On Jacket Matrices Based on Weighted Hadamard Matrices

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Abstract

Jacket matrices which are defined to be $n \times n$ matrices $A=(a_{jk})$ over a field F with the property $AA^{\dagger}=nI_n$ where $A^{\dagger}$ is the transpose matrix of elements inverse of $A, i.e., A^{\dagger}=(a_{kj}^{-})$, was introduced by Lee in 1984 and are used for signal processing and coding theory, which generalized the Hadamard matrices and Center Weighted Hadamard matrices. In this paper, some properties and constructions of Jacket matrices are extensively investigated and small orders of Jacket matrices are characterized, also present the full rate and the 1/2 code rate complex orthogonal space time code with full diversity.

Key words : Hadamard Matrix, Jacket Matrix, Complex Hadamard Matrix, Center Weighted Hadamard Matrix, Code Rate, Full Diversity, Space-Time Code.

I. Introduction

Sylvester in his paper[1] introduced self-reciprocal matrices, which are defined as a square array of elements of which each is proportional to its first minor. This wide of class matrices includes these with orthogonal rows and columns. In 1983, Hadamard[2] showed that such matrices attained the largest value of the determinant among all matrices with elements bounded by unity. Since the publication of this paper, the matrices whose entries equal to ±1 and mutually orthogonal rows and columns have been called Hadamard matrices.

Investigations of Hadamard matrices were connected with liner algebra problem, such as finding maximum of determinant. Later on the applications of Hadamard matrices are connected with information transfer by nonline electromagnetic waves, with automation training and coding theory.

Real Hadamard matrices have been generalized in various ways. Onehand, Turyn[3] investigate the class of matrices with entries ±1, $\pm i (i^2=-1)$ and pairwise orthogonal rows and columns, which are now called *Turyn-type* Hadamard matrices. On the other hand, Butson in [4] considered a more general class of Hadamard matrices. A Butson-type Hadamard matrix is a $n \times n$ square matrix with all its rows and columns mutually orthogonal and with all its elements are powers of *q-th* root of unity, denoted by $H(q, n)$. Thus $H(2, n)$ represents real Hadamard matrices, while $H(4, n)$ represents Turyn-type Hadamard matrices. In 1996, while studying orthogonal maximal abelian *-subalgebras of the $n \times n$ matrices, Haagerup in [5] introduced complex Hadamard matrices which are mutually orthogonal rows and columns and modular of their entries equal to 1. Complex Hadamard matrices are closely related to various mathematical and theoretical physical problems, such as construction of some *-subalgebras in finite von Neumann algebras, constructing error correcting codes and spin model. Moreover, Lee in [6] proposed the idea of center weighted Hadamard matrices and center weighted Hadamard transform. Further, Lee in [7] proposed the following new class of matrices: Jacket matrices which generalize real Hadamard matrices, Turyn-type Hadamard matrices, Butson-type Hadamard matrices, complex Hadamard matrices and center weighted Hadamard matrices.

It is well known that the Walsh-Hadamard transform based on Hadamard matrix is widely used in signal processing, in particular image coding and error-control coding [8]~[12]. Rajan and Lee in [13] obtained a characterization of quasi-cyclic dyadic codes in the Walsh-Hadamard transform. Further, many kinds of transforms based on various generalizations of Hadamard matrices have been proposed and investigated [9]~[12]. Lee in [6] and Lee and Lee[14] introduced center weighted Hadamard transform. Similarly, Lee, Rajan and Park in [15] and Lee and Lee in [14], [16] proposed the Jacket transform and reverse Jacket transform The further investigations of these Jacket transform and relative problems may be referred to [17] ~[21].

The Hadamard transform and its generalizations in

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various ways have been proposed and used for audio and video coding since these transforms are highly practical value for representing the signal and images[8]. Due to the ease and efficiency of these transforms that are widely used for signal and image representations and compression. The most advantage of these transforms is lied to their inverse transforms that are easily obtained. In order to offer quality of representations over the central region of the image and to retain the simplicity of Hadamard transform, the center weighted transform was proposed and studied. With the rapid development of communication systems that require more transmission and storage capacities of multilevel cased in co-channels for numerous clients, recently, the Jacket transform and reverse Jacket transform based on Jacket matrix have extensively proposed and investigated. For more applications the readers may be referred to [20], [21], [23], [24], [28]. With more increasing applications of Jacket matrices and Jacket transforms, it will be more interesting and important to study further properties of Jacket matrices and their construction.

This paper is structured as follows. Section 2 discusses the some properties of Jacket matrices. Section 3 presents the relation among Hadamard matrices, complex Hadamard matrices and Jacket matrices. Section 4 presents the preliminary properties of Jacket matrices. Section 5 presents the catalogue of Jacket matrices of small orders. Section 6 presents the construction of Jacket matrices based on center weighted Hadamard matrices. Section 7 presents the *J*-Hurwitz Radon of complex orthogonal Jacket space time code. Finally, some conclusions are drawn in section 8.

Ⅱ. Definition of Jacket Matrices

Definition 2-1:

Let $A=(a_{kt})$ be an $n\times n$ matrix whose elements are in a field F (including real fields, complex fields and finite fields, etc). Denote by $A^{\dagger}$, the transpose matrix of elements inverse of A,*i.e.*$A^{\dagger}=(a_{kt}^{-})$. A is called a Jacket matrix if $AA^{\dagger}=A^{\dagger}A=nI_n$ where I_n is the identity matrix over a field F.

Example 2.1

$$A=\begin{pmatrix} a & \sqrt{ac} \\ \sqrt{ac} & -c \end{pmatrix},\quad A^{-1}=\begin{pmatrix} \frac{1}{a} & \frac{1}{\sqrt{ac}} \\ \frac{1}{\sqrt{ac}} & -\frac{1}{c} \end{pmatrix}, \tag{1}$$

so A is 2×2 Jacket matrix when $a=c=1$, it is a 2×2 Hadamard matrix.

