A Delay-Dependent Approach to Robust Filtering for LPV Systems with Discrete and Distributed Delays using PPDQ Functions

Hamid Reza Karimi, Boris Lohmann, and Christof Büskens

Abstract: This paper presents a delay-dependent approach to robust filtering for linear parameter-varying (LPV) systems with discrete and distributed time-invariant delays in the states and outputs. It is assumed that the state-space matrices affinely depend on parameters that are measurable in real-time. Some new parameter-dependent delay-dependent stability conditions are established in terms of linear matrix inequalities (LMIs) such that the filtering process remains asymptotically stable and satisfies a prescribed H_∞ performance level. Using polynomially parameter-dependent quadratic (PPDQ) functions and some Lagrange multiplier matrices, we establish the parameter-independent delay-dependent conditions with high precision under which the desired robust H_∞ filters exist and derive the explicit expression of these filters. A numerical example is provided to demonstrate the validity of the proposed design approach.

Keywords: Delay, LPV systems, H_∞ filtering, LMI, polynomially parameter-dependent quadratic functions.

1. INTRODUCTION

In the past decade, a number of papers have attempted to develop robust filters that are capable of guaranteeing satisfactory estimation in the presence of modeling errors and unknown signal statistics [1]. Concerning the energy bounded deterministic noise inputs, the H_∞ filtering theory has been developed which minimizes the worst-case energy gain from the energy-bounded disturbances to the estimation errors [2]. Furthermore, the H_∞ filtering problem has recently received considerable attention. The aim of this problem is to pursue the enforcement of the upper bound constraint on the H_∞ norm where the system is affected by parameter uncertainties [3].

The stability analysis and control design of linear parameter-varying (LPV) systems have received considerable attention recently [4-7]. To investigate the stability of LPV systems one needs to resort the use of parameter-dependent Lyapunov functions to achieve necessary and sufficient conditions of system stability [8-14]. Development of some conditions for robust stability analysis of LPV systems with polytopic uncertain parameters in terms of solvability of some linear matrix inequalities (LMIs) without conservatism is investigated in [10]. Concerning unknown parameter vector, an adaptive method has been presented for robust stabilization with H_∞ performance of LPV systems in [6]. The existence of a polynomially parameter-dependent quadratic (PPDQ) Lyapunov function for parameter-dependent systems, which are robustly stable, is stated in [15]. Recently, sufficient conditions for robust stability of the linear state-space models affected by polytopic uncertainty have been provided in [16] using homogeneous PPDQ Lyapunov functions.

On the other hand, in addition to the system uncertainties, it is well known that the time-delay is also often the main cause of instability and poor performance of dynamical systems [17,18]. The stability and the performance issues of the LPV state-delayed systems are then both theoretically and practically important and are a field of intense research. Recently, some appreciable works have been performed to analyze and synthesize LPV time-delay systems (e.g., see [19-21]). More recently, a systematic way for the use of PPDQ functions in the state feedback control of LTI parameter-dependent systems with time-delay in state was proposed in [22].

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It is noted that the above paper introduces a delay-independent stability criterion which is a source of conservativeness in comparison with the present work. On the other hand, it is known that the conservatism of the delay-dependent stability conditions stems from two causes: one is the model transformation used and the other is the inequality bounding technique [23, Lemma 1] employed for some cross terms encountered in the analysis [24]. Considering these, in [21], a model which is equivalent to the original LPV time-delayed system was proposed and the bounding technique was used. However, conservatism still remains in these results, which motivates the present study. Generally, LPV time-delayed systems may arise from simplification of some partial differential equations. Furthermore, LPV time-delayed systems are used to model dynamical systems in engineering such as a milling process [21].

The problem of filter design for uncertain time-delay systems has received much less attention although they are important in control design and signal processing applications. Recently, the problem of H_∞ filtering for LTI and linear time-varying systems with time-delay measurements was investigated in [25-27]. It is also worth citing that few studies have been done for the design of H_∞ filters for LPV systems [28,29]. However, the H_∞ filtering problem for LPV systems with delayed states and output has not been fully investigated and remains to be important and challenging.

This paper presents a delay-dependent method to robust filtering problem for a class of LPV systems with discrete and distributed time-invariant delays in the states and outputs. It is assumed that the state-space data affinely depend on parameter vector that is measurable in real-time. Using Leibniz-Newton formula and some free weighting matrices, some new delay dependent stability conditions are established with less conservative and the filtering process remains stable and satisfies a prescribed H_∞ performance level. Moreover, using PPDQ functions and some Lagrange multiplier matrices, the parameter-dependent delay-dependent conditions are relaxed to the parameter-independent delay-dependent conditions with high precision under which the desired H_∞ filters exist; then, the explicit expression of these filters is derived. Eventually, an illustrative example is given to show the qualification of our method.

Notations: The symbol $*$ denotes the elements below the main diagonal of a symmetric block matrix. Also, the symbol $\otimes$ denotes Kronecker product, the power of Kronecker products being used with the natural meaning $M^{0\otimes}=1$, $M^{p\otimes}:=M^{(p-1)\otimes}\otimes M$ and $\otimes_{i=m}^{1}\rho_i^{[k]}:=\rho_m^{[k]}\otimes\cdots\otimes\rho_1^{[k]}$. Let $\{\hat{J}_k,\tilde{J}_k\}\in\Re^{k\times(k+1)}$, $\hat{J}_{n,n}\in\Re^{n\times 2n}$ and $\vartheta^{[k]}$ with scalar ϑ be defined by $\hat{J}_k:=[I_k\ 0_{k\times 1}]$, $\tilde{J}_k:=[0_{k\times 1}\ I_k]$, $\hat{J}_{n,n}:=[I_n\ 0_{n\times n}]$ and $\vartheta^{[k]}:=col\{1,\vartheta,\cdots,\vartheta^{k-1}\}$, respectively, which have essential roles for polynomial manipulations. Finally, $\|x(t)\|_2$ denotes the L_2 norm of $x(t)$.

2. PROBLEM DESCRIPTION

Consider a class of LPV systems with discrete and distributed delays in the states and outputs as

$$\dot{x}(t)=A(\rho)x(t)+A_d(\rho)x(t-d)+A_{d\tau}(\rho)\int_{t-\tau}^{t}x(s)ds+E_1(\rho)w(t), \tag{1a}$$

$$x(t)=\phi(t),\qquad t\in[-d,0], \tag{1b}$$

$$z(t)=L(\rho)x(t)+L_d(\rho)x(t-d)+E_3(\rho)w(t), \tag{1c}$$

$$y(t)=C(\rho)x(t)+C_d(\rho)x(t-d)+C_{d\tau}(\rho)\int_{t-\tau}^{t}x(s)ds+E_2(\rho)w(t), \tag{1d}$$

where $x(t)\in\Re^n$, $w(t)\in L_2^s[0,\infty)$, $z(t)\in\Re^z$, and $y(t)\in\Re^p$ are state, disturbance, estimated output and measured output, respectively. $\phi(t)$ is continuous vector valued initial function. Moreover, the parameters d and τ are constant delays and the vector $\rho=col\{\rho_1,\rho_2,\cdots,\rho_m\}\in\zeta\subset\Re^m$ is uncertain but the parameters ρ_i are measurable in real- time with ζ being a compact set.

In (1), the parameter-dependent matrices are unknown real continuous matrix functions, which affinely depend on the vector ρ, that are

$$\begin{bmatrix} A(\rho) & A_d(\rho) & A_{d\tau}(\rho) & E_1(\rho)\\ L(\rho) & L_d(\rho) & 0 & E_3(\rho)\\ C(\rho) & C_d(\rho) & C_{d\tau}(\rho) & E_2(\rho)\end{bmatrix}=\begin{bmatrix} A_0 & A_{0d} & A_{0d\tau} & E_{01}\\ L_0 & L_{0d} & 0 & E_{03}\\ C_0 & C_{0d} & C_{0d\tau} & E_{02}\end{bmatrix}+\sum_{j=1}^{m}\rho_j\begin{bmatrix} A_j & A_{jd} & A_{jd\tau} & E_{j1}\\ L_j & L_{jd} & 0 & E_{j3}\\ C_j & C_{jd} & C_{jd\tau} & E_{j2}\end{bmatrix}. \tag{2}$$

