A Robust Adaptive Controller for Markovian Jump Uncertain Nonlinear Systems with Wiener Noises of Unknown Covariance

Jin Zhu, Hong-sheng Xi*, Hai-bo Ji, and Bing Wang

Abstract: A robust adaptive controller design for a class of Markovian jump parametric -strict-feedback systems is given. The disturbances considered herein include both uncertain nonlinearities and Wiener noises of unknown covariance. And they satisfy some bound-conditions. By using stochastic Lyapunov method in Markovian jump systems, a switching robust adaptive controller was obtained that guarantees global uniform ultimate boundedness of the closed-loop jump system.

Keywords: Markovian jump nonlinear systems, robust adaptive control, Wiener noise, uncertain nonlinearity.

1. INTRODUCTION

The passed decades have witnessed substantial research activities in the development of Markovian jump systems, and much effort is directed towards jump linear systems [1]. With many linear problems (Kalman filtering [2,3], LQG [4,5], Slide-mode control [6] etc.) solved, more attention is focused on the study of Markovian jump non-linear systems. Some results can be found in the works of Aliyu [7] and Zhu [8]. However, noise and disturbance are not considered in these works.

On the other hand, many physical systems do be disturbed by noises. Thus Markovian jump nonlinear systems disturbed by Wiener noise (or Brown motion) have been the subject of numerous studies in recent years. For this class of jump systems, Mao [9] gives the sufficient condition to ensure existence and uniqueness of the solution; Yuan [10,11] introduce the notions of asymptotic stability and robust stability; Boukas [12] presents the notions of mean square stability. However, their work mainly focus on the notions and definitions of stochastic stability, not the practical controller design in jump systems. At the knowledge of the authors, the practical control design for Markovian jump nonlinear systems with stochastic noises has received little attention in literature.

In this paper, we are interested in the practical switching robust adaptive controller design for Markovian jump nonlinear systems disturbed by Wiener noise. As parametric-strict-feedback systems could rep-resent many practical systems in real world, and many other nonlinear systems could be converted to this form via mathematical transformation [13]. We choose the Markovian jump parametric-strict-feedback systems as the research model. Considering that modeling errors, parametric uncertainty and time variations may exist, the systems structure uncertainty is thus taken into account. Here the covariance of Wiener noise is assumed to be unknown but bounded, and the structure uncertainty is assumed to satisfy some growth conditions [14]. With the control law and the parameter adaptive law designed, all signals of the closed-loop system are globally uniformly ultimately bounded.

The rest of this paper is organized as follows: Section 2 briefly introduces some mathematical notions and the Markovian jump nonlinear system model. The robust adaptive controller for the system is then proposed in Section 3. In Section 4, an example is shown to illustrate the validity of the controller design. Finally, conclusions are drawn in Section 5.

2. PROBLEM AND PRELIMINARIES

2.1. Notation

Throughout the paper, unless otherwise specified, we denote by $(\Omega, \mathbb{F}, \{\mathbb{F}_t\}_{t\geq 0}, P)$, a complete probability space with a filtration $\{\mathbb{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., it is right continuous and $\mathbb{F}_0$ contains all p-null sets). Let $|\cdot|_p$ stand for the p-th Euclidean

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norm for vectors. The superscript T will denote transpose and we refer to $Tr(\cdot)$ as the trace for matrix. In addition we use $L^2(P)$ to denote the space of Lebesgue square integrable vector.

Let $r(t), t \geq 0$, be a right continuous Markov chain on the probability space taking values in finite state space $S = \{1, 2, \cdots, N\}$, and we introduce $\Phi(t) = [\Phi_1(t), \Phi_2(t) \cdots, \Phi_N(t)]^T$ the indicator process for the regime $r(t)$, as:

$$\Phi_j(t) = \begin{cases} 1 & r(t) = j \\ 0 & r(t) \neq j, j \in S \end{cases} \tag{1}$$

and $\Phi(t)$ satisfies the following equation:

$$\Phi(t) = \Phi(t_0) + \Pi \int_{t_0}^{t} \Phi(s) ds + M(t) \tag{2}$$

with $M(t) = [M_1(t), M_2(t), \cdots, M_N(t)]^T$, an N-dimensional $\mathbb{F}_t$-martingale satisfying $M(t) \in L^2(P)$ and $\Pi = [\pi_{kj}]$ the chain generator, an $N \times N$ matrix. The entries $\pi_{kj}, k, j = 1, 2, \cdots, N$ are interpreted as transition rates such that

$$P(r(t+dt) = j \mid r(t) = k) = \begin{cases} \pi_{kj} dt + o(dt) & if \quad k \neq j \\ 1 + \pi_{kj} dt + o(dt) & if \quad k = j, \end{cases} \tag{3}$$

where $dt>0$. Here $\pi_{kj} > 0 (k \neq j)$ is the transition rate from k to j. Notice that the total probability axiom imposes π_{kk} negative and

$$\sum_{j=1}^{N} \pi_{kj} = 0, \forall k \in S.$$

Consider a stochastic differential equation with Markovian switching of the form

$$dx = f(x, t, r(t))dt + g(x, t, r(t))d\omega \tag{4}$$

on $t \geq 0$ with initial data $x(0) = x_0 \in R^n$ and $r(0) = k_0 \in S$, where $f: R^n \times R_+ \times S \to R^n$, $g: R^n \times S \to R^{n\times m}$, $x = x(t)$ is system state vector, $\omega(t) = (\omega_{t1}, \omega_{t2}, \cdots, \omega_{tm})^T$ is an independent m-dimensional Wiener noise defined on the probability space, with covariance $E\{d\omega \cdot d\omega^T\} = \Upsilon(t)\Upsilon(t)^T dt$, where $\Upsilon(t)$ is an unknown boundedmatrix-valued function. Furthermore, we assume that the Wiener noise $\omega(t)$ is independent of the Markov chain $r(t)$. For the existence and uniqueness of the solution, we shall impose a hypothesis [9]:

(H) Both f and g satisfy the local Lipschitz condition and the linear growth condition. That is for each $h = 1, 2, \cdots$, there is an $L_h > 0$ such that

$$|f(x,t,k) - f(y,t,k)| \vee |g(x,t,k) - g(y,t,k)| \leq L_h |x - y|$$

for all $(t,k) \in R_+ \times S$ and those $x, y \in R^n$ with $|x| \vee |y| \leq h$.