On one hand, from the definition of Jacket matrices, it is easy to see that the class of Jacket matrices contains complex Hadamard matrices, on the other hands; Lee in [6], [15] defined the center weighted Hadamard matrices W as following:

$$W_4\equiv\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -w & w & -1 \\ 1 & w & -w & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix},\quad W_4^{-1}=\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -\frac{1}{w} & \frac{1}{w} & -1 \\ 1 & \frac{1}{w} & -\frac{1}{w} & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}, \tag{2}$$

and

$$W_{2n}=W_n\otimes H_2, \tag{3}$$

where,

$$H_2=\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

is a 2×2 Hadamard matrix and n=4, 8, ···, 4k, *and* $\otimes$ is the kronecker product[30]. Clearly, by simple calculations, $W_4W_4^{\dagger}=4I_4$. Hence W_4 is a Jacket matrix. In particular, if w=1, it is a Hadamard matrix and if w=2 it is a special center weighted Hadamard matrix. Furthermore, there exists a permutation matrix P(each row and column of P has exactly 1) such that $PW_{2n}P^T=P(W_n\otimes H_2)P^T=H_2\otimes W_n$, where P^T is the transpose matrix of P. Hence

$$\begin{aligned} W_{2n}W_{2n}^{\dagger}&=P^T(H_2\otimes W_n)PP^T(H_2\otimes W_n)P \\ &=P^T\begin{pmatrix} W_n & W_n \\ W_n & W_n \end{pmatrix}\begin{pmatrix} W_n & W_n \\ W_n & W_n \end{pmatrix}^{\dagger}P=2nI_{2n}. \end{aligned} \tag{4}$$

Hence the Center Weighted matrices are also Jacket matrices.

Ⅲ. Relations among Hadamard Matrices, Complex Hadamatd Matrices and Jacket Matrices

In this section, we present some relations among real Hadamard matrices, Turyn-type Hadamard matrices, Butson-type Hadamard matrices, complex Hadamard matrices and Jacket matrices. From their definitions, its is easy to see that set of all real Hadamard matrices belong to the set of all Turyn-type Hadamard matrices which belongs to the set of all Butson-type Hadamard matrices which belongs to the set of all complex Hadamard matrices. Further, the set of all complex Hadamard matrices belongs to the set of all Jacket

matrices.

Propositions 3-1

Let A be a complex Hadamard matrix. Then A is a Jacket matrix.

Proof: Let $A=(a_{jk})$ be an $n\times n$ complex Hadamard matrix. Then $AA^{*}=nI_n$. Moreover, $a_{jk}\overline{a_{jk}}=1/a_{jk}$. Therefore, $A^{\dagger}=A^{*}$ *and* $AA^{\dagger}=AA^{*}=nI_n$.

Now, we present some examples that illustrate their converse do not hold.

Example 3.1: Let $i^2=-1$ and

$$A=\begin{pmatrix}1&1&1&1\\1&i&i^2&i^3\\1&i^2&i^4&i^6\\1&i^3&i^6&i^9\end{pmatrix},\quad A^{-1}=\begin{pmatrix}1&1&1&1\\1&\frac{1}{i}&\frac{1}{i^2}&\frac{1}{i^3}\\1&\frac{1}{i^2}&\frac{1}{i^4}&\frac{1}{i^6}\\1&\frac{1}{i^3}&\frac{1}{i^6}&\frac{1}{i^9}\end{pmatrix}. \tag{5}$$

Hence A is a Turyn-type Hadamard matrix and also a DFT matrix, but not a real Hadamard matrix.

Example 3.2: Let w is a third root of unity, i.e., $w=e^{2\pi i/3}$, $i^2=-1$ and

$$B=\begin{pmatrix}1&1&1\\1&w&w^2\\1&w^2&w\end{pmatrix},\quad B^{-1}=\begin{pmatrix}1&1&1\\1&\frac{1}{w}&\frac{1}{w^2}\\1&\frac{1}{w^2}&\frac{1}{w}\end{pmatrix} \tag{6}$$

It is easy to see that

$$BB^{*}=\begin{pmatrix}1&1&1\\1&w&w^2\\1&w^2&w\end{pmatrix}\begin{pmatrix}1&1&1\\1&w^2&w\\1&w&w^2\end{pmatrix}=3I_3. \tag{7}$$

So B is a Buston-type Hadamard matrix, but it is not a Turyn-type Hadamard matrix.

Example 3.3: Let $i^2=-1$, w be a third unit of unity and be a 6×6 complex matrix with the modular of each entry equal to 1. By a simple calculation, it is easy to see that

$$C=\begin{pmatrix}1&1&1&1&1&1\\1&\frac{1}{w}&\frac{1}{iw}&\frac{1}{w^2}&\frac{1}{iw^2}&\frac{1}{i}\\1&\frac{1}{w^2}&\frac{1}{iw^2}&\frac{1}{w}&\frac{1}{iw}&\frac{1}{i}\\1&\frac{1}{w^2}&-\frac{1}{iw^2}&\frac{1}{w}&-\frac{1}{iw}&-\frac{1}{i}\\1&\frac{1}{w}&-\frac{1}{iw}&\frac{1}{w^2}&-\frac{1}{iw^2}&-\frac{1}{i}\\1&1&-\frac{1}{i}&1&-\frac{1}{i}&-\frac{1}{i}\end{pmatrix} \tag{9}$$

and $CC^{*}=6I_6$. Then C is a complex Hadamard matrix, but not a Butson-type Hadamard matrix. Finally, the center weighted Hadamard matrix is a Jacket a matrix from the last section, but not a complex Hadamard matrix.

