PLL Equivalent Augmented System Incorporated with State Feedback Designed by LQR

Somsak Wanchana, Taworn Benjanarasuth, Noriyuki Komine, and Jongkol Ngamwiwit*

Abstract: The PLL equivalent augmented system incorporated with state feedback is proposed in this paper. The optimal value of filter time constant of loop filter in the phase-locked loop control system and the optimal state feedback gain designed by using linear quadratic regulator approach are derived. This approach allows the PLL control system to employ the large value of the phase-frequency gain K_d and voltage control oscillator gain K_o. In designing, the structure of phase-locked loop control system will be rearranged to be a phase-locked loop equivalent augmented system by including the structure of loop filter into the process and by considering the voltage control oscillator as an additional integrator. The designed controller consisting of state feedback gain matrix K and integral gain k_I is an optimal controller. The integral gain k_I related to weighting matrices q and R will be an optimal value for assigning the filter time constant of loop filter. The experimental results in controlling the second-order lag pressure process using two types of loop filters show that the system response is fast without steady-state error, the output disturbance effect rejection is fast and the tracking to step changes is good.

Keywords: Linear quadratic regulator, phase-locked loop, PLL equivalent augmented system, process control system.

1. INTRODUCTION

In the industry, root-locus technique and frequency-response method are mainly used to design the control system to meet the desired performances. They require the transfer function for designing an acceptable performance system but cannot be applicable for designing the optimal control system. For linear optimal control system, the system is expressed in state-space representation. The most fundamental control system design approach is the linear quadratic regulator (LQR) approach where the process or plant is assumed linear and the controller is constrained to be linear [1].

On the other hand, the phase-locked loop (PLL) technique has been extensively applied in various fields. The standard PLL is composed of a phase frequency detector (PFD), a loop filter (LF) and voltage control oscillator (VCO) [2]. In control system applications, it is well known that the PLL techniques give the result of controlling the system accurately when system is in locked state as stated in [3].

In literature, most of PLL control systems are designed by root-locus technique which exhibits the following drawbacks.

1) Response speed of the PLL control system for process with several first-order lags is slow, when the value of filter time constant of LF is large.
2) The value of gain $K_d K_o$ must be kept to be small value for stable system. This will cause the slow system response.

Many researchers proposed several techniques in order to improve the response speed of PLL control system. The first technique has been proposed by using two-mode control scheme as reported in [4] and [5]. The condition of changing the control mode is based on the error signal between the reference and output signal of the system. As shown in [4], the utilization of PLL control and the advantage of a fuzzy controller in a single system have been employed for improving the performance of induction motor speed drives. In [5], PLL control incorporated with PI controller that gives fast response of water flow rate has also been presented. However, slow rejection of the disturbance effect still occurs.

Manuscript received September 20, 2005; revised August 11, 2006 and November 9, 2006; accepted January 30, 2007. Recommended by Editorial Board member Myotaeg Lim under the direction of Editor Tae-Woong Yoon.

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The second technique has been reported in [6] and [7] in order to improve either the speed of transient response or the speed of disturbance effect rejection. This technique is based on the adaptive gain changed according to the error signal between reference and output signals. The adaptive PLL for process control system giving fast rejection of disturbance effect and satisfactory response speed has been proposed in [6]. By adding a feedforward path to the system reported in [6], the improvement of response speed and fast disturbance effect rejection has been obtained as reported in [7].

However, both techniques as reported are still using trial and error method in selecting the proper value of filter time constant of LF when the root-locus technique is employed. This leads to time consuming in PLL control system design. Consequently, LQR technique is employed to assign the value of filter time constant of PLL process control system as proposed recently in [8] and [9]. In [8], the filter time constant for a first order lag filter can be obtained by assigning the values of natural frequency ω_n, weighting matrices q and R. However, in [9], the filter time constant for an active-PI filter can be obtained from the value of $K_d K_o$, q and R where K_d is the phase-frequency gain and K_o is the voltage control oscillator gain. According to these two papers, the large value of $K_d K_o$ can be assigned. This implies that the response of the control system is fast.

In PLL control, the general structure of LF called first-order low-pass filter is used, and its filter time constant must be assigned carefully but should still be obtained easily for practical use. Therefore, this paper presents the state feedback design of PLL equivalent augmented system using LQR approach which leads to the proper tuning of those filter time constants.

The implementation results in controlling the second-order lag pressure process by using PLL control designed by proposed method are shown.

