A Class of Binary Cocyclic Quasi-Jacket Block Matrices

Moon Ho Lee · Subash Shree Pokhrel · Seung Je Choi · Chang Joo Kim

Abstract

In this paper, we present a quasi-Jacket block matrices over binary matrices which all are belong to a class of cocyclic matrices is the same as the Hadamard case and are useful in digital signal processing, CDMA, and coded modulation. Based on circular permutation matrix(CPM) cocyclic quasi block low-density matrix is introduced in this paper which is useful in coding theory. Additionally, we show that the fast algorithm of quasi-Jacket block matrix.

Key words : Cocyclic Matrices, Jacket Matrices, Hadamard Matrices, Low-Density Matrices.

Ⅰ. Introduction

The Walsh-Hadamard matrix is widely used for the Walsh representation of the data sequence in image coding and for Hadamard-Walsh orthogonal-sequence generator in code division multiple access(CDMA), spread-spectrum communications[1]~[6], coded modulation [7],[8]. Their basic functions are sampled Walsh functions which can be expressed in terms of the Hadamard $[H]_N$ matrices. Using the orthogonality of Hadamard matrices, more general matrices have been developed[9]~[18]. These matrices are called Jacket matrices. In a general definition, a square matrix $J_{N\times N}=[j_{i,l}]_{N\times N}$ is called Jacket matrix if $[J]^{-1}_{N\times N}=\frac{1}{C}[j_{jl}^{-1}]^T_{N\times N}$, for $1\le i, j\le N$, where C is a nonzero constant and $(\cdot)^T$ denotes the transpose of the matrix. Since the inverse of the Jacket matrix can be calculated easily, it is very helpful to employ this kind of matrix in the signal processing[11], encoding[16], multiple-input multiple-output(MIMO) designs[4], sequence design[3], mobile communication[2], and cryptography[18]. In addition, the Jacket matrices are associated with many kind of matrices, such as unitary matrices, and Hermitian matrices which are very important in signal processing, communication (e.g., encoding), orthogonal code design, mathematics and physics.

The outline of the paper is as follows. Section 2 presents a construction of cocyclic quasi-Jacket block matrices over binary matrices and fast algorithm of these quasi-Jacket block matrices. The block matrices may have important application in signal processing, communication, quantum information, etc. A generalized cocyclic quasi-Jacket block low-density matrix based on circular permutation matrix is defined and is constructed in Section 3. Finally, some conclusions are presented in Section 4.

Ⅱ. Cocyclic Quasi-Jacket Block Matrices

Many interesting matrices, such as Hadamard, WHT, DFT, and its variations matrices are all belong to the class of Jacket family. In many applications, cocyclic matrices are very useful. The definition of cocyclic matrix is in the following[15].

If G is a finite group of order v and C is a finite Abelian group of order w, a (two-dimensional) cocycle is a mapping $\varphi: G\times G\rightarrow C$ satisfying the cocycle equation

$$\varphi(g,h)\varphi(gh,k)=\varphi(g,hk)\varphi(h,k),\quad \forall g,h,k\in G. \tag{1}$$

This implies

$$\varphi(g,1)=\varphi(1,h)=\varphi(1,1). \quad \forall g,h\in G$$

Then, $\varphi(1,1)=1$, we call it normalized-cycle[15].

A cocycle associated with the groups G and C is naturally represented as a square matrix of order $v\times v$, whose rows and columns are indexed by the elements of the group G under some fixed ordering and whose entry in position (g, h) is $\varphi(g,h)$. We call such matrices G-cocyclic matrices. We represent a G-cocyclic matrix as

$$M_\varphi=\left[\varphi(g,h)\right]_{g,h\in G}. \tag{2}$$

If the cocycle Ψ is symmetric then M_Ψ is a sy-

Manuscript received December 28, 2006 ; revised March 8, 2007. (ID No. 20061228-035J)
Institute of Information and Communication, Chonbuk National University, Chonbuk, Korea.

mmetric matrix.

