Design of Multi-loop PID Controllers Based on the Generalized IMC-PID Method with Mp Criterion

Truong Nguyen Luan Vu, Jietae Lee, and Moonyong Lee*

Abstract: A new method of designing multi-loop PID controllers is presented in this paper. By using the generalized IMC-PID method for multi-loop systems, the optimization problem involved in finding the PID parameters is efficiently simplified to find the optimum closed-loop time constant in a reduced search space. A weighted sum Mp criterion is proposed as a performance cost function to cope with both the performance and robustness of a multi-loop control system. Several illustrative examples are included to demonstrate the improved performance of the multi-loop PID controllers obtained by the proposed design method.

Keywords: Generalized IMC-PID method, MIMO system, *Mp* criterion, multi-loop PID controller, PID controller tuning, process interaction.

1. INTRODUCTION

Multivariable or multi-input multi-output (MIMO) systems are frequently encountered in the chemical and process industries. Despite the considerable work that has been done on advanced multivariable controllers for MIMO systems, multi-loop PID controllers (sometimes known as decentralized PID controllers) are still much more favored in most commercial process control applications, because of their satisfactory performance along with their simple, failure tolerant, and easy to understand structure [1,2]. Multi-loop PID controllers are made up of individual PID controllers acting in a multi-loop fashion and tuned mainly on a single loop basis. However, due to the process interactions in MIMO systems, this approach cannot guarantee stability when all of the loops are closed simultaneously [3]. This is because the closing of one loop affects the dynamics of the other loops and can make them worse or even unstable. This complex interactive nature of MIMO systems makes the proper tuning of multi-loop PID controllers quite difficult.

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For this reason, despite the wide popularity of multi-loop PID controllers, the number of applicable tuning methods is relatively limited. Furthermore, many of the existing methods of designing multi-loop PID controllers are computationally intensive and/or require solving a large scale optimization problem and, therefore, are less appealing to the practitioners. Most existing tuning methods for multi-loop PID controllers are similar in that they first use the single loop tuning rules by ignoring process interactions and then detune the individual loops to preserve stability [4-7] or adjust them to meet some performance specification [8]. However, since these approaches are based on single loop tuning rules, they often lead to local optimum solutions or too conservative responses.

In this article, we consider the design of multi-loop PID controllers for square multivariable systems. The proposed method combines the internal model control (IMC) based PID design method [9] with a frequency domain performance criterion, *Mp*, defined as the maximum magnitude of the closed-loop frequency response. In addition to its superior performance, the proposed approach has several important advantages. Firstly, the optimization problem is significantly simplified to the finding of only one design parameter, *i.e.*, the closed-loop time constant of each loop. Secondly, there is no restriction on the process model type. A simulation study for several well-known distillation column models is carried out to illustrate the usefulness of the proposed method.

2. PROPOSED DESIGN METHOD

2.1. Generalized IMC-PID method for multi-loop PID controller design [9]

In the $n \times n$ multi-loop feedback control system shown

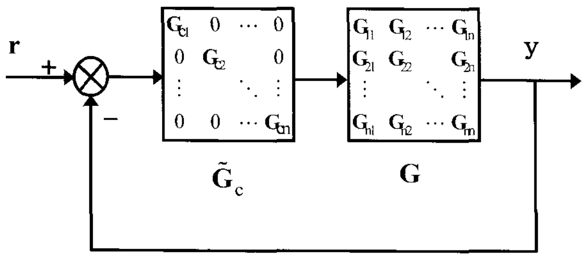

Fig. 1. Block diagram for multi-loop control system.

in Fig. 1, the closed-loop response to the set-point change is represented by

$$\mathbf{y}(s)=\mathbf{H}(s)\mathbf{r}(s)=\left(\mathbf{I}+\mathbf{G}(s)\tilde{\mathbf{G}}_c(s)\right)^{-1}\mathbf{G}(s)\tilde{\mathbf{G}}_c(s)\mathbf{r}(s), \quad (1)$$

where **H**(s) is the closed-loop transfer function; **G**(s) is the process transfer function which is open-loop stable; $\tilde{\mathbf{G}}_c(s)$ is the multi-loop controller with diagonal elements only; **y**(s) and **r**(s) are the controlled variable and the set-point, respectively.