2. AUGMENTED SYSTEM FOR PLL CONTROL SYSTEM

The structure of PLL control system will be described in this section first. Then how to construct a PLL equivalent augmented system for PLL control system will be described later.

2.1. PLL control system

A basic block diagram of the process control system using PLL technique is illustrated in Fig. 1. It is known that the PLL is a feedback-controlled system maintaining a constant phase/frequency difference between a reference input signal and a feedback output signal [2]. It composes of two exactly matches VCO, a PFD, a LF and a process. One VCO is employed to provide a phase or frequency ω_r according to the reference voltage V_r and another one converts the output voltage V_o to feedback output phase or frequency ω_o. The PFD compares the phase or frequency ω_r with the phase or frequency ω_o. A phase detector output pulse of the PFD is generated in proportion to that phase difference. This output pulse is smoothed by passing it through the LF. The resulting dc component from output of LF is used as the input voltage for controlling the process variable. So the process output voltage V_o is related to the phase difference. The output frequency ω_o is fed back to the PFD input for comparison, which in turn controls the VCO oscillating frequency to minimize the phase difference. Therefore, both of frequency and phase are regulated until the synchronization, i.e., $\theta_r = \theta_o$ and $\omega_r = \omega_o$, or namely the phase and frequency of the VCO and the reference signal source are in a locked state.

The transfer function of the VCO is represented by $\frac{K_o}{s}$, where K_o is the gain in rad/sec/V. K_d is the constant gain in V/rad for the PFD. From the Fig. 1, LF has a major role in determining the characteristics of the PLL response. The transfer function of first-order low-pass filter can generally be expressed as

$$\frac{Y_F(s)}{U(s)} = F(s) = \frac{\tau_{F2}s+1}{\tau_{F1}s+b}, \tag{1}$$

where generally $\tau_{F1} \gg \tau_{F2}$ and $b=1$ or $b=0$. This means that pole of LF is located near the origin more than zero of LF. The most commonly used LF are a lag filter ($b=1$), an active-PI filter ($b=0$) and a first-order lag filter ($b=1$ and $\tau_{F2}=0$), In order to meet the desired performance, a filter time constant τ_{F1} and τ_{F2} of LF must be specified properly. Since LF is a low-pass filter type, it implies that the value of the filter time constant τ_{F2} should be less than the appropriate value of the filter time constant τ_{F1}.

From Fig. 1, the open-loop transfer function of the

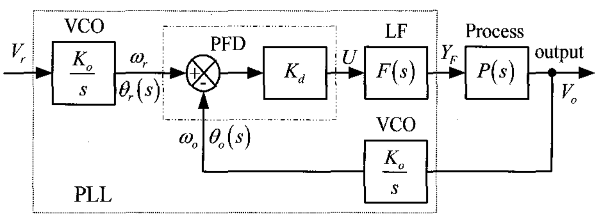

Fig. 1. PLL control system.

PLL control system with LF in (1) can be expressed as

$$\frac{V_o(s)}{V_r(s)} = \frac{K_d K_o}{s} \cdot \frac{\tau_{F2}s+1}{\tau_{F1}s+b} \cdot P(s). \tag{2}$$

If root-locus technique is employed in designing the PLL control system, the response speed may be slow. For instance, if $P(s)$ is a type 0 system with no zero. It is seen from (2) that there is one pole located at the origin. In order to maintain the stability at the large value of gain $K_d K_o$, it is necessary to locate the zero of LF near the origin than process poles. However, this will result in large τ_{F1} which still causes the slow closed-loop response [3,5-7]. To overcome this limitation, the state feedback designed by LQR is employed in this paper to improve the speed of the response. The suitable filter time constant τ_{F1} will also be directly assigned from the proposed procedure.

2.2. PLL equivalent augmented system arrangement

In this sub-section, a PLL equivalent augmented system obtained from the PLL control system shown in Fig. 1 is described first. The state feedback gain matrix and the integral gain relating to filter time constant assigning for LF of the PLL control are then described later.

The process $P(s)$ to be controlled is a SISO system and its state-space form can be represented as

$$\dot{x}_p(t) = A_p x_p(t) + B_p u_p(t), \tag{3}$$

$$y_p(t) = C_p x_p(t), \tag{4}$$

where $x_p(t) \in \mathbf{R}^{(n-1)}$ is the state vector, $u_p(t) \in \mathbf{R}^1$ is the input, $y_p(t) \in \mathbf{R}^1$ is the output, and where $A_p \in \mathbf{R}^{(n-1)\times(n-1)}$, $B_p \in \mathbf{R}^{(n-1)\times 1}$ and $C_p \in \mathbf{R}^{1\times(n-1)}$ are the matrices of the process.