Definition 2.1: Let $[J]_{N\times N} = \left[(j)_{i,l}\right]_{N\times N}$ be a block matrix if $[J]^{-1}_{N\times N} = \frac{1}{N}\left[(j)_{i,l}^{-1}\right]^{T}_{N\times N}$, for $1 \le i,\ 1 \le N$, then the matrix $[J]_{N\times N}$ is a quasi-Jacket block matrix, where C is a nonzero constant, and $(\cdot)^T$ denotes the transpose of the matrix.

In a quasi-Jacket block matrix, any element of the block Jacket matrices is a common matrix or block matrix which is denoted $(j)_{i,l}$. The elements of the quasi-Jacket block matrix $[J]_{N\times N}$ are substituted with a matrix or even a block matrix. Especially, the inverse of the quasi-Jacket block matrix $[J]_{N\times N}$ may be obtained by the block-wise elements.

Let us consider the following binary matrices:

$$\alpha \triangleq \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix},\quad \beta \triangleq \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \tag{3}$$

It can be easily checked that

$$\alpha^2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \beta^2, \tag{4}$$

$$\begin{aligned} \alpha\beta + \beta\alpha &= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} = I_2, \end{aligned} \tag{5}$$

and

$$\alpha\beta \times \beta\alpha = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} = I_2, \tag{6}$$

i.e.

$$\begin{cases} \alpha = \alpha^{-1},\ \beta = \beta^{-1},\ \alpha^2 + \beta^2 = O_2, \\ \alpha^2 = \beta^2 = I_2,\ \alpha\beta + \beta\alpha = I_2, \\ (\alpha\beta)^{-1} = \beta\alpha,\ (\beta\alpha)^{-1} = \alpha\beta. \end{cases} \tag{7}$$

Furthermore,

$$\begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} \times \begin{bmatrix} \beta & \alpha \\ \alpha & \beta \end{bmatrix} = \begin{bmatrix} \alpha\beta + \beta\alpha & \alpha^2 + \beta^2 \\ \beta^2 + \alpha^2 & \beta\alpha + \alpha\beta \end{bmatrix} = \begin{bmatrix} I_2 & O_2 \\ O_2 & I_2 \end{bmatrix}, \tag{8}$$

Then we can write,

$$\begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}^{-1} = \begin{bmatrix} \beta & \alpha \\ \alpha & \beta \end{bmatrix}. \tag{9}$$

Let

$$[QJ]_4 \triangleq \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} = \left[\begin{array}{cc|cc} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ \hline 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right], \tag{10}$$

Its inverse matrix is

$$\begin{aligned} [QJ]_4^{-1} = [QJ]_4^T = \begin{bmatrix} \beta & \alpha \\ \alpha & \beta \end{bmatrix} &= \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ \hline 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{array}\right] \\ &= \begin{bmatrix} O_2 & I_2 \\ I_2 & O_2 \end{bmatrix} \times \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} = [QJ]^T, \end{aligned} \tag{11}$$

From definition 2.1,$[QJ]_4$ is a cocyclic quasi-Jacket block matrix. Similarly, we can construct the large quasi-Jacket block matrix based on kronecker product.

