. Our approach for this case will be useful in some of

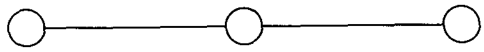

Fig. 3. Best grooming pattern for $g = 2$ and $t = 1$.

the solutions presented for other cases.

A.1.a $t = 1$. In this case, we can groom upto 2 edges onto a wavelength. The pattern that achieves the best ADM efficiency is the one that uses 3 ADMs to groom 2 edges as shown in Fig. 3. Also, the full capacity of a wavelength is used up by this pattern. Thus, a grooming solution that covers all the edges with the pattern shown in Fig. 3 would result in both $W_{\min}$ as well as $A_{\min}$. Theorem 2 guarantees that optimal solutions can be obtained for all-to-all, uniform traffic.

Theorem 1: In a connected undirected graph $G(V, E)$ with an even number of edges, E can be partitioned into k pairs of edges E_i, $1 \leq i \leq k = |E|/2$ such that every E_i is of the type shown in Fig. 3, i.e., is made up of consecutive edges.

Proof: See Appendix. □

Now, if the connection graph G has an even number of edges (i.e., if C is even), then the above theorem directly guarantees that there exists a grooming solution that achieves $A_{\min} = \frac{3C}{2}$ and $W_{\min} = \frac{C}{2}$.

If G has an odd number of edges, then we can always remove an edge from G such that the remaining graph is still connected. To see this, consider the case when G has a leaf node. Then, we remove the edge that connects to the leaf node. If G does not have a leaf node, then there exists a cycle in G. Remove an arbitrary edge from this cycle to form G'. Now, optimal grooming for the edges in G' can be done by using the above theorem, and the remaining edge must be groomed onto a separate wavelength and would require 2 more ADMs. It is easily seen that these are the minimum possible values for odd C, which are given by $W_{\min} = \lceil \frac{C}{2} \rceil$ and $A_{\min} = 3\lfloor \frac{C}{2} \rfloor + 2$.

A.1.b $t > 1$. In this case, $\lfloor t/2 \rfloor$ pairs of circuits between the same pair of nodes can be groomed onto one wavelength each. This can be done for the circuits between all the node pairs. The remaining set of ungroomed circuits would then form an all-to-all traffic pattern with $t = 1$, for which the optimal grooming solution is given above. Again in this case, $W_{\min}$ as well as $A_{\min}$ are achieved as all the wavelengths (except possibly the last one, if t is odd) are fully utilized and the best ADM efficieny patterns are used. Note that in this case the best ADM efficiency pattern is not the one shown in Fig. 3, but the one in which two circuits between the same pair of nodes are groomed onto a wavelength with two ADMs.

A.2 $g = 3$

A.2.a $t = 1$. Here, we can groom upto three circuits onto a wavelength. Thus, $W_{\min} = \lceil \frac{C}{3} \rceil$. The best possible ADM efficiency of one is achieved when the three circuits among three nodes are groomed together onto a wavelength using 3 ADMs, giving $A_{\min} = C$. Also, this pattern utilizes the wavelength to its full capacity. Thus, if it were possible to cover all the circuits by using only this grooming pattern, we could achieve $A_{\min}$ as well as $W_{\min}$ simultaneously, as for the case of $g = 2$. Our algorithm below shows that this can always be done when $N = 3^k$ for integer $k \geq 1$.

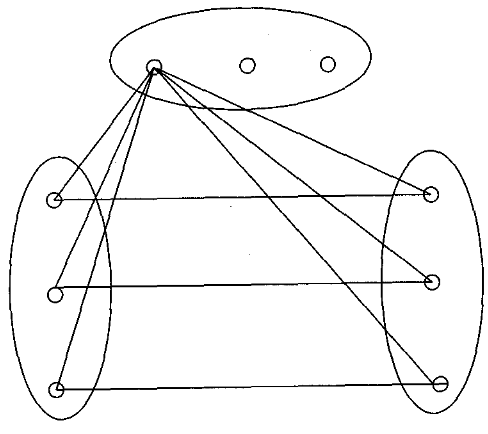

Fig. 4. Trianguar pattern formation for $g = 3$, when $m = 3, x = 0$.

We now give the grooming solution for an arbitrary number of nodes N. We divide the node set of size N into four disjoint sets S_1, S_2, S_3, and S_4 as follows: S_1, S_2, and S_3 being arbitrary (but disjoint) sets of nodes of cardinality $m = \lfloor \frac{N}{3} \rfloor$ each, and S_4 of size $x = N \bmod 3$. Let us number the nodes in each set from 1. We now focus on the cross-circuits between S_1, S_2, and S_3. These are $3m^2$ in number. The m^2 circuits between S_1 and S_2 can be arranged into m sets of size m each, such that each of the sets of edges represents an m-matching between the nodes of the two sets. Consider the first matching between S_1 and S_2, and the first node of S_3. There are m cross-circuits between node 1 of S_3 and each of the nodes of S_1 and S_2 for a total of $2m$. It is clear from Fig. 4 that these $2m$ circuits along with the m circuits from the first matching form m triangle patterns. None of these triangles have an edge in common with any other. For every triangle thus obtained, we groom its edges onto a wavelength. Each triangle requires three ADMs. We then take the next matching formed out of the cross-circuits between S_1 and S_2 and the second node from S_3 and repeat the above procedure forming another set of m triangular patterns. Note that these m new triangles do not have any edge in common with the m triangles formed in the previous step, thus ensuring that a circuit is considered only once in the grooming solution. This procedure is continued until all the m nodes in S_3 and all the m matchings between S_1 and S_2 are considered. Thus, a total of m^2 triangular patterns can be formed this way covering all the cross-ciruits between S_1, S_2, and S_3.