Moreover there is an $\nu > 0$ such that:

$$|f(x,t,k)| \vee |g(x,t,k)| \leq \nu(1 + |x|)$$

for all $(x,t,k) \in R^n \times R_+ \times S$.

In general, the hypothesis (H) will guarantee a unique local solution to (4).

Let $C^{2,1}(R^n \times R_+ \times S)$ denote the family of all functions $F(x,t,k)$ on $R^n \times R_+ \times S$ which are continuously twice differentiable in x and once in t. Furthermore, we will given the stochastic differrentiable equation of $F(x,t,k)$:

Fix any $(x_0, t_0, k) \in R^n \times R_+ \times S$ and suppose $x(t)$ is the unique solution to (4). By the generalized Ito formula, we have

$$\begin{aligned} F(x,t,r(t)) &= F(x_0,t_0,k) + \int_{t_0}^{t} \frac{\partial F(x,s,r(s))}{\partial s} ds \\ &+ \int_{t_0}^{t} \frac{\partial F(x,s,r(s))}{\partial x} f(x,s,r(s)) ds \\ &+ \int_{t_0}^{t} \frac{1}{2} Tr[\Upsilon^T g^T(x,s,r(s)) \frac{\partial^2 F(x,s,r(s))}{\partial x^2} g(x,s,r(s))\Upsilon] ds \\ &+ \int_{t_0}^{t} \frac{\partial F(x,s,r(s))}{\partial x} g(x,s,r(s)) d\omega \\ &+ \int_{t_0}^{t} \sum_{j=1}^{N} [F(x,s,j) - F(x,s,k)] d\Phi_j(s). \end{aligned} \tag{5}$$

According to (2), the differential equation of the indicator $\Phi(t)$ is as following:

$$d\Phi(t) = \Pi\Phi(t)dt + dM(t). \tag{6}$$

Submit (6) into (5) and notice that

$$\sum_{j=1}^{N} \pi_{kj} F(x,t,k) = 0. \quad \forall k \in S.$$

Therefore, the stochastic differentiable equation of $F(x,t,k)$ is given by the following:

$$dF(x,t,k) = \frac{\partial F(x,t,k)}{\partial t} dt + \frac{\partial F(x,t,k)}{\partial x} f(x,t,k) dt$$

$$+\frac{1}{2}Tr[\Upsilon^T g^T(x,t,k)\frac{\partial^2 F(x,t,k)}{\partial x^2}g(x,t,k)\Upsilon]dt$$
$$+\sum_{j=1}^{N}\pi_{kj}F(x,t,j)dt+\frac{\partial F(x,t,k)}{\partial x}g(x,t,k)d\omega \quad (7)$$
$$+\sum_{j=1}^{N}[F(x,t,j)-F(x,t,k)]dM_j(t).$$

We take the expectation in (7), so that the infinitesimal generator produces [9]

$$LF(x,t,k)=\frac{\partial F(x,t,k)}{\partial t}+\frac{\partial F(x,t,k)}{\partial x}f(x,t,k)$$
$$+\frac{1}{2}Tr[\Upsilon^T g^T(x,t,k)\frac{\partial^2 F(x,t,k)}{\partial x^2}g(x,t,k)\Upsilon] \quad (8)$$
$$+\sum_{j=1}^{N}\pi_{kj}F(x,t,j).$$

Lemma 1 (Martingale Representation) [15]: Let $B(t)=[B_1(t),B_2(t),\cdots,B_N(t)]$ be N-dimensional standard Wiener noise. Suppose $M(t)$ is an $\mathbb{F}_t^N$-martingale (w.r.t. P) and that $M(t)\in L^2(P)$ for all $t\geq 0$. Then there exists a stochastic process $\Psi\in L^2(\mathbb{F}_t,P)$, such that

$$dM(t)=\Psi\cdot dB(t). \quad (9)$$

Lemma 2 (Young's inequality): For any two vectors $x,y\in R^n$, the following holds

$$x^T y\leq\frac{\varepsilon^p}{p}|x|^p+\frac{1}{q\varepsilon^q}|y|^q, \quad (10)$$

where $\varepsilon>0$ and the constants $p>1,q>1$ satisfy $(p-1)(q-1)=1$.

2.2. Problem description

Consider the following Markovian jump uncertain nonlinear systems with Wiener noises:

$$\begin{cases} dxi=x_{i+1}dt+\varphi_i(\bar{x}_i,t,r(t))^T\theta^* dt+\Delta_i(\bar{x}_i,t,r(t))dt \\ \quad +g_i(\bar{x}_i,t,r(t))^T d\omega \\ dx_n=u\,dt+\varphi_n(x,t,r(t))^T\theta^* dt+\Delta_n(x,t,r(t))dt \\ \quad +g_n(x,t,r(t))^T d\omega \\ y=x_1, \quad i=1,2,\cdots,n-1, \end{cases} \quad (11)$$

where $x=(x_1,x_2,\cdots,x_n)^T\in R^n$ is the state vector, here $\bar{x}_i\triangleq(x_1,x_2,\cdots,x_i)^T$, $u\in R$ is the input, and $y\in R$ is the output of the system. $\theta^*\in R^P$ is a vector of unknown constant parameters; The Markov chain r(t) and m-dimensional Wiener noise ω are as defined in Section 2.1. $\varphi_i(\bar{x}_i,t,r(t))$, $g_i(\bar{x}_i,t,r(t))$ are vector-valued smooth functions. $\Delta_i(\bar{x}_i,t,r(t))$ is an unknown function which could be due to modeling errors, parametric uncertainty, time variations in the systems, or a combination of these. And it maybe different with each regime $r(t)$. It is assumed that the control designer has, at least, partial knowledge of bounds for the function uncertainty $\Delta_i(\bar{x}_i,t,r(t))$. In particular, we assume that

$$\begin{aligned}&\Delta_i(\bar{x}_i,t,r(t)=k)\leq\psi_i^* p_i(\bar{x}_i,r(t)=k),\\ &\forall\bar{x}_i\in R^i,\forall t\in R_+,\forall k\in S,\end{aligned} \quad (12)$$

where $p_i(\bar{x}_i,r(t))\in C^1(R^i\times S,R^+)$ is a known smooth function and $\psi_i^*\geq 0$ is a constant parameter, which is not necessarily known. Note that ψ_i^* is not unique, since any $\overline{\psi}_i^*>\psi_i^*$ satisfies in-equality (12). To avoid confusion, we define ψ_i^* the smallest(non-negative) constant such that (12) is satisfied. In this paper, the equilibrium x=0 is assumed a common one for all the regimes, which means $\varphi_i(0,t,k)=0$, $g_i(0,t,k)=0$, $\forall k\in S$. With $\varphi_i(\bar{x}_i,t,r(t))$, $g_i(\bar{x}_i,t,r(t))$, $\Delta_i(\bar{x}_i,t,r(t))$ satisfying hypothesis (H), Markovian jump system (11) has a unique solution.

