Intersymbol Decorrelating Detector for Asynchronous CDMA Systems

Gaonan Zhang, Guoan Bi, and Alex Chichung Kot

Abstract: **Estimated channel information, especially multipath length and multipath channel of the desired user, is necessary for most previously reported linear blind multiuser detectors in code division multiple access (CDMA) systems. This paper presents a new blind intersymbol decorrelating detector in asynchronous CDMA systems, which uses the cross correlation matrix of the consecutive symbols. The proposed detector is attractive for its simplicity because no channel estimation is required except the synchronization of the desired user. Compared with other reported multiuser detectors, simulation results show that the proposed detector provides a good performance when the active users have significant intersymbol interference and the multipath length is short.**

Index Terms: **Cross correlation matrix, decorrelating detector, intersymbol information, multiuser detection.**

I. INTRODUCTION

Blind multiuser detection has been studied extensively for code-division multiple-access (CDMA) systems because of its superiority to combat the detrimental effect of multiple access interference (MAI) and intersymbol interference (ISI). Without considering multipath, a blind minimum output energy (MOE) detector was reported in [1] and afterwards the canonical subspace representation of the decorrelating detector and the minimum mean square error (MMSE) detector were generalized in [2]. By using array processing techniques, a reduced-rank MOE detector was proposed in [3]. However, these detectors can not work properly when ISI can not be ignored and the multipath channel is not known. In [4]–[6], a number of subspace approaches were developed based on the use of channel estimation. A reduced computational constrained optimization solution was proposed in [7] and [8] with some sacrifice in performance (i.e., inferior to MMSE detector). However, all these methods still need to estimate the multipath length and the multipath channel of the desired user. In practical wireless communication systems, the position and the environment of the mobile users are often changing with time, which leads to variations of channel parameters. Therefore, accurate and timely estimation of channel parameters is always a troublesome problem for multiuser detection.

In this paper, we attempt to avoid channel estimation based on a new multiuser detection scheme that exploits the intersymbol information. It is noted that all previously mentioned detectors treat ISI as noise and do not make use of the intersymbol relationship. In fact, the current received symbol has certain correlation with its preceeding and succeeding symbols because of the existence of ISI and MAI. Utilization of such correlation would be beneficial for multiuser detection. We first construct a cross correlation matrix from the consecutively received symbols to obtain the subspace of interfering users. By exploiting this subspace, we develop a new intersymbol multiuser detector for asynchronous CDMA systems with multipath. Since our proposed detector is based on asynchronous signal model, the detector can be applied to CDMA uplink while multipath channel response estimation for each known user can be avoided. It is shown that the proposed detector, which is equivalent to the decorrelating detector in [9], can be implemented conveniently without the need for the estimation of the background noise, the multipath length, and the multipath channel of the desired user. It should be pointed out that the proposed detector is only applicable to short-code CDMA systems. Simulation results show that the proposed detector achieves a promising performance with a subspace-based method when the active users have significant intersymbol interference and the multipath length is short.

The rest of paper is organized as follows. Section II presents the asynchronous multipath CDMA signal model and Section III proposes the intersymbol decorrelating detector. In Section IV, simulation examples are provided to demonstrate the performance of the proposed detector. Section V contains the conclusion.

II. SIGNAL MODEL

We consider a DS-CDMA system that has K users and a normalized spreading factor of N chips per symbol. The transmitted signal of user k is given by

$$y_k(t) = A_k \sum_{i=-\infty}^{\infty} b_k[i] s_k(t - iT) \tag{1}$$

where T is the symbol duration, A_k and $b_k[i] \in \{+1, -1\}$ are, respectively, the amplitude and the symbol stream of the k-th user, and the signature waveform $s_k(t)$ is of the form

$$s_k(t) = \sum_{j=0}^{N-1} c_k[j]\psi(t - jT_c), \quad 0 \le t \le T \tag{2}$$

where $c_k[j] = \pm 1 \ (0 \le j \le N-1)$ is the spreading sequence allocated to the k-th user and $\psi(t)$ is the normalized chip waveform with a duration $T_c = T/N$. The discrete-time expression of the transmitted signal of user k at the chip rate is obtained by a multirate convolution

$$y_k(n) = \sum_{i=-\infty}^{\infty} b_k(i)\bar{c}_k(n - iN) \tag{3}$$

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where $\bar{c}_k(n) = A_k c_k(n)$ for $n = 0, \cdots, N-1$. By propagating through the asynchronous multipath channel that is assumed to have a maximum length of M $(M < N)$ in terms of chip duration, the received discrete-time signal $r_k(n)$ due to user k is given by [7]