The structure of LF shown in (1) is now expressed as follow

$$\dot{x}_F(t) = -\frac{b}{\tau_{F1}} x_F(t) + \frac{1-(\tau_{F2}/\tau_{F1})b}{\tau_{F1}} u(t), \tag{5}$$

$$y_F(t) = x_F(t) + \frac{\tau_{F2}}{\tau_{F1}} u(t), \tag{6}$$

where $x_F(t) \in \mathbf{R}^1$ is the state variable of LF. Merging the above LF into the process, the new state equation and output equation can be respectively given as

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{7}$$

$$y(t) = Cx(t), \tag{8}$$

where $x(t) = \begin{bmatrix} x_p(t) \\ x_F(t) \end{bmatrix} \in \mathbf{R}^n$ is the state vector of the process including LF, $u(t) \in \mathbf{R}^1$ is the control input and $y(t) \in \mathbf{R}^1$ is the output, and where $A = \begin{bmatrix} A_p & B_p \\ 0 & -\dfrac{b}{\tau_{F1}} \end{bmatrix}$, $B = \begin{bmatrix} B_p a \\ \dfrac{1-ab}{\tau_{F1}} \end{bmatrix}$, $C = \begin{bmatrix} C_p & 0 \end{bmatrix}$ and $a = \dfrac{\tau_{F2}}{\tau_{F1}}$.

In order to express the PLL control system into PLL equivalent augmented system, the summing point of the block PFD in Fig. 1 is first moved to the front of the block VCO. This will make the process including LF be an augmented system. Therefore, PLL equivalent augmented system arranged from the PLL control system with

$$\dot{e}(t) = r(t) - y(t) = r(t) - Cx(t) \tag{9}$$

can be expressed as

$$\dot{x}_a(t) = A_a x_a(t) + B_a u(t) + F_a r(t), \tag{10}$$

$$y(t) = C_a x_a(t), \tag{11}$$

where $x_a(t) = \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} \in \mathbf{R}^{(n+1)}$ is the state vector of PLL equivalent augmented system, $e(t) \in \mathbf{R}^1$ is the error signal, $r(t) \in \mathbf{R}^1$ is the reference signal, and where $A_a = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}$, $B_a = \begin{bmatrix} B \\ 0 \end{bmatrix}$, $F_a = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, and $C_a = \begin{bmatrix} C & 0 \end{bmatrix}$. The block diagram of the feedback system of the PLL equivalent augmented system can be illustrated as Fig. 2.

If the system (10) is completely controllable, then the control law

$$u(t) = -\begin{bmatrix} K & -k_I \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} = -R^{-1} \begin{bmatrix} B \\ 0 \end{bmatrix}^T \begin{bmatrix} p_{11} & p_{12} \\ p_{12}^T & p_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} \tag{12}$$

that minimizes the performance index

$$J = \int_0^\infty \left(\begin{bmatrix} x(t) \\ e(t) \end{bmatrix}^T \begin{bmatrix} Q & 0 \\ 0 & q \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} + u^T(t) R u(t) \right) dt \tag{13}$$

can be found, where $\begin{bmatrix} Q & 0 \\ 0 & q \end{bmatrix} \geq 0$, $R > 0$ and the matrix $\begin{bmatrix} p_{11} & p_{12} \\ p_{12}^T & p_{22} \end{bmatrix} > 0$ is the unique solution of the

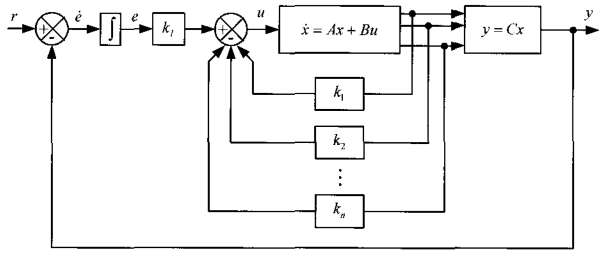

Fig. 2. Feedback system of the PLL equivalent augmented system.