In this paper, we focus on the design of a full order delay-free H_∞ filter with the following equations

$$\dot{\hat{x}}(t) = F(\rho)\hat{x}(t) + G(\rho)y(t), \tag{3a}$$

$$\hat{x}(t) = 0, \qquad t \in [-d, 0], \tag{3b}$$

$$\hat{z}(t) = L(\rho)\hat{x}(t) + E_3(\rho)w(t), \tag{3c}$$

where the state-space parameter-dependent matrices $F(\rho)$, $F_d(\rho)$ and $G(\rho)$ of the appropriate dimensions are the filter design objectives to be determined. In the absence of $w(t)$, it is required that

$$\|x(t) - \hat{x}(t)\|_2 \to 0 \ \ as \ \ t \to \infty,$$

where $\hat{x}(t) \in \Re^n$ and $\hat{z}(t)$ are the estimation of $x(t)$ and of $z(t)$, respectively, and $e(t) = z(t) - \hat{z}(t)$ is the estimation error. Now, we obtain the following state-space model, namely filtering error system:

$$\dot{X}(t) = A_{e\rho}X(t) + A_{ed\rho}X(t-d) + A_{ed\tau\rho}\int_{t-\tau}^{t} X(s)ds + E_{e\rho}w(t), \tag{4a}$$

$$X(t) = [\phi(t)^T \quad \phi(t)^T]^T, \quad t \in [-d, 0], \tag{4b}$$

$$e(t) = L_{e\rho}X(t) + L_{ed\rho}X(t-d), \tag{4c}$$

where $X(t) = col\{x(t), \tilde{x}(t)\}$ with $\tilde{x}(t) = x(t) - \hat{x}(t)$ and

$$A_{e\rho} := A_e(\rho) = \begin{bmatrix} A_\rho & 0 \\ A_\rho - F_\rho - G_\rho C_\rho & F_\rho \end{bmatrix},$$

$$A_{ed\rho} := A_{ed}(\rho) = \begin{bmatrix} A_{d\rho} \\ A_{d\rho} - G_\rho C_{d\rho} \end{bmatrix} \hat{J}_{n,n},$$

$$A_{ed\tau\rho} := A_{ed\tau}(\rho) = \begin{bmatrix} A_{d\tau\rho} \\ A_{d\tau\rho} - G_\rho C_{d\tau\rho} \end{bmatrix} \hat{J}_{n,n},$$

$$E_{e\rho} := E_e(\rho) = \begin{bmatrix} E_{1\rho} \\ E_{1\rho} - G_\rho E_{2\rho} \end{bmatrix},$$

$$L_{e\rho} := L_e(\rho) = \begin{bmatrix} 0 & L_\rho \end{bmatrix}, L_{ed\rho} := L_{ed}(\rho) = \begin{bmatrix} L_{d\rho} & 0 \end{bmatrix}.$$

Remark 1: In the case $X(t) = col\{x(t), \hat{x}(t)\}$ and in the absence of the distributed delay in (1), the formulation of the filtering error system (4) can be obtained directly from the results of [25,27].

Definition 1: The robust H_∞ filter of the type (3) is said to guarantee robust disturbance attenuation if

$$\sup_{\rho \in \zeta} \sup_{\|w\|_2 \neq 0} \frac{\|z(t) - \hat{z}(t)\|_2}{\|w(t)\|_2} \le \gamma$$

holds under zero initial condition for all bounded energy disturbances and a prescribed positive value γ.

The main objective of the paper is to seek the state-space parameter-dependent matrices of the robust filter (3) guarantees a prescribed H_∞ performance for the augmented system (4). To investigate the Lyapunov-based stability of the augmented system, one important role will be played by the search for PPDQ functions chosen within the following class.

Definition 2 [10]: We call a polynomially parameter-dependent quadratic (PPDQ) function any quadratic function $x^T(t)S(\rho)x(t)$ on $\Re^n$ such that

$$S(\rho) := (\otimes_{i=m}^{1} \rho_i^{[k]} \otimes I_n)^T S_k (\otimes_{i=m}^{1} \rho_i^{[k]} \otimes I_n)$$

for a certain $S_k \in \Re^{k^m n \times k^m n}$. The integer $k-1$ is called the degree of the PPDQ function $S(\rho)$.

Notice that a very important recent result in [30] provides upper bounds on the degree of the PPDQ function.

3. ROBUST H_∞ FILTERING

In the following, it will be assumed that the robust H_∞ filter (3) is known and the delay-dependent stability conditions will be investigated under which the augmented system (4) is stable and satisfies the prescribed H_∞ performance for all vectors $\rho \in \zeta$.

The approach employed here is to investigate the delay-dependent stability analysis of the augmented system (4) using quadratic Lyapunov-Krasovskii functionals [17,31] in the presence of the disturbance. Now, we choose a Lyapunov-Krasovskii functional candidate for the LPV system (1) as

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t), \tag{5}$$

where

$$V_1(t) = X(t)^T P_\rho X(t),$$

$$V_2(t) = \int_{t-d}^{t} X(\sigma)^T \hat{J}_{n,n}^T Z_{1\rho} \hat{J}_{n,n} X(\sigma) d\sigma,$$

$$V_3(t) = \int_{t-\tau}^{0} \left(\int_{s}^{t} X(\theta)^T \hat{J}_{n,n}^T d\theta \right) Z_{2\rho} \left(\int_{s}^{t} \hat{J}_{n,n} X(\theta) d\theta \right) ds,$$

$$V_4(t) = \int_{0}^{\tau} \int_{t-\beta}^{t} (\alpha - t + \beta) X(\alpha)^T \hat{J}_{n,n}^T Z_{2\rho} \hat{J}_{n,n} X(\alpha) d\alpha d\beta,$$

$$V_5(t) = \int_{-d}^{0} \int_{t+\beta}^{t} \dot{X}(\alpha)^T \hat{J}_{n,n}^T Z_{3\rho} \hat{J}_{n,n} \dot{X}(\alpha) d\alpha d\beta$$

with the positive definite matrices

$$P_\rho := P(\rho) = \begin{bmatrix} P_{1\rho} & 0 \\ * & P_{2\rho} \end{bmatrix} \in \Re^{2n\times 2n}, \quad (6)$$

where the PPDQ functions $\{P_{j\rho}\}_{j=1}^{2}$ and $\{Z_{l\rho}\}_{l=1}^{3}$ satisfying the following representation forms:

$$P_{j\rho} := P_j(\rho) = (\otimes_{i=m}^{1} \rho_i^{[k]} \otimes I_n)^T P_{j,\,k} (\otimes_{i=m}^{1} \rho_i^{[k]} \otimes I_n),$$
$$Z_{l\rho} := Z_l(\rho) = (\otimes_{i=m}^{1} \rho_i^{[k]} \otimes I_n)^T Z_{l,\,k} (\otimes_{i=m}^{1} \rho_i^{[k]} \otimes I_n), \quad (7)$$

with parameter-independent positive definite matrices $\{P_{j,k}, Z_{l,\,k}\} \in \Re^{k^m n \times k^m n}$ of the order $k-1$.

Now, let us define a Hamiltonian function $H(w,\rho)$ as:

$$H(w,\rho) = \frac{d}{d\ t} V(t) + (z-\hat{z})^T (z-\hat{z}) - \gamma^2 w^T w. \quad (8)$$

It is known that the inequity

$$H(w,\rho) < 0 \quad (9)$$

implies the following inequality

$$\int_0^T (z-\hat{z})^T (z-\hat{z}) dt < \gamma^2 \int_0^T w^T w dt$$
$$+V(X(0)) - V(X(T)) < \gamma^2 \int_0^T w^T w dt,$$

that is identical to the performance specification in Definition 1.

Using the Leibniz-Newton formula, i.e., $X(t-d) = X(t) - \int_{t-d}^{t} \dot{X}(\sigma) d\sigma$, for any appropriately dimensioned matrices Y_ρ, T_ρ, and S_ρ, we have

$$2(X(t)^T Y_\rho + X(t-d)^T T_\rho + w(t)^T S_\rho) \times (X(t) - X(t-d) - \int_{t-d}^{t} \dot{X}(\sigma) d\sigma) = 0, \quad (10)$$

which is added to the Hamiltonian function $H(w,\rho)$. On the other hand, for any semi-positive definite matrix

$$X_\rho = \begin{bmatrix} X_{11\rho} & X_{12\rho} & X_{13\rho} & X_{14\rho} \\ * & X_{22\rho} & X_{23\rho} & X_{24\rho} \\ * & * & X_{33\rho} & X_{34\rho} \\ * & * & * & X_{44\rho} \end{bmatrix} \geq 0 \quad (11)$$

the following holds.

$$d\xi^T(t) X_\rho \xi(t) - \int_{t-d}^{t} \xi^T(t) X_\rho \xi(t) d\sigma = 0, \quad (12)$$

where $\xi(t) = col\{X(t), X(t-d), \int_{t-\tau}^{t} X(s)ds, w(t)\}$.

The time derivative of $\{V_i(t)\}_{i=1}^{5}$ along the solution of the system (4) can be given, respectively, by

$$\dot{V}_1(t) = 2X(t)^T P_\rho (A_{e\rho} X(t) + A_{ed\rho} X(t-d) + A_{ed\tau\rho} \int_{t-\tau}^{t} X(s)ds + E_{e\rho} w(t)),$$

$$\dot{V}_2(t) = X(t)^T \hat{J}_{n,n}^T Z_{1\rho} \hat{J}_{n,n} X(t) - X(t-d)^T \hat{J}_{n,n}^T Z_{1\rho} \hat{J}_{n,n} X(t-d),$$

$$\dot{V}_3(t) = -\left(\int_{t-\tau}^{t} X(\theta)^T \hat{J}_{n,n}^T d\theta\right) Z_{2\rho} \left(\int_{t-\tau}^{t} \hat{J}_{n,n} X(\theta) d\theta\right) +2\int_{t-\tau}^{0} X(t)^T \hat{J}_{n,n}^T Z_{2\rho} \int_{s}^{t} \hat{J}_{n,n} X(\theta) d\theta ds$$
$$\leq -\left(\int_{t-\tau}^{t} X(\theta)^T \hat{J}_{n,n}^T d\theta\right) Z_{2\rho} \left(\int_{t-\tau}^{t} \hat{J}_{n,n} X(\theta) d\theta\right) + \frac{\tau^2}{2} X(t)^T \hat{J}_{n,n}^T Z_{2\rho} \hat{J}_{n,n} X(t) + \int_{t-\tau}^{t} (\theta - t + \tau) X(\theta)^T \hat{J}_{n,n}^T Z_{2\rho} \hat{J}_{n,n} X(\theta) d\theta,$$

$$\dot{V}_4(t) = \frac{\tau^2}{2} X(t)^T \hat{J}_{n,n}^T Z_{2\rho} \hat{J}_{n,n} X(t) - \int_{t-\tau}^{t} (\alpha - t + \tau) X(\alpha)^T \hat{J}_{n,n}^T Z_{2\rho} \hat{J}_{n,n} X(\alpha) d\alpha,$$

$$\dot{V}_5(t) = d\dot{X}(t)^T \hat{J}_{n,n}^T Z_{3\rho} \hat{J}_{n,n} \dot{X}(t) - \int_{t-d}^{t} \dot{X}(\alpha)^T \hat{J}_{n,n}^T Z_{3\rho} \hat{J}_{n,n} \dot{X}(\alpha) d\alpha.$$