$$r_k(n) = \sum_{m=-\infty}^{\infty} y_k(m) g_k(n - d_k - m) \tag{4}$$

where $g_k(m)$, the m-th complex multipath parameter for user k, is nonzero for $0 \le m \le M$, and $0 \le d_k < N$ is the delay of user k in terms of chip duration. Based on (3) and (4), we obtain

$$r_k(n) = \sum_{m=-\infty}^{\infty} b_k(m) h_k(n - d_k - mN) \tag{5}$$

$$h_k(n) = \sum_{j=-\infty}^{\infty} \bar{c}_k(j) g_k(n-j). \tag{6}$$

Then, the total received signal is the superposition of all K users plus additive white Gaussian noise, i.e.,

$$r(n) = \sum_{k=1}^{K} r_k(n) + v(n) \tag{7}$$

where $v(n)$ is a zero mean complex Gaussian process with a variance of σ^2. From (6) and denoting $\mathbf{h}_{k,all} = [h_k(0), h_k(1), \cdots, h_k(N-1+M)]^H$ as the signature vector of user k, we obtain

$$\mathbf{h}_{k,all} = \bar{\mathbf{C}}_k \mathbf{g}_k \tag{8}$$

where

$$\bar{\mathbf{C}}_k = \begin{bmatrix} \bar{\mathbf{c}}_k(0) & & \mathbf{0} \\ \vdots & \ddots & \bar{\mathbf{c}}_k(0) \\ \bar{\mathbf{c}}_k(N-1) & \ddots & \vdots \\ \mathbf{0} & \ddots & \bar{\mathbf{c}}_k(N-1) \end{bmatrix}, \quad \mathbf{g}_k = \begin{bmatrix} g_k(0) \\ \vdots \\ g_k(M) \end{bmatrix}. \tag{9}$$

In (9), $\mathbf{g}_k$ is the kth user's multipath channel vector. From (5) and (8), we denote

$$\underline{\mathbf{r}}_k(n) = \begin{bmatrix} r_k(nN) \\ \vdots \\ r_k(nN+N-1) \end{bmatrix}, \quad \underline{\mathbf{h}}_k = \begin{bmatrix} \mathbf{0} \\ h_k(0) \\ \vdots \\ h_k(N-d_k-1) \end{bmatrix},$$

$$\underline{\bar{\mathbf{h}}}_k = \begin{bmatrix} h_k(N-d_k) \\ \vdots \\ h_k(N+M-1) \\ \mathbf{0} \end{bmatrix}.$$

The nth received symbol vector for user k can be given by

$$\underline{\mathbf{r}}_k(n) = \underline{\mathbf{h}}_k b_k(n) + \underline{\bar{\mathbf{h}}}_k b_k(n-1) \tag{10}$$

and the total received users' signal vector $\underline{\mathbf{r}}(n) = [r(nN), \cdots, r(nN+N-1)]^T$ is given by

$$\underline{\mathbf{r}}(n) = \sum_{k=1}^{K} \underline{\mathbf{r}}_k(n) + \underline{\mathbf{v}}(n) = \underline{\mathbf{H}}\mathbf{b}(n) + \underline{\mathbf{v}}(n) \tag{11}$$

where $\underline{\mathbf{H}} = [\underline{\mathbf{h}}_1, \cdots, \underline{\mathbf{h}}_K, \underline{\bar{\mathbf{h}}}_1, \cdots, \underline{\bar{\mathbf{h}}}_K]$ is the signature matrix of all users with full column rank $2K$, $\mathbf{b}(n) = [b_1(n), \cdots, b_K(n), b_1(n-1), \cdots, b_K(n-1)]^T$ contains all received bits, and $\underline{\mathbf{v}}(n) = [v(nN), \cdots, v(nN+N-1)]^T$ is the independent white Gaussian noise vector.

III. INTERSYMBOL DECORRELATING DETECTOR

Let us assume without loss of generality that user 1 is the desired user, and the receiver is synchronized to user 1 with respect to time, i.e., $d_1 = 0$. Then, the signal in (11) can be written as

$$\underline{\mathbf{r}}(n) = \underline{\mathbf{h}}_1 b_1(n) + \underline{\bar{\mathbf{h}}}_1 b_1(n-1) + \underline{\tilde{\mathbf{H}}}\tilde{\mathbf{b}}(n) + \underline{\mathbf{v}}(n) \tag{12}$$

where $\underline{\tilde{\mathbf{H}}} = [\underline{\mathbf{h}}_2, \cdots, \underline{\mathbf{h}}_K, \underline{\bar{\mathbf{h}}}_2, \cdots, \underline{\bar{\mathbf{h}}}_K]$ and $\tilde{\mathbf{b}}(n) = [b_2(n), \cdots, b_K(n), b_2(n-1), \cdots, b_K(n-1)]^T$ are, respectively, the signature matrix and the received bits of the interfering users. It is noted from (12) that only the first M entries of $\underline{\bar{\mathbf{h}}}_1$ are non-zero. Thus, the ISI caused by $\underline{\bar{\mathbf{h}}}_1$ can be removed by truncating the first $G \ge M$ entries of $\underline{\mathbf{r}}(n)$ where G is the length of the truncation of the received signal. For convenience and without loss of generality, we choose $G = M$ to obtain