At this point, we are left with the cross-circuits between the nodes of S_4 and all the nodes in S_1, S_2 and S_3, as well as the (non-cross) circuits between the nodes of each of the sets. Now consider $S_1 \cup S_4$ (formed by circuits that have not yet been groomed) of size $m + x$. The traffic pattern among this set of nodes is all-to-all. We apply the above algorithm for this set. Next, consider $S_2 \cup S_4$ and $S_3 \cup S_4$. The traffic for these sets is not all-to-all as some circuits have already been groomed when $S_1 \cup S_4$ was considered. But the ADM requirement is no more than that required for all-to-all traffic within these sets of nodes. Thus we may obtain an upper bound on the ADM requirement by assuming the traffic to be all-to-all for $S_2 \cup S_4$ and $S_3 \cup S_4$

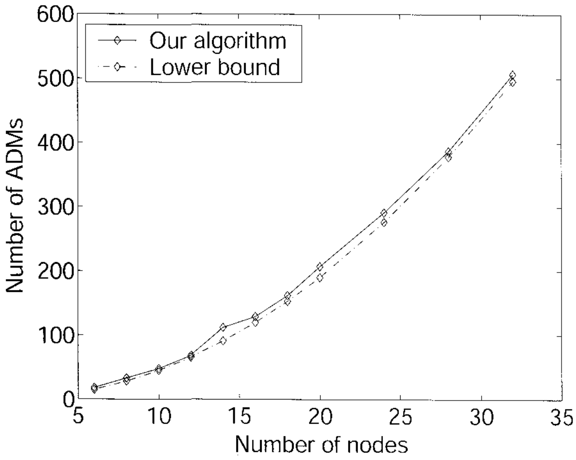

Fig. 5. Comparison of algorithm for $g = 3$ vs. the lower bound $A_{\min}$ for various values of N.

as well.

Let $N = 3m + x$, where $m \geq 1$ and $0 \leq x \leq 2$. Denoting the number of ADMs sufficient for grooming in an N-node ring by $A(N)$, we have, $A(N) = 3m^2 + 3A(m + x)$. For small values of N, one may obtain the best grooming solutions by hand quite easily so that $A(1) = 0, A(2) = 2, A(3) = 3$ and $A(4) = 7$. We compare the number of ADMs obtained by our algorithm with the lower bound $A_{\min}$ for various values of N in Fig. 5. As can be seen, the grooming algorithm is near-optimal.

We also present the following asymptotic upper bound on the number of ADMs required by our algorithm.

Theorem 2: For large N, the algorithm above requires no more than about $\frac{10A_{\min}}{9}$ ADMs.

Proof: See Appendix. □

We now show that the algorithm is actually optimal for $N = 3^k$.

Lemma 1: For $N = 3^k$, $k \geq 1$, integer, the grooming solution is optimal.

Proof: See Appendix. □

A.2.b $t = 2$. Now, every edge in the connection graph actually stands for two circuits. First, suppose N is odd. Then, the connection graph is an Eulerian graph and we can find an Euler cycle. To arrive at the grooming solution, we traverse the Euler cycle and form the pattern shown in Fig. 6 for every pair of edges. This pattern is the best in terms of ADM efficiency for $g = 3$, $t = 2$, and at the same time utilizes the wavelength fully. (Note that the triangle pattern we used for $t = 1$ also gives an ADM efficiency of one, but there may be cases when triangles cannot be formed but the pattern of consecutive edges can.) Therefore, if we are able to cover all the circuits by forming only this pattern, then the grooming solution thus obtained will be optimal in both the number of wavelengths and ADMs. If C is divisible by 3, then all the circuits can be covered by using only the pattern shown in Fig. 6, i.e., the circuits over three consecutive edges in G can be covered completely by two instances of the pattern.

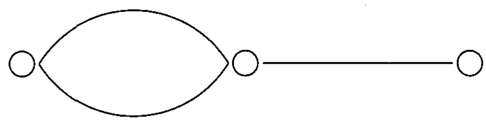

Fig. 6. Grooming pattern for the n-circuits when $g = 3$ and $t = 2$.

Otherwise, we simply apply the above for $\lfloor C/3 \rfloor$ consecutive edges of the Euler cycle, and there will be one or two remaining edges (consisting of two or four circuits, respectively). These circuits are then groomed separately. This achieves the minimum number of ADMs as well as wavelengths. Thus, we have $W_{\min} = \lceil \frac{2C}{3} \rceil$ and $A_{\min} = 6\lfloor \frac{C}{3} \rfloor + 2(C \bmod 3)$.

If N is even, then the connection graph for a subset of $N - 1$ nodes must have an Euler cycle. We apply the above procedure to this Euler cycle. However, the left-over circuits on this Euler cycle are not groomed separately as in the case for N odd, but are considered with other circuits as explained below. From the node which was not included in the subset, say node 0, there are $2(N - 1)$ circuits to the other nodes, yet to be groomed. If $N - 1$ is divisible by three, all these circuits can be covered by using only the pattern shown in Fig. 6 by organizing the $N - 1$ edges between node 0 and the other nodes in groups of three. Otherwise there will be either 2 (1) or 4 (2) circuits (edges) left. These left-over circuits are chosen so that they, along with the remaining circuits from the first procedure (based on Euler cycle), form a connected graph. This graph will have at most 4 edges or 8 circuits. If there are three or four edges, we can group three of them together and cover the associated edges by using only the pattern shown in the figure. The remaining edges are either one or two in number. If there is one edge, the two ciruits are groomed separately onto a wavelength. If there are two edges, we groom each of the two circuits between the same pair of nodes onto a wavelength. This algorithm is again optimal in both W as well as A as we pack every wavelength (except at most two) fully and use the most ADM-efficient pattern to cover all circuits (except for two or four).

A.2.c $t > 2$. In this case, we groom each of the $\lfloor \frac{t}{3} \rfloor$ sets of 3 circuits each between every node pair onto a wavelength. The remaining number of circuits between every node pair will then be either one or two for which optimal grooming solutions have been given above.

A.3 $g = 4$

A.3.a $t = 1$. A grooming solution for the case of $t = 1$ that is optimal in both the number of wavelengths and ADMs, (i.e., $A_{\min}$ achieved for $W_{\min}$) is given in [11].

A.3.b $t = 2$. The algorithm and $W_{\min}$ and $A_{\min}$ for $t = 2$ is identical to the case of $g = 2, t = 1$.