Then, we have

$$\dot{V}(t) \leq 2X(t)^T P_\rho (A_{e\rho} X(t) + A_{ed\rho} X(t-d) + A_{ed\tau\rho} \int_{t-\tau}^{t} X(s)ds + E_{e\rho} w(t)) + X(t)^T \hat{J}_{n,n}^T \times (Z_{1\rho} + \tau^2 Z_{2\rho}) \hat{J}_{n,n} X(t) - X(t-d)^T \hat{J}_{n,n}^T \times Z_{1\rho} \hat{J}_{n,n} X(t-d) - \left(\int_{t-\tau}^{t} X(\theta)^T \hat{J}_{n,n}^T d\theta\right) Z_{2\rho} \times \left(\int_{t-\tau}^{t} \hat{J}_{n,n} X(\theta) d\theta\right) + d(A_{e\rho} X(t) + A_{ed\rho} X(t-d)$$

$$+A_{ed\tau\rho}\int_{t-\tau}^{t} X(s)ds$$
$$+E_{e\rho}w(t))^T \hat{J}_{n,n}^T Z_{3\rho}\hat{J}_{n,n}(A_{e\rho}X(t)$$
$$+A_{ed\rho}X(t-d)+A_{ed\tau\rho}\int_{t-\tau}^{t} X(s)ds+E_{e\rho}w(t))$$
$$-\int_{t-d}^{t}\dot{X}(\alpha)^T \hat{J}_{n,n}^T Z_{3\rho}\hat{J}_{n,n}\dot{X}(\alpha)d\alpha. \tag{13}$$

From (13) and adding the equations (10) and (12) to (8), results in

$$H(w,\rho)\le \xi(t)^T \Xi_\rho \xi(t) - \int_{t-d}^{t} \zeta(t,\sigma)^T \Omega_\rho \zeta(t,\sigma)d\sigma, \tag{14}$$

where

$$\zeta(t,\sigma)=col\{X(t),X(t-d),\int_{t-\tau}^{t} X(s)ds, w(t), \dot{X}(\sigma)\}$$

and

$$\Omega_\rho=\begin{bmatrix} X_{11\rho} & X_{12\rho} & X_{13\rho} & X_{14\rho} & Y_\rho \\ * & X_{22\rho} & X_{23\rho} & X_{24\rho} & T_\rho \\ * & * & X_{33\rho} & X_{34\rho} & 0 \\ * & * & * & X_{44\rho} & S_\rho \\ * & * & * & * & \hat{J}_{n,n}^T Z_{3\rho}\hat{J}_{n,n} \end{bmatrix},$$

$$\Xi_\rho=\begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & \Delta_{14} \\ * & \Delta_{22} & \Delta_{23} & \Delta_{24} \\ * & * & \Delta_{33} & \Delta_{34} \\ * & * & * & \Delta_{44} \end{bmatrix}$$

with

$$\Delta_{11}=P_\rho A_{e\rho}+A_{e\rho}^T P_\rho + Y_\rho + Y_\rho^T$$
$$+\hat{J}_{n,n}^T(Z_{1\rho}+\tau^2 Z_{2\rho})\hat{J}_{n,n}+L_{e\rho}^T L_{e\rho}$$
$$+dA_{e\rho}^T \hat{J}_{n,n}^T Z_{3\rho}\hat{J}_{n,n}A_{e\rho}+dX_{11\rho},$$
$$\Delta_{13}=P_\rho A_{ed\tau\rho}+dX_{13\rho}+dA_{e\rho}^T \hat{J}_{n,n}^T Z_{3\rho}\hat{J}_{n,n}A_{ed\tau\rho},$$
$$\Delta_{14}=P_\rho E_{e\rho}+S_\rho^T+dX_{14\rho}+dA_{e\rho}^T \hat{J}_{n,n}^T Z_{3\rho}\hat{J}_{n,n}E_{e\rho},$$
$$\Delta_{23}=dX_{23\rho}+dA_{ed\rho}^T \hat{J}_{n,n}^T Z_{3\rho}\hat{J}_{n,n}A_{ed\tau\rho},$$
$$\Delta_{22}=-\hat{J}_{n,n}^T Z_{1\rho}\hat{J}_{n,n}-T_\rho-T_\rho^T+dX_{22\rho}$$
$$+dA_{ed\rho}^T \hat{J}_{n,n}^T Z_{3\rho}\times \hat{J}_{n,n}A_{ed\rho}+L_{ed\rho}^T L_{ed\rho},$$
$$\Delta_{24}=-S_\rho^T+dA_{ed\rho}^T \hat{J}_{n,n}^T Z_{3\rho}\hat{J}_{n,n}E_{e\rho}+dX_{24\rho},$$
$$\Delta_{44}=-\gamma^2 I_s+dX_{44\rho}+dE_{e\rho}^T \hat{J}_{n,n}^T Z_{3\rho}\hat{J}_{n,n}E_{e\rho},$$
$$\Delta_{33}=-\hat{J}_{n,n}^T Z_{2\rho}\hat{J}_{n,n}+dX_{33\rho}+dA_{ed\tau\rho}^T \hat{J}_{n,n}^T Z_{3\rho}\hat{J}_{n,n}A_{ed\tau\rho},$$
$$\Delta_{34}=dX_{34\rho}+dA_{ed\tau\rho}^T \hat{J}_{n,n}^T Z_{3\rho}\hat{J}_{n,n}E_{e\rho}.$$

According to partitioning the existing matrices if $\Xi_\rho<0$ and $\Omega_\rho\ge 0$, then $H(w,\rho)<0$ for any $\xi(t)\ne 0$. Applying the Schur complement Lemma shows that inequality $\Xi_\rho<0$ implies

$$\hat{\Pi}_\rho=\begin{bmatrix} \hat{\Delta}_{11} & \hat{\Delta}_{12} & \hat{\Delta}_{13} & \hat{\Delta}_{14} & d\ A_{e\rho}^T \hat{J}_{n,n}^T\ Z_{3\rho} \\ * & \hat{\Delta}_{22} & d\ X_{23\rho} & -S_\rho^T+d\ X_{24\rho} & d\ A_{ed\rho}^T J_{n,n}^T\ Z_{3\rho} \\ * & * & \hat{\Delta}_{33} & d\ X_{34\rho} & d\ A_{ed\tau\rho}^T J_{n,n}^T\ Z_{3\rho} \\ * & * & * & -\gamma^2 I_s+d\ X_{44\rho} & d\ E_{e\rho}^T \hat{J}_{n,n}^T\ Z_{3\rho} \\ * & * & * & * & -d\ Z_{3\rho} \end{bmatrix}<0 \tag{15}$$

with

$$\hat{\Delta}_{11}=P_\rho A_{e\rho}+A_{e\rho}^T P_\rho+Y_\rho+Y_\rho^T$$
$$+\hat{J}_{n,n}^T(Z_{1\rho}+\tau^2 Z_{2\rho})\hat{J}_{n,n}+d\ X_{11\rho}+L_{e\rho}^T L_{e\rho},$$
$$\hat{\Delta}_{12}=P_\rho A_{ed\rho}-Y_\rho+T^T+d\ X_{12\rho}+L_{e\rho}^T L_{ed\rho},$$
$$\hat{\Delta}_{13}=P_\rho A_{ed\tau\rho}+d\ X_{13\rho},$$
$$\hat{\Delta}_{14}=P_\rho E_{e\rho}+S_\rho^T+d\ X_{14\rho},$$
$$\hat{\Delta}_{22}=-\hat{J}_{n,n}^T Z_{1\rho}\hat{J}_{n,n}-T_\rho-T_\rho^T+dX_{22\rho}+L_{ed\rho}^T L_{ed\rho},$$
$$\hat{\Delta}_{33}=-\hat{J}_{n,n}^T Z_{2\rho}\hat{J}_{n,n}+dX_{33\rho}.$$

Notice that the matrix inequality (15) includes multiplication of filter matrices and Lyapunov matrices. In the literature, more attention has been paid to the problems having this nature, which called bilinear matrix inequality (BMI) problems [32]. In the sequel, it is shown that, by a suitable change of variables, the filtering problem can be converted into convex programming problems written in terms of LMIs.