$$\begin{aligned} [QJ]_{16} &\triangleq \left[\begin{array}{cc|cc} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ \hline 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right] \otimes \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} \\ &= \left[\begin{array}{cc|cc|cc|cc} \alpha & \beta & O_2 & O_2 & \alpha & \beta & \alpha & \beta \\ \beta & \alpha & O_2 & O_2 & \beta & \alpha & \beta & \alpha \\ \hline \alpha & \beta & \alpha & \beta & O_2 & O_2 & \alpha & \beta \\ \beta & \alpha & \beta & \alpha & O_2 & O_2 & \beta & \alpha \\ \hline \alpha & \beta & \alpha & \beta & \alpha & \beta & O_2 & O_2 \\ \beta & \alpha & \beta & \alpha & \beta & \alpha & O_2 & O_2 \\ \hline O_2 & O_2 & \alpha & \beta & \alpha & \beta & \alpha & \beta \\ O_2 & O_2 & \beta & \alpha & \beta & \alpha & \beta & \alpha \end{array}\right], \\ &= \begin{bmatrix} [QJ]_4 & O_4 & [QJ]_4 & [QJ]_4 \\ [QJ]_4 & [QJ]_4 & O_4 & [QJ]_4 \\ [QJ]_4 & [QJ]_4 & [QJ]_4 & O_4 \\ O_4 & [QJ]_4 & [QJ]_4 & [QJ]_4 \end{bmatrix}, \\ &= \left[\begin{array}{cccc|cccc|cccc|cccc} 1&0&1&1&0&0&0&0&1&0&1&1&1&0&1&1 \\ 1&1&0&1&0&0&0&0&1&1&0&1&1&1&0&1 \\ 1&1&1&0&0&0&0&0&1&1&1&0&1&1&1&0 \\ 0&1&1&1&0&0&0&0&0&1&1&1&0&1&1&1 \\ \hline 1&0&1&1&1&0&1&1&0&0&0&0&1&0&1&1 \\ 1&1&0&1&1&1&0&1&0&0&0&0&1&1&0&1 \\ 1&1&1&0&1&1&1&0&0&0&0&0&1&1&1&0 \\ 0&1&1&1&0&1&1&1&0&0&0&0&0&1&1&1 \\ \hline 1&0&1&1&1&0&1&1&1&0&1&1&0&0&0&0 \\ 1&1&0&1&1&1&0&1&1&1&0&1&0&0&0&0 \\ 1&1&1&0&1&1&1&0&1&1&1&0&0&0&0&0 \\ 0&1&1&1&0&1&1&1&0&1&1&1&0&0&0&0 \\ \hline 0&0&0&0&1&0&1&1&1&0&1&1&1&0&1&1 \\ 0&0&0&0&1&1&0&1&1&1&0&1&1&1&0&1 \\ 0&0&0&0&1&1&1&0&1&1&1&0&1&1&1&0 \\ 0&0&0&0&0&1&1&1&0&1&1&1&0&1&1&1 \end{array}\right]. \end{aligned} \tag{12}$$

Then

$$[QJ]_4 \times [QJ]_4^{-1} = I_{16}, \tag{13}$$

where

$$[QJ]_{16}^{-1} = \left[\begin{array}{cc|cc|cc|cc} \beta & \alpha & \beta & \alpha & \beta & \alpha & O_2 & O_2 \\ \alpha & \beta & \alpha & \beta & \alpha & \beta & O_2 & O_2 \\ \hline O_2 & O_2 & \beta & \alpha & \beta & \alpha & \beta & \alpha \\ O_2 & O_2 & \alpha & \beta & \alpha & \beta & \alpha & \beta \\ \hline \beta & \alpha & O_2 & O_2 & \beta & \alpha & \beta & \alpha \\ \alpha & \beta & O_2 & O_2 & \alpha & \beta & \alpha & \beta \\ \hline \beta & \alpha & \beta & \alpha & O_2 & O_2 & \beta & \alpha \\ \alpha & \beta & \alpha & \beta & O_2 & O_2 & \alpha & \beta \end{array}\right] = [QJ]^T, \tag{14}$$

The matrix $[QJ]_{16}^{-1}$ can be written as follows:

$$[QJ]_{16}^{-1} = \begin{bmatrix} [QJ]_4^{-1} & [QJ]_4^{-1} & [QJ]_4^{-1} & O_4 \\ O_4 & [QJ]_4^{-1} & [QJ]_4^{-1} & [QJ]_4^{-1} \\ [QJ]_4^{-1} & [O]_4 & [QJ]_4^{-1} & [QJ]_4^{-1} \\ [QJ]_4^{-1} & [QJ]_4^{-1} & O_4 & [QJ]_4^{-1} \end{bmatrix},$$