A.3.c $t = 3$. The solution for this case is again based on Euler cycle. We assume that C is divisible by four. We form an Euler cycle. Partition the cycle into sets of four consecutive edges. For each of the set of four consective edges, say e_1, e_2, e_3, and e_4 (which stand for 12 circuits), form three patterns as explained below. The three circuits over e_1 and a single circuit over e_2 are groomed onto a wavelength. The remaining two circuits from e_2 are groomed with two circuits from e_3. Then, the remaining

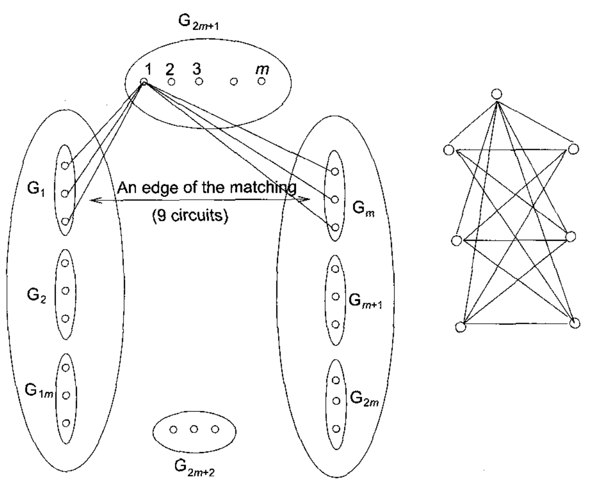

Fig. 7. (a) Schematic of our algorithm for $g = 16$ and $t = 1$, and (b) our grooming pattern for $g = 16$ and $t = 1$.

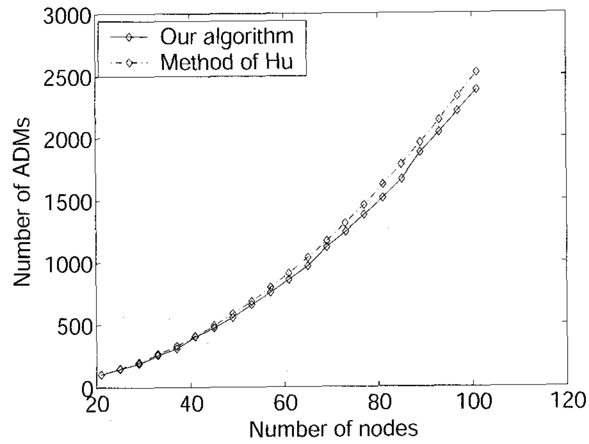

Fig. 8. Comparison of our algorithm and Hu's algorithm for $g = 16$ and $t = 1$.

one circuit on e_3 is groomed with the three circuits on e_4. This solution is optimal in W as well as A because we use the best ADM-efficient pattern whose wavelength capacity is also fully utilized. Clearly, $A_{\min} = \frac{9C}{4}$.

A.3.d $t > 3$. There are $\lfloor \frac{t}{4} \rfloor$ sets of four circuits each and t mod 3 circuits between each pair of nodes. We groom all the four circuits in every set of four circuits onto a wavelength. The remaining circuits between all node pairs are then groomed as shown for the cases of $t = 1, 2$, or 3 above, as is appropriate.

A.4 $g = 16$

We only consider the case $t = 1$ here. In [11], a heuristic algorithm was presented for this case. We now give another algorithm that performs better or no worse in most of the cases, and very slightly worse in a small number of cases we have looked at.

Let $N = 7m + x$, $0 \leq x \leq 6$. Divide the nodes into $2m$ groups $G_1, G_2, \cdots, G_{2m}$ of size 3 each, one group G_{2m+1} of size m, and the last group G_{2m+2} of size x. Let $S_1 = G_1 \cup G_2 \cup \cdots \cup G_m$ and $S_2 = G_{m+1} \cup G_{m+2} \cup \cdots \cup G_{2m}$. Then, there are $9m^2$ cross-circuits between the $3m$ nodes of S_1 and the $3m$ nodes of S_2. Form a bipartite graph between S_1 and S_2 such that the nodes are the sets G_x with edges from every set in S_1 to S_2, as shown in Fig. 7. Each such edge represents 9 cross-circuits between the nodes (the G_x sets) they terminate at. Now, we do a procedure similar to the one we proposed for $g = 3$. We generate m different matchings between S_1 and S_2, each of cardinality m. Consider one such matching and take the first node from G_{2m+1} and consider its cross-circuits with the nodes in S_1 and S_2. A pattern consisting of 15 circuits encompassing 7 nodes can be formed by one edge from the matching (which represents 9 circuits between two G_x sets and encompasses 6 nodes of the same two G_x sets) and the 6 cross-circuits that the first node of G_{2m+1} has with these 6 nodes as shown in Fig. 7(b). By considering another edge of the matching we can form another such pattern. We can form m such patterns from a single matching. These m patterns require $7m$ ADMs and m wavelengths. We then change the matching and consider the next node from G_{2m+1} and repeat the procedure. Thus, by considering each of the m nodes in G_{2m+1} in turn, we can groom all these cross-circuits using $7m^2$ ADMs and m^2 wavelengths. Note that for this step we have used a pattern that gives an ADM efficiency of $\frac{15}{7}$, i.e., 7 ADMs serve 15 circuits. The reason why our algorithm performs better than the one in [11] is that, we use a more ADM-efficient pattern than the one used in [11] (which uses 8 ADMs to serve 16 circuits) for a considerable number of circuits.

At the end of the above procedure, all the cross-circuits between S_1 and S_2, and between the set G_{2m+1} and the sets S_1 and S_2, will have been groomed. Now the traffic within $S_1 \cup G_{2m+2}$, $S_2 \cup G_{2m+2}$ and $G_{2m+1} \cup G_{2m+2}$ will be all-to-all. Note that the circuits within G_{2m+2} can be considered for grooming only in one of the three union sets. However, an upper bound on the ADM requirement can be obtained by finding the number of ADMs for all the three union sets. We apply the same algorithm as outlined above to these sets. At any step of this algorithm, if the size of a set being considered is less than twenty, we apply the mixed-integer-linear-programming (MILP) solution given in [11] for the all-to-all traffic within it. Let us denote the ADM requirement of our algorithm by $A(N)$. Then, we have $A(7m + x) = 7m^2 + A(m + x) + 2A(3m + x)$. Our algorithm provides a way for forming grooming solutions for $N > 20$ as the CPLEX program used to solve the MILP in [11] cannot find solutions for large N within reasonable time. We varied N from 21 to 100 and found the ADM requirement for our algorithm as well as Hu's algorithm [11]. Only for the case of $N = 26$ did our algorithm perform worse than Hu's algorithm; our algorithm required 159 ADMs wherease Hu's algorithm gives 156 ADMs. For 3 other cases, namely, $N = 21, 23$, and 34, both algorithms gave the same number of ADMs – 102, 120, and 272, respectively. In all other cases, our algorithm performed better, with the average percentage reduction in the number of ADMs being 4.8% and the best-case being 8.1%. Such a saving can be significant, as the absolute number of ADMs required can be quite large for large N. Fig. 8 compares our algorithm with that given by Hu as N is varied from 21 to 101.