Remark 2: Considering the parameter-dependent BMI (15) in addition to partitioning the existing matrices P_ρ and $\{Z_{j\rho}\}_{j=1}^3$ and assuming

$$[W_{1\rho},W_{2\rho}]=P_{2\rho}[F_\rho,G_\rho], \tag{16}$$

where $W_{1\rho} \in \Re^{n\times n}$ and $W_{2\rho} \in \Re^{n\times p}$ leads to

$$\Pi_\rho = \begin{bmatrix} \bar{\Delta}_{11} & \bar{\Delta}_{12} & \bar{\Delta}_{13} & \bar{\Delta}_{14} & \bar{\Delta}_{15} & dX_{13,12\rho} & \bar{\Delta}_{17} & dA_\rho^T Z_{3\rho} \\ * & \bar{\Delta}_{22} & \bar{\Delta}_{23} & \bar{\Delta}_{24} & \bar{\Delta}_{25} & dX_{13,22\rho} & \bar{\Delta}_{27} & 0 \\ * & * & \bar{\Delta}_{33} & \bar{\Delta}_{34} & d\ X_{23,11\rho} & dX_{23,12\rho} & \bar{\Delta}_{37} & dA_{d\rho}^T Z_{3\rho} \\ * & * & * & \bar{\Delta}_{44} & dX_{23,21\rho} & dX_{23,22\rho} & \bar{\Delta}_{47} & 0 \\ * & * & * & * & \bar{\Delta}_{55} & dX_{33,12\rho} & d\ X_{34,1\rho} & dA_{d\tau\rho}^T Z_{3\rho} \\ * & * & * & * & * & dX_{33,22\rho} & dX_{34,2\rho} & 0 \\ * & * & * & * & * & * & \bar{\Delta}_{77} & dE_{1\rho}^T Z_{3\rho} \\ * & * & * & * & * & * & * & -d\ Z_{3\rho} \end{bmatrix} < 0 \quad (17)$$

with

$$\bar{\Delta}_{11} = A_\rho^T P_{1\rho} + P_{1\rho}A_\rho + Y_{11\rho} + Y_{11\rho}^T + Z_{1\rho} + \tau^2 Z_{2\rho} + dX_{11,11\rho} + L_\rho^T L_\rho,$$

$$\bar{\Delta}_{12} = C_\rho^T W_{2\rho}^T + Y_{12\rho} + Y_{21\rho}^T + dX_{11,12\rho} - L_\rho^T L_\rho,$$

$$\bar{\Delta}_{13} = P_{1\rho}A_{d\rho} - Y_{11\rho} + T_{11\rho}^T + dX_{12,11\rho} + L_\rho^T L_{d\rho},$$

$$\bar{\Delta}_{14} = -Y_{12\rho} + T_{21}^T + dX_{12,12\rho},$$

$$\bar{\Delta}_{15} = P_{1\rho}A_{d\tau\rho} + dX_{13,11\rho},$$

$$\bar{\Delta}_{17} = P_{1\rho}E_{1\rho} + S_1^T + dX_{14,1\rho},$$

$$\bar{\Delta}_{22} = W_{1\rho} + W_{1\rho}^T + Y_{22\rho} + Y_{22\rho}^T + dX_{11,22\rho} + L_\rho^T L_\rho,$$

$$\bar{\Delta}_{23} = W_{2\rho}C_{d\rho} - Y_{21\rho} + T_{12\rho}^T + dX_{12,21\rho} - L_\rho^T L_{d\rho},$$

$$\bar{\Delta}_{24} = -Y_{22\rho} + T_{12\rho}^T + dX_{12,22\rho},$$

$$\bar{\Delta}_{25} = W_{2\rho}C_{d\tau\rho} + dX_{13,21\rho},$$

$$\bar{\Delta}_{27} = W_{2\rho}E_{2\rho} + S_2^T + dX_{14,2\rho},$$

$$\bar{\Delta}_{33} = -Z_{1\rho} - T_{11\rho} - T_{11\rho}^T + dX_{22,11\rho} + L_{d\rho}^T L_{d\rho},$$

$$\bar{\Delta}_{34} = -T_{12\rho} - T_{21\rho}^T + dX_{22,12\rho},$$

$$\bar{\Delta}_{37} = -S_{1\rho}^T + dX_{24,1\rho},$$

$$\bar{\Delta}_{44} = -T_{22\rho} - T_{22\rho}^T + dX_{22,22\rho},$$

$$\bar{\Delta}_{47} = -S_{2\rho}^T + hX_{24,2\rho},$$

$$\bar{\Delta}_{55} = -Z_{2\rho} + dX_{33,11\rho},$$

$$\bar{\Delta}_{77} = -\gamma^2 I_s + dX_{44\rho},$$

where $Y_\rho = \begin{bmatrix} Y_{11\rho} & Y_{12\rho} \\ Y_{21\rho} & Y_{22\rho} \end{bmatrix}$, $T_\rho = \begin{bmatrix} T_{11\rho} & T_{12\rho} \\ T_{21\rho} & T_{22\rho} \end{bmatrix}$, $S_\rho = col\{S_{1\rho}, S_{2\rho}\}$, $X_{ij\rho} = \begin{bmatrix} X_{ij,11\rho} & X_{ij,12\rho} \\ * & X_{ij,22\rho} \end{bmatrix}$, and $X_{i4\rho} = col\{X_{i4,1\rho}, X_{i4,2\rho}\}$ for $i,j = 1,2,3$.

Theorem 1: The filtering error system (4) obtained from the interconnection of the plant (1) and the filter (3) is stable and achieves the H_∞ performance for a given performance bound γ in the sense of Definition 1 if there exist the parameter-dependent positive definite matrices $\{P_{i\rho}\}_{i=1}^2$ and $\{Z_{j\rho}\}_{j=1}^3$, a symmetric semi-positive definite matrix X_ρ, the parameter-dependent matrixes $\{W_{i\rho}\}_{i=1}^2$ and any appropriately dimensioned matrices Y_ρ, T_ρ and S_ρ such that the parameter-dependent LMIs $\Pi_\rho < 0$ and $\Omega_\rho \geq 0$ are satisfied, respectively.

Remark 3: It is noted that the proposed delay-dependent stability conditions in Theorem 1 is obtained without resorting to any model transformations [17] and bounding techniques [25,27] for some cross terms, thus reducing the conservatism in the derivation of the stability conditions.

4. PARAMETER-DEPENDENT LMI RELAXATIONS

This section is devoted to solve the parameter-dependent LMIs to finding the parameter-dependent state-space matrices F_ρ and G_ρ. These parameter-dependent LMIs are corresponded to infinite-dimensional convex problems. In the literature, there are some attempts to obtain a finite-dimensional optimization problem such the parameter-dependent Lyapunov functions are approximated using a finite set of basis functions [19,20,33,34]. The main approach employed here is using the PPDQ functions as the basis functions to relax parameter-dependent LMIs into parameter-independent LMI forms by utilizing some Lagrange multiplier matrices.

Lemma 1: Let the degree of the PPDQ function $P_{1\rho}$ be $k-1$. A PPDQ function of degree k for parameter-dependent matrix $P_{1\rho}\mathrm{T}_\rho$ is given by

$$P_{1\rho}\mathrm{T}_\rho := (\otimes_{i=m}^1 \rho_i^{[k+1]} \otimes I_n)^T S_k (\otimes_{i=m}^1 \rho_i^{[k+1]} \otimes I_q),$$

where $T_\rho = T_0 + \sum_{i=1}^{m} \rho_i T_i$ and $T_i \in \Re^{n\times q}$, then the parameter-independent matrix $S_k \in \Re^{((k+1)^m n)\times((k+1)^m q)}$ which depends on the parameter-independent matrix P_{1k} linearly is defined as

$$S_k = (\hat{J}_k^{m\otimes} \otimes I_n)^T P_{1,k} \times \left((\hat{J}_k^{m\otimes} \otimes T_0) + \sum_{i=1}^{m} (\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(i-1)\otimes} \otimes T_i) \right).$$

Proof: See [13,22]. □

According to Lemma 1 for the parameter-dependent matrices $E_{1\rho}$, A_ρ, $A_{d\rho}$ and $A_{d\tau\rho}$, we obtain

$$P_{i\rho} A_\rho = (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n)^T F_{1,k}^{(i)} (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n),$$
$$P_{i\rho} A_{d\rho} = (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n)^T F_{1d,k}^{(i)} (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n),$$
$$P_{i\rho} A_{d\tau\rho} = (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n)^T F_{1d\tau,k}^{(i)} (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n),$$
$$P_{i\rho} E_{1\rho} = (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n)^T \Xi_{1,k}^{(i)} (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_s),$$
$$Z_{3\rho} A_\rho = (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n)^T F_{2,k} (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n),$$
$$Z_{3\rho} A_{d\rho} = (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n)^T F_{2d,k} (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n),$$
$$Z_{3\rho} A_{d\tau\rho} = (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n)^T F_{2d\tau,k} (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n),$$
$$Z_{3\rho} E_{1\rho} = (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n)^T \Xi_{2,k} (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_s),$$

where $F_{1,k}^{(i)}$, $F_{1d,k}^{(i)}$, $F_{1d\tau,k}^{(i)}$, $\Xi_{1,k}^{(i)}$, $F_{2,k}$, $F_{2d,k}$, $F_{2d\tau,k}$ and $\Xi_{2,k}$ are represented in the following forms:

$$F_{1,k}^{(i)} = (\hat{J}_k^{m\otimes} \otimes I_n)^T P_{i,k} \times \left(\hat{J}_k^{m\otimes} \otimes A_0 + \sum_{j=1}^{m} \hat{J}_k^{(m-j)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(j-1)\otimes} \otimes A_j \right),$$

$$F_{1d,k}^{(i)} = (\hat{J}_k^{m\otimes} \otimes I_n)^T P_{i,k} \times \left(\hat{J}_k^{m\otimes} \otimes A_{0d} + \sum_{j=1}^{m} \hat{J}_k^{(m-j)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(j-1)\otimes} \otimes A_{jd} \right),$$