3. CONTROL DESIGN

Now we begin to design a robust adaptive controller for system (11), where the parameter θ^* and ψ_i^* need to be estimated. Denote the estimation of θ^* with θ, and the estimation of ψ_i^* with ψ_i. First we employ a coordinate transformation:

$$z_i=x_i-\alpha_{i-1}(\bar{x}_{i-1},\theta,\psi_i,t,r(t)=k), \quad (13)$$

where $\alpha_0=0$, $\forall k\in S$, and the new coordinate is $Z=(z_1,z_2,\cdots,z_n)$. For simplicity, we just denote $\alpha_{i-1}(\bar{x}_{i-1},\theta,\psi_i,t,k)$, $\varphi_i(\bar{x}_i,t,k)$, $g_i(\bar{x}_i,t,k)$, $\Delta_i(\bar{x}_i,t,k)$ by $\alpha_{i-1}(k)$, $\varphi_i(k)$, $g_i(k)$, and $\Delta_i(k)$. According to (7), the (13) can be written as:

$$\begin{aligned}dz_i&=[x_{i+1}+\varphi_i^T(k)\theta^*+\Delta_i(k)]dt+g_i^T(k)d\omega-d\alpha_{i-1}(k)\\ &=\{z_{i+1}+\alpha_i(k)+\varphi_i^T(k)\theta^*+\Delta_i(k)\}dt\\ &\quad-\frac{\partial\alpha_{i-1}(k)}{\partial t}dt-\frac{\partial\alpha_{i-1}(k)}{\partial\theta}\dot{\theta}dt-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(k)}{\partial\psi_j}\psi_j dt\end{aligned}$$

$$-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(k)}{\partial x_j}[x_{j+1}+\varphi_j^T(k)\theta^*+\Delta_j(k)]dt$$
$$-\frac{1}{2}\sum_{p,q=1}^{i-1}\frac{\partial^2\alpha_{i-1}(k)}{\partial x_p\partial x_q}g_p^T(k)\Upsilon\Upsilon^T g_q(k)dt$$
$$-\sum_{j=1}^{N}\pi_{kj}\alpha_{i-1}(j)dt+[g_i^T(k)-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(k)}{\partial x_j}g_j^T(k)]d\omega$$
$$+\sum_{j=1}^{N}[\alpha_{i-1}(k)-\alpha_{i-1}(j)]dM_j(t). \tag{14}$$

By Lemma 1, there exists a $\Psi\in L^2(\mathbb{F}_t,P)$ and an N-dimensional standard Wiener noise $B(t)$, such that $dM(t)=\Psi dB(t)$, where $E\{\Psi\Psi^T\}=\phi\phi^T\le Q<\infty$ and Q is a positive bounded constant. Moreover, we define an N-dimensional vector

$$\Gamma_i(k)\triangleq[\alpha_{i-1}(k)-\alpha_{i-1}(1),\alpha_{i-1}(k)-\alpha_{i-1}(2),\cdots,\alpha_{i-1}(k)-\alpha_{i-1}(N)].$$

Therefore (14) is as following:

$$dz_i=[z_{i+1}+\alpha_i(k)+\varphi_i^T(k)\theta^*+\Lambda_i(k)]dt$$
$$-\frac{\partial\alpha_{i-1}(k)}{\partial t}dt-\frac{\partial\alpha_{i-1}(k)}{\partial\theta}\dot{\theta}dt-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(k)}{\partial\psi_j}\psi_j dt$$
$$-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(k)}{\partial x_j}[x_{j+1}+\varphi_j^T(k)\theta^*]dt$$
$$-\frac{1}{2}\sum_{p,q=1}^{i-1}\frac{\partial^2\alpha_{i-1}(k)}{\partial x_p\partial x_q}g_p^T(k)\Upsilon\Upsilon^T g_q(k)dt$$
$$-\sum_{j=1}^{N}\pi_{kj}\alpha_{i-1}(j)dt+[g_i^T(k)-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(k)}{\partial x_j}g_j^T(k)]d\omega$$
$$+\Gamma_i(k)\Psi dB, \tag{15}$$

where $\Lambda_i(k)$ is:

$$\Lambda_i(k)\triangleq\Delta_i(k)-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(k)}{\partial x_j}\Delta_j(k)$$

and according to inequality (12), it is easily seen that there exist a series of continuous function $\bar{p}_i(\bar{x}_i,k)\in C(R_i\times S,R_+)$, such that

$$|\Lambda_i(k)|\le\psi_i^*\bar{p}_i(\bar{x}_i,k),\forall\bar{x}_i\in R^i,\forall t\in R_+,\forall k\in S. \tag{16}$$

Choose a Lyapunov function of the form

$$V=\frac{1}{4}\sum_{i=1}^{n}z_i^4+\frac{1}{2\gamma}\tilde{\theta}^T\tilde{\theta}+\sum_{i=1}^{n}\frac{1}{2\sigma_i}\chi_i^2, \tag{17}$$

where $\gamma>0,\sigma_i>0$ are constants. $\tilde{\theta}=\theta^*-\theta$ and $\chi_i=\psi_i^M-\psi_i$ are the parameter estimation errors, where $\psi_i^M\triangleq max\{\psi_i^*,\psi_i^0\}$, and ψ_i^0 are given positive constants.