$$= \left[\begin{array}{cccc|cccc|cccc|cccc} 1&1&1&0&1&1&1&0&1&1&1&0&0&0&0&0 \\ 0&1&1&1&0&1&1&1&0&1&1&1&0&0&0&0 \\ 1&0&1&1&1&0&1&1&1&0&1&1&0&0&0&0 \\ 1&1&0&1&1&1&0&1&1&1&0&1&0&0&0&0 \\ \hline 0&0&0&0&1&1&1&0&1&1&1&0&1&1&1&0 \\ 0&0&0&0&0&1&1&1&0&1&1&1&0&1&1&1 \\ 0&0&0&0&1&0&1&1&1&0&1&1&1&0&1&1 \\ 0&0&0&0&1&1&0&1&1&1&0&1&1&1&0&1 \\ \hline 1&1&1&0&0&0&0&0&1&1&1&0&1&1&1&0 \\ 0&1&1&1&0&0&0&0&0&1&1&1&0&1&1&1 \\ 1&0&1&1&0&0&0&0&1&0&1&1&1&0&1&1 \\ 1&1&0&1&0&0&0&0&1&1&0&1&1&1&0&1 \\ \hline 1&1&1&0&1&1&1&0&0&0&0&0&1&1&1&0 \\ 0&1&1&1&0&1&1&1&0&0&0&0&0&1&1&1 \\ 1&0&1&1&1&0&1&1&0&0&0&0&1&0&1&1 \\ 1&1&0&1&1&1&0&1&0&0&0&0&1&1&0&1 \end{array}\right]. \tag{15}$$

This is also cocyclic quasi-Jacket block matrix according to definition 2.1.

Definition 2.2: Let $[QJ]_4$ be defined by (2.5). For $m \geq 2$, Let,

$$\begin{aligned} [QJ]_{4^m} &\triangleq [QJ]_4 \otimes [QJ]_{4^{m-1}} \\ &= \prod_{i=0}^{m-1} \left([I]_{4^{m-i-1}} \otimes [QJ]_4 \otimes [I]_{4^i} \right), \\ &= \prod_{i=1}^{m} \left([I]_{4^{m-i}} \otimes [QJ]_4 \otimes [QJ]_{4^{i-1}} \right). \end{aligned} \tag{16}$$

when $m=2$

$$= [QJ]_4 \otimes [QJ]_4,$$

$$= \left(\begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} \otimes [I]_4 \right) \left([I]_4 \otimes \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} \right).$$

$$= \begin{bmatrix} \alpha & 0 & 0 & 0 & \beta & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & 0 & \beta & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & \alpha & 0 & 0 & 0 & \beta \\ \beta & 0 & 0 & 0 & \alpha & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 & 0 & \alpha & 0 & 0 \\ 0 & 0 & \beta & 0 & 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & \beta & 0 & 0 & 0 & \alpha \end{bmatrix} \times \begin{bmatrix} \alpha & \beta & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta & \alpha & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & \beta & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta & \alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha & \beta & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta & \alpha & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha & \beta \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta & \alpha \end{bmatrix}. \tag{17}$$

The signal flow graph corresponding to (17) is shown in Fig. 1. It requires 16 additions and 32 multiplications for computation.

In general, the computational complexity of the fast quasi-Jacket block fast algorithm is shown in Table 1. In $GF(2)$ the matrix order $N=2^m$ requires $N(\log_2 N-1)$ additions and $2N(\log_2 N-1)$ multiplications.

Ⅲ. Construction of Cocyclic Quasi-Jacket Block Low-Density Matrices Based on Circular Permutation Matrix

In this section, we present low density cocyclic quasi block Jacket matrices which are over GF (2), i.e., binary matrices. These types of low density quasi block Jacket matrices are useful in coding theory such

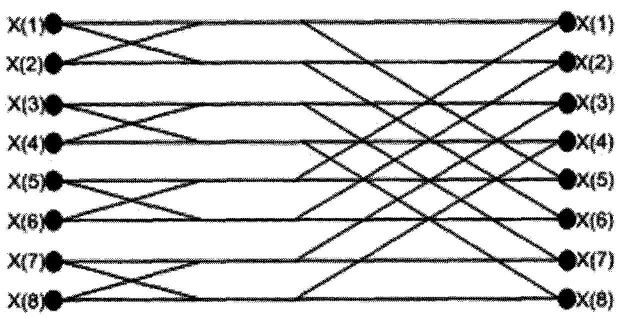


Fig. 1. Quasi-Jacket Transform signal flow graph for N=8.