following Riccati equation

$$\begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}^T \begin{bmatrix} p_{11} & p_{12} \\ p_{12}^T & p_{22} \end{bmatrix} + \begin{bmatrix} p_{11} & p_{12} \\ p_{12}^T & p_{22} \end{bmatrix} \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}$$
$$- \begin{bmatrix} p_{11} & p_{12} \\ p_{12}^T & p_{22} \end{bmatrix} \begin{bmatrix} B \\ 0 \end{bmatrix} R^{-1} \begin{bmatrix} B \\ 0 \end{bmatrix}^T \begin{bmatrix} p_{11} & p_{12} \\ p_{12}^T & p_{22} \end{bmatrix} + \begin{bmatrix} Q & 0 \\ 0 & q \end{bmatrix} = 0, \tag{14}$$

where T denotes transpose operation. Consequently, the stable closed-loop system can be given as

$$\begin{bmatrix} \dot{x}(t) \\ \dot{e}(t) \end{bmatrix} = \begin{bmatrix} A - BK & Bk_I \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t), \tag{15}$$

where $K = \begin{bmatrix} k_1 & k_2 & \cdots & k_n \end{bmatrix}$ is the state feedback gain matrix and k_I is the integral gain. The integral gain k_I will play an important role to assign the filter time constant of LF.

3. CONTROLLER DESIGN

In order to design a controller by LQR approach, the procedure in assigning the filter time constant of LF and feedback gain matrix obtaining for the PLL equivalent augmented system which is rearranged from the PLL control system will be described in this section.

3.1. Assignment of loop filter time constant

In assigning the filter time constant of LF, the following assumptions are given:

Assumption 1: The process parameter matrices of the system (3) and (4) satisfy the following assumptions:

1) All column vectors of matrix A_p are linearly independent with the column vector B_p.
2) All row vectors of matrix A_p are linearly independent with the row vector C_p.
3) The pair (A_p, B_p) is completely controllable.

Assumption 2: The integral gain k_I obtained from LQR approach is equal to $K_d K_o$ divided by natural frequency ω_n of PLL because the unit of $K_d K_o$ is in frequency.

Based on the Assumption 1 and Assumption 2, the following lemmas and theorem can be obtained.

Lemma 1: System (7) is completely controllable if and only if the pair (A_p, B_p) is completely controllable and $ab \neq 1$.

Proof: The proof is shown in Appendix A.

Lemma 2: The matrix $\begin{bmatrix} A & B \\ -C & 0 \end{bmatrix}$ is maximum rank if the Assumption 1 is satisfied.

Proof: Since $\begin{bmatrix} A & B \\ -C & 0 \end{bmatrix} = \begin{bmatrix} A_p & B_p & B_p a \\ 0 & -\dfrac{b}{\tau_{F1}} & \dfrac{1-ab}{\tau_{F1}} \\ -C_p & 0 & 0 \end{bmatrix}$, the Assumption 1 clearly guarantees that all row vectors and all column vectors are linearly independent. Therefore, the matrix $\begin{bmatrix} A & B \\ -C & 0 \end{bmatrix}$ is maximum rank.

Lemma 3: The PLL equivalent augmented system (10) is completely controllable if and only if the pair (A, B) is completely controllable and the matrix $\begin{bmatrix} A & B \\ -C & 0 \end{bmatrix}$ is maximum rank.

Proof: The proof is shown in Appendix B.

Lemma 4: If the system (10) is completely controllable, then the optimal integral gain is

$$k_I = \sqrt{\frac{q}{R}}.$$

Proof: From (14), it can be found that

$$\left(p_{12}^T B\right) R^{-1} \left(B^T p_{12}\right) = \frac{\left(B^T p_{12}\right)^2}{R} = q,$$

and from (12), the integral gain k_I is equal to

$$k_I = R^{-1} B^T p_{12} = \frac{\left(B^T p_{12}\right)}{R}.$$

Solving these two equations yield $k_I = \sqrt{\dfrac{q}{R}}$.

Theorem: If the system (10) is completely controllable and Assumption 2 holds, the optimal value of filter time constant τ_{F1} of PLL equivalent augmented system is

$$\tau_{F1} = \frac{1}{K_d K_o} \frac{q}{R},$$

when ω_n is defined by $\omega_n = \sqrt{\frac{K_d K_o}{\tau_{F1}}}$ [10].

Proof: Since system (10) is completely controllable, the optimal integral gain can be found by Lemma 4 as $k_I = \sqrt{\frac{q}{R}}$ while Assumption 2 gives the relation $k_I = \frac{K_d K_o}{\omega_n}$. Equating these equations yield $\omega_n = K_d K_o \sqrt{\frac{R}{q}}$. From definition of ω_n, the optimal value of time constant τ_{F1} of LF (1) can be obtained as

$$\tau_{F1} = \frac{1}{K_d K_o} \frac{q}{R}.$$

In order to have fast system response, the value of filter time constant τ_{F1} must be less than $10^3 \sim 10^6$ times of the process time constant. Finally, the filter time constant τ_{F2} could be assigned with its value is much less than τ_{F1} to have a lag filter characteristic for loop filter.