$$\mathbf{r}(n) = \mathbf{h}_1 b_1(n) + \tilde{\mathbf{H}}\tilde{\mathbf{b}}(n) + \mathbf{v}(n) \tag{13}$$

where $\mathbf{r}(n)$ is an $(N-G) \times 1$ vector which consists of the last $N-G$ entries of $\underline{\mathbf{r}}(n)$. Similarly, $\mathbf{h}_1$, $\tilde{\mathbf{H}}$, and $\mathbf{v}(n)$ are composed of the last $N-G$ rows of $\underline{\mathbf{h}}_1$, $\underline{\tilde{\mathbf{H}}}$, and $\underline{\mathbf{v}}(n)$, respectively. The signal in (13) is a truncation of the signal in (12) which indicates that the ISI of the desired user is eliminated. In (13), the ISI of the interfering users still exists as their non-zero delays. This is an advantageous property for the following proposed intersymbol cross correlation matrix.

We first construct the auto-correlation matrix of the received signal $\mathbf{r}(n)$ in (13), given by

$$\mathbf{R} = E\{\mathbf{r}(n)\mathbf{r}(n)^H\} = \mathbf{h}_1\mathbf{h}_1^H + \tilde{\mathbf{H}}\tilde{\mathbf{H}}^H + \sigma^2 \mathbf{I}_{N-G} \tag{14}$$

where $\mathbf{I}_{N-G}$ is an $(N-G) \times (N-G)$ unitary matrix. In order to obtain the useful intersymbol information, we construct the cross correlation matrix of the received signal $\mathbf{r}(n)$, which is given by

$$\begin{aligned} \mathbf{R}_+ &= E\{\mathbf{r}(n)\mathbf{r}(n+1)^H\} = E\{\tilde{\mathbf{H}}\tilde{\mathbf{b}}(n)\tilde{\mathbf{b}}(n+1)^T\tilde{\mathbf{H}}^H\} \\ &= \tilde{\mathbf{H}} \begin{bmatrix} \mathbf{0} & \mathbf{I}_{K-1} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \tilde{\mathbf{H}}^H \end{aligned} \tag{15}$$

$$\begin{aligned} \mathbf{R}_- &= E\{\mathbf{r}(n)\mathbf{r}(n-1)^H\} = E\{\tilde{\mathbf{H}}\tilde{\mathbf{b}}(n)\tilde{\mathbf{b}}(n-1)^T\tilde{\mathbf{H}}^H\} \\ &= \tilde{\mathbf{H}} \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{I}_{K-1} & \mathbf{0} \end{bmatrix} \tilde{\mathbf{H}}^H \end{aligned} \tag{16}$$

$$\bar{\mathbf{R}} = \mathbf{R}_+ + \mathbf{R}_- = \tilde{\mathbf{H}} \begin{bmatrix} \mathbf{0} & \mathbf{I}_{K-1} \\ \mathbf{I}_{K-1} & \mathbf{0} \end{bmatrix} \tilde{\mathbf{H}}^H. \tag{17}$$

In (15), $\mathbf{R}_+$ is the correlation matrix between the current symbol and the next symbol and $\mathbf{R}_-$ in (16) is the correlation matrix between the current symbol and the previous symbol. Based on (13) and (17), it is clear that $\bar{\mathbf{R}}$ is only composed of the signature matrix of all interfering users without the desired user because the ISI of the desired user has been truncated in (13).

By performing an eigendecomposition of the matrix $\mathbf{R}$ and $\bar{\mathbf{R}}$, we obtain

$$\mathbf{R} = [\mathbf{U}_s, \mathbf{U}_n] \begin{bmatrix} \Lambda_s & \mathbf{0} \\ \mathbf{0} & \Lambda_n \end{bmatrix} [\mathbf{U}_s, \mathbf{U}_n]^H = \mathbf{U}_s \Lambda_s \mathbf{U}_s^H + \mathbf{U}_n \Lambda_n \mathbf{U}_n^H \quad (18)$$

$$\bar{\mathbf{R}} = [\bar{\mathbf{U}}_s, \bar{\mathbf{U}}_n] \begin{bmatrix} \bar{\Lambda}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} [\bar{\mathbf{U}}_s, \bar{\mathbf{U}}_n]^H = \bar{\mathbf{U}}_s \bar{\Lambda}_s \bar{\mathbf{U}}_s^H \quad (19)$$