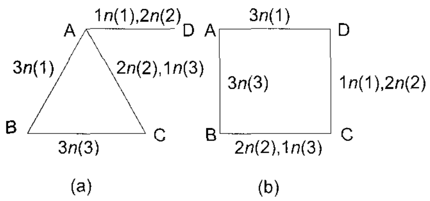

Fig. 9. Optimal grooming patterns for $g = 4$, and grooming patterns for the n-circuits for achieving $W_{\min}$ when $g = 4, t = 4$, and $s = 1$.

B. Survivable Traffic Grooming Results

In this section, we will provide solutions for survivable traffic grooming, i.e., when some of the connections are survivable while others are not. Our aim is to find $A_{\min}$, $W(A_{\min})$, $W_{\min}$, $A(W_{\min})$, and the number of ADMs needed for a given $W \geq W_{\min}$. When $s = 0$ or $s = t$, the problem reduces to that of the traditional grooming problem; so we have to consider only $s = 1, 2 \ldots, t$. Note that the number of wavelengths for $s = t$ is twice the number of wavelengths required for $s = 0$ since every working wavelength must be protected.

At this time, we do not have lower bounds for $A(W)$, the number of ADMs required for a grooming solution that uses no more than W wavelengths, and hence are unable to state that the results we give below are optimal. Nevertheless, it is our conjecture that they are so. First, we present a grooming solution that achieves $W_{\min}$.

B.1 $g = 2, t = 2, s = 1$

To achieve $W_{\min}$, we need to groom all the s-circuits together as this reduces the overhead in the number of wavelengths set aside for protection as pointed out earlier. We use the grooming solution that was obtained by us for non-survivable grooming and $g = 2$, for all the s-circuits. We use the same grooming pattern for all the n-circuits as well. If C is odd, then one n-circuit and one s-circuit will be left behind after covering all the other circuits as shown in Fig. 3. Then, these two circuits can be groomed onto one wavelength. Thus, $W_{\min} = 2\lceil \frac{C}{2} \rceil + \lfloor \frac{C}{2} \rfloor$ because $\lceil \frac{C}{2} \rceil$ wavelengths carry survivable traffic and must be protected, and $\lfloor \frac{C}{2} \rfloor$ wavelengths carry only n-circuits that do not need protection. 0Clearly $A(W_{\min}) = 6\lfloor \frac{C}{2} \rfloor + 2(C \bmod 2)$, as $3\lfloor \frac{C}{2} \rfloor$ ADMs are needed for grooming the n-circuits and an equal number for the s-circuits. If C is odd, then the n-circuit and the s-circuit which are groomed together adds two more ADMs.

When wavelengths are not a constraint, we should groom two circuits that have the same source-destination pair onto a wavelength as this pattern is the most ADM-efficient one for $g = 2$ when $t = 2$. This gives $A_{\min} = W(A_{\min}) = 2C$.

From the above, we can see that by using approximately $\frac{C}{2}$ additional wavelengths, C ADMs can be saved. Let us now try to get the number of ADMs that can be reduced from $A(W_{\min})$ by using $W_{\min} + k$ wavelengths, where $1 \leq k \leq \frac{C}{2}$. Consider initially the grooming solution for achieving $W_{\min}$. We take one wavelength onto which two s-circuits are groomed and another wavelength onto which two n-circuits are groomed. We break these two grooming patterns and form two new grooming patterns in which the two s-circuits and the two n-circuits, respectively, are groomed together. The two new patterns require four wavelengths while the two old patterns used three wavelengths. The new patterns require four ADMs while the old patterns used six ADMs. Thus, by trading off a wavelength, we are able to reduce two ADMs. We can continue doing this procedure until all the patterns shown in Fig. 3 are exhausted. We thus have a linear relationship between W and A. This relationship is given below for even C. Results for odd values of C can be similarly obtained. Thus, $A(W_{\min} + k) = A(W_{\min}) - 2k$ for $0 \leq k \leq \frac{C}{2}$. We conjecture that the above tradeoff is optimal, i.e., for a given W, the value of A given by the tradeoff relationship is the minimum one. These results can be generalized in a straightforward way to other combinations of t and s.

B.2 $g = 4$

We now give a comprehensive solution for all-to-all uniform traffic for this case.

B.2.a $t = 4, s = 1$. We first consider the case when C is a multiple of 4. Between any given node pair, there are three n-circuits and one s-circuit. To minimize the number of wavelengths, we have to groom all the s-circuits together. Thus, the grooming of s-circuits alone is nothing but the ordinary grooming problem for $g = 4$. An algorithm that minimizes both W and A simultaneously for $g = 4, t = 1$, and $s = 0$ was given by Hu in [11]. We can apply that algorithm to obtain the optimal grooming solution for the s-circuits. Hu's algorithm is such that the only two grooming patterns that are used in this case are the square and a triangle with an extra edge as shown in Fig. 9. We then groom the n-circuits as follows. Consider the 12 n-circuits associated with the four edges of every grooming pattern for the s-circuits. These must be groomed onto three wavelengths as shown in Fig. 9. The number in parantheses represents the wavelength number. Thus, three n-circuits from one edge and one n-circuit from an adjacent edge are groomed onto a wavelength. The two remaining n-circuits from the adjacent edge are groomed with two n-circuits from its adjacent edge, and so on, as shown in Fig. 9. The s-circuits require $\frac{C}{4}$ working wavelengths and $\frac{C}{4}$ protection wavelengths. The n-circuits require $\frac{3C}{4}$ wavelengths. This gives us $W_{\min} = \frac{5C}{4}$. It is easy to see that the s-circuits require C ADMs and the n-circuits require $\frac{9C}{4}$ ADMs, hence $A(W_{\min}) = \frac{13C}{4}$.

To achieve $A_{\min}$ we have to groom all the four circuits (three n-circuits and one s-circuit) between each node pair together onto a wavelength as this achieves the best ADM efficiency. This gives $A_{\min} = W(A_{\min}) = 2C$.