3.2. Controller for PLL equivalent augmented system

The steps for assigning the state feedback gain matrix K and integral gain k_I of the controller are as follows:

1) Choose Q, q and R. Then, from the values of q, R and $K_d K_o$, find the filter time constant τ_{F1} of the LF stated in the theorem.
2) Construct the PLL equivalent augmented system (10) and (11) by using the matrix A, B and C.
3) Find the control law $u(t)$ of (12).

4. EXPERIMENT

In this section, the structure of pressure process in laboratory is described first and the experimental results of the proposed control system using two types of LF will be investigated later.

4.1. Structure of pressure process

A second-order lag pressure process illustrated in Fig. 3 is employed and controlled by the proposed controller. Hence, the corresponding state equation and output equation of the process are expressed as

$$\begin{bmatrix} \dot{x}_{p1}(t) \\ \dot{x}_{p2}(t) \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_2 C_2} & \frac{1}{R_2 C_2} \\ \frac{1}{R_2 C_1} & -\left(\frac{R_1 + R_2}{R_1 R_2 C_1}\right) \end{bmatrix} \begin{bmatrix} x_{p1}(t) \\ x_{p2}(t) \end{bmatrix}$$

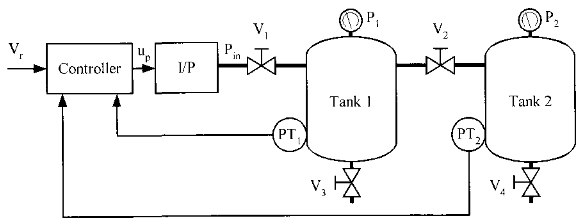

Fig. 3. Pressure process control system.

$$+\begin{bmatrix} 0 \\ \frac{K_p}{R_1 C_1} \end{bmatrix} u_p(t) \tag{16}$$

and

$$y_p(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_{p1}(t) \\ x_{p2}(t) \end{bmatrix}, \tag{17}$$

where K_p is the process gain, R_1 and R_2 are the gas flow resistance of valve V_1 and V_2, and C_1 and C_2 are the capacitance of pressure tank 1 and tank 2 respectively, and where $u_p(t)$ is the input of the process, $y_p(t)$ is the output pressure of tank 2, $x_{p1}(t)$ is the pressure at tank 2 and $x_{p2}(t)$ is the pressure at tank 1.

4.2. Experimental results

The unknown values K_p, R_1, R_2, C_1, and C_2 of the process shown in Fig. 3 can be found from the experiment, where the capacity of the two tanks are set to be the same value, i.e., $C_1 = C_2 = C$. The pressure process parameters in the experiments are shown in Table 1. The frequency of the feedback system of the PLL equivalent augmented system is assigned to lock at 225.73kHz which corresponds to the pressure of 7.5psi. At this pressure, the two pressure transmitters PT_1 and PT_2 are calibrated to give a voltage of 2.5 volts. The IC chip No. CD4046BE is employed for PFD and VCO. Furthermore, the values of K_d and K_o are respectively assigned to be 1.4324V/rad and 3.4548×10^4rad/sec/V.

In the following case studies, the two types of LF, namely, the lag filter and the active-PI filter of which their filter time constant τ_{F1} values obtained from the theorem will be used in the experiments.

4.2.1 Case study 1: PLL control using lag filter

By choosing the values of weighting matrices as $diag\begin{bmatrix} Q & q \end{bmatrix} = diag\begin{bmatrix} 0 & 0 & 0 & 0.01 \end{bmatrix}$ and $R = 0.1$, and adopting the theorem, the value of time constant τ_{F1}

of the lag filter with the transfer function

$$F(s)=\frac{\tau_{F2}s+1}{\tau_{F1}s+1}$$

is obtained as 2×10^{-6} second. As the value of τ_{F2} must be smaller than τ_{F1}, τ_{F2} used in this experiment is selected to be less than 10 times of τ_{F1} or τ_{F2} is 2×10^{-7} second. Consequently, the PLL equivalent augmented system (10) is obtained as