We set out to choose the function $\alpha_{i-1}(k)$ and adaptive functions to make LV non-positive. Along the solutions of (15), we have

$$LV=\sum_{i=1}^{n}z_i^3\{z_{i+1}+\alpha_i(k)+\varphi_i^T(k)\theta^*-\frac{\partial\alpha_{i-1}(k)}{\partial t}$$
$$-\frac{\partial\alpha_{i-1}(k)}{\partial\theta}\dot{\theta}-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(k)}{\partial\psi_j}\psi_j-\sum_{j=1}^{N}\pi_{kj}\alpha_{i-1}(j)$$
$$-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(k)}{\partial x_j}[x_{j+1}+\varphi_j^T(k)\theta^*]+\Lambda_i(k)$$
$$-\frac{1}{2}\sum_{p,q=1}^{i-1}\frac{\partial^2\alpha_{i-1}(k)}{\partial x_p\partial x_q}g_p^T(k)\Upsilon\Upsilon^T g_q(k)\}$$
$$+\frac{3}{2}\sum_{i=1}^{n}z_i^2[g_i^T(k)-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(k)}{\partial x_j}g_j^T(k)]\Upsilon\Upsilon^T$$
$$[g_i(k)-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(k)}{\partial x_j}g_j(k)]$$
$$+\frac{3}{2}\sum_{i=1}^{N}z_i^2\Gamma_i(k)\phi\phi^T\Gamma_i^T(k)$$
$$-\frac{1}{\gamma}\tilde{\theta}^T\dot{\theta}-\sum_{i=1}^{n}\frac{1}{\sigma_i}\chi_i\dot{\psi}_i$$
$$\le\sum_{i=1}^{N}z_i^3\{(\frac{3}{4}\delta_i^{\frac{4}{3}}+\frac{1}{4\delta_{i-1}^4})z_i+\alpha_i(k)-\frac{\partial\alpha_{i-1}(k)}{\partial t}$$
$$-\frac{\partial\alpha_{i-1}(k)}{\partial\theta}\dot{\theta}-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(k)}{\partial\psi_j}\psi_j+\tau_i^T(k)\theta$$
$$-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(k)}{\partial x_j}x_{j+1}-\sum_{j=1}^{N}\pi_{kj}\alpha_{i-1}(j)$$
$$+\lambda z_i^3\sum_{p,q=1}^{i-1}[\frac{\partial^2\alpha_{i-1}(k)}{\partial x_p\partial x_q}]^2 g_p^T(k)g_p(k)$$
$$g_q^T(k)g_q(k)+\mu_1 z_i[\rho_i^T(k)\rho_i(k)]^2$$
$$+\sum_{i=1}^{n}\mu_2 z_i[\Gamma_i(k)\Gamma_i^T(k)]^2\}+\sum_{i=1}^{n}\frac{9}{16\mu_2}Q^2+\iota|\Upsilon|^4$$
$$-\tilde{\theta}^T[\frac{1}{\gamma}\dot{\theta}-\sum_{i=1}^{n}z_i^3\tau_i(k)]-\sum_{i=1}^{n}[\frac{1}{\sigma_i}\chi_i\dot{\psi}_i-z_i^3\Lambda_i(k)] \tag{18}$$

with

$$\tau_i(k)=\varphi_i(k)-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(k)}{\partial x_j}\varphi_j(k),$$

$$\rho_i(k) = g_i(k) - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}(k)}{\partial x_j} g_j(k).$$

In (18), the following inequalities are used, which can be reduced from Young's inequalities and norm inequalities with the help of changing the order of summations or exchanging the indices of the summations:

$$\sum_{i=1}^{n} z_i^3 z_{i+1} \le \frac{3}{4}\sum_{i=1}^{n-1} \delta^{\frac{4}{3}} z_i^4 + \frac{1}{4}\sum_{i=1}^{n-1} \frac{1}{\delta_i^4} z_{i+1}^4 = \sum_{i=1}^{n} (\frac{3}{4}\delta_i^{\frac{4}{3}} + \frac{1}{4\delta_{i-1}^4}) z_i^4,$$

where $\delta_0 = \infty$, $\delta_n = 0$, and $\delta_i > 0$, $i = 1, 2, \cdots, n-1$.

$$\begin{aligned}
&\frac{1}{2}\sum_{i=1}^{n} z_i^3 \sum_{p,q=1}^{i-1} \frac{\partial^2 \alpha_{i-1}(k)}{\partial x_p \partial x_q} g_p^T(k) \Upsilon\Upsilon^T g_q(k) \\
&\le \sum_{i=1}^{n} \lambda z_i^6 \sum_{p,q=1}^{i-1} [\frac{\partial^2 \alpha_{i-1}(k)}{\partial x_p \partial x_q}]^2 |g_p(k)|^2 |g_q(k)|^2 \\
&\quad + \sum_{i=1}^{n}\sum_{p,q=1}^{i-1} \frac{1}{16\lambda} |\Upsilon\Upsilon^T|^2 \\
&= \sum_{i=1}^{n} \lambda z_i^6 \sum_{p,q=1}^{i-1} (\frac{\partial^2 \alpha_{i-1}(k)}{\partial x_p \partial x_q})^2 g_p^T(k) g_p(k) g_q^T(k) g_q(k) \\
&\quad + \frac{|\Upsilon\Upsilon^T|^2}{96\lambda}(n-1)n(2n-1)
\end{aligned}$$

$$\begin{aligned}
&\frac{3}{2}\sum_{i=1}^{n} z_i^2 [g_i^T(k) - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}(k)}{\partial x_j} g_j^T(k)] \\
&\quad \Upsilon\Upsilon^T [g_i(k) - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}(k)}{\partial x_j} g_j(k)] \\
&\le \sum_{i=1}^{n} \mu_1 z_i^4 \{[g_i(k) - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}(k)}{\partial x_j} g_j(k)]^T \\
&\quad [g_i(k) - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}(k)}{\partial x_j} g_j(k)]\}^2 + \sum_{i=1}^{n} \frac{9}{16\mu_1} |\Upsilon\Upsilon^T|^2.
\end{aligned}$$

$$\begin{aligned}
&\frac{3}{2}\sum_{i=1}^{n} z_i^2 \Gamma_i(k) \phi\phi^T \Gamma_i^T(k) \\
&\le \frac{3}{2}\sum_{i=1}^{n} z_i^2 \Gamma_i(k) Q \Gamma_i^T(k) \\
&\le \sum_{i=1}^{n} \mu_2 z_i^4 [\Gamma_i(k)\Gamma_i^T(k)]^2 + \sum_{i=1}^{n} \frac{9}{16\mu_2} Q^2,
\end{aligned}$$

where $\lambda > 0, \mu_1 > 0, \mu_2 > 0$ are design parameters,

$$\iota = \frac{(n-1)n(2n-1)}{96\lambda} + \frac{9n}{16\mu_1}. \tag{19}$$

According to [14], we suggest the following adaptive laws:

$$\dot{\theta} = \gamma[\sum_{i=1}^{n} z_i^3 \tau_i(k) - l(\theta - \theta^0)], \tag{20}$$

$$\dot{\psi}_i = \sigma_i[z_i^3 \varpi_i(k) - m_i(\psi_i - \psi_i^0)], \tag{21}$$

$$\varpi_i(k) = \bar{p}_i(k) \cdot tanh[\frac{z_i^3 \bar{p}_i(k)}{\varepsilon_i}]. \tag{22}$$