$$F_{1d\tau,k}^{(i)} = (\hat{J}_k^{m\otimes} \otimes I_n)^T P_{i,k} \times \left(\hat{J}_k^{m\otimes} \otimes A_{0d\tau} + \sum_{j=1}^{m} \hat{J}_k^{(m-j)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(j-1)\otimes} \otimes A_{jd\tau} \right),$$

$$\Xi_{1,k}^{(i)} = (\hat{J}_k^{m\otimes} \otimes I_n)^T P_{i,k} \times \left(\hat{J}_k^{m\otimes} \otimes E_{01} + \sum_{j=1}^{m} \hat{J}_k^{(m-j)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(j-1)\otimes} \otimes E_{j1} \right),$$

$$F_{2,k} = (\hat{J}_k^{m\otimes} \otimes I_n)^T Z_{3,k} \times \left(\hat{J}_k^{m\otimes} \otimes A_0 + \sum_{j=1}^{m} \hat{J}_k^{(m-j)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(j-1)\otimes} \otimes A_j \right),$$

$$F_{2d,k} = (\hat{J}_k^{m\otimes} \otimes I_n)^T Z_{3,k} \times \left(\hat{J}_k^{m\otimes} \otimes A_{0d} + \sum_{j=1}^{m} \hat{J}_k^{(m-j)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(j-1)\otimes} \otimes A_{jd} \right),$$

$$F_{2d\tau,k} = (\hat{J}_k^{m\otimes} \otimes I_n)^T Z_{3,k} \times \left(\hat{J}_k^{m\otimes} \otimes A_{0d\tau} + \sum_{j=1}^{m} \hat{J}_k^{(m-j)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(j-1)\otimes} \otimes A_{jd\tau} \right),$$

$$\Xi_{2,k} = (\hat{J}_k^{m\otimes} \otimes I_n)^T Z_{3,k} \times \left(\hat{J}_k^{m\otimes} \otimes E_{01} + \sum_{j=1}^{m} \hat{J}_k^{(m-j)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(j-1)\otimes} \otimes E_{j1} \right).$$

Remark 4: For the matrix $R_{1\rho} := A_\rho^T P_{1\rho} + P_{1\rho} A_\rho$ the PPDQ function of degree k is given by

$$R_{1\rho} = (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n)^T R_{1,k} (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n),$$

and from Lemma 1, the positive definite matrices $R_{1,k} \in \Re^{(k+1)^m n \times (k+1)^m n}$ which depends on the parameter-independent matrix P_k linearly is obtained as

$$R_{1,k} = \left(\hat{J}_k^{m\otimes} \otimes A_0 + \sum_{i=1}^{m} \hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(i-1)\otimes} \otimes A_i \right)^T P_{1,k} \times (\hat{J}_k^{m\otimes} \otimes I_n) + (\hat{J}_k^{m\otimes} \otimes I_n)^T P_{1,k} \times \left(\hat{J}_k^{m\otimes} \otimes A_0 + \sum_{i=1}^{m} \hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(i-1)\otimes} \otimes A_i \right).$$

The parameter-dependent matrices $\{W_{j\rho}\}_{j=1}^{2}$ in (16) can be expressed in the forms

$$W_{1\rho} = (\otimes_{i=m}^{1} \rho_i^{[k]} \otimes I_n)^T W_{1,k} \; (\otimes_{i=m}^{1} \rho_i^{[k]} \otimes I_n),$$
$$W_{2\rho} = (\otimes_{i=m}^{1} \rho_i^{[k]} \otimes I_n)^T W_{2,k} \; (\otimes_{i=m}^{1} \rho_i^{[k]} \otimes I_p),$$

with $W_{1,k} \in \Re^{(k^m n)\times(k^m n)}$ and $W_{2,k} \in \Re^{(k^m n)\times(k^m p)}$. Then, we have

$$W_{2\rho} C_\rho = (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n)^T \bar{W}_{2,k} (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_p),$$
$$W_{2\rho} C_{d\rho} = (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n)^T \bar{W}_{2d,k} (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_p),$$
$$W_{2\rho} C_{d\tau\rho} = (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n)^T \bar{W}_{2d\tau,k} (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_p),$$

$$W_{2\rho}E_{2\rho} = (\otimes_{i=m}^{1}\rho_i^{[k+1]}\otimes I_n)^T \tilde{W}_{2,k}(\otimes_{i=m}^{1}\rho_i^{[k+1]}\otimes I_p),$$

where $\{\bar{W}_{2,k},\bar{W}_{2d,k},\bar{W}_{2d\tau,k},\tilde{W}_{2,k}\}\in\Re^{((k+1)^m n)\times((k+1)^m p)}$ are defined, respectively, as

$$\bar{W}_{2,k} = (\hat{J}_k^{m\otimes}\otimes I_n)^T W_{2,k}\times \left(\hat{J}_k^{m\otimes}\otimes C_0 + \sum_{i=1}^{m}\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes\hat{J}_k^{(i-1)\otimes}\otimes C_i\right),$$

$$\bar{W}_{2d,k} = (\hat{J}_k^{m\otimes}\otimes I_n)^T W_{2,k}\times \left(\hat{J}_k^{m\otimes}\otimes C_{0d} + \sum_{i=1}^{m}\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes\hat{J}_k^{(i-1)\otimes}\otimes C_{id}\right),$$

$$\bar{W}_{2d\tau,k} = (\hat{J}_k^{m\otimes}\otimes I_n)^T W_{2,k}\times \left(\hat{J}_k^{m\otimes}\otimes C_{0d\tau} + \sum_{i=1}^{m}\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes\hat{J}_k^{(i-1)\otimes}\otimes C_{id\tau}\right),$$

$$\tilde{W}_{2,k} = (\hat{J}_k^{m\otimes}\otimes I_n)^T W_{2,k}\times \left(\hat{J}_k^{m\otimes}\otimes E_{02} + \sum_{i=1}^{m}\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes\hat{J}_k^{(i-1)\otimes}\otimes E_{i2}\right).$$

Similarly, the matrices $L_\rho^T L_\rho$, $L_{d\rho}^T L_{d\rho}$ and I_s can be represented, respectively, by

$$L_\rho^T L_\rho = (\otimes_{i=m}^{1}\rho_i^{[k+1]}\otimes I_n)^T(\hat{J}_k^{m\otimes}\otimes I_n)^T\bar{L}_k(\hat{J}_k^{m\otimes}\otimes I_n)\times(\otimes_{i=m}^{1}\rho_i^{[k+1]}\otimes I_n),$$

$$L_{d\rho}^T L_{d\rho} = (\otimes_{i=m}^{1}\rho_i^{[k+1]}\otimes I_n)^T(\hat{J}_k^{m\otimes}\otimes I_n)^T\bar{L}_{d,k}(\hat{J}_k^{m\otimes}\otimes I_n)\times(\otimes_{i=m}^{1}\rho_i^{[k+1]}\otimes I_n),$$

$$I_s = (\otimes_{i=m}^{1}\rho_i^{[k+1]}\otimes I_s)^T(\hat{J}_k^{m\otimes}\otimes I_s)^T\bar{I}_k(\hat{J}_k^{m\otimes}\otimes I_s)\times(\otimes_{i=m}^{1}\rho_i^{[k+1]}\otimes I_s),$$

where $\bar{L}_k\in\Re^{(k^m n)\times(k^m n)}$, $\bar{L}_{d,k}\in\Re^{(k^m n)\times(k^m n)}$ and $\bar{I}_k\in\Re^{(k^m s)\times(k^m s)}$ are given by

$$\bar{L}_k = Block\ diag\left(\begin{bmatrix}L_0^T\\ L_1^T\\ \vdots\\ L_m^T\end{bmatrix}\begin{bmatrix}L_0 & L_1 & \cdots & L_m\end{bmatrix},\ \underbrace{0_n,\cdots,0_n}_{(k^m-m-1)\ elements}\right),$$

$$\bar{L}_{d,k} = Block\ diag\left(\begin{bmatrix}L_{0d}^T\\ L_{1d}^T\\ \vdots\\ L_{md}^T\end{bmatrix}\begin{bmatrix}L_{0d} & L_{1d} & \cdots & L_{md}\end{bmatrix},\ \underbrace{0_n,\cdots,0_n}_{(k^m-m-1)\ elements}\right),$$

$$\bar{I}_k = diag\left(I_s,\ \underbrace{0_s,\cdots,0_s}_{(k^m-1)\ elements}\right).$$

We are now in the position to state our main results in the following theorem.