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\\ \dot{x}_3(t)\\ \dot{e}(t)\end{bmatrix}=\begin{bmatrix}-0.1 & 0.1 & 0 & 0\\ 0.1 & -0.2 & 0.1 & 0\\ 0 & 0 & -4.9487\times10^5 & 0\\ -1 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\\ x_3(t)\\ e(t)\end{bmatrix}+\begin{bmatrix}0\\ 0.01\\ 4.4538\times10^5\\ 0\end{bmatrix}u(t)+\begin{bmatrix}0\\0\\0\\1\end{bmatrix}r(t).$$

After solving (14), the state feedback gain matrix K and integral gain k_I of the control law (12) for the PLL equivalent augmented system can be obtained as

$$\begin{bmatrix}K & -k_I\end{bmatrix}=\begin{bmatrix}3.0550 & 1.1794 & 0.0000 & -0.3159\end{bmatrix}.$$

1) PLL equivalent augmented system response

When apply the step reference signal at 7.5psi to the augmented system rearranged from PLL control system using LQR approach, its response is shown in Fig. 4. It is found that the response is fast, percent overshoot P_o is 1.9%, rise time is 14.8seconds approximately, settling time t_s (±2% of final value) is 19.5seconds and steady-state error is zero.

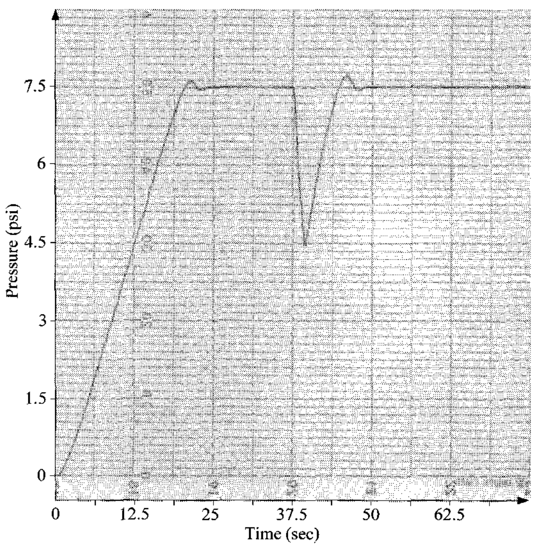

Fig. 4. Response of PLL equivalent augmented system.

In order to investigate the effectiveness of the proposed control system in term of output disturbance rejection, the valve V_4 is opened at 37.5seconds and closed when the pressure drops to 4.5psi. The effect of the disturbance is also shown in Fig. 4. It is seen that the effect of the output disturbance can also be fast rejected and converge to its reference signal again without steady-state error.

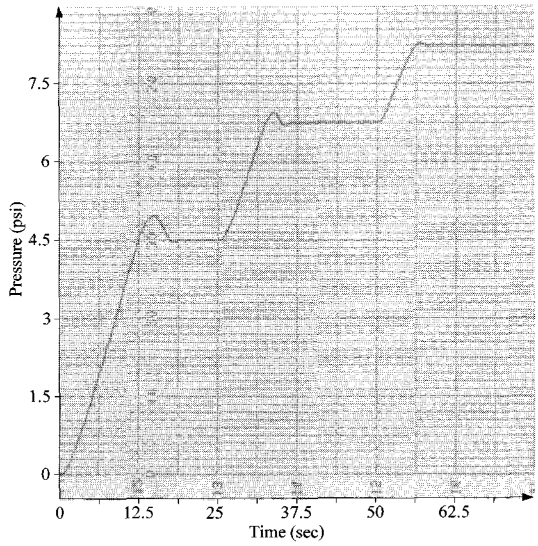

Fig. 5. Tracking capability.

2) Tracking capability

The capability of tracking of the proposed control system is investigated here. The step reference signal is considered to change from 0psi to 4.5psi, from 4.5 psi to 6.75psi and from 6.75psi to 8.25psi of the interval 25seconds respectively.

The experimental result of the proposed control system is shown in Fig. 5. It is seen that the output of the system can track the changed reference signal properly without steady-sate error. However, there is a remarkable maximum overshoot occurred at the first tracking.