Here $l > 0, m_i > 0, \varepsilon_i > 0, \theta^0 \in R^p$ are given constants. Denote

$$\beta_i(k) = \psi_i \cdot \varpi_i(k). \tag{23}$$

Substituting (20),(21),(23) into (18), and we suggest the virtual control as

$$\begin{aligned}
\alpha_i(k) = &-c_i z_i - (\frac{3}{4}\delta_i^{\frac{4}{3}} + \frac{1}{4\delta_{i-1}^4}) z_i + \frac{\partial \alpha_{i-1}(k)}{\partial t} \\
&-\lambda z_i^3 \sum_{p,q=1}^{i-1} [\frac{\partial^2 \alpha_{i-1}(k)}{\partial x_p \partial x_q}]^2 g_p^T(k) g_p(k) g_q^T(k) g_q(k) \\
&-\mu_1 z_i [\rho_i^T(k)\rho_i(k)]^2 - \mu_2 z_i [\Gamma_i(k)\Gamma_i^T(k)]^2 \\
&+\frac{\partial \alpha_{i-1}(k)}{\partial \theta}\dot{\theta} + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}(k)}{\partial \psi_j}\psi_j(k) - \tau_i^T(k)\theta \\
&+\sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}(k)}{\partial x_j} x_{j+1} + \sum_{j=1}^{N} \pi_{kj}\alpha_{i-1}(j) - \beta_i(k).
\end{aligned} \tag{24}$$

However, if adaptive law (20) is adopted, $\dot{\theta}$ concerning with $z_1, \cdots, z_n$ exists in (24). Therefore it is impossible to get $\alpha_i(k)$ directly. For this reason, the following transitions are necessary:

$$\begin{aligned}
&\sum_{i=1}^{n} z_i^3 \frac{\partial \alpha_{i-1}(k)}{\partial \theta}\dot{\theta} \\
&= \sum_{i=1}^{n} z_i^3 \frac{\partial \alpha_{i-1}(k)}{\partial \theta} \gamma[\sum_{j=1}^{i} z_j^3 \tau_j(k) + \sum_{j=i+1}^{n} z_j^3 \tau_j(k) - l(\theta - \theta^0)] \\
&= \sum_{i=1}^{n} z_i^3 \frac{\partial \alpha_{i-1}(k)}{\partial \theta} \sum_{j=1}^{i} z_j^3 \tau_j(k) \\
&\quad + \sum_{j=1}^{n}\sum_{i=1}^{j-1} z_i^3 \frac{\partial \alpha_{i-1}(k)}{\partial \theta} \gamma z_j^3 \tau_j(k) \\
&\quad - \sum_{i=1}^{n} z_i^3 \frac{\partial \alpha_{i-1}(k)}{\partial \theta} \gamma l(\theta - \theta^0) \\
&= \gamma \sum_{i=1}^{n} z_i^3 [\frac{\partial \alpha_{i-1}(k)}{\partial \theta} \sum_{j=1}^{i} z_j^3 \tau_j(k) \\
&\quad + (\sum_{j=1}^{i-1} z_j^3 \frac{\partial \alpha_{j-1}(k)}{\partial \theta}) \tau_i(k) - l(\theta - \theta^0)].
\end{aligned} \tag{25}$$

Substituting (25) into (24), and the virtual control design is

$$\alpha_i(k)=-c_iz_i-(\frac{3}{4}\delta_i^{\frac{4}{3}}+\frac{1}{4\delta_{i-1}^4})z_i+\frac{\partial\alpha_{i-1}(k)}{\partial t}$$
$$+\gamma\frac{\partial\alpha_{i-1}(k)}{\partial\theta}\sum_{j=1}^{i}z_j^3\tau_j(k)+\gamma\sum_{j=1}^{i-1}z_j^3\frac{\partial\alpha_{j-1}(k)}{\partial\theta}\tau_i(k)$$
$$-\gamma l\frac{\partial\alpha_{i-1}(k)}{\partial\theta}(\theta-\theta^0)+\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(k)}{\partial\psi_j}\psi_j$$
$$-\tau_i^T(k)\theta-\mu_1z_i[\rho_i^T(k)\rho_i(k)]^2-\mu_2z_i[\Gamma_i(k)\Gamma_i^T(k)]^2$$
$$-\lambda z_i^3\sum_{p,q=1}^{i-1}[\frac{\partial^2\alpha_{i-1}(k)}{\partial x_p\partial x_q}]^2g_p^T(k)g_p(k)g_q^T(k)g_q(k)$$
$$+\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(k)}{\partial x_j}x_{j+1}+\sum_{j=1}^{N}\pi_{kj}\alpha_{i-1}(j)-\beta_i(k), \tag{26}$$

where $\alpha_0(k)=0, c_i>0, i=1,2,\cdots,n$ with the actual control :

$$u(k)=\alpha_n(k), \tag{27}$$

then the infinitesimal generator of V becomes :

$$LV\le-\sum_{i=1}^{n}c_iz_i^4-\sum_{i=1}^{n}[z_i^3\beta_i(k)+z_i^3\chi_i\varpi_i(k)-z_i^3\Lambda_i(k)$$
$$-\chi_im_i(\psi_i-\psi_i^0)]+l\tilde{\theta}^T(\theta-\theta^0)+\frac{9n}{16\mu_2}Q^2+\tau|\Delta|^4$$
$$=-\sum_{i=1}^{n}c_iz_i^4+l\tilde{\theta}^T(\theta-\theta^0)+\frac{9n}{16\mu_2}Q^2+\tau|\Delta|^4$$
$$+\sum_{i=1}^{n}[z_i^3\Lambda_i(k)-(\psi_i+\chi_i)z_i^3\varpi_i(k)]+\sum_{i=1}^{n}m_i\chi_i(\psi_i-\psi_i^0)$$
$$=-\sum_{i=1}^{n}c_iz_i^4+l\tilde{\theta}^T(\theta-\theta^0)+\sum_{i=1}^{n}[z_i^3\Lambda_i(k)-\psi_i^Mz_i^3\varpi_i(k)]$$
$$+\sum_{i=1}^{n}m_i\chi_i(\psi_i-\psi_i^0)+\frac{9n}{16\mu_2}Q^2+\tau|\Delta|^4. \tag{28}$$