Suppose that the desired closed-loop response of the diagonal elements in the multi-loop system is given by

$$\tilde{\mathbf{R}}(s)=diag[R_1,R_2,...,R_n]. \quad (2)$$

According to the design strategy of the IMC controller [6], the desired closed-loop response R_i of the *i*th loop is given by

$$\frac{y_i(s)}{r_i(s)}=R_i(s)=\frac{G_{ii+}(s)}{(\lambda_i s+1)^{n_i}}, \quad (3)$$

where G_{ii+} is the non-minimum part of G_{ii} and chosen to be the all pass form; λ_i is an adjustable constant for system performance and robustness; n_i is chosen for the IMC controller to be realizable.

The multi-loop controller $\tilde{\mathbf{G}}_c(s)$ with integral term can be expressed in a Maclaurin series as

$$\tilde{\mathbf{G}}_c(s)=\frac{1}{s}\left[\tilde{\mathbf{G}}_{c0}+\tilde{\mathbf{G}}_{c1}s+\tilde{\mathbf{G}}_{c2}s^2+O(s^3)\right], \quad (4)$$

where $\tilde{\mathbf{G}}_{c0},\tilde{\mathbf{G}}_{c1},\tilde{\mathbf{G}}_{c2}$ can be considered as the integral, proportional, and derivative terms of the multi-loop PID controller, respectively.

As can be gleaned from (4), the impact of the proportional and derivative terms (i.e., $\tilde{\mathbf{G}}_{c1},\tilde{\mathbf{G}}_{c2}$) dominates at high frequencies and, thus, they should be designed based on the process characteristics at high frequencies. On the other hand, the integral term $\tilde{\mathbf{G}}_{c0}$ dominates at low frequencies and, thus, needs to be designed based on the process characteristics at low frequencies.

At high frequencies, the magnitude of the open loop gain becomes $\left|\mathbf{G}(j\omega)\tilde{\mathbf{G}}_c(j\omega)\right|\ll 1$ and thus **H**(s) can be approximated to

$$\mathbf{H}(s)=(\mathbf{I}+\mathbf{G}(s)\tilde{\mathbf{G}}_c(s))^{-1}\mathbf{G}(s)\tilde{\mathbf{G}}_c(s)\approx\mathbf{G}(s)\tilde{\mathbf{G}}_c(s). \quad (5)$$

This indicates that $\tilde{\mathbf{G}}_{c0}$ and $\tilde{\mathbf{G}}_{c1}$ can be designed by considering only the diagonal elements in **G**(s), which means that the generalized IMC-PID method for the SISO system [10] can be directly applied to the design of the proportional and derivative terms in the multi-loop PID controller. Therefore, at high frequencies, the ideal multi-loop feedback controller giving the desired closed-loop response $\tilde{\mathbf{R}}(s)$ is designed by

$$\tilde{\mathbf{G}}_c(s)=\tilde{\mathbf{G}}^{-1}(s)\tilde{\mathbf{R}}(s)(\mathbf{I}-\tilde{\mathbf{R}}(s))^{-1}, \quad (6)$$

where $\tilde{\mathbf{G}}(s)=diag[G_{11},G_{22},...,G_{nn}]$.

Accordingly, the ideal multi-loop controller of the *i*th loop can be designed by

$$G_{ci}(s)=\frac{(G_{ii-}(s))^{-1}}{(\lambda_i s+1)^{n_i}-G_{ii+}(s)}, \quad (7)$$

where G_{ii-} is the minimum part of G_{ii}.

Since $G_{ii+}(0)=1$, (7) can be rewritten in a Maclaurin series with an integral term as follows

$$G_{ci}(s)=\frac{1}{s}(f_i(0)+f_i'(0)s+\frac{f_i''(0)}{2}s^2+0(s^3)), \quad (8)$$

where $f_i(s)=G_{ci}(s)s$.

The standard PID control algorithm is given by

$$G_{ci}(s)=K_{ci}(1+\frac{1}{\tau_{Ii}}s+\tau_{Di}s). \quad (9)$$

Comparing (8) with (9) gives the analytical tuning rules for the proportional gain and the derivative time constant of the multi-loop PID controller as follows

$$K_{ci}=f_i'(0)\ ;\ \tau_{Di}=\frac{f_i''(0)}{2K_{ci}}. \quad (10)$$

At low frequencies, the interaction effect between the control loops cannot be neglected. The expansion of **G**(s) in a Maclaurin series gives

$$\mathbf{G}(s)=\mathbf{G}_0+\mathbf{G}_1 s+\mathbf{G}_2 s^2+O(s^3), \quad (11)$$

where $\mathbf{G}_0=\mathbf{G}(0)$; $\mathbf{G}_1=\mathbf{G}'(0)$; $\mathbf{G}_2=\mathbf{G}''(0)/2$.