Table 1. Computational compexity of fast algorithm.

	Directed Computation	Proposed $N=2^m$
Additions	$N(N-1)$	$N(\log_2 N-1)$
Multiplications	$N\times N$	$2N(\log_2 N-1)$

LDPC encoding, information theory, and orthogonal code design.

Let p be a prime, and let

$$E^h=\left[e_{i,j}\right]_{p\times p}, \tag{18}$$

and

$$e_{i,j}=\begin{cases}1 & for\, i=\langle j+h\rangle \\ 0 & otherwise\end{cases}. \tag{19}$$

where $\langle j+h\rangle = j+h \bmod p\, and\, 0\le i,j,h\le p-1$.

The matrices E^h, for $0\leq h\leq p-1$ are refer to circulant permutation matrices(CPM). It can be easily seen that $\{I, E, \cdots, E^{p-1}\}$ form an Abelian group with traditional matrix multiplication and $I=E^p$.

Example: Let p=5 have

$$I=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix},\quad E=\begin{bmatrix}0&0&0&0&1\\1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\end{bmatrix},$$

$$E^2=\begin{bmatrix}0&0&0&1&0\\0&0&0&0&1\\1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\end{bmatrix},\quad E^3=\begin{bmatrix}0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\\1&0&0&0&0\\0&1&0&0&0\end{bmatrix},$$

$$E^4=\begin{bmatrix}0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\\1&0&0&0&0\end{bmatrix}. \tag{20}$$

Let

$$\Lambda\triangleq\begin{bmatrix}I&O\\E^h&I\end{bmatrix},\quad \Omega\triangleq\begin{bmatrix}I&E^{-h}\\O&I\end{bmatrix}. \tag{21}$$

$$\Lambda^2=\begin{bmatrix}I&O\\E^h&I\end{bmatrix}\times\begin{bmatrix}I&O\\E^h&I\end{bmatrix}=I_{2p}=\begin{bmatrix}I&E^{-h}\\O&I\end{bmatrix}\times\begin{bmatrix}I&E^{-h}\\O&I\end{bmatrix}=\Omega^2, \tag{22}$$

$$(\Lambda\Omega+\Omega\Lambda)=\begin{bmatrix}I&O\\E^h&I\end{bmatrix}\times\begin{bmatrix}I&E^{-h}\\O&I\end{bmatrix}+\begin{bmatrix}I&E^{-h}\\O&I\end{bmatrix}\times\begin{bmatrix}I&O\\E^h&I\end{bmatrix}=$$

$$\begin{bmatrix}I&I\\I&O\end{bmatrix}+\begin{bmatrix}O&I\\I&I\end{bmatrix}=I_{2p}, \tag{23}$$

and

$$(\Lambda\Omega)\times(\Omega\Lambda)=\begin{bmatrix}I&I\\I&O\end{bmatrix}\times\begin{bmatrix}O&I\\I&I\end{bmatrix}=\begin{bmatrix}I&O\\O&I\end{bmatrix}=I_{2p}, \tag{24}$$

i.e.

$$\begin{cases}\Lambda=\Lambda^{-1},\Omega=\Omega^{-1},\Lambda^2=\Omega^2=I_{2p},\\ \Lambda\Omega+\Omega\Lambda=I_{2p},\\ (\Lambda\Omega)^{-1}=\Omega\Lambda,(\Omega\Lambda)^{-1}=\Lambda\Omega.\end{cases} \tag{25}$$

Furthermore,

$$\begin{bmatrix}\Lambda&\Omega\\\Omega&\Lambda\end{bmatrix}\times\begin{bmatrix}\Omega&\Lambda\\\Lambda&\Omega\end{bmatrix}=\begin{bmatrix}\Lambda\Omega+\Omega\Lambda&\Lambda^2+\Omega^2\\\Omega^2+\Lambda^2&\Omega\Lambda+\Lambda\Omega\end{bmatrix}=\begin{bmatrix}I&O\\O&I\end{bmatrix}. \tag{26}$$

i.e.