Consider (16), (22), we get

$$z_i^3\Lambda_i(k)-\psi_i^Mz_i^3\varpi_i(k)\le\psi_i^*|z_i^3p_i(k)|-\psi_i^Mz_i^3\varpi_i(k)$$
$$\le\psi_i^M|z_i^3p_i(k)|-\psi_i^Mz_i^3p_i(k)tanh[\frac{z_i^3p_i(k)}{\varepsilon_i}]$$

and according to

$$0\le|\eta|-\eta tanh(\frac{\eta}{\varepsilon})\le0.2785\varepsilon<\frac{1}{2}\varepsilon,$$

such that

$$z_i^3\Lambda_i(k)-\psi_i^Mz_i^3\varpi_i(k)\le\frac{1}{2}\psi_i^M\varepsilon_i. \tag{29}$$

By using the following inequalities:

$$l\tilde{\theta}^T(\theta-\theta^0)=-\frac{1}{2}l\tilde{\theta}^T\tilde{\theta}-\frac{1}{2}l(\theta-\theta^0)^T(\theta-\theta^0)$$
$$+\frac{1}{2}l(\theta^*-\theta^0)^T(\theta^*-\theta^0)$$
$$\le-\frac{1}{2}l\tilde{\theta}^T\tilde{\theta}+\frac{1}{2}l(\theta^*-\theta^0)^T(\theta^*-\theta^0)$$
$$m_i\chi_i(\psi_i-\psi_i^0)=-\frac{1}{2}m_i\chi_i^2-\frac{1}{2}m_i(\psi_i-\psi_i^0)^2$$
$$+\frac{1}{2}m_i(\psi_i^M-\psi_i^0)$$
$$\le-\frac{1}{2}m_i\chi_i^2+\frac{1}{2}m_i(\psi_i^M-\psi_i^0)^2,$$

therefore

$$LV\le-\sum_{i=1}^{n}c_iz_i^4-\frac{1}{2}l\tilde{\theta}^T\tilde{\theta}+\frac{1}{2}l(\theta^*-\theta^0)^T(\theta^*-\theta^0)$$
$$+\frac{1}{2}\sum_{i=1}^{n}\varepsilon_i\psi_i^M-\frac{1}{2}m\sum_{i=1}^{n}\chi_i^2$$
$$+\frac{1}{2}\sum_{i=1}^{n}m_i(\psi_i^M-\psi_i^0)^2+\frac{9n}{16\mu_2}Q^2+\iota|\Delta|^4$$
$$\le-cV+\kappa, \tag{30}$$

where

$$m=min(m_i),\quad c=min(4c_i,l\gamma,m\sigma_i)$$
$$\kappa=\frac{1}{2}\sum_{i=1}^{n}\varepsilon_i\psi_i^M+\frac{1}{2}\sum_{i=1}^{n}m_i(\psi_i^M-\psi_i^0)^2+\iota|\Delta|^4$$
$$+\frac{1}{2}l(\theta^*-\theta^0)^T(\theta^*-\theta^0)+\frac{9n}{16\mu_2}Q^2.$$

Theorem 1: Considering Markovian jump nonlinear system(11), if adaptive law (20), (21) and controller (27) is adopted, the equilibrium of the closed-loop system is globally uniformly ultimately bounded in the 4th-moment. Furthermore, for any given $\varepsilon>0$, there is

$$\lim_{t\to\infty}E(|Z|_4^4)<\varepsilon. \tag{31}$$

Proof: According to the conclusion in [11], we have

$$EV\le e^{-ct}[V(x_0,t_0,r_0)-\frac{\kappa}{c}]+\frac{\kappa}{c}$$

and there is

$$V=\frac{1}{4}\sum_{i=1}^{n}z_i^4+\frac{1}{2\gamma}\tilde{\theta}^T\tilde{\theta}+\sum_{i=1}^{n}\frac{1}{2\sigma_i}\chi_i^2\ge\frac{1}{4}\sum_{i=1}^{n}z_i^4.$$

Take expectation in the above equation

$$E(|Z|_4^4) \le 4EV \le 4e^{-ct}[V(x_0,t_0,r_0) - \frac{\kappa}{c}] + \frac{4\kappa}{c}, \quad (32)$$

which means that $Z = (z_1, z_2, \cdots, z_n)$ is globally uniformly bounded in 4th-moment, thus $x = (x_1, x_2, \cdots, x_n)$ is globally uniformly bounded in 4th-moment. Moreover, $\exists T > 0$, if $t \ge T$, there is $4e^{-ct}[V(x_0, t_0, r_0) - \frac{\kappa}{c}] \le \frac{4\kappa}{c}$, and $E|Z|_4^4 \le \frac{8\kappa}{c}$. So for any given $\varepsilon > 0$, appropriate control design parameters c_i, l, m_i can be chosen to guarantee $\frac{8\kappa}{c} < \varepsilon$.

Therefore when $t \ge T$,

$$E|Z|_4^4 \le \frac{8\kappa}{c} < \varepsilon. \quad (33)$$

Remark 1: From Theorem 1 above, it could be seen that all signals in the closed-loop system are globally uniformly ultimately bounded in the 4th-moment. According to the knowledge of stochastic stability [15], the closed-loop jump system (11) is thus stable in probability.

Remark 2: According to (26), only the bound of noise ω and uncertainty Δ_i are needed, and we do not care for the detailed information of them. Meanwhile the increase of $c_i, l, \gamma, m, \sigma_i$ will reduce the effect of noise and structure uncertainty. As long as these design parameters are large enough, the system output $y = x_1$ could be as close to the equilibrium as possible, disregarding the difference of system equations caused by regime $r(t)$ switching stochastically.