4.2.2 Case study 2: PLL control using active-PI filter

Similarly to the case study 1, the value of time constant τ_{F1} of LF

$$F(s)=\frac{\tau_{F2}s+1}{\tau_{F1}s}$$

can be obtained as 1×10^{-4} second when the values of the weighting matrices are selected as $diag\begin{bmatrix}Q & q\end{bmatrix}$ $=diag\begin{bmatrix}0 & 0 & 0 & 0.5\end{bmatrix}$ and $R=0.1$. In this experiment, the value of filter time constant τ_{F2} is selected to be

1×10^{-5} second. From the pressure process parameters shown in Table 1 for this case, the PLL equivalent augmented system (10) can be obtained as

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\\ \dot{x}_3(t)\\ \dot{e}(t)\end{bmatrix}=\begin{bmatrix}-0.05 & 0.05 & 0 & 0\\ 0.05 & -0.1 & 0.05 & 0\\ 0 & 0 & 0 & 0\\ -1 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\\ x_3(t)\\ e(t)\end{bmatrix}+\begin{bmatrix}0\\ 0.005\\ 1\times10^4\\ 0\end{bmatrix}u(t)+\begin{bmatrix}0\\ 0\\ 0\\ 1\end{bmatrix}r(t)$$

and the state feedback gain matrix K and integral gain k_I of the control law (12) can be found as

$$\begin{bmatrix}K & -k_I\end{bmatrix}=\begin{bmatrix}2.0875 & 0.0489 & 0.0007 & -2.2361\end{bmatrix}.$$

1) PLL equivalent augmented system response

The response of the PLL equivalent augmented system rearranged from PLL control system using LQR approach to the step reference signal at 7.5psi is shown in Fig. 6. It is found that the response is fast, percent overshoot P_o is 6%, rise time t_r is 37.4 seconds, settling time t_s (±2% of final value) is 45 seconds and steady-state error is zero.

The effectiveness of the proposed control system in term of output disturbance rejection is investigated. By opening the valve V_4 at 37.5seconds and closing it when the pressure drops to 4.5psi, the effect of the disturbance is also shown in Fig. 6. It is seen that the effect of the output disturbance can also be fast rejected and converge to its reference signal again.

2) Tracking capability

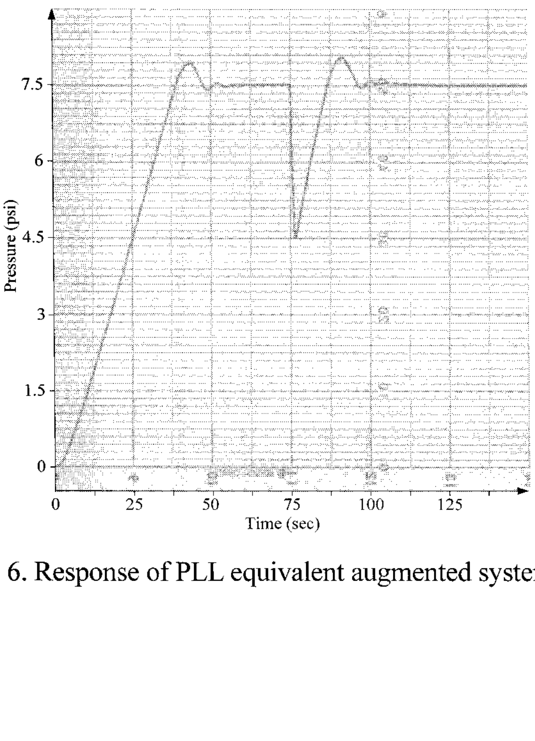

Fig. 6. Response of PLL equivalent augmented system.

Table 1. System performances of PLL equivalent augmented system.

Type of LF	Process Parameter			Performance Criteria		
	K_p	R_1C sec.	R_2C sec.	P_o %	t_r sec.	t_s sec.
Lag	1	10	10	1.9	14.8	19.5
Active-PI	1	20	20	6	37.4	45

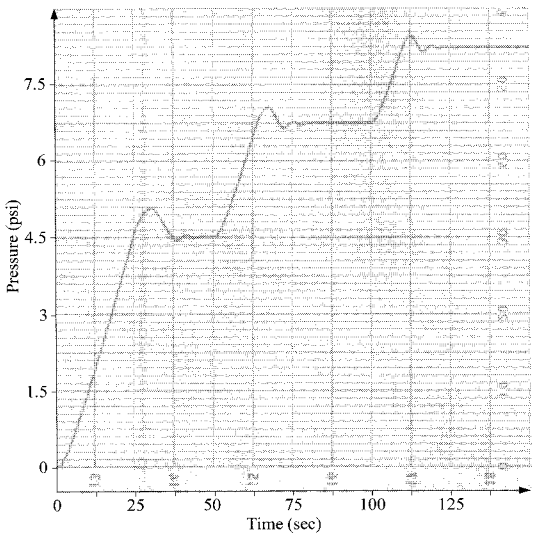

Fig. 7. Tracking capability.