where $\Lambda_s = \mathrm{diag}(\lambda_1, \cdots, \lambda_{2K-1})$ contains $(2K-1)$ largest eigenvalues of the signal subspace in $\mathbf{R}$ and $\mathbf{U}_s$ contains the corresponding orthonormal eigenvectors. Both $\Lambda_n = \sigma^2 \mathbf{I}_{N-G-2K+1}$ and $\mathbf{U}_n$ are, respectively, eigenvalues and orthonormal eigenvectors of the noise subspace. Similarly, $\bar{\Lambda}_s = \mathrm{diag}(\bar{\lambda}_1, \cdots, \bar{\lambda}_{2K-2})$ contains $(2K-2)$ non-zero eigenvalues and $\bar{\mathbf{U}}_s$ contains the corresponding eigenvectors. Obviously, $\bar{\mathbf{R}}$ contains ISI and MAI of all interfering users. The following proposition will show the relationship between the subspace spanned by $\bar{\mathbf{R}}$ and the subspace spanned by $\tilde{\mathbf{H}}$.

Proposition 1: The null space of $\bar{\mathbf{U}}_s$ in (19) is equivalent to the null space of $\tilde{\mathbf{H}}$, i.e., null$(\bar{\mathbf{U}}_s)$ = null$(\tilde{\mathbf{H}})$.

Proof: See Appendix A. □

Based on Proposition 1, it is easy to obtain

$$\mathrm{null}(\tilde{\mathbf{H}}) = \mathrm{null}(\bar{\mathbf{U}}_s) = \mathrm{range}(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \quad (20)$$

where $\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H$ projects any signal onto the null space of $\tilde{\mathbf{H}}$. For an optimal linear solution, it is reasonable to explore the orthogonal space of $\tilde{\mathbf{H}}$ as a constraint for minimization of interference since $\tilde{\mathbf{H}}$ contains the ISI and MAI of all interfering users. By projecting the signal in (13) onto $\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H$ directly, we obtain

$$\begin{aligned} \tilde{\mathbf{r}}(n) &= (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)\mathbf{r}(n) \\ &= (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)[\mathbf{h}_1 b_1(n) + \tilde{\mathbf{H}}\tilde{\mathbf{b}}(n) + \mathbf{v}(n)] \quad (21) \\ &= (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)[\mathbf{h}_1 b_1(n) + \mathbf{v}(n)] \end{aligned}$$

where the third equality follows from Proposition 1, i.e., $(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)\tilde{\mathbf{H}} = \mathbf{0}$. It is clear from (21) that the MAI and ISI of all interfering users are removed from $\tilde{\mathbf{r}}(n)$. Then, we consider the following optimization problem:

$$\mathbf{w}_{opt}(n) \triangleq \arg \min_{\mathbf{w}} E\{|b_1[n] - \mathbf{w}^H \tilde{\mathbf{r}}[n]|^2\}. \quad (22)$$

Denoting $\tilde{\mathbf{R}} \triangleq E\{\tilde{\mathbf{r}}(n)\tilde{\mathbf{r}}^H(n)\}$ and performing the derivative of (22) with respect to $\mathbf{w}$, we have

$$\tilde{\mathbf{R}}\mathbf{w} = (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)\mathbf{h}_1. \quad (23)$$

We next premultiply $(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)\mathbf{h}_1$ by $\tilde{\mathbf{R}}$ to obtain

$$\begin{aligned} &\tilde{\mathbf{R}}(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)\mathbf{h}_1 \\ &= [(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)\mathbf{h}_1 \mathbf{h}_1^H (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \\ &\quad + \sigma^2 (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)](\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)\mathbf{h}_1 \quad (24) \\ &= [\mathbf{h}_1^H (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)\mathbf{h}_1 + \sigma^2](\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)\mathbf{h}_1 \end{aligned}$$

where the first equality follows from (21). It is seen from (24) that $(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)\mathbf{h}_1$ is an eigenvector of $\tilde{\mathbf{R}}$ with the corresponding eigenvalue $\mathbf{h}_1^H (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)\mathbf{h}_1 + \sigma^2$. Hence, the solution to (23) is given by

$$\mathbf{w}_{opt} = [\mathbf{h}_1^H (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)\mathbf{h}_1 + \sigma^2]^{-1} (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)\mathbf{h}_1. \quad (25)$$

Since $[\mathbf{h}_1^H (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)\mathbf{h}_1 + \sigma^2]^{-1}$ is only a positive real scalar factor which is irrelevant to the performance of the detector, $\mathbf{w}_{opt}$ can be rewritten as

$$\mathbf{w}_{opt} = (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)\mathbf{h}_1. \quad (26)$$

Proposition 2: $(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)\mathbf{h}_1$ is the decorrelating detector for user 1 in (13).