4. EXMAPLE

Consider a 2-order Markovian jump nonlinear system with the regime transition space $S = \{1,2\}$, and the transition rate matrix is

$$\Pi = \begin{bmatrix} -2 & 2 \\ 5 & -5 \end{bmatrix}.$$

The system is as follows:

$$\begin{cases} dx_1 &= x_2 dt + \xi_1(x_1,t,r(t))\theta^* dt + \Delta(x_1,t,r(t))dt, \\ dx_2 &= u\,dt + \xi_2(x,t,r(t))\theta^* dt + g_2(x,t,r(t))d\omega, \\ y &= x_1. \end{cases}$$

Here

$$\xi_1(x_1,t,1) = x_1^2, \quad \xi_1(x_1,t,2) = x_1,$$
$$\xi_2(x,t,1) = x_1, \quad \xi_2(x,t,2) = x_1 sinx_2,$$
$$g_2(x,t,1) = sinx_2, \quad g_2(x,t,2) = 1 - 2cosx_2,$$

where θ^* is an unknown parameter and $\Delta(x_1,t,r(t))$ is an unknown bounded disturbance. For simulation purposes, we let $\theta^* = 2$, $\Delta(x_1,t,1) = 0.6sin2t$, $\Delta(x_1,t,2) = cos4t$, and the noise covariance $\Upsilon = 2$.

The control law and the adaptive law are taken as follows (here $\delta_1 = 1$)

Case 1: The system regime is 1:

$$\alpha_1(1) = -(c_1 + \frac{3}{4})z_1 - \tau_1(1)\theta - \beta_1(1),$$
$$\alpha_2(1) = -(c_2 + \frac{1}{4})z_2 - \tau_2(1)\theta - x_1^2\gamma[z_1^3\tau_1(1) + z_2^3\tau_2(1)]$$
$$+ \gamma l(\theta - \theta^0)x_1^2 - \beta_2(1)$$
$$+ \gamma l(\theta - \theta^0)x_1^2 - \beta_2(1)$$
$$-\frac{\partial\alpha_1(1)}{\partial x_1}x_2 - \varpi_1(1)\dot{\psi}_1 + \pi_{11}\alpha_1(1) + \pi_{12}\alpha_1(2),$$
$$-\mu_1 z_1(sinx_2)^4 - \mu_2 z_2[\alpha_1(1) - \alpha_1(2)]^4,$$
$$\dot{\theta} = \gamma[z_1^3\tau_1(1) + z_2^3\tau_2(1) - l(\theta - \theta^0)],$$
$$\dot{\psi}_1 = \sigma_1[z_1^3 w_1(1) - m_1(\psi_1 - \psi_1^0)],$$
$$\dot{\psi}_2 = \sigma_2[z_2^3 w_2(1) - m_2(\psi_2 - \psi_2^0)],$$

where

$$z_1 = x_1, \quad z_2 = x_2 - \alpha_1(1),$$
$$\tau_1(1) = x_1^2, \quad \overline{p}_1(1) = 1,$$
$$\varpi_1(1) = \overline{p}_1(1)tanh(\frac{z_1^3\overline{p}_1(1)}{\varepsilon_1}), \quad \beta_1(1) = \psi_1\varpi_1(1),$$
$$\overline{p}_2(1) = |\frac{\partial\alpha_1(1)}{\partial x_1}| = |c_1 + \frac{3}{4} + 2\theta x_1 + \frac{3z_1^2\psi_1}{\varepsilon_1}[1 - \varpi_1^2(1)]|,$$
$$\tau_2(1) = x_1 - \frac{\partial\alpha_1(1)}{\partial x_1}x_1^2,$$
$$\varpi_2(1) = \overline{p}_2(1)tanh(\frac{z_2^3\overline{p}_2(1)}{\varepsilon_2}),$$
$$\beta_2(1) = \psi_2\varpi_2(1)$$

Case 2: The system regime is 2:

$$\alpha_1(2) = -(c_1 + \frac{3}{4})z_1 - \tau_1(2)\theta - \beta_1(2),$$
$$\alpha_2(2) = -(c_2 + \frac{1}{4})z_2 - \tau_2(2)\theta - x_1\gamma[z_1^3\tau_1(2) + z_2^3\tau_2(2)]$$
$$+ \gamma l(\theta - \theta^0)x_1 - \beta_2(2)$$
$$-\frac{\partial\alpha_1(2)}{\partial x_1}x_2 - \varpi_1(2)\dot{\psi}_1 + \pi_{21}\alpha_1(1)$$

$$+\pi_{22}\alpha_1(2) - 4\mu_1 z_1(1-2cosx_2)^4$$
$$-\mu_2 z_2[\alpha_1(1)-\alpha_1(2)]^4,$$
$$\dot{\theta} = \gamma[z_1^3\tau_1(2) + z_2^3\tau_2(2) - l(\theta-\theta^0)],$$
$$\psi_1 = \sigma_1[z_1^3 w_1(2) - m_1(\psi_1 - \psi_1^0)],$$
$$\psi_2 = \sigma_2[z_2^3 w_2(2) - m_2(\psi_2 - \psi_2^0)],$$

where

$$z_1 = x_1,\ z_2 = x_2 - \alpha_1(2),$$
$$\tau_1(2) = x_1,\ \bar{p}_1(2) = 1,$$
$$\varpi_1(2) = \bar{p}_1(2)tanh(\frac{z_1^3\bar{p}_1(2)}{\varepsilon_1}),$$
$$\beta_1(2) = \psi_1\varpi_1(2),$$
$$\bar{p}_2(2) = |\frac{\partial\alpha_1(2)}{\partial x_1}| = |c_1 + \frac{3}{4} + \theta + \frac{3z_1^2\psi_1}{\varepsilon_1}[1-\varpi_1^2(2)]|,$$
$$\tau_2(2) = x_1 sinx_2 - \frac{\partial\alpha_1(2)}{\partial x_1}x_1,$$
$$\varpi_2(2) = \bar{p}_2(2)tanh(\frac{z_2^3\bar{p}_2(2)}{\varepsilon_2}),$$
$$\beta_2(2) = \psi_2\varpi_2(2).$$

In computation we take design constants $c_1 = c_2 = 5$, $\theta^0 = 1$, $\psi_1^0 = \psi_2^0 = 1$, $\sigma_1 = \sigma_2 = \gamma = 1$, $m_1 = m_2 = l = 1$, $\mu_1 = 50$, $\mu_2 = 25$, $\varepsilon_1 = \varepsilon_2 = 0.4$, choose the initial values as $x_1 = 8.7$, $x_2 = -2$, $\theta(0) = 0$, $\psi_1(0) = \psi_2(0) = 0$ and the time step is 0.01s.