By substituting (4) and (11) into (1), one can obtain **H**(s) as

$$\mathbf{H}(s)=\mathbf{I}-(\mathbf{G}_0\tilde{\mathbf{G}}_{c0})^{-1}s+O(s^2). \quad (12)$$

Furthermore, the desired closed-loop response $\tilde{\mathbf{R}}$ can also be written in a Maclaurin series as

$$\tilde{\mathbf{R}}(s) = \tilde{\mathbf{R}}(0) + \tilde{\mathbf{R}}'(0)s + O(s^2), \tag{13}$$

where $\tilde{\mathbf{R}}(0) = \mathbf{I}$ because $G_{ii+}(0) = 1$.

By comparing the diagonal element of **H**(s) in (12) and $\tilde{\mathbf{R}}(s)$ in (13) for the first-order s term, one can obtain the analytical tuning rule for the integral time constant of the multi-loop PID controller as follows

$$\tau_{Ii} = -\frac{(G'_{ii+}(0) - n_i\lambda_i)K_{ci}}{(\mathbf{G}^{-1}(0))_{ii}}. \tag{14}$$

The tuning formulae by (10) and (14) provide an important advantage in solving the optimization problem used for finding the PID parameter values: for a given process, all of the PID parameters can be expressed by a single design parameter, λ_i.

2.2. Weighted sum *Mp* criterion for multi-loop controller tuning

The resonant peak *Mp* in a SISO system is defined as the maximum magnitude of the closed-loop frequency response (i.e., the maximum magnitude of the complementary sensitivity function). The *Mp* is widely accepted as a good index to address both the performance and robustness of a control system. An empirical study showed that the value of *Mp* should lie between 1.1 and 1.4 in SISO systems [11]. In a MIMO system, the closed-loop transfer function consists of the individual closed-loop transfer functions as

$$\mathbf{H}(s) = \{H_{ij}\}, \tag{15}$$

where H_{ij} represents the closed-loop transfer function of the *ith* loop to the set-point change in the *jth* loop.

The overall control performance depends not only on the closed-loop transfer functions associated with the diagonal elements, but also on those of the off-diagonal elements in the closed-loop transfer function matrix **H**. Thus, the *Mp* specification for the diagonal elements does not guarantee good performance by itself. For well-balanced closed-loop performance, the off-diagonal H_{ij} should be close to zero as well as the diagonal H_{ii} being close to unity. To take these requirements into account in the design of the controller, a weighted sum of the individual Mp values is proposed for the objective function used to find the optimum PID parameters, as follows.

$$\min_{\lambda}\left[(1-w)\sum_i\sum_{j\neq i} Mp_{ij} + w\sum_i Mp_{ii}\right] \tag{16}$$

subject to $Mp_{ii} \geq Mp_{low}$,

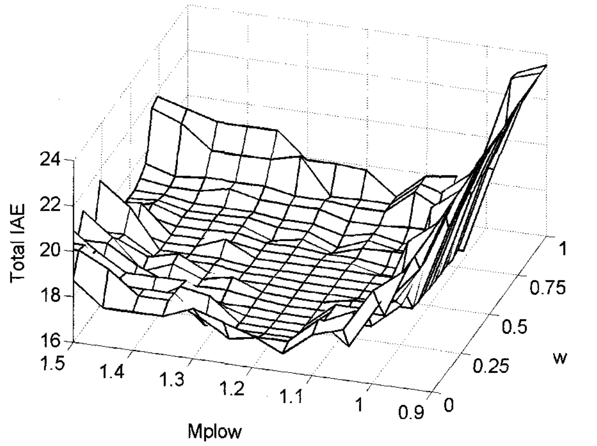

Fig. 2. Effect of Mp_{low} and w on the IAE: WB column example.