Proof: See Appendix B. □

Proposition 2 shows that $\mathbf{w}_{opt}$ is equivalent to the decorrelating detector for the desired user. We now consider the implementation of $\mathbf{w}_{opt}$. Recalling that $\mathbf{w}_{opt}$ is an eigenvector of $\tilde{\mathbf{R}}$ with the corresponding eigenvalue $\mathbf{h}_1^H (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)\mathbf{h}_1 + \sigma^2$, we consider the other eigenvectors of $\tilde{\mathbf{R}}$. Denoting $\mathbf{u}$ as an eigenvector of $\tilde{\mathbf{R}}$ which is orthogonal to $\mathbf{w}_{opt}$, i.e., $\mathbf{u}^H (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)\mathbf{h}_1 = 0$, we have

$$\begin{aligned} \tilde{\mathbf{R}}\mathbf{u} =& [(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)\mathbf{h}_1 \mathbf{h}_1^H (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \\ &+ \sigma^2 (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)]\mathbf{u} \\ =& \sigma^2 (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)\mathbf{u} \leq \sigma^2 \mathbf{u} \end{aligned} \quad (27)$$

where the second equality follows from $\mathbf{u}^H (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)\mathbf{h}_1 = 0$. It is clear from (27) that $\mathbf{h}_1^H (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)\mathbf{h}_1 + \sigma^2$ is the largest eigenvalue of $\tilde{\mathbf{R}}$, and all other eigenvalues of $\tilde{\mathbf{R}}$ are less than σ^2. It means that the proposed detector $\mathbf{w}_{opt}$ is equivalent to the principal eigenvector of $\tilde{\mathbf{R}}$ up to a complex phase factor α, i.e.,

$$\arg \max \tilde{\mathbf{R}} = \alpha (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)\mathbf{h}_1 = \alpha \mathbf{w}_{opt}. \quad (28)$$

In general, the phase of α will degrade the performance of the detector. Fortunately, such ambiguity can be resolved by differential detection [10].

Table 1 summarizes the implementation of the proposed detector. For the estimation of the subspace of $\bar{\mathbf{R}}$, we assume the number of the active users K is known. It is seen from Table 1 that the realization of the detector is very simple. We do not need any channel information on background noise level, multipath channel and even the spreading sequence of the desired user. In contrast, some of these parameters must be assumed to be known or estimated for the detectors in [4], [7], and [8].

Remark: In Table 1, the proposed detector utilizes the eigendecomposition of the cross-correlation matrix $\bar{\mathbf{R}}$ to null out the ISI and MAI of all interfering users. According to (13), $\bar{\mathbf{R}}$ has $2(K-1)$ eigenvectors spanned by $\mathbf{h}_k \bar{\mathbf{h}}_k^H + \bar{\mathbf{h}}_k \mathbf{h}_k^H$ for $k = 2, \cdots, K$. However, if the k-th interfering user has similar delays to that of the desired user, i.e., $d_k \approx 0$, the eigenvalues corresponding to the k-th user in $\bar{\mathbf{R}}$ will be small since only the first d_k entries of $\bar{\mathbf{h}}_k$ are non-zero. Because it is usually difficult

Table 1. Intersymbol decorrelating detector for asynchronous CDMA systems with multipath

Signal frame: $\mathbf{r}(0), \mathbf{r}(1), \cdots, \mathbf{r}(P)$
Step 1. Compute autocorrelation and cross correlation
$\mathbf{R} = \frac{1}{P+1}\sum_{n=0}^{P}\mathbf{r}(n)\mathbf{r}(n)^H$
$\bar{\mathbf{R}} = \frac{1}{P}(\sum_{n=0}^{P-1}\mathbf{r}(n)\mathbf{r}(n+1)^H + \sum_{n=1}^{P}\mathbf{r}(n)\mathbf{r}(n-1)^H)$
Step 2. Compute eigendecomposition of $\bar{\mathbf{R}}$
$\bar{\mathbf{R}} = \bar{\mathbf{U}}_s\bar{\Lambda}_s\bar{\mathbf{U}}_s^H$
Step 3. Form the detector
$\hat{\mathbf{w}} =$ Max-eigenvector
$[\tilde{\mathbf{R}} = (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)\mathbf{R}(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)]$
Step 4. Perform differential detection
$z_1(n) = \hat{\mathbf{w}}^H\mathbf{r}(n), \quad \hat{\beta}_1(n) = \text{sign}\,(\Re\{z_1(n)z_1(n-1)^*\})$

to obtain the small eigenvalue's subspace accurately, more errors will occur on the estimation of the k-th user's eigenvectors at step 2 for small d_k. In this case, the estimated $\bar{\mathbf{U}}_s$ may lose a part of the k-th user's information, and the proposed detector can not totally null out the interference of the k-th user. Then, the performance of the proposed detector will be deteriorated, which are shown in the next section. In addition, the algorithm in Table 1 is valid based on the assumption that the length of the multipath channel response of all users are within one symbol period. If the length of the multipath channel response is longer than one symbol period, the proposed algorithm would not be applicable.