It is well known that any pair columns of a real Hadamard matrix is orthogonal, but pair columns of a Jacket matrix may not be a orthogonal, for example, the center weighted Hadamard matrix. In fact, even a real matrix is both Jacket and row orthogonal which means any its pair columns are orthogonal.

Example 3.4: Let

$$A=\begin{pmatrix}1&1&1&1\\2&-2&2&-2\\3&3&-3&-3\\\frac{1}{2}&-\frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\end{pmatrix},\quad A^{-1}=\frac{1}{4}\begin{pmatrix}1&\frac{1}{2}&\frac{1}{3}&2\\1&-\frac{1}{2}&\frac{1}{3}&-2\\1&\frac{1}{2}&-\frac{1}{3}&-2\\1&-\frac{1}{2}&-\frac{1}{3}&2\end{pmatrix}. \tag{10}$$

Clearly, any pair of rows of A are orthogonal and A is a Jacket matrix since $AA^{\dagger}=4I_4$ but it is not a real Hadamard matrix.

Ⅳ. Preliminary Properties of Jacket Matrices

We begin with some preliminary properties of Jacket matrices in this section.

Proposition 4-1

An $n\times n$ matrix $A=(a_{ij})$ over a field F is a Jacket matrix if and only if

$$\text{for all } j \neq k \qquad \sum_{i=1}^{n} \frac{a_{ji}}{a_{ki}} = 0$$

$$\text{for all } j \neq k \qquad \sum_{i=1}^{n} \frac{a_{ji}}{a_{ik}} = 0 \tag{11}$$

Proof: It follows from $AA^{\dagger}=nI_n$ if and only if $A^{\dagger}A=nI_n$ that the assertion holds.

Proposition 4-2

For any integer n, there exists at least a Jacket matrix of order n.

Proof: We construct an $n\times n$ matrix $A=(a_{jk})$ as follows: $a_{jk} = \exp\left\{\frac{2\pi i}{n}(j-1)(k-1)\right\}$ *for* $j,k = 1,\ldots,n$, where $i^2=-1$. It is easy to see that

$$\sum_{t=1}^{n} \frac{a_{jt}}{a_{kt}} = \sum_{t=1}^{n} \exp\left\{\frac{2\pi i}{n}(j-k)(t-1)\right\} \tag{12}$$

for all $j\neq k$. Hence A is a Jacket matrix of order n.

Remark: There exists a Jacket matrix of any order, although the existence of real Hadamard matrices is a very tough equation. In fact, there is a conjecture on the existence orders of real Hadamard matrices; there exists Hadamard matrix of order $4n$ for any positive integer n.

Proposition 4-3

Let $A=(a_{jk})$ be $n\times n$ Jacket matrix.

(1) If $|a_{jk}|=1$ *for all* $j,k=1,\ldots,n$ then A is a complex Hadamard matrix.

(2) If a_{jk} is real and $a_{ik}^2=1$ *for all* $j,k=1,\ldots,n$, then A is a Hadamard matrix.

Proof : (1) if $|a_{jk}|=1$, then $a_{jk}\overline{a_{jk}}=1$ and $\overline{a_{jk}}=\frac{1}{a_{jk}}$. Hence $A^{*}=A^{\dagger}$ and $AA^{*}=AA=nI_n$. (2) It is obvious that $a_{jk}=\pm 1$ and $AA^{T}=AA^{\dagger}=nI_n$.

Proposition 4-4

Let A be a Jacket matrix.

(1) Then $A^{\dagger}$, A^{-1} and A^{T} are also Jacket matrices.

(2) (det A)(det $A^{\dagger}$)$=n^n$.

Proof: Since A is a Jacket matrix, $AA^{\dagger}=nI_n$. $SoA^{-1}=\frac{1}{n}A$. Hence $A^{\dagger}(A^{\dagger})^{\dagger}=A^{\dagger}A=nI_n$. Thus $A^{\dagger}$ is a Jacket matrix. Similarly, A^{-1} and A^{T} are also Jacket Matrices. (2) follows from that $n^n=\det AA^{\dagger}=\det A\det A^{\dagger}$.

Proposition 4-5

Let A be $n\times n$ Jacket matrix and let D and E be diagonal matrices. Then DAE is also a Jacket matrix.

Proof: $A=(a_{jk}), D=diag(d_1,\ldots,d_n)$, $E=diag(e_1,\ldots,e_n)$ and $E=DAE=(b_{jk})$. Thus and $b_{jk}=d_j a_{jk} e_k$

$$\sum_{k=1}^{n} \frac{b_{ik}}{b_{jk}} = \frac{e_i}{e_j}\sum_{k=1}^{n} \frac{a_{ik}}{a_{jk}} = 0 \tag{13}$$

for all $j\neq j$. Hence DAE is a Jacket a matrix.

Proposition 4-6

Let A be $n\times n$ Jacket matrix and let be $n\times n$ Pand Q permutation matrices. Then PAQ is also a Jacket matrix.

Proof: Since $(PAQ)^{\dagger}=Q^{T}A^{\dagger}P^{T}$ and $AA=nI_n$,

$$(PAQ)(PAQ)^{\dagger}=PAQQ^{\dagger}A^{\dagger}P^{T}=nI_n. \tag{14}$$

Combining with the above propositions, we have the following:

Theorem 4-7

Let A be $n\times n$ Jacket matrices if D and E are diagonal matrices and P and Q are permutation matrices, then $PDAEQ$ is a Jacket matrix.

