Partial Pole Assignment via Constant Gain Feedback in Two Classes of Frequency-domain Models

Guo-Sheng Wang, Guo-Zhen Yang, and Guang-Ren Duan

Abstract: The design problem of partial pole assignment (PPA) in two classes of frequency-domain MIMO models by constant gain feedback is investigated in this paper. Its aim is to design a constant gain feedback which changes only a subset of the open-loop eigenvalues, while the rest of them are kept unchanged in the closed-loop system. A near general parametric expression for the feedback gain matrix in term of a set of design parameter vectors and the set of the closed-loop poles, and a simple parametric approach for solving the proposed problem are presented. The set of poles do not need to be previously prescribed, and can be set undetermined and treated together with the set of parametric vectors as degrees of design freedom provided by the approach. An illustrative example shows that the proposed parametric method is simple and effective.

Keywords: Feedback control, frequency-domain models, parameterization, pole assignment.

1. INTRODUCTION

It is well known that the systems' transient responses are determined mainly by the locations of the systems' eigenvalues. As an important design method associated with eigenvalues and eigenvectors in control theory, pole assignment and eigenstructure assignment are very attractive and important problems in control system designs and has been extensively studied by many authors (see, for instance, [1-13]). One type of approach for eigenstructure assignment is the parametric approach, which parameterizes all the solutions to the problem, such as [6-8]. This method presents complete, explicit and parametric expressions of all the feedback gain matrices and the closed-loop eigenvector matrices. Moreover, this method offers all the design degrees of freedom, which can be further utilized to satisfy some additional performances, such as robustness [9-13].

In some situations, it is desirable to change only a subset of the open-loop eigenvalues, while the rest of them are remained the same in the closed-loop system. This idea is called partial pole assignment (PPA), and also has been investigated by a few researchers [14-17] in time-domain models via constant gain feedback, e.g., the constant state gain feedback. However, for PPA in MIMO frequency-domain models, there have been few results.

In this paper the design problem of PPA in two classes of frequency-domain MIMO models by constant gain feedback is investigated, which is suspired by the time-domain parametric partial eigenstructure assignment results in [18]. A very simple parametric approach for solving the problem is proposed and a near general parametric expressions for the gain matrix is presented in term of a set of design parameter vectors and the set of closed-loop poles. Therefore, the set of poles do not need to be previously prescribed, and can be set undetermined and treated together with the set of parametric vectors as degrees of design freedom provided by the approach. In addition, utilizing the proposed method can eliminate the computations.

This paper is organized as follows. The next section gives the description of partial pole assignment via constant gain feedback in a class of frequency-domain models. Section 3 proposes the parametric result of partial pole assignment via constant gain feedback in frequency-domain. The partial pole assignment via constant gain feedback in another class of frequency-domain models, which is dual with the models in Section 2, is considered, and a simple and effective algorithm is developed in Section 4. Section 5 presents an illustrative example to show the simplicity and effectiveness of the proposed algorithm.

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Concluding remarks are drawn in Section 6.

For convenience, in the sequential sections, we use $R^{m\times n}[s]$ to denote the set of all $m\times n$ polynomial matrices with real coefficient matrices, $\det(A)$ the determinant of matrix A, $\deg(P(s))$ the degree of polynomial $P(s)$, and $\text{root}(P(s))$ the set of roots of polynomial $P(s)$.

2. PROBLEM FORMULATION

Consider the following plant

$$G_r(s) = N(s)D^{-1}(s), \tag{1}$$

with $N(s)\in R^{n\times r}[s]$ and $D(s)\in R^{r\times r}[s]$ satisfies the following assumptions:

Assumption A1: $N(s)$ and $D(s)$ are right coprime polynomial matrices;

Assumption A2: $\deg\det(D(s)) = n$.

It is clear to see that the poles of system (1) can be given by the roots of the following characteristic polynomial

$$\Delta_D(s)=\det(D(s)). \tag{2}$$

Due to Assumption A2, system (1) obviously possesses n poles.