IV. SIMULATION RESULTS

We test the proposed method in an asynchronous CDMA system with eight users and spreading gain $N = 31$. The spreading sequences for all users are generated by Gold codes. For all cases, we simulate a severe near-far case in which the power of each interfering user is at least 10 dB more than that of the desired user. The multipath gains in each user's channel, which have been normalized with equal power, are randomly chosen and kept fixed, i.e., $\|\mathbf{g}_k\|^2 = 1$. The desired user is assumed to be synchronized and the length of the signal frame is $P = 500$. In each example, the truncating window of the proposed detector is $G = 5$, i.e., the first five entries of the received signal vector are truncated to remove the ISI of the desired user.

We compare the performance of five detectors, namely: The true decorrelating detector, the proposed decorrelating detector, the blind zero-forcing (decorrelating) detector in [6], the constraint optimized detector in [7] and the conventional matched-filter.

Fig. 1 illustrates the bit-error-rates (BER) of these detectors with multipath length $M = 3$. In this example, the initial delay d_k of each interfering user is assumed to be randomly distributed between 8 and 31 in terms of the chip cycle T_c. It is seen from this figure that the matched filter performs poorly compared with the other four linear detectors because it is near-far limited. It is also observed that the performance of the proposed decorrelating detector is close to the performance of the zero-forcing (decorrelating) detector in [6], and is a little better than that of the constraint optimized detector in [7]. In addition, there

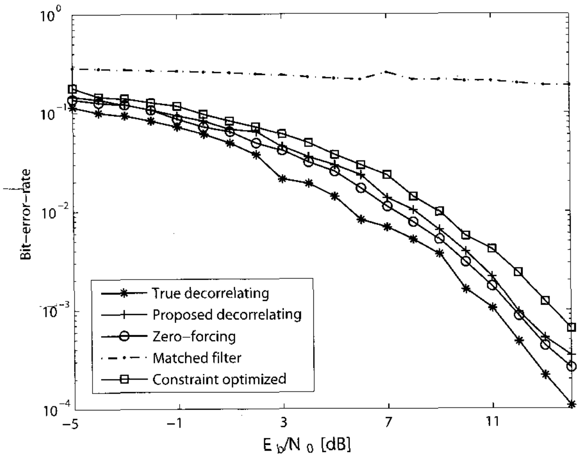

Fig. 1. Bit-error-rate comparison of various linear detectors in an asynchronous CDMA system. $M = 3$, $G = 5$, $K = 8$, and $d_k \geq 8$ ($k = 2, \cdots, 8$).

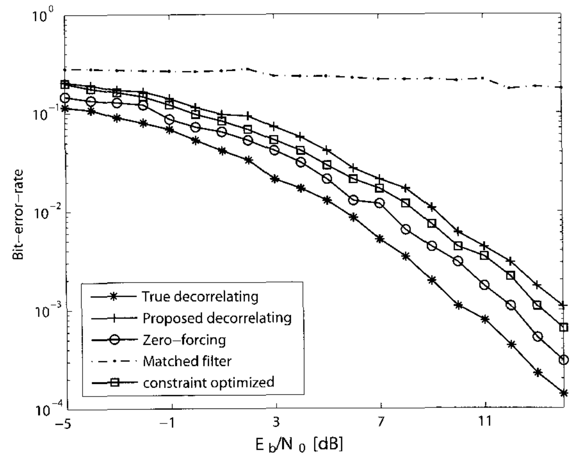

Fig. 2. Bit-error-rate comparison of various linear detectors in an asynchronous CDMA system. $M = 5$, $G = 5$, $K = 8$, $d_2 = 3$, $d_3 = 2$, and $d_k \geq 8$ ($k = 4, \cdots, 8$).

is no significant performance difference between the proposed detector and the true decorrelating detector. Clearly, our method performs well in this case because the signal channel is ideal for the proposed detector, i.e., the ISI of the desired user is totally removed because $M < G$, and all interfering users have significant intersymbol interference as the delay $d_k \geq 8$.