$$\begin{bmatrix}\Lambda&\Omega\\\Omega&\Lambda\end{bmatrix}^{-1}=\begin{bmatrix}\Omega&\Lambda\\\Lambda&\Omega\end{bmatrix}=\begin{bmatrix}\Omega^T&\Lambda^T\\\Lambda^T&\Omega^T\end{bmatrix}=\begin{bmatrix}\Lambda&\Omega\\\Omega&\Lambda\end{bmatrix}^T. \tag{27}$$

Let,

$$J\triangleq\begin{bmatrix}\Lambda&\Omega\\\Omega&\Lambda\end{bmatrix}=\left[\begin{array}{cc|cc}I&O&I&E^{-h}\\E^h&I&O&I\\\hline I&E^{-h}&I&O\\O&I&E^h&I\end{array}\right]. \tag{28}$$

It can be easily checked that

$$J\times J^{-1}=I_{2p}. \tag{29}$$

where

$$J^{-1}\triangleq\begin{bmatrix}\Omega&\Lambda\\\Lambda&\Omega\end{bmatrix}=\left[\begin{array}{cc|cc}I&E^{-h}&I&O\\O&I&E^h&I\\\hline I&O&I&E^{-h}\\E^h&I&O&I\end{array}\right]=J^T. \tag{30}$$

or

$$J^{-1}=\begin{bmatrix}O_2&I_2\\I_2&O_2\end{bmatrix}\times\begin{bmatrix}\Lambda&\Omega\\\Omega&\Lambda\end{bmatrix}=\begin{bmatrix}O_2&I_2\\I_2&O_2\end{bmatrix}\times\begin{bmatrix}\Lambda^{-1}&\Omega^{-1}\\\Omega^{-1}&\Lambda^{-1}\end{bmatrix}. \tag{31}$$

From the Definition 2.1, J is a quasi-Jacket which can be seen that J is a cocyclic matrix. Now we consider the density of 0's and 1's in $4p\times4p$ binary matrix denoted by

$$S_{4p}=\frac{N}{N_0+N}. \tag{32}$$

where N_0 and N_1 the numbers of 0's and 1 are respectively. The total number of 0's and 1's in $4p\times4p$ binary matrix is $16p^2$. There are $12I$, E^{-h}, and E^h. Each of I, E^{-h}, and E^h contains only p 1's. The total number of 1's is 12p. Thus the density of 1's in J is

$$S_{4p}=\frac{12p}{16p^2}=\frac{3}{4p}. \tag{33}$$

It means that this quasi-Jacket-matrix is low density matrix. For example, let p=5 and h=2, then, size of the matrix well be 20×20, as shown (34)~(36),

$$\begin{aligned}[J]_{20}&=[J]_5\otimes[J]_4,\\ &=\left([I]_4\otimes[J]_5\right)\left([J]_4\otimes[I]_5\right).\end{aligned} \tag{34}$$

Remark: J is a (3,3)-low density matrix, i.e., each row and each column of J contains exactly three 1's. The quasi-Jacket block matrix of order 20×20 contains four 1's in the corner of the matrix, we can say it as a cycle-4 or girth-4. Generally, short cycles have negative impact on the decoding algorithm for LDPC [19]. To use these kinds of matrices in LDPC, once should remove the cycle-4 or girth-4 of the matrices.

Ⅳ. Conclusion

In this paper, we have presented cocyclic quasi-Jacket block matrices over binary matrices which all are belong to a class of cocyclic matrices. We have

$$J_{20}=\left[\begin{array}{ccccc|ccccc|ccccc|ccccc}
1&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&1&0&0\\
0&1&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&1&0\\
0&0&1&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&1\\
0&0&0&1&0&0&0&0&0&0&0&0&0&1&0&1&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&1&0&1&0&0&0\\ \hline
0&0&0&1&0&1&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&1&0&1&0&0&0&0&0&0&0&0&0&1&0&0&0\\
1&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&1&0&0\\
0&1&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&1&0\\
0&0&1&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&1\\ \hline
1&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&0\\
0&0&0&1&0&1&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&1&0&1&0&0&0&0&0&0&0&1&0&0&0&0&0\\ \hline
0&0&0&0&0&1&0&0&0&0&0&0&0&1&0&1&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&1&0&1&0&0&0\\
0&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&1
\end{array}\right] \tag{35}$$