Now we may define that two Jacket matrices A and B are equivalent if there exist diagonal matrices and permutation matrices P and Q such that $B=PDAEQ$. These equivalence relations may be regarded as a generalization of the Hadamard equivalences relations in the class of Hadamard matrices, in which permutations and multiplying-1 of row and columns are allowed. Hence for any Jacket matrix A, there exist diagonal matrices D and E and permutation matrices P and Q such that the entries of the first row and column of $PDAEQ$ are equal to 1. So, that Jacket matrices are called normalized Jacket matrices. If a normalized Jacket matrix A is written as $A=\begin{pmatrix}1 & e^{T}\\ e & A_1\end{pmatrix}$, where e a column vector of all one is, A_1 is called the core of the Jacket matrix A.

V. Catalogue of Jacket Matrices of Small Orders

It is interesting questions for us to determine all Jacket matrices of order n, since it will help us to construct some real Hadamard or complex Hadamard

matrices of order n and to prove their non-existence. We will describe all Jacket matrices of order $n \leq 4$ in this section.

Theorem 5-1

(1) Any Jacket matrix A of order 2 is equivalent to the following matrix

$$J_2 = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \tag{15}$$

(2) Any Jacket matrix A of order 3 is equivalent to the following Jacket matrix

$$J_3 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & w & w^2 \\ 1 & w^2 & w \end{pmatrix}. \tag{16}$$

Proof: Let $A = \begin{pmatrix} 1 & 1 \\ 1 & a_{22} \end{pmatrix}$ be a normalized Jacket matrix. Then, from proposition 4.1, $1+a_{222}=0$. Hence (1) holds. Let

$$B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & b_{22} & b_{23} \\ 1 & b_{32} & b_{33} \end{pmatrix}, \quad B^{-1} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \frac{1}{b_{22}} & \frac{1}{b_{32}} \\ 1 & \frac{1}{b_{32}} & \frac{1}{b_{33}} \end{pmatrix} \tag{17}$$

be a normalized Jacket matrix and its inverse matrix. By Proposition 4.1, we have

$$1 + b_{22} + b_{23} = 0, \tag{18}$$

$$1 + \frac{1}{b_{22}} + \frac{1}{b_{23}} = 0, \tag{19}$$

$$1 + b_{32} + b_{33} = 0, \tag{20}$$

$$1 + \frac{1}{b_{32}} + \frac{1}{b_{33}} = 0, \tag{21}$$

$$1 + \frac{b_{22}}{b_{32}} + \frac{b_{23}}{b_{33}} = 0, \tag{22}$$

$$1 + \frac{b_{32}}{b_{22}} + \frac{b_{33}}{b_{23}} = 0. \tag{23}$$

From equation (19), we have

$$1 + \frac{b_{22} + b_{23}}{b_{22} b_{23}} = 0. \tag{24}$$

Then by (18), $b_{22}b_{23} - 1 = 0$ which yields $b_{23} = \frac{1}{b_{22}}$. Hence by (18), we have, $b_{22}^2 + b_{22} + 1 = 0$ So, $b_{22}=w$ or $b_{22}=w^2$. Similarly, by (20) and (21), $b_{32}=w$ or $b_{33}=w^2$.

By (22) and (23), if $b_{22}=w$, then $b_{32}=b_{23}=w^2$. If $b^{22}=w^2$, then, $b_{32}=b_{23}=w$ Therefore

$$B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & w & w^2 \\ 1 & w^2 & w \end{pmatrix} or \; B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & w^2 & w \\ 1 & w & w^2 \end{pmatrix}. \tag{25}$$

Hence the result holds.

Theorem 5-2

Any Jacket matrix of order 4 is equivalent to the following Jacket matrix

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -w & w & -1 \\ 1 & w & -w & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}. \tag{26}$$

Proof: Let B be a Jacket matrix of order 4. Thus B is equivalent to the following Jacket matrix:

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & a_{22} & a_{23} & a_{24} \\ 1 & a_{32} & a_{33} & a_{34} \\ 1 & a_{42} & a_{43} & a_{44} \end{pmatrix}. \tag{27}$$

Then by Proposition 4.1,

$$1 + a_{22} + a_{23} + a_{24} = 0, \tag{28}$$

$$1 + \frac{1}{a_{22}} + \frac{1}{a_{23}} + \frac{1}{a_{24}} = 0, \tag{29}$$

$$1 + \frac{a_{22}}{a_{32}} + \frac{a_{23}}{a_{33}} + \frac{a_{24}}{a_{34}} = 0, \tag{30}$$

$$1 + \frac{a_{32}}{a_{22}} + \frac{a_{33}}{a_{23}} + \frac{a_{34}}{a_{24}} = 0, \tag{31}$$

$$1 + \frac{a_{22}}{a_{42}} + \frac{a_{23}}{a_{43}} + \frac{a_{24}}{a_{44}} = 0. \tag{32}$$

From equations (28) and (29), we have

$$a_{22}a_{23}a_{24} + a_{23}a_{24} + a_{22}a_{24} + a_{22}a_{23} + (a_{23}a_{24} + a_{23} + a_{24} + 1) = 0. \tag{33}$$

Hence

$$(a_{22}+1)(a_{23}+1)(a_{24}+1)=0. \tag{34}$$

Similarly, we have the following equations

$$(a_{32}+1)(a_{33}+1)(a_{34}+1)=0. \tag{35}$$

$$(a_{42}+1)(a_{43}+1)(a_{44}+1)=0. \tag{36}$$

$$\left(\frac{a_{22}}{a_{32}}+1\right)\left(\frac{a_{23}}{a_{33}}+1\right)\left(\frac{a_{24}}{a_{34}}+1\right)=0. \tag{37}$$

From equation (34), $a_{22}=-1$, or $a_{23}=-1$, or $a_{24}=-1$. Without loss of generality, we may assume that $a_{22}-1$. From (35) and (36), we only need consider the following four cases.