The simulation figures are as follows:

In simulation, Fig. 1 shows the regime switching with time; Fig. 2 shows the corresponding input control u, and Fig. 3 shows the time response of the state variation (solid line for state x_1 or output y, and dashed line for x_2). As could be seen from the figures, the system regime may switch stochastically

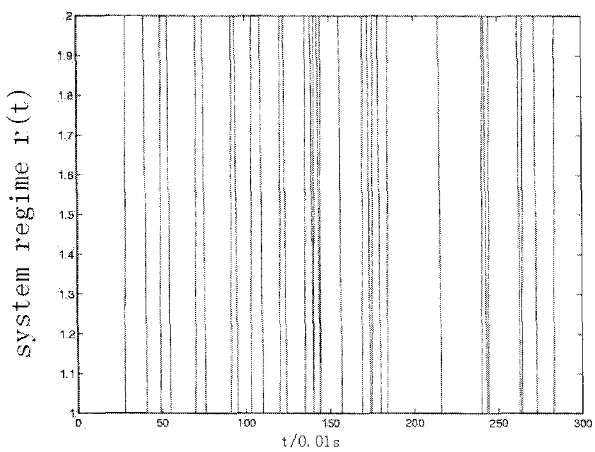

Fig. 1 Regime switching with time t.

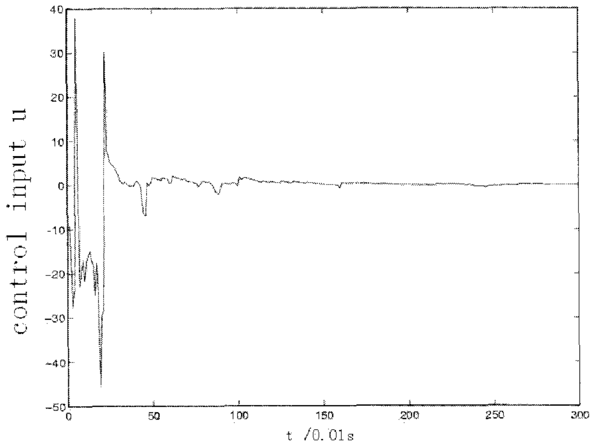

Fig. 2. System control input u with time t.

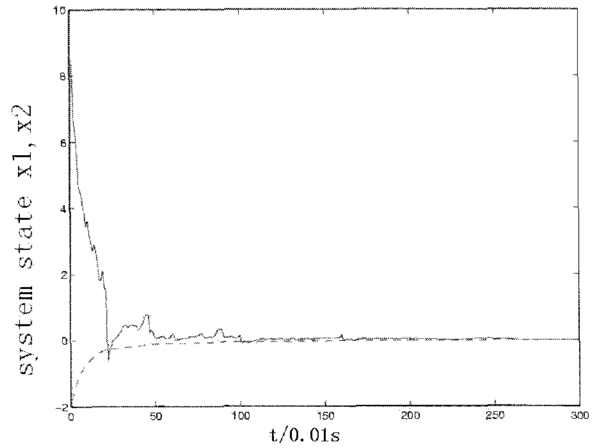

Fig. 3. System states variation with time t.

such that the control input will vary with the regime, thus they are not continuous to time t. However, at the time-point of regime switching, the system states will have an abrupt "jump", but they are still continuous to time t, and this is an important quality of Markovian jump systems. The simulation results illustrate the global uniform ultimate boundedness of the closed-loop system.

5. CONCLUSION

The robust adaptive control of Markovian jump uncertain nonlinear parametric-strict-feedback systems disturbed by Wiener noises of unknown covariance was investigated. A robust adaptive control scheme was obtained by using a stochastic Lyapunov method and backstepping techniques, which guarantees that the closed-loop system is globally uniformly ultimately bounded.

Backstepping design for non-jump stochastic system has been developed before by Deng [16]. In contrast to the design of Deng, the main difference is the introduction of the Markovian stochastic switching, thus the difficulties in controller design lie in follows:

1. The introduction of stochastic Markovian switching added the complexity of system analysis largely. Consider an *N-regime* Markovian jump systemsand it consists of N different subsystems. For each regime k, the actual controller $u(k)$ would present a different expression. Therefore the regime-dependent controller $u(k)$ represents the stochastic switching controller between the N different subsystem. In the infinitesimal generator for Markovian jump systems(11), the coupling of regime π_{kj} is added, thus the switching controller $u(k)$ concerns not only with the current regime k, but with all the regime $j = 1,2,\cdots,N$.
2. Since the generalized Ito formula only deals with problems of stochastic systems perturbed by Wiener noise, it would be hard to handle the martingale process caused by Markovian switching. In this paper the martingale is converted to the correspondent Wiener noise $B(t)$ so that infinitesimal generator could be applied. Thus in the switching controller $u(k)$, the term $\mu_2 z_i[\Gamma_i(k)\Gamma_i^T(k)]^2$ is included to reduced the effect of martingale.

From above, it could been seen Deng's controller would regarded as a specific example of our work in which system regime $N \equiv 1$. Thus this paper extends the controller design for stochastic nonlinear systems to a more general form. And the design accuracy could be ensured with appropriate parameters chosen.

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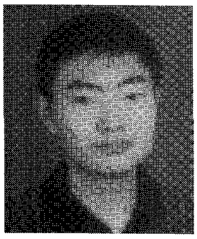

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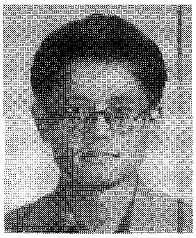

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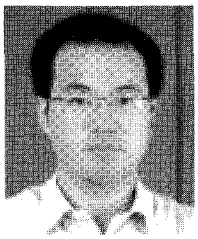

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