We next consider a dispersive channel with multipath length $M = 5$. In this channel, two users are assumed to have similar delays to that of user 1, i.e., $d_2 = 3$ and $d_3 = 2$, while the other five interfering users' delays are still no less than $8T_c$. The performance of various detectors is shown in Fig. 2. It is seen that all detectors except the matched filter have certain performance deterioration as the length of multipath increases. However, although the ISI of the desired user is also removed in this case ($M = G$), the proposed detector suffers more performance loss and its BER is a little higher than that of the constraint optimized detector in [7]. As explained in the previous section, this is because the eigenvalues corresponding to user 2 and user 3 in $\bar{\mathbf{R}}$ are close to zero due to their small delays. Thus, more errors will

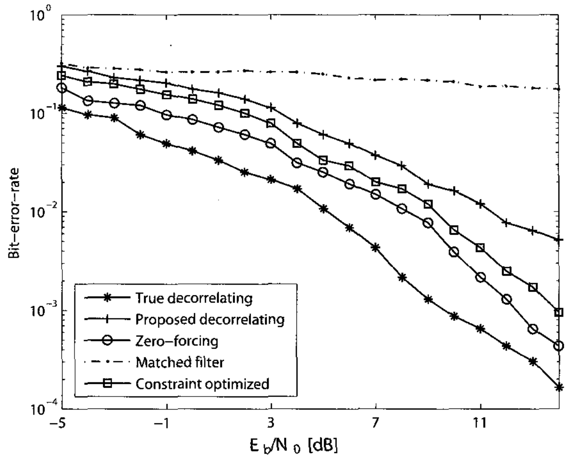

Fig. 3. Bit-error-rate comparison of various linear detectors in an asynchronous CDMA system. $M = 7$, $G = 5$, $K = 8$, $d_2 = 3$, $d_3 = 2$, and $d_k \geq 8$ $(k = 4, \cdots, 8)$.

occur on the estimation of their eigenvectors within the limited signal frame, which leads to the deterioration of the proposed detector. In Fig. 3, the multipath length increases to $M = 7$ and the delays of the interfering users are the same as the delays in Fig. 2. It is seen that the performance deterioration of the proposed detector is more apparent compared with Fig. 2, and the proposed detector is obviously inferior to the decorrelating detector in [6] and the constraint optimized detector in [7] although it is still significantly superior to the matched filter. The reason for such deterioration is that the signal in (13) can not totally remove the ISI of the desired user since $M > G$. Hence, the remanent ISI of the desired user in $\bar{\mathbf{R}}$ also influences the accuracy of the proposed detector. In general, our method is more sensitive to the multipath length and the delays of the interfering users compared with the decorrelating detector in [6] and the constraint optimized detector in [7]. However, our method is very simple which only requires the delay of the desired user, while the decorrelating detector in [6] requires knowledge of the delay and the signature sequence of the desired user as well as the estimation of the noise level σ^2 and the desired user's multipath channel $\mathbf{g}_1$. The detector in [7] also requires information of the delay, the signature sequence and the channel estimation of the desired user.

V. CONCLUSIONS

A new intersymbol decorrelating detector is proposed for asynchronous CDMA systems. The detector makes use of an important cross correlation matrix between adjacent symbols to mitigate the influence of the interfering users. It is shown that the proposed detector can be easily implemented without the estimations of the noise level, the multipath channel and the multipath length while a satisfying performance is achieved as shown in the simulation results.

APPENDIX A: PROOF OF PROPOSITION 1

Denote an $(N - G) \times 1$ vector $\mathbf{x}$. The proof is equivalent to the proof of the following items.

1. If $\mathbf{x}^H\tilde{\mathbf{H}} = \mathbf{0} \implies \mathbf{x}^H\bar{\mathbf{U}}_s = \mathbf{0}$.
2. If $\mathbf{x}^H\bar{\mathbf{U}}_s = \mathbf{0} \implies \mathbf{x}^H\tilde{\mathbf{H}} = \mathbf{0}$.

We now consider $\bar{\mathbf{R}}\bar{\mathbf{R}}^H$. By denoting

$$\mathbf{W} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_{K-1} \\ \mathbf{I}_{K-1} & \mathbf{0} \end{bmatrix} \tilde{\mathbf{H}}^H\tilde{\mathbf{H}} \begin{bmatrix} \mathbf{0} & \mathbf{I}_{K-1} \\ \mathbf{I}_{K-1} & \mathbf{0} \end{bmatrix},$$