Case 1: $a_{32}=-1$ and $a_{42}=-1$. By (28), $a_{24}=-a_{23}$ Similarly, $a_{34}=-a_{33}$ and $a_{44}=-a_{43}$. Hence by (30), $a_{33}=-a_{23}$ and by (32) we have $a_{43}=-a_{23}$. Therefore

$$\frac{1}{1}+\frac{a_{32}}{a_{42}}+\frac{a_{33}}{a_{43}}+\frac{a_{34}}{a_{44}}=4. \tag{38}$$

It is a contradiction. Therefore this case is impossible.

Case 2: $a_{32}=-1$ and $a_{42}\neq -1$ We may consider the following two subcases.

Subcase 2.1: $a_{43}=-1$. From (28), $a_{24}=-a_{23}$. Similarly, we have $a_{34}=-a_{33}$ and $a_{44}=-a_{42}$. Then by (32),

$$(1-a_{23})(1-a_{42})=0. \tag{39}$$

Hence a_{42}=1. Otherwise, a_{23}=1, which yields a_{33}=1 So,

$$\frac{1}{1}+\frac{a_{32}}{a_{42}}+\frac{a_{33}}{a_{43}}+\frac{a_{34}}{a_{44}}=2-\frac{2}{a_{42}}\neq 0. \tag{40}$$

It is a contradiction. Therefore by (31), $a_{32}=-a_{23}$, $a_{34}=a_{23}$. It is easy to this matrix is equivalent to the form (26).

Subcase 2.2: $a_{43}\neq -1$. Then by (36), $a_{44}-1$. Hence from (32), it is easy to see that $(1+a_{23})(a_{42}-1)=0$. Similarly, from (35), we have $(1-a_{23})(a_{42}-1)=0$. Therefore a_{42}=1. So it is easy to see that it is equivalent to the form (26).

Case 3: $a_{32}\neq -1$ and $a_{42}\neq 1$. By similar to the method of Case 2, we may show that A is equivalent to the form (26).

Case 4: $a_{32}=-1$ and $a_{42}\neq -1$. We only need consider the following two subcases.

Subcase 4.1: $a_{33}\neq -1$. We claim that $a_{43}=-1$. Otherwise, $a_{44}=-1$ by (36). Then by $1+a_{42}+a_{43}+a_{44}=0$ and

$$\frac{1}{1}+\frac{a_{32}}{a_{42}}+\frac{a_{33}}{a_{43}}+\frac{a_{34}}{a_{44}}=0, \tag{41}$$

we have $(1+a_{32})(1+a_{42})=0$. Hence $a_{32}=-1$. It is a contradiction. Now from (31) and (32), we have $(1-a_{23})(a_{32}-1)=0$ and $(1-a_{23})(a_{42}-1)=0$. Hence a_{23}=1 and it is easy to see that A is equivalent to form (26).

Subcase 4.2: $a_{33}\neq -1$. Then $a_{34}\neq -1$. By similar to the method of subscase 4.1, we may show that A is equivalent to form (26).

Remark: In Theorem 5.2, if w=1, then (26) is a real Hadamard matrix; if w=1, then (26) is a turn-type Hadamard matrix; if w=2, then (26) is a center weighted Hadamard matrix. Furthermore, from theorem 5.1 and 5.2, we may see that Jacket matrices of order 2, 3, and 4 are unique under equivalent relationship. It is natural to ask whether Jacket matrices of order 5 are unique. Let φ be fifth prime root of unity, i.e., $\varphi=e^{2\pi i/5}$, $a=\dfrac{-5+\sqrt{5}}{2}$ and

$$A=\begin{pmatrix}1&1&1&1&1\\1&\varphi&\varphi^2&\varphi^3&\varphi^4\\1&\varphi^2&\varphi^4&\varphi&\varphi^3\\1&\varphi^3&\varphi&\varphi^4&\varphi^2\\1&\varphi^4&\varphi^3&\varphi^2&\varphi^2\end{pmatrix}, \tag{42}$$

$$B=\begin{pmatrix}a+1&1&1&1&1\\1&a+1&1&1&1\\1&1&a+1&1&1\\1&1&1&a+1&1\\1&1&1&1&a+1\end{pmatrix}. \tag{43}$$

By a simple calculation,

$$AA^{\dagger}=\begin{pmatrix}1&1&1&1&1\\1&\varphi&\varphi^2&\varphi^3&\varphi^4\\1&\varphi^2&\varphi^4&\varphi&\varphi^3\\1&\varphi^3&\varphi&\varphi^4&\varphi^2\\1&\varphi^4&\varphi^3&\varphi^2&\varphi^2\end{pmatrix}\begin{pmatrix}1&1&1&1&1\\1&\varphi^4&\varphi^3&\varphi^2&\varphi\\1&\varphi^3&\varphi&\varphi^4&\varphi^2\\1&\varphi^2&\varphi^4&\varphi&\varphi^3\\1&\varphi&\varphi^2&\varphi^3&\varphi^4\end{pmatrix}=5I_5 \tag{44}$$

and

$$BB^{\dagger}=\begin{pmatrix}a+1&1&1&1&1\\1&a+1&1&1&1\\1&1&a+1&1&1\\1&1&1&a+1&1\\1&1&1&1&a+1\end{pmatrix}\begin{pmatrix}\frac{1}{a+1}&1&1&1&1\\1&\frac{1}{a+1}&1&1&1\\1&1&\frac{1}{a+1}&1&1\\1&1&1&\frac{1}{a+1}&1\\1&1&1&1&\frac{1}{a+1}\end{pmatrix}=5I_5 \tag{45}$$

Hence A and B are Jacket matrix of order 5. However, A is not equivalent to B. Moreover, there are at least two Jacket matrices of any order $n \geq 5$ which are not equivalent.