where Mp_{ij} is defined by $\max_{\omega\geq 0}|H_{ij}(j\omega,\lambda)|$; Mp_{low} is the lower bound of the diagonal Mp needed to secure the minimum speed required; w is the weighting factor for the diagonal and the off-diagonal closed-loop responses and ranges from 0 to 1. Mp_{low} and w can also be considered as optimizing variables to minimize the integral absolute error (IAE) of the closed-loop time response. However, since it appears that the effect of these parameters on the IAE is sufficiently small over a wide range of values of Mp_{low} and w, they can be fixed as constant values within the given range. Fig. 2 shows an example of the impact of Mp_{low} and w on the overall performance in the WB column [14] case. As can be seen from this figure, the shape of the minimum IAE surface is quite flat for a wide range of values of Mp_{low} and w. Our extensive simulation study shows that the desirable value of w lies between 0.5 and 0.75 and that of Mp_{low} is from 1.1 to 1.4 for well-balanced closed-loop performance.

3. CASE STUDIES

To evaluate the performance of the proposed method, the closed-loop responses by the proposed method were compared with those by several well-known existing design methods, such as the biggest log modulus tuning (BLT) method [4], the decentralized λ tuning (DLT) method [12], and the sequential auto tuning (SAT) method [13].

Example 1: Consider the Wood and Berry (WB) column model for separating methanol and water [14].

$$G(s) = \begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \tag{17}$$

In the simulation study, step changes in the set-

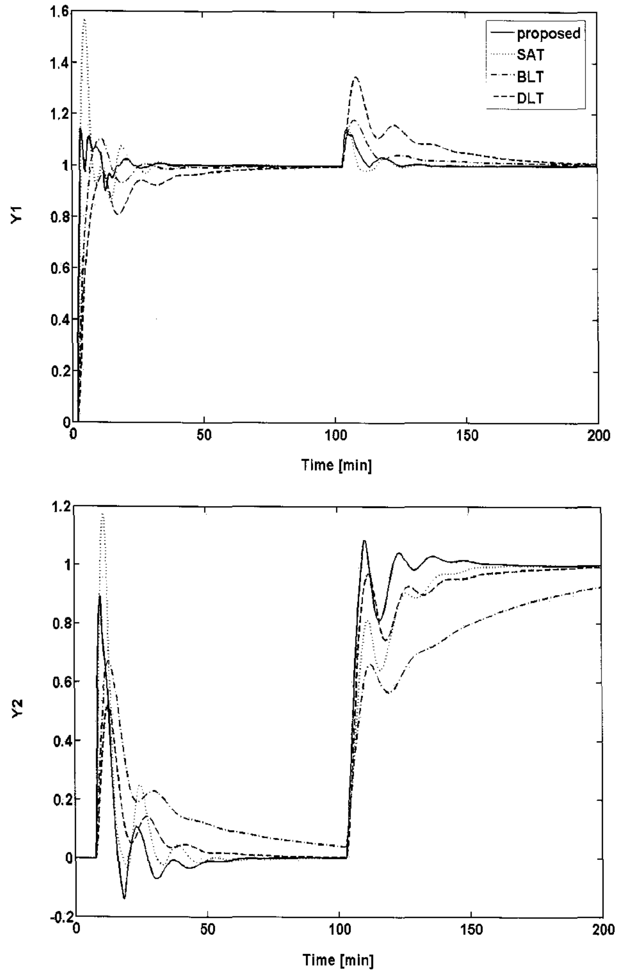

Fig. 3. Closed-loop responses to sequential step changes in set-point for WB column.