Next, we obtain the tradeoff between the the number of wavelengths and ADMs. From the above we can see that $\frac{3C}{4}$ wavelengths can be traded off for $\frac{5C}{4}$ ADMs. Let us call the grooming patterns used in the grooming solution for achieving $W_{\min}$ as old patterns. We break one old grooming pattern for the s-circuits which is either a square or a triangle with one extra edge as shown in Fig. 9, and one grooming pattern for the 12 n-circuits laid over the same edges as the pattern of the s-circuits just considered. These patterns alone require a total of 5 wavelengths (two for the s-circuits and two for the n-circuits) and 13 ADMs (4 for the s-circuits and 9 for the n-circuits). We now

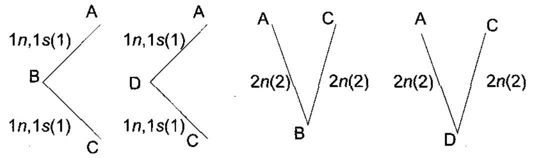

Fig. 10. Grooming pattern for 6 wavelengths and 12 ADMs for $g = 4$. $t = 4$, $s = 1$.

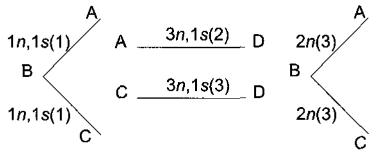

Fig. 11. Grooming pattern for 7 wavelengths and 10 ADMs for $g = 4$. $t = 4$, $s = 1$.

form new patterns from these two old patterns which require 6, 7, and 8 wavelengths and 12, 10, and 8 ADMs, respectively. The patterns requiring six and seven wavelengths are shown in Figs. 10 and 11 (assuming the square pattern; a simple relabelling of the nodes will work for the pattern in Fig. 9(a)). For forming the pattern that requires 8 wavelengths, we just groom all the four circuits (three n-circuits and one s-circuit) associated with a given edge onto a wavelength. Thus, there will be four such patterns corresponding to the four edges in the grooming patterns of Fig. 9.

The above procedure of breaking up two old patterns and forming several new patterns with varying wavelength and ADM requirements can be continued until all the old patterns are exhausted. Thus, we may obtain the relationship between A and W as, $A(W_{\min} + 3k) = A(W_{\min}) - 5k$, $A(W_{\min} + 3k + 1) = A(W_{\min}) - 5k - 1$, $A(W_{\min} + 3k + 2) = A(W_{\min}) - 5k - 2$, for $0 \leq k \leq \frac{C}{4}$.

Now, let $C = 4m + 1$. We first form the grooming solution for achieving $W_{\min}$. The optimal grooming solution for the s-circuits will, in addition to having the patterns shown in Fig. 9, also have three triangular patterns in which three circuits are groomed together using three ADMs as shown by the wavelength 1 of Fig. 12(a). This is because every wavelength can have only either three or four circuits with either three or four ADMs respectively. We set the grooming pattern for the n-circuits as in the case of C being divisible by four except for the circuits over the edges of these three triangles. Since the triangle has only three circuits while a wavelength can accommodate four circuits, we can also groom one n-circuit along an edge of a triangle together with the other three circuits of the triangle. We do this for each of the three triangles. We are now left with eight circuits over the edges of each triangle. These are groomed as shown in Fig. 12(a) for each of the three triangles. Thus, $W_{\min} = 2\frac{(C-9)}{4} + 12 = \lceil \frac{C}{2} \rceil + 7$ and $A_{\min} = C + 9(C-9)/4 + 18$. To obtain $A_{\min}$, all the four circuits between any node pair have to be groomed onto a wavelength. This gives $A_{\min} = W(A_{\min}) = 2C$.

Thus, we see that approximately $\frac{C}{2} - 7$ wavelengths can be traded off for approximately $\frac{5C}{4}$ ADMs. Next, we obtain the tradeoff between W and A. We know that by breaking a grooming pattern consisting of four s-circuits subsequently as shown

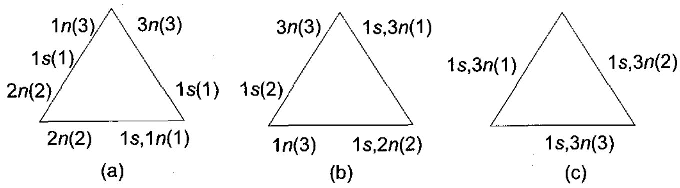

Fig. 12. Breaking a triangle for $g = 4$, $t = 4$, $s = 1$.

for the case when C is divisible by four, we can trade off 1 wavelength for 1 ADM, then another wavelength for 2 more ADMs, and then another wavelength for another 2 ADMs. Consider the tradeoff obtained by breaking the triangular patterns. Initially, one triangular pattern requires 4 wavelengths and 9 ADMs. By breaking the pattern, two subsequent new patterns can be formed as shown in Fig. 12(b) and (c) requiring 5 wavelengths and 8 ADMs, and 6 wavelengths and 6 ADMs, respectively. To obtain the tradeoff, we first break the patterns of s-circuits consisting of four circuits as shown in Fig. 9. This trades off one wavelength for an ADM, then one more wavelength for two more ADMs, and again one more wavelength for two more ADMs. We repeat this procedure until all the patterns for s-circuits consisting of four edges are exhausted, i.e., $(C-9)/4$ times. We then break the triangles and form new patterns as explained above. When we are done, we end up in the grooming solution for $A_{\min}$. This gives the complete tradeoff.

If $C = 4m + 2$ or $C = 4m + 3$, then we will either have only two triangular patterns or one triangular pattern in the optimal grooming solution for the s-circuits. The procedure obtaining the tradeoff for these cases is then exactly the same as explained for the case when $C = 4m + 1$.

From now on, we explain the results only for the case when C is divisible by 4 to avoid complexity. If C were not divisible by 4, it is not difficult to modify the procedures in light of the discussion given above.

B.2.b $t = 4, s = 2$. This scenario is similar to the case when $g = 2, s = 2$ and $t = 1$. We have $W_{\min} = 2\lceil \frac{C}{2} \rceil + \lfloor \frac{C}{2} \rfloor$, $A(W_{\min}) = 6\lfloor \frac{C}{2} \rfloor + 2(N \bmod 2)$ and $A_{\min} = W(A_{\min}) = 2C$.