we have

$$\begin{aligned} \bar{\mathbf{R}}\bar{\mathbf{R}}^H &= \tilde{\mathbf{H}} \begin{bmatrix} \mathbf{0} & \mathbf{I}_{K-1} \\ \mathbf{I}_{K-1} & \mathbf{0} \end{bmatrix} \tilde{\mathbf{H}}^H\tilde{\mathbf{H}} \begin{bmatrix} \mathbf{0} & \mathbf{I}_{K-1} \\ \mathbf{I}_{K-1} & \mathbf{0} \end{bmatrix} \tilde{\mathbf{H}}^H \\ &= \tilde{\mathbf{H}}\mathbf{W}\tilde{\mathbf{H}}^H \\ &= \bar{\mathbf{U}}_s\bar{\Lambda}_s\bar{\mathbf{U}}_s^H\bar{\mathbf{U}}_s\bar{\Lambda}_s\bar{\mathbf{U}}_s^H \\ &= \bar{\mathbf{U}}_s\bar{\Lambda}_s^2\bar{\mathbf{U}}_s^H \end{aligned} \tag{29}$$

where the third equality follows from (19). To prove the first item, we have

$$\begin{aligned} \mathbf{x}^H\tilde{\mathbf{H}} = \mathbf{0} &\implies \mathbf{x}^H\tilde{\mathbf{H}}\mathbf{W}\tilde{\mathbf{H}}^H\mathbf{x} = 0 \\ &\implies \mathbf{x}^H\bar{\mathbf{U}}_s\bar{\Lambda}_s^2\bar{\mathbf{U}}_s^H\mathbf{x} = 0 \\ &\implies \mathbf{x}^H\bar{\mathbf{U}}_s = \mathbf{0} \end{aligned} \tag{30}$$

where the last step follows from the fact that $\bar{\Lambda}_s^2$ is a positive diagonal matrix. For the second item, the proof is as follows

$$\begin{aligned} \mathbf{x}^H\bar{\mathbf{U}}_s = \mathbf{0} &\implies \mathbf{x}^H\bar{\mathbf{U}}_s\bar{\Lambda}_s^2\bar{\mathbf{U}}_s^H\mathbf{x} = 0 \\ &\implies \mathbf{x}^H\tilde{\mathbf{H}}\mathbf{W}\tilde{\mathbf{H}}^H\mathbf{x} = 0 \\ &\implies \mathbf{x}^H\tilde{\mathbf{H}} = \mathbf{0}. \end{aligned} \tag{31}$$

Similarly, the last step in (31) is achieved because $\mathbf{W}$ is also positive and of full rank. Combining items 1 and 2, we arrive at the desired result.

APPENDIX B: PROOF OF PROPOSITION 2

According to the definition in [9], the decorrelating detector for the desired user must be in the range of user subspace and orthogonal to all interfering users except the desired one. Therefore, based on (13) and the eigendecomposition of (18), we need to verify that

$$\begin{aligned} &\mathbf{d}_1 = (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)\mathbf{h}_1 \in \text{range}(\mathbf{U}_s), \\ &\text{and} \begin{cases} \mathbf{d}_1^H\mathbf{h}_1 > 0 \\ \mathbf{d}_1^H\tilde{\mathbf{H}} = \mathbf{0} \end{cases} \end{aligned} \tag{32}$$

According to (18), the proof of the first condition is equivalent to the proof of $\mathbf{U}_n^H\mathbf{d}_1 = \mathbf{0}$ since range$(\mathbf{U}_s)$ = null$(\mathbf{U}_n)$. It is easily achieved by

$$\begin{aligned} \mathbf{U}_n^H\mathbf{d}_1 &= \mathbf{U}_n^H(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)\mathbf{h}_1 \\ &= \mathbf{U}_n^H\mathbf{h}_1 - \mathbf{U}_n^H\bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H\mathbf{h}_1 \\ &= \mathbf{0} \end{aligned} \tag{33}$$

where the third equality follows from the fact that $\mathbf{h}_1 \in \text{range}(\mathbf{U}_s)$ is orthogonal to $\mathbf{U}_n$, and $\text{range}(\mathbf{U}_n) = \text{null}(\mathbf{h}_1, \ \tilde{\mathbf{H}})$ belongs to the subspace of $\text{null}(\tilde{\mathbf{H}}) = \text{null}(\bar{\mathbf{U}}_s)$, i.e., $\mathbf{U}_n^H \bar{\mathbf{U}}_s = \mathbf{0}$. Now, we consider the second condition. Based on (19), (20), and Proposition 1, it is easy to obtain

$$\begin{aligned} \mathbf{d}_1^H \mathbf{h}_1 &= \mathbf{h}_1^H (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{h}_1 > 0 \\ \mathbf{d}_1^H \tilde{\mathbf{H}} &= \mathbf{h}_1^H (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \tilde{\mathbf{H}} = \mathbf{0} \end{aligned} \tag{34}$$

Therefore, $\mathbf{d}_1$ is the decorrelating detector for user 1.

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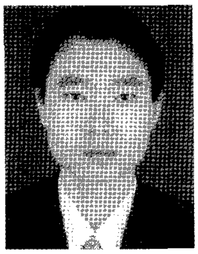

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