Ⅵ. Proposed Jacket Matrices Based on Center Weighted Hadmard Matrices

In this section, we construct some Jacket matrices based on Hadamard matrices. We first show the following propositions.

Proposition 6-1

The Kronecker product of two Jacket matrices is also a Jacket matrix.

Proof: Let A be $m \times m$ Jacket matrix and B be $n \times n$ Jacket matrix. Then $AA^{\dagger}=mI_m$ and $BB=nI_n$. Clearly, $(A \otimes B)^{\dagger}=A^{\dagger} \otimes B^{\dagger}$[30]. Hence

$$(A \otimes B)(A \otimes B)^{\dagger}=(A \otimes B)(A^{\dagger} \otimes B^{\dagger}) \\ =(AA^{\dagger}) \otimes (BB^{\dagger})=mnI_{mn}. \tag{46}$$

So $A \otimes B$ is a Jacket matrix.

Proposition 6-2

Let A and B be two $n \times n$ Jacket matrices. Then

$$\begin{pmatrix} A & \lambda B \\ A & -\lambda B \end{pmatrix} \tag{47}$$

is also Jacket matrix of order $2n$, where $\lambda \neq 0$.

Proof: It follows from the definition of Jacket matrix

Example 6.1: Let

$$A=\begin{pmatrix} 1 & 1 & 1 \\ 1 & w & w^2 \\ 1 & w^2 & w \end{pmatrix}, B=\begin{pmatrix} 1 & 1 & 1 \\ 1 & w^2 & w \\ 1 & w & w^2 \end{pmatrix}. \tag{48}$$

If $\lambda=1$, then

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & w & w^2 & 1 & w^2 & w \\ 1 & w^2 & w & 1 & w & w^2 \\ 1 & 1 & 1 & -1 & -1 & -1 \\ 1 & w & w^2 & -1 & -w^2 & -w \\ 1 & w^2 & w & -1 & -w & -w^2 \end{pmatrix} \tag{49}$$

Is a Jacket matrix of order 6. If $\lambda=1$, then

$$A=\begin{pmatrix} 1 & 1 & 1 & 2 & 2 & 2 \\ 1 & w & w^2 & 2 & 2w^2 & 2w \\ 1 & w^2 & w & 2 & 2w & 2w^2 \\ 1 & 1 & 1 & -2 & -2 & -21 \\ 1 & w & w^2 & -2 & -2w^2 & -2w \\ 1 & w^2 & w & -2 & -2w & -2w^2 \end{pmatrix} \tag{50}$$

is also a Jacket matrix, since

$$A^{-1}=\frac{1}{6}A=\frac{1}{6}\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \frac{1}{w} & \frac{1}{w^2} & 1 & \frac{1}{w} & \frac{1}{w^2} \\ 1 & \frac{1}{w^2} & \frac{1}{w} & 1 & \frac{1}{w^2} & \frac{1}{w} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2w^2} & \frac{1}{2w} & -\frac{1}{2} & -\frac{1}{2w^2} & -\frac{1}{2w} \\ \frac{1}{2} & \frac{1}{2w} & \frac{1}{2w^2} & -\frac{1}{2} & -\frac{1}{2w} & -\frac{1}{2w^2} \end{pmatrix}. \tag{51}$$

Theorem 6-3

Let A_1, B_1, C_1 and D_1 be the core of Jacket matrices of A, B, C, D of order n, respectively. Then $AC^{\dagger}=BD^{\dagger}$ if and only if

$$X=\begin{pmatrix} 1 & e^T & e^T & 1 \\ e & A_1 & B_1 & e \\ e & C_1 & -D_1 & -e \\ 1 & e^T & -e^T & -1 \end{pmatrix} \tag{52}$$

is a Jacket matrix of order $2n$, where e is a column vector of all one.

Proof: since $A=\begin{pmatrix} 1 & e^T \\ e & A_1 \end{pmatrix}$ is a Jacket matrix,

$$\begin{pmatrix} 1 & e^T \\ e & A_1 \end{pmatrix}\begin{pmatrix} 1 & e^T \\ e & A_1^{\dagger} \end{pmatrix}=\begin{pmatrix} 1 & e^T \\ e & A_1^{\dagger} \end{pmatrix}\begin{pmatrix} 1 & e^T \\ e & A_1 \end{pmatrix}=nI_n. \tag{53}$$

Hence

$$e^T+e^TA_1=0,\ (A_1+I)e=0,\ ee^T+A_1A_1^{\dagger}=nI_{n-1}. \tag{54}$$

Similarly, we have

$$e^T+e^TB_1=0,\ (B_1+I)e=0,\ ee^T+B_1B_1^{\dagger}=nI_{n-1}. \tag{55}$$

$$e^T+e^TC_1=0,\ (C_1+I)e=0,\ ee^T+C_1C_1^{\dagger}=nI_{n-1}. \tag{56}$$

$$e^T+e^TD_1=0,\ (D_1+I)e=0,\ ee^T+D_1D_1^{\dagger}=nI_{n-1}. \tag{57}$$

Hence, one onehand,

$$AC^{\dagger}=\begin{pmatrix}1 & e^T\\ e & A_1\end{pmatrix}\begin{pmatrix}1 & e^T\\ e & C_1^{\dagger}\end{pmatrix}=\begin{pmatrix}n & 0\\ 0 & ee^T+A_1C_1^{\dagger}\end{pmatrix} \tag{58}$$

and

$$BD^{\dagger}=\begin{pmatrix}1 & e^T\\ e & B_1\end{pmatrix}\begin{pmatrix}1 & e^T\\ e & D_1^{\dagger}\end{pmatrix}=\begin{pmatrix}n & 0\\ 0 & ee^T+B_1D_1^{\dagger}\end{pmatrix}. \tag{59}$$