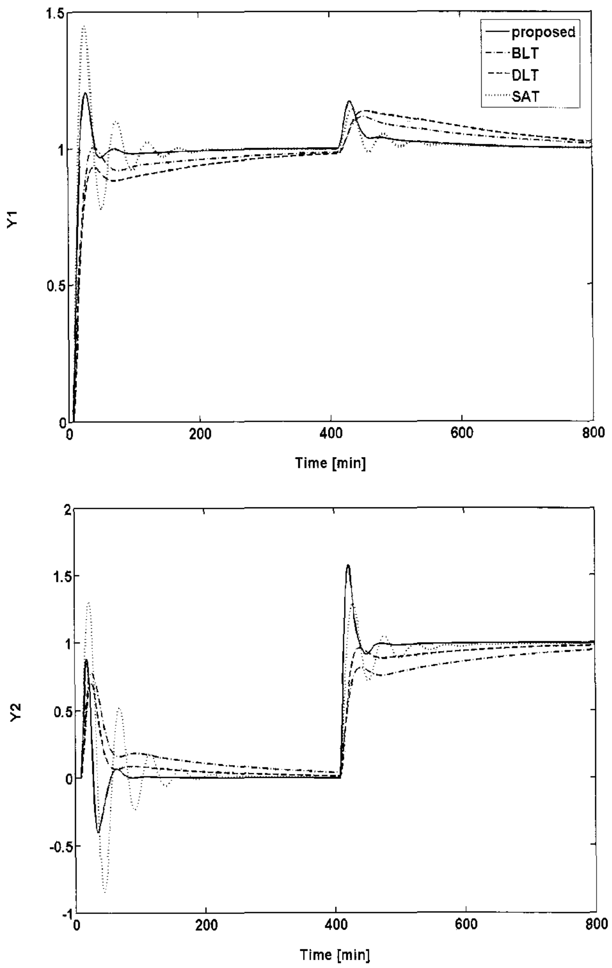

Fig. 4. Closed-loop responses to sequential step changes in set-point for WW column.

point were sequentially made in the individual loops. n_i in (3) was chosen as 1 for all loops. The optimum λ values based on (16) were found to be 0.029 and 3.227 for loops 1 and 2, respectively. All of the control parameters used in the example are listed in Table 1, including the IAE values of the closed-loop responses. Values of 1.2 and 0.75 were chosen for Mp_{low} and w, respectively.

These Mp_{low} and w values were also applied to all other examples in this paper. Fig. 3 shows the closed-loop responses obtained using the various design methods. As seen in this figure, the proposed method provides more well-balanced and faster responses in comparison to the other methods. The superiority of the proposed method is also illustrated by its IAE values in Table 1. The IAE values for which all of the parameters in the actual process are assumed to be changed by +10% and -10% are also listed in Table 1. One can see that the controller obtained by the proposed method provides more robust performance than those obtained using the other methods.

Example 2: Consider the Wardle and Wood (WW) column studied by Luyben [4].

$$G(s) = \begin{bmatrix} \dfrac{0.126\, e^{-6s}}{60s+1} & \dfrac{-0.101\, e^{-12s}}{(48s+1)(45s+1)} \\ \dfrac{0.094\, e^{-8s}}{38s+1} & \dfrac{-0.12\, e^{-8s}}{35s+1} \end{bmatrix} \tag{18}$$

The optimum λ values based on (16) were found to be 5.25 and 0.019 for loops 1 and 2, respectively. All of the control parameters and the resulting IAE values are also listed in Table 1. Fig. 4 compares the closed-loop response obtained using the various design methods. The superior performance of the proposed method is apparent from this figure and the IAE values listed in Table 1.

Example 3: Consider the Vinante and Luyben (VL) column studied by Luyben [4].

$$G(s)= \begin{bmatrix} \dfrac{-2.2\, e^{-s}}{7s+1} & \dfrac{1.3\, e^{-0.3s}}{7s+1} \\ \dfrac{-2.8e^{-1.8s}}{9.5s+1} & \dfrac{4.3e^{-0.35s}}{9.2s+1} \end{bmatrix} \tag{19}$$

The λ values were calculated to be 0.388 and 0.051

Table 1. PID parameter and IAE values obtained using the various methods.

Process		Proposed	BLT	DLT	SAT
WB	K_c	1.30	0.38	0.34	0.87
		-0.13	-0.08	-0.14	-0.09
	τ_I	8.55	3.25	17.20	8.29
	τ_D	0.48	-	0.49	-
		0.67	-	1.36	-
	IAE	16.55	57.99	32.72	23.31
	$IAE_{(+10\%)}$	19.83	51.89	30.14	25.19
	$IAE_{(-10\%)}$	16.62	61.26	36.07	25.43
WW	K_c	43.26	27.40	31.25	48.10
		-40.42	-13.30	-18.06	-25.4
	τ_I	22.92	41.40	63.00	18.99
		14.47	52.90	39.00	26.30
	τ_D	1.541	-	2.86	-
		3.718	-	3.59	-
	IAE	66.26	212.16	146.97	114.39
	$IAE_{(+10\%)}$	77.79	142.08	130.74	158.28
	$IAE_{(-10\%)}$	60.89	239.39	165.04	94.78
VL	K_c	-2.43	-1.07	-3.66	-1.35
		5.44	1.97	2.94	3.36
	τ_I	4.53	7.1	12.36	3.00
		5.75	2.58	14.36	1.33
	τ_D	0.35	-	2.21	-
		0.15	-	2.20	-
	IAE	4.79	8.61	14.47	7.18
	$IAE_{(+10\%)}$	5.16	8.40	14.72	7.80
	$IAE_{(-10\%)}$	4.53	9.03	14.27	6.62