Theorem 2: Let the positive integer $k-1$ as the degree of the PPDQ functions be given. Consider the LPV system (1) with the discrete and distributed time-delay parameters d and τ, respectively. For a given performance bound γ, if there exist the set of parameter-independent matrices $\{W_{1,k}, W_{2,k}, X_{11,12k}, X_{12,11k}, X_{12,12k}, X_{12,21k}, X_{12,22k}, X_{13,11k}, X_{13,12k}, X_{13,21k}, X_{13,22k}, X_{22,12k}, X_{23,11k}, X_{23,12k}, X_{23,22k}, X_{33,12k}, X_{14,1k}, X_{14,2k}, X_{24,1k}, X_{24,2k}, X_{34,1k}, X_{34,2k}, Y_{11,k}, Y_{12,k}, Y_{21,k}, Y_{22,k}, T_{11,k}, T_{12,k}, T_{21,k}, T_{22,k}, S_{1,k}, S_{2,k}\}$, the set of parameter-independent positive definite matrices $\{P_{1,k}, P_{2,k}, Z_{1,k}, Z_{2,k}, Z_{3,k}, X_{11,11k}, X_{11,22k}, X_{22,11k}, X_{22,22k}, X_{33,11k}, X_{33,22k}, X_{44k}\}$ and the set of positive definite Lagrange multipliers $\{\hat{Q}_{i,k}^{(1)},\cdots,\hat{Q}_{i,k}^{(8)},\tilde{Q}_{i,k}^{(1)},\cdots,\tilde{Q}_{i,k}^{(8)}\}_{i=1}^{m}$ to the following LMIs,

$$\Omega_{m,k} = \begin{bmatrix}
\Psi_{11} & X_{11,12k} & X_{12,11k} & X_{12,12k} & X_{13,11k} & X_{13,12k} & X_{14,1k} & Y_{11,k} & Y_{12,k}\\
* & \Psi_{22} & X_{12,21k} & X_{12,22k} & X_{13,21k} & X_{13,22k} & X_{14,2k} & Y_{21,k} & Y_{22,k}\\
* & * & \Psi_{33} & X_{22,12k} & X_{23,11k} & X_{23,12k} & X_{24,1k} & T_{11,k} & T_{12,k}\\
* & * & * & \Psi_{44} & X_{23,21k} & X_{23,22k} & X_{24,2k} & T_{21,k} & T_{22,k}\\
* & * & * & * & \Psi_{55} & X_{33,12k} & X_{34,1k} & 0 & 0\\
* & * & * & * & * & \Psi_{66} & X_{34,2k} & 0 & 0\\
* & * & * & * & * & * & \Psi_{77} & S_{1,k} & S_{2,k}\\
* & * & * & * & * & * & * & \Psi_{88} & 0\\
* & * & * & * & * & * & * & * & 0
\end{bmatrix} \geq 0, \quad (18)$$

$$\Pi_{m,k}=\begin{bmatrix}\Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & \Sigma_{15} & \begin{pmatrix}d(\hat{J}_k^{m\otimes}\otimes I_n)^T X_{13,12k}\\(\hat{J}_k^{m\otimes}\otimes I_n)\end{pmatrix} & \Sigma_{17} & dF_{2,k}^T\\ * & \Sigma_{22} & \Sigma_{23} & \Sigma_{24} & \Sigma_{25} & \begin{pmatrix}d(\hat{J}_k^{m\otimes}\otimes I_n)^T X_{13,22k}\\(\hat{J}_k^{m\otimes}\otimes I_n)\end{pmatrix} & \Sigma_{27} & 0\\ * & * & \Sigma_{33} & \Sigma_{34} & \begin{pmatrix}d(\hat{J}_k^{m\otimes}\otimes I_n)^T X_{23,11k}\\(\hat{J}_k^{m\otimes}\otimes I_n)\end{pmatrix} & \begin{pmatrix}d(\hat{J}_k^{m\otimes}\otimes I_n)^T X_{23,12k}\\(\hat{J}_k^{m\otimes}\otimes I_n)\end{pmatrix} & \Sigma_{37} & dF_{2d,k}^T\\ * & * & * & \Sigma_{44} & \begin{pmatrix}d(\hat{J}_k^{m\otimes}\otimes I_n)^T X_{23,21k}\\(\hat{J}_k^{m\otimes}\otimes I_n)\end{pmatrix} & \begin{pmatrix}d(\hat{J}_k^{m\otimes}\otimes I_n)^T X_{23,22k}\\(\hat{J}_k^{m\otimes}\otimes I_n)\end{pmatrix} & \Sigma_{47} & 0\\ * & * & * & * & \Sigma_{55} & \begin{pmatrix}d(\hat{J}_k^{m\otimes}\otimes I_n)^T X_{33,12k}\\(\hat{J}_k^{m\otimes}\otimes I_n)\end{pmatrix} & \Sigma_{57} & dF_{2d\tau,k}^T\\ * & * & * & * & * & \Sigma_{66} & \Sigma_{67} & 0\\ * & * & * & * & * & * & \Sigma_{77} & d\Xi_{2,k}^T\\ * & * & * & * & * & * & * & \Sigma_{88}\end{bmatrix}<0,\quad(19)$$

where

$$\Psi_{11}=X_{11,11k}-\sum_{i=1}^{m}\left(\hat{J}_k^{(m-i+1)\otimes}\otimes I_{(k+1)^{i-1}n}\right)^T\times\hat{Q}_{i,k}^{(1)}\left(\hat{J}_k^{(m-i+1)\otimes}\otimes I_{(k+1)^{i-1}n}\right)+\sum_{i=1}^{m}\left(\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes I_{(k+1)^{i-1}n}\right)^T\times\hat{Q}_{i,k}^{(1)}\left(\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes I_{(k+1)^{i-1}n}\right),$$

$$\Psi_{22}=X_{11,22k}-\sum_{i=1}^{m}\left(\hat{J}_k^{(m-i+1)\otimes}\otimes I_{(k+1)^{i-1}n}\right)^T\times\hat{Q}_{i,k}^{(2)}\left(\hat{J}_k^{(m-i+1)\otimes}\otimes I_{(k+1)^{i-1}n}\right)+\sum_{i=1}^{m}\left(\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes I_{(k+1)^{i-1}n}\right)^T\times\hat{Q}_{i,k}^{(2)}\left(\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes I_{(k+1)^{i-1}n}\right),$$

$$\Psi_{33}=X_{22,11k}-\sum_{i=1}^{m}\left(\hat{J}_k^{(m-i+1)\otimes}\otimes I_{(k+1)^{i-1}n}\right)^T\times\hat{Q}_{i,k}^{(3)}\left(\hat{J}_k^{(m-i+1)\otimes}\otimes I_{(k+1)^{i-1}n}\right)+\sum_{i=1}^{m}\left(\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes I_{(k+1)^{i-1}n}\right)^T\times\hat{Q}_{i,k}^{(3)}\left(\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes I_{(k+1)^{i-1}n}\right),$$

$$\Psi_{44}=X_{22,22k}-\sum_{i=1}^{m}\left(\hat{J}_k^{(m-i+1)\otimes}\otimes I_{(k+1)^{i-1}n}\right)^T\times\hat{Q}_{i,k}^{(4)}\left(\hat{J}_k^{(m-i+1)\otimes}\otimes I_{(k+1)^{i-1}n}\right)+\sum_{i=1}^{m}\left(\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes I_{(k+1)^{i-1}n}\right)^T\times\hat{Q}_{i,k}^{(4)}\left(\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes I_{(k+1)^{i-1}n}\right),$$

$$\Psi_{55}=X_{33,11k}-\sum_{i=1}^{m}\left(\hat{J}_k^{(m-i+1)\otimes}\otimes I_{(k+1)^{i-1}s}\right)^T\times\hat{Q}_{i,k}^{(5)}\left(\hat{J}_k^{(m-i+1)\otimes}\otimes I_{(k+1)^{i-1}s}\right)+\sum_{i=1}^{m}\left(\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes I_{(k+1)^{i-1}s}\right)^T\times\hat{Q}_{i,k}^{(5)}\left(\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes I_{(k+1)^{i-1}s}\right),$$

$$\Psi_{66}=X_{33,22k}-\sum_{i=1}^{m}\left(\hat{J}_k^{(m-i+1)\otimes}\otimes I_{(k+1)^{i-1}n}\right)^T\times\hat{Q}_{i,k}^{(6)}\left(\hat{J}_k^{(m-i+1)\otimes}\otimes I_{(k+1)^{i-1}n}\right)+\sum_{i=1}^{m}\left(\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes I_{(k+1)^{i-1}n}\right)^T\times\hat{Q}_{i,k}^{(6)}\left(\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes I_{(k+1)^{i-1}n}\right),$$

$$\Psi_{77}=X_{44k}-\sum_{i=1}^{m}\left(\hat{J}_k^{(m-i+1)\otimes}\otimes I_{(k+1)^{i-1}n}\right)^T\times\hat{Q}_{i,k}^{(7)}\left(\hat{J}_k^{(m-i+1)\otimes}\otimes I_{(k+1)^{i-1}n}\right)$$

$$
\begin{aligned}
&+\sum_{i=1}^{m}\left(\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes I_{(k+1)^{i-1}n}\right)^T\\
&\times\hat{Q}_{i,k}^{(7)}\left(\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes I_{(k+1)^{i-1}n}\right),
\end{aligned}
$$

$$
\begin{aligned}
\Psi_{88}={}&Z_{3k}-\sum_{i=1}^{m}\left(\hat{J}_k^{(m-i+1)\otimes}\otimes I_{(k+1)^{i-1}n}\right)^T\\
&\times\hat{Q}_{i,k}^{(8)}\left(\hat{J}_k^{(m-i+1)\otimes}\otimes I_{(k+1)^{i-1}n}\right)\\
&+\sum_{i=1}^{m}\left(\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes I_{(k+1)^{i-1}n}\right)^T\\
&\times\hat{Q}_{i,k}^{(8)}\left(\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes I_{(k+1)^{i-1}n}\right),
\end{aligned}
$$