B.2.c $t = 4, s = 3$. For achieving $W_{\min}$, we groom in the same way as we did for $t = 4$ and $s = 1$ but with the n-circuits and s-circuits interchanged. This gives $W_{\min} = \frac{7C}{4}$, $A(W_{\min}) = \frac{13C}{4}$ and $A_{\min} = W(A_{\min}) = 2C$. Thus, $\frac{C}{4}$ wavelengths can be traded off for $\frac{5C}{4}$ ADMs. To obtain the complete tradeoff relationship, we do the following. We break one pattern for the n-circuits which is like either of the two patterns shown in Fig. 9. We then choose one pattern consisting of 12 s-circuits whose edges are the same as the above pattern. We break these two old patterns to form four new patterns, in each of which the four circuits associated with a given node pair are groomed onto the same wavelength. It is easy to see that the two old patterns required a total of 7 wavelengths and 13 ADMs while the four new patterns require 8 wavelengths and 8 ADMs. We continue this procedure until all the old patterns are exhausted, whereupon we reach the grooming solution which achieves $A_{\min}$. Thus, we obtain the number of ADMs required for every wavelength W, such that $W_{\min} \leq W \leq W(A_{\min})$ as $A(W_{\min} + k) = A(W_{\min}) - 5k$, for $0 \leq k \leq \frac{C}{4}$.

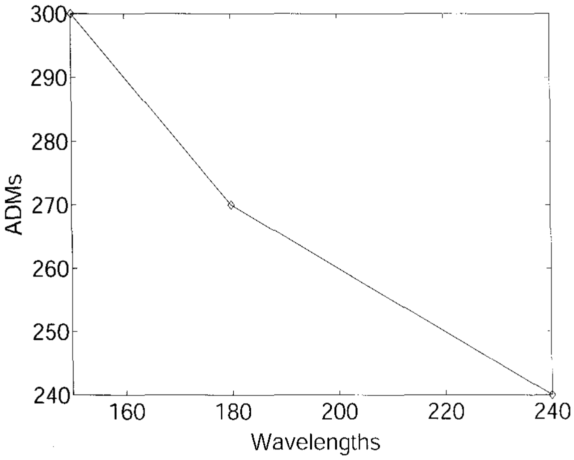

Fig. 13. W vs A for $g = 4, t = 3, s = 2$ for $N = 16$.

B.2.d $t = 3$, $s = 2$. In this case, every edge has an s-circuit and two n-circuits. To achieve $W_{\min}$, we groom all the n-circuits using the patterns shown in Fig. 9. The grooming solution for the s-circuits is the same as that for the case when $g = 2$, $t = 1$, and $s = 0$, i.e., each pattern is a pair of consecutive edges. Also, we choose each edge pair from a single grooming pattern for the n-circuits, which is one of the two patterns in Fig. 9. This gives $W_{\min} = \frac{5C}{4}$ and $A(W_{\min}) = \frac{5C}{2}$. To achieve $A_{\min}$, we groom all the three circuits between every node pair onto the same wavelength. This gives $A_{\min} = W_{\min} = 2C$. Thus, $\frac{3C}{4}$ wavelengths can be traded off for $\frac{C}{2}$ ADMs. To obtain the complete tradeoff relationship between W and A, consider one grooming pattern for the n-circuits and the two corresponding grooming patterns for the s-circuits. These patterns require 5 wavelengths and 10 ADMs. These can be broken to form a set of new patterns as shown in Fig. 14(a). This requires 6 wavelengths and 9 ADMs. Thus, by using one extra wavelength we can save one ADM. This procedure can be repeated until all the old patterns are exhausted. Since there are C edges totally and each initial pattern consists of 4 edges, we can do the above procedure $\frac{C}{4}$ times from the initial grooming solution for $W_{\min}$.

To reduce the number of ADMs even further we take one complete set of patterns formed by the above procedure (i.e., all the three patterns corresponding to the 3 wavelengths in Fig. 14(a)) and form a new pattern in which all the three circuits between the same node pair are groomed together. This new pattern requires 8 wavelengths and 8 ADMs. Thus, we can save one ADM by increasing the number of wavelengths by 2. This procedure can be done until all the $\frac{C}{4}$ patterns are exhausted. At the end of this above procedure, we end up with the grooming solution for $A_{\min}$.

Fig. 13 shows the relationship between W and A for $N = 16$, $t = 3$, and $s = 2$. (We note that the tradeoff for the range $\frac{3C}{2}$ was obtained by looking at the number of ADMs saved by increasing the number of wavelengths by $3k$, $1 \leq k \leq \frac{C}{6}$. We have done a linear interpolation of these results to show the tradeoff for the entire range of wavelengths in Fig. 13).

B.2.e $t = 3, s = 1$. We first assume that C is even for simplicity. For achieving $W_{\min}$, we groom all the s-circuits together in patterns as shown in Fig. 9. The grooming of the n-circuits is then the same as the grooming for the case of $g = 2$, $t = 1$. Let us choose the patterns for the n-circuits such that two of the patterns have the same edges as one of the patterns for the s-circuits. This way, we have $W_{\min} = C$ and $A(W_{\min}) = \frac{5C}{2}$. For achieving $A_{\min}$, we groom the three circuits (two n-circuits and 1 s-circuit) between every node pair together onto a wavelength. This gives $A_{\min} = W(A_{\min}) = 2C$. Thus, C wavelengths can be traded off for $\frac{C}{2}$ ADMs.

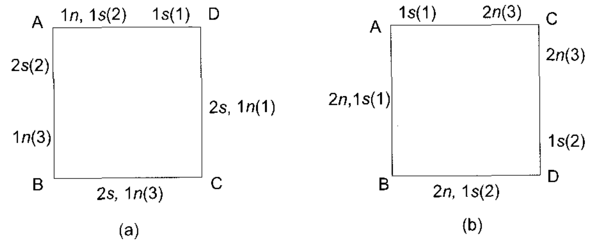

Fig. 14. Pattern requiring (a) 6 wavelengths and 9 ADMs for $g = 4, t = 3, s = 2$, and (b) 5 wavelengths and 9 ADMs for $g = 4, t = 3, s = 1$.