$$J_{20}^{-1}=\left[\begin{array}{ccccc|ccccc|ccccc|ccccc}
1&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&0\\
0&0&0&1&0&1&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&1&0&1&0&0&0&0&0&0&0&1&0&0&0&0&0\\ \hline
0&0&0&0&0&1&0&0&0&0&0&0&0&1&0&1&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&1&0&1&0&0&0\\
0&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&1\\ \hline
1&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&1&0&0\\
0&1&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&1&0\\
0&0&1&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&1\\
0&0&0&1&0&0&0&0&0&0&0&0&0&1&0&1&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&1&0&1&0&0&0\\ \hline
0&0&0&1&0&1&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&1&0&1&0&0&0&0&0&0&0&0&0&1&0&0&0\\
1&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&1&0&0\\
0&1&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&1&0\\
0&0&1&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&1
\end{array}\right] \tag{36}$$

also shown how to construct low-density cocyclic quasi-Jacket matrices based on circular permutation matrix. These cocyclic quasi-Jacket block matrices and low-density matrices are useful in digital signal processing, CDMA, coding theory, orthogonal code design, and coded modulation [1]~[3],[6].

This work was supported by Minister of Information and Communication(MIC) supervised by IIFA and ITRC supervised by IITA, ITSOC and International Cooperative Research Program of the Ministry of Science and Technology, KOTEF, and 2^{nd} stage BK21, Chonbuk National University, Korea.

References

[1] W. K. Pratt, J. Kane, and H. C. Andrews, "Hadamard transform image coding", *Proc. IEEE*, vol. 37, pp. 58-68, 1996.

[2] F. Adachi, M. Sawahashi, and K. Okawa, "Tree-structured generation of orthogonal spreading codes with different length for forward link of DS-CDMA mobile radion", *IEEE Electronics Letters*, vol. 33, no. 1, pp. 27-28, Jan. 1997.

[3] Oppermann, "Orthogonal complex-valued spreading sequences with a wide range of correlation properties", *IEEE. Trans. Commun.*, vol. 45, no. 11, pp. 1379-1380, Nov. 1997.

[4] S. M. Alamouti, "Simple transmit diversity techniques for wireless communications", *IEEE J.* on *Selected Areas in Communications*, vol. 16, no. 8, pp. 1451-1458, Oct. 1998.

[5] A. J. Viterbi, *CDMA Principles of Spread Spectrum Communications*, Addison-Wesley, 1995.

[6] A. M. Viterbi, A. J., "Erlang capacity of a power

controlled CDMA system", *IEEE J. Select. Areas Commun.*, vol. 11, no. 8, pp. 892-900, Aug. 1993.

[7] A. J. Goldsmith, S. G. Chua, "Adaptive coded modulation for fading channels", *IEEE Trans. On Communications*, vol. 46, no. 5, pp. 595-602, May 1998.

[8] K. J. Hole, G. E. Oien, "Spectral efficiency of adaptive coded modulation in urban microcellular networks", *IEEE Trans. On Vehicular Technology*, vol. 50, no. 1, pp. 205-222, Jan. 2001.

[9] Moon Ho Lee, *Jacket Matrix Principles and Applications: with Applications to Communications and Signal/Image Processing,* Reprint book, 2004.

[10] Moon Ho Lee, "The center weighted hadamard transform", *IEEE Trans. Circuits Syst.*, vol. 36, no. 9, pp. 1247-1249, Sep. 1989.

[11] Moon Ho Lee, "Fast complex reverse Jacket transform", *Proc. 22nd Symp. Information Theory and Its Applications*(SITA99), Yuzawa, Niigata, no. 9, Nov.-Dec. 1999.

[12] Moon Ho Lee, "A new reverse Jacket transform based on hadamard matrix", *Proc. Int. Symp. Information Theory*, Sorrento, Jun. 25-30, 2000.