Therefore $AC^{\dagger}=BD^{\dagger}$ if and only if $A_1C_1^{\dagger}=B_1D_1^{\dagger}$. On the other hand, by proposition 3.4, $AC^{\dagger}CA^{\dagger}=n_2I_n=BD^{\dagger}DB^{\dagger}$ Hence by $AC^{\dagger}=BD^{\dagger}$, we have $CA^{\dagger}=DB^{\dagger}$, Since A,B,C†,D† are invertible. Therefore, $AC^{\dagger}=BD^{\dagger}$ if and only if $A_1C_1^{\dagger}=B_1D_1^{\dagger}$ and $CA^{\dagger}=DB^{\dagger}$ if and only if $XX^{\dagger}=2_nI_{2n}$by a simple calculation.

Remark: In [22], they only gave the special case of this form matrix to be Jacket matrices.

Corollary 6-4

Let A and B be Jacket matrices of order n with core A_1 and B_1 respectively. Then

$$X=\begin{pmatrix}1 & e^T & e^T & 1\\ e & A_1 & B_1 & e\\ e & A_1 & -B_1 & -e\\ 1 & e^T & -e^T & -1\end{pmatrix},\quad Y=\begin{pmatrix}1 & e^T & e^T & 1\\ e & A_1 & A_1 & e\\ e & A_1^{H} & -A_1 & -e\\ 1 & e^T & -e^T & -1\end{pmatrix} \tag{60}$$

are Jacket matrices of order $2n$

Proof: It follows from Theorem 6.3 by $C=A$, $D=B$; and $B=A$, $C=D=A^{\dagger}$ respectively that the assertion holds.

Ⅶ. J-Hurwitz Radon of Complex Orthogonal Jacket Space Time Code

Conventionally the Hurwitz-Radon (HR) family of matrices was defined in [25]~[28] for deriving the orthogonal STBC design. In this section, we present its extension theory, called j-Hurwitz-Radon units [29] and j-sip units [7], [14] as follows:

Definition 7-1

A $n\times n$ complex matrix S is called a size n j-rotation Hurwitz-Radon (HR) unit if $S^HS=I$ and $S^H=-jS$, where H is the Hermitian of the complex matrix.

Definition 7-2

For $n\times n$ complex matrix as Z is called a size n-sip unit when $Z^HZ=I$, $\det(Z^H)=-\det(Z)$, and $Z^H=Z^*$, where det() is the determinant of matrix, and * denotes the conjugate of the matrix.

In equation (2), we have changed the j instead of w in center part [6], then [15] by taking the diagonal terms of this $[J]_4$,

$$[J]_4=\left(\begin{array}{cc:cc}1 & 1 & 1 & 1\\ 1 & -j & j & -1\\ 1 & j & -j & -1\\ 1 & -1 & -1 & 1\end{array}\right) \text{ and } [J]_4^{-1}=\begin{pmatrix}1 & 1 & 1 & 1\\ 1 & -\frac{1}{j} & \frac{1}{j} & -1\\ 1 & \frac{1}{j} & -\frac{1}{j} & -1\\ 1 & -1 & -1 & 1\end{pmatrix} \tag{61}$$

we can obtain the two j-HR units $\begin{bmatrix}0 & 1\\ j & 0\end{bmatrix},\begin{bmatrix}0 & j\\ 1 & 0\end{bmatrix}$ and two j-sip units $\begin{bmatrix}1 & 0\\ 0 & -j\end{bmatrix},\begin{bmatrix}-j & 0\\ 0 & 1\end{bmatrix}$. Next, based on these forms, we may denote the full rate two antennas j-rotation orthogonal STBC as

$$A_1=\begin{bmatrix}s_1 & js_2\\ s_2^* & -js_1^*\end{bmatrix}=\underbrace{\begin{bmatrix}1 & 0\\ 0 & -j\end{bmatrix}\begin{bmatrix}s_1 & 0\\ 0 & s_1^*\end{bmatrix}}_{j-sip}+\underbrace{\begin{bmatrix}0 & j\\ 1 & 0\end{bmatrix}\begin{bmatrix}s_2^* & 0\\ 0 & s_2\end{bmatrix}}_{j-HR}, \tag{62}$$

and

$$A_2=\begin{bmatrix}-js_1 & s_2\\ js_2^* & s_1^*\end{bmatrix}=\underbrace{\begin{bmatrix}-j & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}s_1 & 0\\ 0 & s_1^*\end{bmatrix}}_{j-sip}+\underbrace{\begin{bmatrix}0 & 1\\ j & 0\end{bmatrix}\begin{bmatrix}s_2^* & 0\\ 0 & s_2\end{bmatrix}}_{j-HR}, \tag{63}$$

where, A_1 and A_2 satisfy the STBC design criterion as [25]~[27],

$$A_1^HA_1=\begin{bmatrix}s_1^* & s_2\\ -js_2^* & js_1\end{bmatrix}\begin{bmatrix}s_1 & js_2\\ s_2^* & -js_1^*\end{bmatrix}=\left(|s_1|^2+|s_2|^2\right)[I]_2, \tag{64}$$

and

$$A_2^HA_2=\begin{bmatrix}js_1^* & -js_2\\ s_2^* & s_1\end{bmatrix}\begin{bmatrix}-js_1 & s_2\\ js_2^* & s_1^*\end{bmatrix}=\left(|s_1|^2+|s_2|^2\right)[I]_2. \tag{65}$$

We can also construct the 1/2 rate complex orthogonal space time codes from [28]. The Jacket matrix can de decomposed as

$$[J]_{\substack{left\\ half}}=\begin{bmatrix}1 & 1\\ 1 & -j\\ 1 & j\\ 1 & -1\end{bmatrix}, \text{ and } [J]_{\substack{right\\ half}}=\begin{bmatrix}1 & 1\\ j & -1\\ -j & -1\\ -1 & -1\end{bmatrix}. \tag{66}$$

thus we can design two kinds of space time block codes.