When a constant gain feedback is applied to system (1), the closed-loop characteristic polynomial is easily obtained as

$$\begin{aligned} G_r^c(s) &= G_r(s)[I-KG_r(s)]^{-1}. \\ &= N(s)D^{-1}(s)[I-KN(s)D^{-1}(s)]^{-1} \\ &= N(s)[D(s)-KN(s)]^{-1}. \end{aligned} \tag{3}$$

Therefore, the closed-loop characteristic polynomial is easily obtained as

$$\Delta_l^c(s)=\det(D(s)-KN(s)). \tag{4}$$

Partition all the poles of the open-loop system (1) as follows

$$\Gamma = \Gamma_r^o \bigcup \Gamma_r^-, \tag{5}$$

with

$$\Gamma_r^o = \{s_i^o,\ i=n_1+1, n_1+2,\cdots,n\}, \tag{6a}$$

and

$$\Gamma_r^- = \{s_i^-,\ i=1,2,\cdots,n_1\}, \tag{6b}$$

where Γ_r^o is formed by the poles of the open-loop system (1) which are satisfactory and to be remained the same in the closed-loop system (3), while Γ_r^- is formed by those unsatisfactory poles of the open-loop system (1) which are to be replaced in the closed-loop system (4).

Recalling the fact that non-defective eigenvalues are less insensitive with respect to parameter perturbations, in this paper we only consider the case that the assigned closed-loop poles are distinct and self-conjugate. With the above preparations, we can now state the problem of partial pole assignment in system (3) by constant gain feedback as follows.

Problem PPA (Partial Pole Assignment): Given system (1) satisfying Assumptions A1 and A2, and a group of distinct and self-conjugate complex scalars s_i^c, $i=1,2,\cdots,n_1$. Further, let the set of poles of the open-loop system (1) be given as in (5), (6), with s_i^o, $i=n_1+1, n_1+2,\cdots,n$ being distinct and self-conjugate. Then determine all the admissible constant gain feedback matrices $K\in R^{r\times n}$ such that equation (3) holds, that is, the following relation holds

$$\text{root}[\det(D(s)-KN(s))] = \Gamma_r^o \bigcup \Gamma_r^c, \tag{7}$$

with Γ_r^o given by (6a) and

$$\Gamma_r^c = \{s_i^c,\ i=1,2,\cdots,n_1\}. \tag{8}$$

3. SOLUTION TO PROBLEM PPA

Regarding solution to Problem PPA, we have the following result.

Theorem 1: Given system (1) satisfying Assumptions A1 and A2, and a group of distinct and self-conjugate complex scalars s_i^c, $i=1,2,\cdots,n_1$. Further, let the set of poles of the open-loop system (1) be given as in (5), (6), with s_i^o, $i=n_1+1, n_1+2,\cdots,n$ being distinct and self-conjugate. Then all the solutions to Problem PPA can be parameterized as follows:

$$K = [W \quad 0][V \quad V^o]^{-1}, \tag{9}$$

where

$$W = [D(s_1^c)f_1 \quad D(s_2^c)f_2 \quad \cdots \quad D(s_{n_1}^c)f_{n_1}], \tag{10}$$

$$V = [N(s_1^c)f_1 \quad N(s_2^c)f_2 \quad \cdots \quad N(s_{n_1}^c)f_{n_1}], \tag{11}$$

and

$$V^o = [N(s_{n_1+1}^o)f_{n_1+1} \quad N(s_{n_1+2}^o)f_{n_1+2} \quad \cdots \quad N(s_n^o)f_n], \tag{12}$$

where $f_i\in C^r$, $i=1,2,\cdots,n$, are a set of parametric vectors satisfying the following constraints:

Constraint C1: $f_i=\overline{f_j}$ if $s_i^c=\overline{s_j^c}$ or $s_i^o=\overline{s_j^o}$, $i,j=1,2,\cdots,n$;

Constraint C2: $\det([V \;\; V^o]) \neq 0$.

Proof: It follows from (4) that condition (7) is equivalent to

$$\det(D(s_i^c) - KN(s_i^c)) = 0,\; i = 1,2,\cdots,n_1, \tag{13}$$

and

$$\det(D(s_i^o) - KN(s_i^o)) = 0,\; i = n_1+1, n_1+2,\cdots,n \tag{14}$$

while (13) and (14) are equivalent to the existence of a set of parameter vectors $f_i \in C^r$, $i = 1,2,\cdots,n$, such that the following equations hold:

$$(D(s_i^c) - KN(s_i^c))f_i = 0,\quad i = 1,2,\cdots,n_1, \tag{15}$$

and

$$(D(s_i^o) - KN(s_i^o))f_i = 0,\quad i = n_1+1, n_1+2,\cdots,n. \tag{16}$$

However, (15) and (16) are equivalent with the following ones:

$$D(s_i^c)f_i = KN(s_i^c)f_i,\; i = 1,2,\cdots,n_1, \tag{17}$$

and

$$D(s_i^o)f_i = KN(s_i^o)f_i,\quad i = n_1+1, n_1+2,\cdots,n. \tag{18}$$

Because $\Gamma_r^o = \{s_i^o,\; i = n_1+1, n_1+2,\cdots,n\}$ is the set of partial poles of the open-loop system (1), there hold