[13] Moon Ho Lee, "A new reverse Jacket transform and its fast algorithm", *IEEE Trans. Circuits Syst. II*, vol. 47, no. 1, pp. 39-47, Jan. 2000.

[14] Moon Ho Lee, B. Sundar Rajan, and J. Y. Park, "A generalized reverse Jacket transform", *IEEE Trans, Circuits Syst. II*, vol. 48, no. 7, pp. 684-690, Jul. 2001.

[15] K. J Horadam, P. Udaya, "Cocylic hadamard codes", *IEEE Trans. on Information Theory*, vol. 46, no. 4, pp. 1545-1550, 2000.

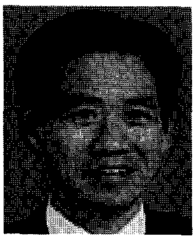
[16] Moon Ho Lee, Ken Finlayson, "A simple elements inverse Jacket transform coding", accepted by *IEEE Signal Processing Letterers*, 2006.

[17] K. Finlayson, Moon Ho Lee, J. Sebery, and M. Yamada, "Jacket matrices constructed from Hadamard matrix and generalized Hadamard matrices", *The Australasian Journal of Combinatorics*, vol. 35, no. 1, pp. 83-88, Jun. 2006.

[18] Jia Hou, Moon Ho Lee, "Cocyclic jacket matrices and its application to cryptosystems", *Springer Berlin/Heidelberg*, LNCS, 2005.

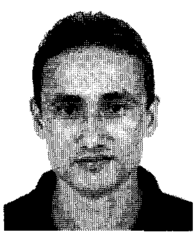
[19] Jose M. F. Moura, Haotian Zhang, "Structured LDPC code with large girth", *IEEE Signal Processing Magazine*, vol. 21, no. 1, pp. 42-55, Jan. 2004.

Moon-Ho Lee



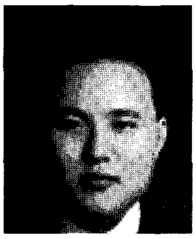
He received the B.S. and M.S. degree both in electrical engineering from the Chonbuk National University, Korea, in 1967 and 1976, respectively, and the Ph. D. degree in electronics engineering from the Chonnam National University in 1984 and the University of Tokyo, Japan, in 1990. From 1970 to 1980, he was a chief engineer with Namyang Moonhwa Broadcasting. Since 1980, he has been a professor with the department of information and communication and a director with the Institute of Information and Communication, both at Chonbuk National University. From 1985 to 1986, he was also with the University of Minnesota, as a Postdoctoral Feller. He has held visiting positions with the University of Hannover, Germany, during 1990, the University of Aachen, Germany, during 1992 and 1996, and the University of Munich, Germany, during 1998. His research interests include the multidimensional source and channel coding, mobile communication, and image processing.

Subash Shree Pokhrel



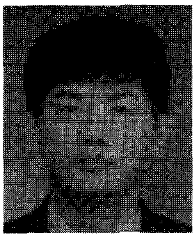
He received the B.S. degree from Tribhuvan University, Nepal in 2000 and M.S. degrees in Information Technology from Manipal University, India in 2004. He is currently in Ph. D. course at Chonbuk National University. His research interests include the areas of mobile communications and channel coding.

Seung Je Choi



received his B.S. degree in computer engineering from Chonbuk National University in 2004, Korea, and M.S. degree in Information & Communications from Chonbuk National University in 2006, Korea. He is now working towards a Ph. D. at the Institute of Information & Communications, Chonbuk National University, Korea. His main research interests are sequences, CDMA mobile communication systems, error coding and space time signal processing.

Chang Joo Kim



He received B.S. degree in avionics engineering from Hankuk Aviation University in 1980, and M.S. and Ph.D. degrees in electrical engineering from KAIST in 1988 and 1993 respectively. From 1980 to 1982 he was engaged as a research engineer at ADD. Since 1983 he has been with the communication field of ETRI, where he is now a director of radio technology research group. His interest involve wireless communications and radio technologies.