$$
\begin{aligned}
\Sigma_{11}={}&R_{1,k}+(\hat{J}_k^{m\otimes}\otimes I_n)^T(Z_{1,k}+\tau^2Z_{2,k}\\
&+dX_{11,11k})(\hat{J}_k^{m\otimes}\otimes I_n)\\
&+\sum_{i=1}^{m}\left(\hat{J}_k^{(m-i+1)\otimes}\otimes I_{(k+1)^{i-1}n}\right)^T\\
&\times\tilde{Q}_{i,k}^{(1)}\left(\hat{J}_k^{(m-i+1)\otimes}\otimes I_{(k+1)^{i-1}n}\right)\\
&-\sum_{i=1}^{m}\left(\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes I_{(k+1)^{i-1}n}\right)^T\\
&\times\tilde{Q}_{i,k}^{(1)}\left(\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes I_{(k+1)^{i-1}n}\right),
\end{aligned}
$$

$$
\begin{aligned}
\Sigma_{12}={}&F_{1,k}^{(2)}-\bar{W}_{2,k}^T+(\hat{J}_k^{m\otimes}\otimes I_n)^T(-W_{1,k}^T+Y_{12,k}\\
&+Y_{21,k}^T+dX_{11,12k})(\hat{J}_k^{m\otimes}\otimes I_n),
\end{aligned}
$$

$$
\begin{aligned}
\Sigma_{13}={}&F_{1d,k}^{(1)}+(\hat{J}_k^{m\otimes}\otimes I_n)^T(-Y_{11,k}+T_{11,k}^T\\
&+dX_{12,11k})(\hat{J}_k^{m\otimes}\otimes I_n),
\end{aligned}
$$

$$
\Sigma_{14}=(\hat{J}_k^{m\otimes}\otimes I_n)^T(-Y_{12,k}+T_{21,k}^T+dX_{12,12k})(\hat{J}_k^{m\otimes}\otimes I_n),
$$

$$
\Sigma_{15}=F_{1d\tau,k}^{(1)}+d(\hat{J}_k^{m\otimes}\otimes I_n)^TX_{13,11k}(\hat{J}_k^{m\otimes}\otimes I_n),
$$

$$
\Sigma_{17}=\Xi_{1,k}^{(1)}+(\hat{J}_k^{m\otimes}\otimes I_n)^T(S_{1,k}^T+dX_{14,1k})(\hat{J}_k^{m\otimes}\otimes I_s),
$$

$$
\begin{aligned}
\Sigma_{22}={}&(\hat{J}_k^{m\otimes}\otimes I_n)^T(-W_{1,k}-W_{1,k}^T+\bar{L}_k+Y_{22,k}\\
&+Y_{22,k}^T+dX_{11,22k})\times(\hat{J}_k^{m\otimes}\otimes I_n)\\
&+\sum_{i=1}^{m}\left(\hat{J}_k^{(m-i+1)\otimes}\otimes I_{(k+1)^{i-1}n}\right)^T\\
&\times\tilde{Q}_{i,k}^{(2)}\left(\hat{J}_k^{(m-i+1)\otimes}\otimes I_{(k+1)^{i-1}n}\right)\\
&-\sum_{i=1}^{m}\left(\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes I_{(k+1)^{i-1}n}\right)^T\\
&\times\tilde{Q}_{i,k}^{(2)}\left(\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes I_{(k+1)^{i-1}n}\right),
\end{aligned}
$$

$$
\begin{aligned}
\Sigma_{23}={}&F_{1d,k}^{(2)}-\bar{W}_{2d,k}+(\hat{J}_k^{m\otimes}\otimes I_n)^T\\
&\times(-Y_{21,k}+T_{12,k}^T+dX_{12,21k}+\tilde{L}_{d,k})(\hat{J}_k^{m\otimes}\otimes I_n),
\end{aligned}
$$

$$
\Sigma_{24}=(\hat{J}_k^{m\otimes}\otimes I_n)^T(T_{12,k}^T-Y_{22,k}+dX_{12,22k})(\hat{J}_k^{m\otimes}\otimes I_n),
$$

$$
\Sigma_{25}=F_{1d\tau,k}^{(2)}-\bar{W}_{2d\tau,k}+d(\hat{J}_k^{m\otimes}\otimes I_n)^TX_{13,21k}(\hat{J}_k^{m\otimes}\otimes I_n),
$$

$$
\Sigma_{27}=\Xi_{1,k}^{(2)}-\tilde{W}_{2,k}+(\hat{J}_k^{m\otimes}\otimes I_n)^T(S_{2,k}^T+dX_{14,2k})(\hat{J}_k^{m\otimes}\otimes I_s),
$$

$$
\begin{aligned}
\Sigma_{33}={}&(\hat{J}_k^{m\otimes}\otimes I_n)^T(-Z_{1,k}-T_{11,k}-T_{11,k}^T+dX_{22,11k}\\
&+\bar{L}_{d,k})(\hat{J}_k^{m\otimes}\otimes I_n)\\
&+\sum_{i=1}^{m}\left(\hat{J}_k^{(m-i+1)\otimes}\otimes I_{(k+1)^{i-1}n}\right)^T\\
&\times\tilde{Q}_{i,k}^{(3)}\left(\hat{J}_k^{(m-i+1)\otimes}\otimes I_{(k+1)^{i-1}n}\right)\\
&-\sum_{i=1}^{m}\left(\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes I_{(k+1)^{i-1}n}\right)^T\\
&\times\tilde{Q}_{i,k}^{(3)}\left(\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes I_{(k+1)^{i-1}n}\right),
\end{aligned}
$$

$$
\Sigma_{34}=-(\hat{J}_k^{m\otimes}\otimes I_n)^T(T_{12,k}+T_{21,k}^T-dX_{22,12k})(\hat{J}_k^{m\otimes}\otimes I_n),
$$

$$
\Sigma_{37}=(\hat{J}_k^{m\otimes}\otimes I_n)^T(-S_{1,k}^T+dX_{24,1k})(\hat{J}_k^{m\otimes}\otimes I_s),
$$

$$
\begin{aligned}
\Sigma_{44}={}&-(\hat{J}_k^{m\otimes}\otimes I_n)^T(T_{22,k}+T_{22,k}^T-dX_{22,22k})(\hat{J}_k^{m\otimes}\otimes I_n)\\
&+\sum_{i=1}^{m}\left(\hat{J}_k^{(m-i+1)\otimes}\otimes I_{(k+1)^{i-1}n}\right)^T\\
&\times\tilde{Q}_{i,k}^{(4)}\left(\hat{J}_k^{(m-i+1)\otimes}\otimes I_{(k+1)^{i-1}n}\right)\\
&-\sum_{i=1}^{m}\left(\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes I_{(k+1)^{i-1}n}\right)^T\\
&\times\tilde{Q}_{i,k}^{(4)}\left(\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes I_{(k+1)^{i-1}n}\right),
\end{aligned}
$$

$$
\Sigma_{47}=(\hat{J}_k^{m\otimes}\otimes I_n)^T(-S_{2,k}^T+dX_{24,2k})(\hat{J}_k^{m\otimes}\otimes I_s),
$$

$$
\begin{aligned}
\Sigma_{55}={}&(\hat{J}_k^{m\otimes}\otimes I_n)^T(-Z_{2,k}+dX_{33,11k})(\hat{J}_k^{m\otimes}\otimes I_n)\\
&+\sum_{i=1}^{m}\left(\hat{J}_k^{(m-i+1)\otimes}\otimes I_{(k+1)^{i-1}n}\right)^T\\
&\times\tilde{Q}_{i,k}^{(5)}\left(\hat{J}_k^{(m-i+1)\otimes}\otimes I_{(k+1)^{i-1}n}\right)\\
&-\sum_{i=1}^{m}\left(\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes I_{(k+1)^{i-1}n}\right)^T\\
&\times\tilde{Q}_{i,k}^{(5)}\left(\hat{J}_k^{(m-i)\otimes}\otimes\tilde{J}_k\otimes I_{(k+1)^{i-1}n}\right),
\end{aligned}
$$

$$
\Sigma_{57}=d(\hat{J}_k^{m\otimes}\otimes I_n)^TX_{34,1k}(\hat{J}_k^{m\otimes}\otimes I_s),
$$

$$
\Sigma_{67}=d(\hat{J}_k^{m\otimes}\otimes I_n)^TX_{34,2k}(\hat{J}_k^{m\otimes}\otimes I_s),
$$

$$\Sigma_{66} = d(\hat{J}_k^{m\otimes} \otimes I_n)^T X_{33,22k} (\hat{J}_k^{m\otimes} \otimes I_n) + \sum_{i=1}^{m} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n} \right)^T \times \tilde{Q}_{i,k}^{(6)} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n} \right) - \sum_{i=1}^{m} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n} \right)^T \times \tilde{Q}_{i,k}^{(6)} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n} \right),$$

$$\Sigma_{77} = (\hat{J}_k^{m\otimes} \otimes I_s)^T (-\gamma^2 \overline{I}_s + dX_{44,k}) (\hat{J}_k^{m\otimes} \otimes I_s) + \sum_{i=1}^{m} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}s} \right)^T \times \tilde{Q}_{i,k}^{(7)} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}s} \right) - \sum_{i=1}^{m} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}s} \right)^T \times \tilde{Q}_{i,k}^{(7)} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}s} \right),$$

$$\Sigma_{88} = -d(\hat{J}_k^{m\otimes} \otimes I_n)^T Z_{3,k} (\hat{J}_k^{m\otimes} \otimes I_n) + \sum_{i=1}^{m} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n} \right)^T \times \tilde{Q}_{i,k}^{(8)} \left(\hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n} \right) - \sum_{i=1}^{m} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n} \right)^T \times \tilde{Q}_{i,k}^{(8)} \left(\hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes I_{(k+1)^{i-1}n} \right),$$

then the state-space parameter-dependent matrices for the robust H_∞ filter of the type (3) which achieve both the asymptotic stability and H_∞ performance in the sense of Definition 1 are given by

$$\left[F_\rho, G_\rho \right] = P_{2\rho}^{-1} \left[W_{1\rho}, W_{2\rho} \right]. \tag{20}$$

Notice that the conditions (18) and (19) are sufficient conditions to both asymptotic stability and H_∞ performance in the sense of Definition 1. Moreover, Theorem 2 gives a sub-optimal solution to the filtering and this result can be reformulated as an optimal filter by solving the following convex optimization problem

$$\text{Min } \lambda$$

$$\text{subject to (18) and (19) with } \lambda := \gamma^2.$$

Remark 5: It is noting that the number of the matrices to be determined in the LMIs (18) and (19) is $16m + 44$. It is also observed that the parameter-independent LMIs (18) and (19) are linear in the set of matrices, which are calculated independently from the vector ρ. Of course, the high dimension of the resulting LMIs will increase the computational complexity of the proposed approach to some extent. The LMIs can be solved by the Matlab LMI Control Toolbox [35], which implements state-of-the-art interior-point algorithms and is significantly faster than classical convex optimization algorithms [36].