The capability of tracking of the proposed control system is investigated. The experimental result is shown in Fig. 7 when the step reference signal is considered to change from 0psi to 4.5psi, from 4.5psi to 6.75psi and from 6.75psi to 8.25psi of the interval 50seconds respectively. It is seen that the output of the system can track the changed reference signal properly without steady-state error. The first tracking response with remarkable maximum overshoot is occurred.

System performances of the PLL equivalent augmented system incorporated with state feedback designed by LQR are summarized in Table 1. It is seen that the responses are fast with small overshoot and no steady-state error. Hence, the scheme of phase-locked loop control system with the large values of the phase-frequency gain K_d and voltage control oscillator gain K_o designed by linear quadratic regulator approach can really be utilized in practice.

5. CONCLUSIONS

The PLL equivalent augmented system incorporated with state feedback designed by LQR approach has been proposed in this paper. The fast response, fast output disturbance effect rejection, good tracking capability are achieved as demonstrated in its application to second-order lag pressure process.

It can be concluded here that the proposed technique allows the designer to easily assign the filter time constant of the LF from the gain $K_d K_o$ and the weighting matrices q and R. Its value is also optimal in sense of LQR.

APPENDIX A

Let $\gamma = \frac{1-ab}{\tau_{F1}}$ and $\beta = -\frac{b}{\tau_{F1}}$, then the controllability matrix Q_c of the system (7) is

$$Q_c = \begin{bmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{bmatrix} = MN, \tag{A1}$$

where

$$M = \begin{bmatrix} A_p a + I\frac{1}{\tau_{F1}} & B_p a \\ 0 & \gamma \end{bmatrix}, \tag{A2}$$

and

$$N = \begin{bmatrix} 0 & B_p & A_pB_p + B_p\beta & A_p^2B_p + \left(A_pB_p + B_p\beta\right)\beta \\ 1 & \beta & \beta^2 & \beta^3 \end{bmatrix.}$$
$$\begin{matrix} \cdots & A_p^{n-2}B_p + \left(A_p^{n-3}B_p + \cdots + \left(A_pB_p + B_p\beta\right)\beta\right)\beta \\ & \beta^{n-1} \end{matrix}\Bigg]. \tag{A3}$$

A.1. In case that LF is a lag filter ($b = 1$ and $a = \frac{\tau_{F2}}{\tau_{F1}} \ll 1$)

It is seen from (A2) and (A3) that the rank of M and N is n. Hence, Q_c is maximum rank. Consequently, the system (7) is completely controllable.

A.2. In case that LF is an active-PI filter ($b = 0$ and $a = \frac{\tau_{F2}}{\tau_{F1}} \ll 1$)

It can be obviously seen from (A2) and (A3) that the rank of M and N is also n. Hence, the system (7) is completely controllable since Q_c is maximum rank.

A.3. In case that LF is a first-order lag filter ($b = 1$ and $a = \frac{\tau_{F2}}{\tau_{F1}} = 0$)

The proof is same as case A.1 and case A.2.

APPENDIX B

Let Q_{ca} be the controllability matrix for the PLL equivalent augmented system (10). Hence

$$\begin{aligned} Q_{ca} &= \begin{bmatrix} B_a & A_aB_a & A_a^2B_a & \cdots & A_a^{n-1}B_a \end{bmatrix} \\ &= \begin{bmatrix} A & B \\ -C & 0 \end{bmatrix}\begin{bmatrix} 0 & Q_c \\ 1 & 0 \end{bmatrix}. \end{aligned} \tag{B1}$$

From Appendix A, the rank of Q_c is n, therefore the rank of $\begin{bmatrix} 0 & Q_c \\ 1 & 0 \end{bmatrix}$ is $n+1$. When $\begin{bmatrix} A & B \\ -C & 0 \end{bmatrix}$ is maximum rank [11], then Q_{ca} is also maximum rank. Therefore, the system (10) is completely controllable.

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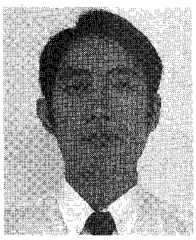

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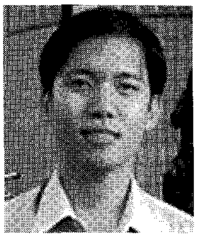

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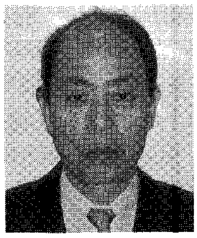

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