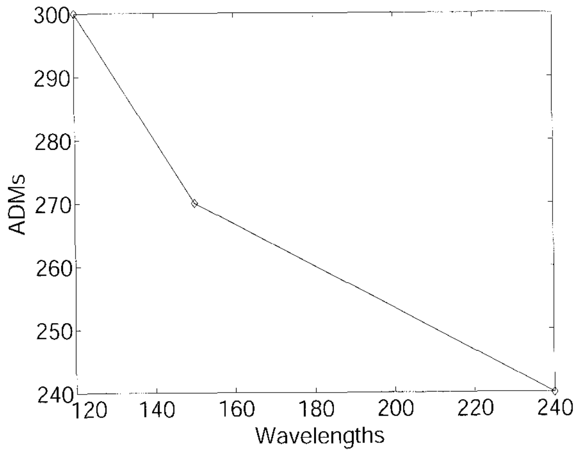

Fig. 15. Tradeoff between W and A for $g = 4, t = 3, s = 1$.

We obtain the complete tradeoff relationship as follows. Initially, let the grooming solution correspond to the one that achieves $W_{\min}$. We take one pattern of the s-circuits and the two corresponding patterns (having 4 edges or 8 circuits) of the n-circuits. These three patterns require 4 wavelengths and 10 ADMs. We break them into three new patterns, as shown in Fig. 14(b), requiring 5 wavelengths and 9 ADMs. To further reduce the number of ADMs, we repeat the above procedure until all the old patterns are exhausted. This procedure can be done $\frac{C}{4}$ times starting from the initial grooming solution for $W_{\min}$, since there are a total of C edges and each pattern of s-circuits in the initial solution has 4 edges. To further reduce the number of ADMs, we take one complete set of new patterns shown in Fig. 14(b) and rearrange the circuits such that all the three circuits between the same node pair are groomed together onto a wavelength. The pattern thus formed requires 8 wavelengths and 8 ADMs. We can continue this procedure until all the patterns formed in the previous step are exhausted. Again, this procedure can be done $\frac{C}{4}$ times as these many new patterns were formed in the first step.

In the first step, we traded off one wavelength for an ADM, while in the second step we traded off 3 wavelengths for an ADM. The resulting tradeoff curve is plotted in Fig. 15 for $N = 16$. Once again, we have employed linear interpolation in Fig. 15 for the range $\frac{5C}{4} \leq W \leq 2C$.

If C is odd, a solution for achieving $W_{\min}$ with minimum number ADMs should have one wavelength in which two n-circuits and one s-circuit between the same pair of nodes are groomed together. This pattern is the one corresponding to the best possible ADM efficiency. Also, its wavelength usage cannot be increased, (i.e., groom four circuits onto the wavelength) without decreasing the wavelength usage of some other wavelength. Thus, we do not break the pattern for this wavelength for obtaining the tradeoff between W and A. We then follow the same procedure given above for the case of C being even for the rest of the circuits.

This completes the discussion of comprehensive survivable traffic grooming solutions for the important case of $g = 4$.

IV. CONCLUSIONS

Traffic grooming is an important practical problem in today's optical networks. While previous work has considered this problem for various architectures and traffic models, survivability of the circuits has not been an element in past research. In this paper, we considered the survivable traffic grooming problem in a unidirectional ring network with all-to-all uniform traffic, and presented the tradeoff between the number of wavelengths and the number of ADMs for two values of the grooming factor g, one of which is the practically important case of $g = 4$. Our results show that this tradeoff is significant and depending on the relative costs of ADMs and wavelengths, the network designer may choose an appropriate grooming solution. We also presented some new results for the non-survivable traffic grooming problem. Among these were an optimal grooming algorithm for $g = 2$, and a near-optimal algorithm for $g = 3$. We also improved upon an earlier algorithm for $g = 16$, besides solving the case of $g = 4$ optimally for uniform all-to-all traffic, in which there is an arbitrary number of circuits $t > 1$ between every pair of nodes. We believe that our work here can serve as a foundation for further developments in the important area of survivable traffic grooming. We summarize our results for non-survivable grooming in Table 1 and survivable grooming in Table 2.

APPENDIX

Some Proofs

Proof of Theorem 1: If the graph G has only two edges, then G must be as shown in Fig. 3 as it is connected. Else, we will show that G can always be expressed as the union of two edge-disjoint graphs G_1 and G_2, both of which are connected and have even number of edges.

Consider an arbitrary edge e in G and a non-leaf node, say a, on which e is incident. Clearly, such a node exists. Consider all the edges of G other than e that are incident on the node a, numbered $1, 2, \ldots, k$. If there exists an edge among these k edges whose removal does not disconnect G, then we pair that edge with e to form G_1 and the rest of the graph becomes G_2.

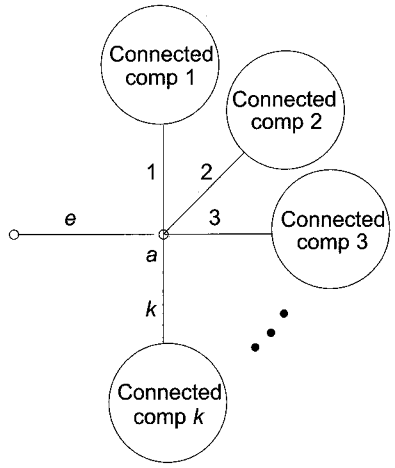

Fig. 16. Representation of the graph G.

Since G_1 consists of only two connected edges it is of the form shown in Fig. 3. Since G_2 has an even number of edges and is connected, we may continue by applying the same procedure to G_2. At any step of this procedure, if any of the decomposed graphs has two edges, we form the pattern shown in Fig. 3 out of it.