$$\det(D(s_i^o)) = 0,\quad i = n_1+1, n_1+2,\cdots,n. \tag{19}$$

For parameter vectors $f_i \in C^r$, $i = n_1+1, n_1+2,\cdots,n$, thus there hold

$$D(s_i^o)f_i = 0,\; i = n_1+1, n_1+2,\cdots,n. \tag{20}$$

Then from (18), there hold

$$D(s_i^o)f_i = KN(s_i^o)f_i = 0,\; i = n_1+1, n_1+2,\cdots,n. \tag{21}$$

The matrices W, V and V^o are defined as in (10)-(12). Regarding the existence of the real gain matrix K, the parameter vectors $f_i \in C^r$, $i = 1, 2,\cdots,n$, must satisfy Constraints C1 and C2. □

From the parametric result of partial pole assignment via constant gain feedback proposed in Theorem 1, the advantages of this parametric method can be given as follows:

Remark 1: This parametric method (9) gives all the solutions to of the constant gain K in Problem PPA;

Remark 2: These parametric vectors $f_i \in \mathbf{C}^r$, $i = 2,\cdots,n$, satisfying Constraints C1 and C2, offer all the design degrees of freedom, and can be utilized to achieve some additional specifications, such as robustness.

Remark 3: If the assigned partial closed-loop eigenvalues are undetermined, they can be viewed as another part of design degrees of freedom in control system designs.

4. THE DUAL PROBLEM

Let the plant be given as

$$G_l(s) = L^{-1}(s)H(s), \tag{22}$$

with $L(s) \in R^{m\times m}[s]$ and $H(s) \in R^{m\times n}[s]$ satisfies the following assumptions:

Assumption A3: $L(s)$ and $H(s)$ are left coprime polynomial matrices;

Assumption A4: $\mathrm{degdet}(L(s)) = n$.

When a constant gain feedback is applied to system (22), the closed-loop system can be given by

$$\begin{aligned} G_l^c(s) &= [I - G_l(s)K]^{-1}G_l(s) \\ &= [I - L^{-1}(s)H(s)K]^{-1}L^{-1}(s)H(s) \\ &= [L(s) - H(s)K]^{-1}H(s). \end{aligned} \tag{23}$$

It is clear to see that the characteristic polynomial of the closed-loop system (23) is easily obtained as

$$\Delta_l^c = \det(L(s) - H(s)K). \tag{24}$$

Similarly, partition all the poles of the open-loop system (22) as follows:

$$\Gamma_l = \Gamma_l^o \bigcup \Gamma_l^-, \tag{25}$$

with

$$\Gamma_l^o = \{s_i^o,\; i = n_1+1, n_1+2,\cdots,n\}, \tag{26}$$

and

$$\Gamma_l^- = \{s_i^-,\; i = 1,2,\cdots,n_1\}, \tag{27}$$

where Γ_l^o is formed by the poles of the open-loop system (22) which are satisfactory and to be remained the same in the closed-loop system (23), while Γ^- is formed by those unsatisfactory poles of the open-loop system (22) which are to be replaced in the closed-loop system (23).

By applying Theorem 1 to the dual system

$$G_l^T(s) = H^T(s)(L^T(s))^{-1}, \tag{28}$$

we can obtain the following result about partial pole assignment in the open-loop system (22) by constant gain feedback. Recalling the fact that non-defective eigenvalues are less insensitive with respect to parameter perturbations, in this part we also consider the case that the assigned closed-loop poles are distinct and self-conjugate.

Theorem 2: Let system (22) satisfy Assumptions A3 and A4, and a group of complex scalars s_i^c, $i=1,2,\cdots,n_1$ be distinct and self-conjugate. Further, let the set of poles of the open-loop system (22) be given as in (25)-(27), with s_i^o, $i=n_1+1,\ n_1+2,\cdots,n$ also being distinct and self-conjugate. Then all the admissible constant gain feedback matrices $K\in R^{m\times n}$, such that equation (23) holds, that is, the following relation holds:

$$\text{root}[\det(L(s)-H(s)K)]=\Gamma_l^o\bigcup\Gamma_l^c, \tag{29}$$

with Γ_l^o given by (26) and

$$\Gamma_l^c=\{s_i^c,\ i=1,2,\cdots,n_1\} \tag{30}$$

can be parameterized as follows:

$$K=\begin{bmatrix}T\\T^o\end{bmatrix}^{-1}\begin{bmatrix}Z\\0\end{bmatrix}, \tag{31}$$