$IAE_{(+10\%)}$ and $IAE_{(-10\%)}$ denote the IAE values under +10% and -10% parametric uncertainty, respectively.
The values of the filter time constant used in the DLT method are (0.38 for loop 1, 0.75 for loop 2), (1.88, 2.22), and (6.54, 6.72) for the WB, WW, and VL processes, respectively.

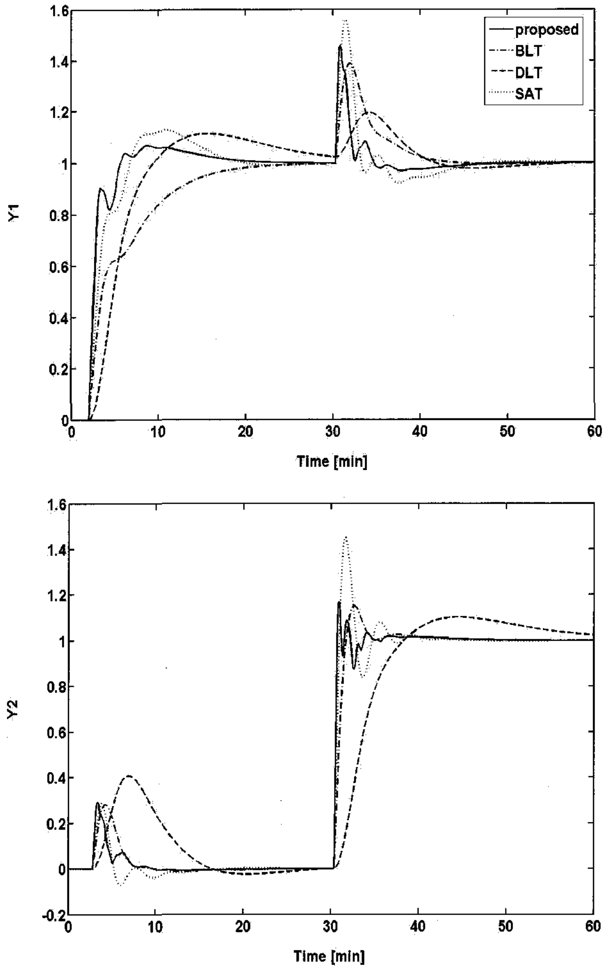

Fig. 5. Closed-loop responses to sequential step changes in set-point for VL column.

for loops 1 and 2, respectively. From Fig. 5 and Table 1, it can be concluded that the proposed method provides a superior response to that of the other methods.

4. CONCLUSIONS

In this paper, we proposed an efficient method of designing multi-loop PID controllers. The proposed method utilizes the one parameter tuning rule to effectively reduce the dimension of the search space used for finding the optimum PID parameters. The generalized IMC-PID method for multi-loop systems was used as the one parameter tuning rule. By using the frequency-dependent property in the closed-loop interactions, the analytical tuning rule can be made to take the interaction effect fully into account in a simple but efficient manner. The weighted sum Mp criterion was proposed as a performance measure to cope with both performance and robustness in multi-loop systems. The superiority of the proposed method in comparison with several well-known existing methods was demonstrated by providing several illustrated examples.

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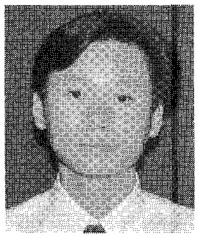

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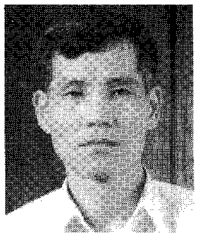

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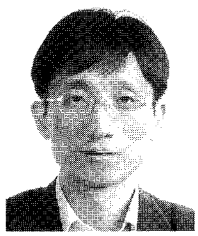

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