Suppose G gets disconnected on removing any of the edges $1, 2, \ldots, k$. Then, the original graph G can be represented as shown in Fig. 16. Consider the component 1. It may have either an even or an odd number of edges. If it has an even number of edges, then we make G_1 as the component 1 and G_2 as the rest of the graph. If the component 1 has an odd number of edges, we take the union of component 1 and the edge numbered 1 shown on the graph and make it G_1 and the rest of the graph as G_2. Clearly, G_1 and G_2 formed as above are connected and have an even number of edges. We again apply this procedure to the reduced graphs G_1 and G_2. Since at every step of the procedure the graph is decomposed into smaller sized graphs, this procedure stops when all the decomposed graphs thus obtained have only two connected edges as in Fig. 3. This completes the proof. □

Proof of Theorem 2: For simplicity, we assume $N = 3m$, where m is a large integer. The number of circuits in all-to-all traffic, $C = 3m(3m-1)/2$, and $A_{\min} = C$. The algorithm first forms m^2 triangular patterns which cover the $3m^2$ cross-circuits using $3m^2$ ADMs. The remaining number of circuits is less than $\frac{3m^2}{2}$. Also, these circuits constitute three all-to-all traffic patterns, each on a set of nodes of size m. For simplicity again, we assume that m is odd and $\frac{m(m-1)}{2}$, which is the number of circuits within each set of nodes, is a multiple of three. Consider the all-of-all traffic that exists among one set of m nodes. Now, in the connection graph corresponding to this all-to-all traffic pattern among m nodes, the degree of each vertex is even because m is odd. Therefore, this is an Eulerian graph and we may form an Euler cycle by traversing each edge exactly once. Since the number of edges is assumed to be a multiple of three, we can pick three consecutive edges on this Euler cycle and groom them together onto a wavelength using 4 ADMs (one for each node in the 3-consecutive-edge segment). Hence, no more than $\frac{4m(m-1)}{(2)(3)}$ ADMs are required for each such all-to-all traf-

Table 1. Summary of non-survivable grooming results.

Parameters	Results
$g = 2, t = 1$	$W_{\min} = \lceil \frac{C}{2} \rceil$, $A_{\min} = 3\lfloor \frac{C}{2} \rfloor + 2(C \bmod 2)$, for any C
$g = 3, t = 1$	Optimal solution available for $N = 3^k$, near optimal solution for other N
$g = 3, t = 2$	$W_{\min} = \lceil \frac{2C}{3} \rceil$, $A_{\min} = 6\lfloor \frac{C}{3} \rfloor + 2(C \bmod 3)$ for odd N, and C a multiple of 3, refer text for other values of N and C
$g = 4, t = 2$	Same results as for $g = 2, t = 1$
$g = 4, t = 3$	$W_{\min} = \frac{3C}{4}$, $A_{\min} = \frac{9C}{4}$ for C a multiple of 4, refer text for other values of C

Table 2. Summary of survivable grooming results. For $g = 4$, C is assumed to be divisible by 4 unless specified otherwise.

Parameters	Results
$g = 2, t = 2, s = 1$	$W_{\min} = 2\lceil \frac{C}{2} \rceil + \lfloor \frac{C}{2} \rfloor$, $A(W_{\min}) = 6\lfloor \frac{C}{2} \rfloor + 2(C \bmod 2)$, (for any C) $A_{\min} = W(A_{\min}) = 2C$, (for any C) $A(W_{\min} + k) = A(W_{\min}) - 2k$ for $0 \leq k \leq \frac{C}{2}$ (for even C)
$g = 4, t = 4, s = 1$	$W_{\min} = \frac{5C}{4}$, $A(W_{\min}) = \frac{13C}{4}$, and $A_{\min} = W(A_{\min}) = 2C$ $A(W_{\min} + 3k) = A(W_{\min}) - 5k$, $A(W_{\min} + 3k + 1) = A(W_{\min}) - 5k - 1$, $A(W_{\min} + 3k + 2) = A(W_{\min}) - 5k - 2$, for $0 \leq k \leq \frac{C}{4}$
$g = 4, t = 4, s = 2$	Same results as for $g = 2, t = 2, s = 1$ given in the first row
$g = 4, t = 4, s = 3$	$W_{\min} = \frac{7C}{4}$, $A(W_{\min}) = \frac{13C}{4}$, and $A_{\min} = W(A_{\min}) = 2C$, $A(W_{\min} + k) = A(W_{\min}) - 5k$, for $0 \leq k \leq \frac{C}{4}$
$g = 4, t = 3, s = 2$	$W_{\min} = \frac{5C}{4}$ and $A(W_{\min}) = \frac{5C}{2}$, and $A_{\min} = W_{\min} = 2C$, $A(W_{\min} + k) = A(W_{\min}) - k$ for $0 \leq k \leq \frac{C}{4}$, $A(W_{\min} + \frac{C}{4} + 2k) = A(W_{\min}) - \frac{C}{4} - k$, for $0 \leq k \leq \frac{C}{4}$
$g = 4, t = 3, s = 1$ (for even C)	$W_{\min} = C$, $A(W_{\min}) = \frac{5C}{2}$, and $A_{\min} = W(A_{\min}) = 2C$ $A(W_{\min} + k) = A(W_{\min}) - k$ for $0 \leq k \leq \frac{C}{4}$, $A(W_{\min} + \frac{C}{4} + 3k) = A(W_{\min}) - \frac{C}{4} - k$, for $0 \leq k \leq \frac{C}{4}$

fic pattern. Considering there are three such patterns and also the $3m^2$ ADMs required for the cross-circuits, we see that the ADM requirement is no more than $5m^2$. Comparing this with $A_{\min} \approx \frac{9m^2}{2}$ gives our desired result. □

Proof of Lemma 1: Let us denote by $A_{\min}(N)$ the minimum number of ADMs required for an N-node ring. For $N = 3^k$, $A_{\min}(N) = 3^k(3^k - 1)/2$. According to our algorithm, the number of ADMs is given by $A(N) = 3m^2 + 3A(m + x)$ for $N = 3m + x$, $0 \leq x \leq 2$. Therefore, for $N = 3^k$, our algorithm gives $A(3^k) = 3^{2k-1} + 3A(3^{k-1})$. To prove optimality, it suffices to show that this last equation is also satisfied if we replace $A(N)$ by $A_{\min}(N)$. It can be easily verified that this is indeed true. □

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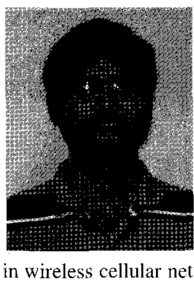

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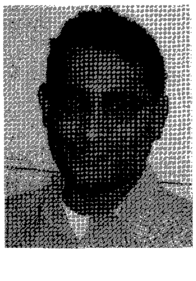

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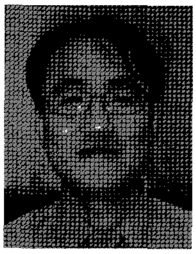

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