Let $A=\begin{bmatrix}1 & 0\\ 0 & -j\end{bmatrix}, B=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}, C=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}, D=\begin{bmatrix}0 & j\\ 1 & 0\end{bmatrix}$.

Then, left hand side 4×2 Jacket matrix C_0 is

$$C_0=\begin{bmatrix}A\\ B\end{bmatrix}\circ\begin{bmatrix}s_1\\ s_1^*\end{bmatrix}+\begin{bmatrix}C\\ D\end{bmatrix}\circ\begin{bmatrix}s_2\\ s_2^*\end{bmatrix}=\begin{bmatrix}As_1\\ Bs_1^*\end{bmatrix}+\begin{bmatrix}Cs_2\\ Ds_2^*\end{bmatrix}$$
$$=\begin{bmatrix}s_1 & 0\\ 0 & -js_1\\ s_1^* & 0\\ 0 & -s_1^*\end{bmatrix}+\begin{bmatrix}0 & s_2\\ s_2 & 0\\ 0 & js_2^*\\ s_2^* & 0\end{bmatrix}=\begin{bmatrix}s_1 & s_2\\ s_2 & -js_1\\ s_1^* & js_2^*\\ s_2^* & -s_1^*\end{bmatrix}, \quad (67)$$

where symbol 0 is the Hadamard product[30].

Therefore,

$$[C_0]^H[C_0]=\begin{bmatrix}s_1^* & s_2^* & s_1 & s_2\\ s_2^* & js_1^* & -js_2 & s_1\end{bmatrix}\begin{bmatrix}s_1 & s_2\\ s_2 & -js_1\\ s_1^* & js_2^*\\ s_2^* & -s_1^*\end{bmatrix}$$
$$=\begin{bmatrix}2\left(|s_1|^2+|s_2|^2\right) & s_1^*s_2-js_2^*s_1+js_2^*s_1-s_1^*s_2\\ s_2^*s_1+js_2s_1^*-js_2s_1^*-s_2^*s_1 & 2\left(|s_1|^2+|s_2|^2\right)\end{bmatrix}$$
$$=2\left(|s_1|^2+|s_2|^2\right)[I]_2. \quad (68)$$

Similarly, from the right hand side 4×2 Jacket matrix C_1 is

$$C_1=\begin{bmatrix}A\\ B\end{bmatrix}\circ\begin{bmatrix}s_1\\ s_1^*\end{bmatrix}+\begin{bmatrix}C\\ D\end{bmatrix}\circ\begin{bmatrix}s_2\\ s_2^*\end{bmatrix}=\begin{bmatrix}As_1\\ Bs_1^*\end{bmatrix}+\begin{bmatrix}Cs_2\\ Ds_2^*\end{bmatrix}$$
$$=\begin{bmatrix}s_1 & 0\\ 0 & -s_1\\ -js_1^* & 0\\ 0 & s_1^*\end{bmatrix}+\begin{bmatrix}0 & s_2\\ js_2 & 0\\ 0 & s_2^*\\ -s_2^* & 0\end{bmatrix}=\begin{bmatrix}s_1 & s_2\\ js_2 & -s_1\\ -js_1^* & s_2^*\\ -s_2^* & s_1^*\end{bmatrix}, \quad (69)$$

we can write,

$$[C_1]^H[C_1]=\begin{bmatrix}s_1^* & -js_2^* & js_1 & -s_2\\ s_2^* & -s_1^* & -s_2 & s_1\end{bmatrix}\begin{bmatrix}s_1 & s_2\\ js_2 & -s_1\\ -js_1^* & -s_2^*\\ -s_2^* & s_1^*\end{bmatrix}$$
$$=2\left(|s_1|^2+|s_2|^2\right)[I]_2. \quad (70)$$

The 1/2 code rate complex orthogonal space time code satisfies the equations (68) and (70) as [28, PP. 82], yielding the full transmits diversity.

Ⅷ. Conclusion

We establish some relations among Hadamard matrices, complex Hadamard matrices and Jacket matrices. At the same time, some properties of Jacket matrices are obtained, in particular, the Jacket matrices of small orders no more than 4 are unique up to equivalent relationship, but there exists two Jacket matrices of all order $n\geq 5$ which are not equivalent and also we designed the full rate and 1/2 rate complex orthogonal space-time code from Jacket matrices. The properties which we are defined in this paper may be used for Jacket matrices to be applied in signal processing, coding theory, and orthogonal code design[18],[20],[21],[23],[24],[29].

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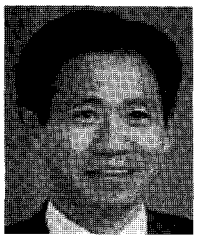

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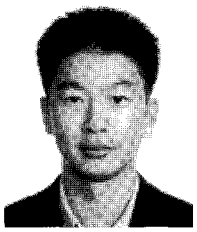
Chang-Hui Choe



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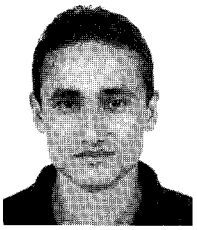
Subash Shree Pokhrel



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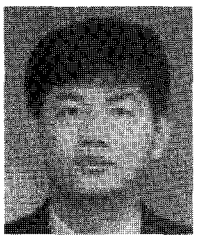

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