Remark 6: A new set of matrices verifying $\Omega_{m,k+1} \geq 0$, $\Pi_{m,k+1} < 0$ and $\Phi_{m,k+1} > 0$ can be generated, with index $k+1$ instead of k in (18) and (19), respectively. In this case, the solvability of $\Omega_{m,k} \geq 0$, $\Pi_{m,k} < 0$ and $\Phi_{m,k} > 0$ implies the same property for the larger values of the index k.

Remark 7: The PPDQ function has polynomial dependence on the vector ρ of a known degree and can be used to derive exact conditions for the stability of LPV systems. It should be pointed out that the question of the lowest degree of the PPDQ function is still open. In practical applications the size of the least k, for which the LMIs are solvable could be difficult to obtain, but only an upper bound is available [30]. In this case the proposed approach in this paper can be applied in robust H_∞ filter design problem for LPV systems.

5. EXAMPLE

Consider the following state-space matrices for the LPV state-delayed system (case $m = 1$ and $r = 1$),

$$A_0 = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}; A_1 = \begin{bmatrix} 0 & 0.2 \\ 0 & 0.1 \end{bmatrix}; A_{0d} = \begin{bmatrix} 0 & 0.1 \\ -0.2 & -0.3 \end{bmatrix};$$

$$A_{1d} = \begin{bmatrix} 0.2 & 0 \\ 0.1 & 0 \end{bmatrix}; A_{0d\tau} = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.3 \end{bmatrix};$$

$$A_{1d\tau} = \begin{bmatrix} -0.04 & 0 \\ 0 & -0.02 \end{bmatrix}; E_{01} = \begin{bmatrix} -0.2 \\ -0.2 \end{bmatrix};$$

$$C_0 = \begin{bmatrix} 0 & 1 \\ 0.5 & 0 \end{bmatrix}; E_{02} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; L_0 = I_2.$$

Assume that the compact set of the parameter ρ is $\zeta = \left[-1, \ 1 \right]$. By Considering $k = 2$ and the performance bound $\gamma = 0.9$, we solve LMIs (18) and (19) using the Matlab LMI Control Toolbox. The solution was obtained after about 50 seconds on a computer with a 2.66 GHz Pentium processor. For instance, parameter-independent positive definite matrices $\{P_{i,2}\}_{i=1}^{2}$, $\{Z_{j,2}\}_{j=1}^{3}$ in (7) are found,

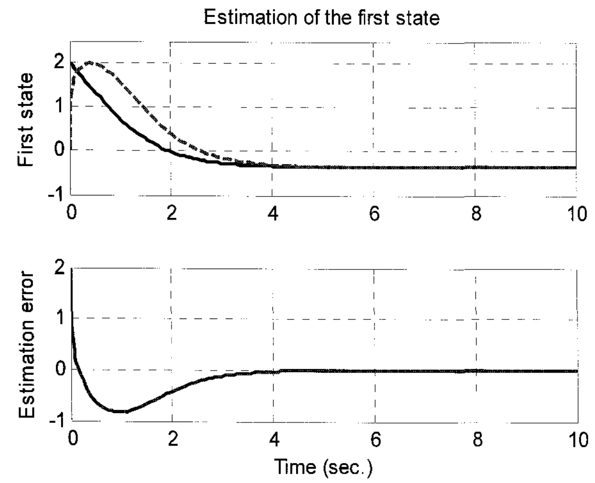

Fig. 1. Estimation results of the first state: with robust H_∞ filter (dotted line), and real state (solid line).

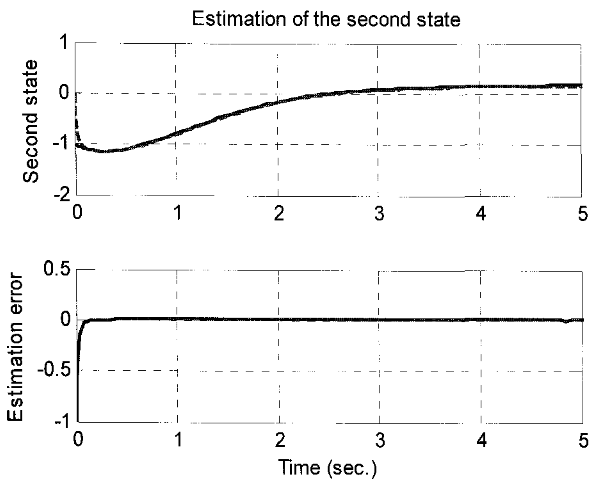

Fig. 2. Estimation results of the second state: with robust H_∞ filter (dotted line), and real state (solid line).

respectively, as

$$P_{1,2} = \begin{bmatrix} 558.1889 & 567.4086 & 3.8314 & -67.1420 \\ * & 584.0305 & 3.2470 & -68.7942 \\ * & * & 60.8586 & 65.3119 \\ * & * & * & 88.9274 \end{bmatrix},$$

$$P_{2,2} = 10^3 \begin{bmatrix} 2.0670 & 2.9107 & 0.4954 & 0.6721 \\ * & 5.5607 & 0.5373 & 0.8268 \\ * & * & 1.1273 & 1.3775 \\ * & * & * & 2.3649 \end{bmatrix},$$

$$Z_{1,2} = \begin{bmatrix} 472.3580 & 488.8646 & -21.4093 & -94.7120 \\ * & 510.5201 & -22.2502 & -97.5753 \\ * & * & 70.3705 & 80.3965 \\ * & * & * & 106.4392 \end{bmatrix},$$

$$Z_{2,2} = 10^4 \begin{bmatrix} 2.0354 & 2.1075 & -0.1309 & -0.4018 \\ * & 2.1930 & -0.1308 & -0.4083 \\ * & * & 0.3252 & 0.3764 \\ * & * & * & 0.4745 \end{bmatrix},$$

$$Z_{3,2} = \begin{bmatrix} 183.3843 & 191.2554 & -9.4432 & -27.9782 \\ * & 203.9704 & -9.2004 & -29.2303 \\ * & * & 42.1449 & 43.0830 \\ * & * & * & 47.8060 \end{bmatrix},$$

and matrices of the filter are

$$W_{1,2} = 10^4 \begin{bmatrix} -0.5741 & -0.2769 & -0.1023 & 0.0650 \\ -0.7336 & -7.3456 & -0.1066 & -0.1784 \\ -0.1232 & 0.0151 & -0.2579 & 0.2513 \\ -0.1160 & -0.1241 & -0.2779 & -3.9703 \end{bmatrix},$$

$$W_{2,2} = 10^5 \begin{bmatrix} -0.8347 & -0.0017 & -0.2189 & -0.0002 \\ 5.8795 & -0.0054 & 0.0077 & -0.0006 \\ -0.1700 & -0.0001 & -0.7119 & -0.0005 \\ 0.0082 & -0.0003 & 3.4226 & -0.0020 \end{bmatrix}.$$

Now, by considering the parameter $\rho = 0.2$, the result of simulations for discrete and distributed delays $d = 0.8$ and $\tau = 0.1$ seconds, respectively, and a unit step disturbance are shown in Figs. 1 and 2. These figures show the plant and filter states trajectory plus their estimation errors. It is observed that the filter is doing well to estimate the plant states.

6. CONCLUSION

The robust filtering problem for a class of LPV systems with discrete and distributed constant delays in the states and outputs has been studied in this paper. By using the Leibniz-Newton formula and a suitable change of variables, some new parameter-dependent delay-dependent stability conditions are established in terms of LMIs such that the filtering process remains asymptotically stable and satisfies a prescribed H_∞ performance level. Moreover, using the PPDQ functions and some Lagrange multiplier matrices, the parameter-independent delay-dependent conditions are developed with high precision under which the desired H_∞ filters exist; then, the explicit expression of these filters is derived. A numerical example has been provided to demonstrate the usefulness of the theory developed.

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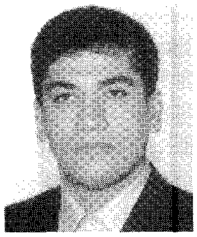

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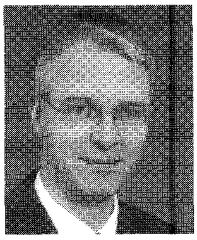

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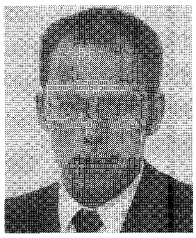

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