with

$$Z=\begin{bmatrix}g_1^TL(s_1)\\g_2^TL(s_2)\\\vdots\\g_{n_1}^TL(s_{n_1})\end{bmatrix}, \tag{32}$$

$$T=\begin{bmatrix}g_1^TL(s_1)\\g_2^TL(s_2)\\\vdots\\g_{n_1}^TL(s_{n_1})\end{bmatrix}, \tag{33}$$

and

$$T^o=\begin{bmatrix}g_{n_1+1}^TL(s_{n_1+1})\\g_{n_1+2}^TL(s_{n_1+2})\\\vdots\\g_n^TL(s_n)\end{bmatrix}, \tag{34}$$

where $g_i\in R^m$, $i=1,2,\cdots,n,$ are a set of parameter vectors satisfying the following constraints:

Constraint C3: $g_i=\overline{g_j}$ if $s_i^c=\overline{s_j^c}$ or $s_i^o=\overline{s_j^o}$, $i,j=1,2,\cdots,n$;

Constraint C4: $\det\left(\begin{bmatrix}T\\T^o\end{bmatrix}\right)\neq 0.$

5. AN ILLUSTRATIVE EXAMPLE

Consider the following transfer function

$$G(s)=\begin{bmatrix}\frac{1}{s^2(s+1)} & \frac{1}{s^2}\\ \frac{1}{s(s+1)} & \frac{1}{s}\\ \frac{1}{s+1} & 0\end{bmatrix}.$$

It is clear to see that $n=3$ and $r=2$. By applying some method for right coprime factorization, we obtain

$$N(s)=\begin{bmatrix}1&0\\s&0\\0&1\end{bmatrix},\ D(s)=\begin{bmatrix}0&s+1\\s^2&-1\end{bmatrix}.$$

We can easily obtain that the poles of the open-loop system are 0, 0 and -1. We assume that $s_3^o=-1$ is a satisfactory pole, which is remained the same in the closed-loop system. In addition, we assume that s_1^c and s_2^c are the poles to be assigned in the closed-loop system.

It can be easily verified that Assumption A1 and A2 are satisfied. Set

$$f_1=\begin{bmatrix}a_1\\b_1\end{bmatrix},\ f_2=\begin{bmatrix}a_2\\b_2\end{bmatrix},\ f_3=\begin{bmatrix}a_3\\b_3\end{bmatrix},$$

where $a_i,b_i\in C$, $i=1,\ 2,\ 3.$ Then according to Theorem 1, for distinct and self-conjugate complex numbers s_1^c, s_2^c and s_3^o, all the constant gains K which the closed-loop poles to s_1^c, s_2^c and s_3^o can be given by

$$K=[W\quad 0][V\quad V^o]^{-1},$$

where

$$W=\begin{bmatrix}(s_1^c+1)b_1 & (s_2^c+1)b_2\\ a_1(s_1^c)^2-b_1 & a_2(s_2^c)^2-b_2\end{bmatrix},$$

$$V=\begin{bmatrix}a_1&a_2\\s_1^ca_1&s_2^ca_2\\b_1&b_2\end{bmatrix},\ V^o=\begin{bmatrix}a_3\\-a_3\\b_3\end{bmatrix}.$$

Specifically, when $s_{1,2}^c=-2\pm 2i,$ and

$$a_1=\overline{a_2}=-i,\quad b_1=\overline{b_2}=-2+i,\quad a_3=3,\quad b_3=2,$$

we have

$$V=\begin{bmatrix}-i&i\\2+2i&2-2i\\-2+i&-2-i\end{bmatrix},$$

$$W = \begin{bmatrix} -5i & 5i \\ -6-i & -6+i \end{bmatrix}, \quad V^o = \begin{bmatrix} 3 \\ -3 \\ 2 \end{bmatrix},$$

and

$$K = [W \quad 0][V \quad V^o]^{-1} = \begin{bmatrix} -0.625 & -1.875 & -1.875 \\ -2.75 & -2.25 & 0.75 \end{bmatrix}.$$

6. CONCLUSION

The design problem of partial pole assignment in two classes of frequency-domain MIMO models by constant gain feedback is investigated. Based on a simple and near general parametric expression for the gain matrix, a very simple parametric approach for solving the problem in term of a set of design parameter vectors and the set of closed-loop poles. Therefore, the set of poles do not need to be previously prescribed, and can be set undetermined and treated together with the set of parametric vectors as degrees of design freedom provided by the approach. In addition, utilizing the proposed method can eliminate the computations. The proposed parametric method is utilized to a numerical example, and the results show that the proposed method is simple and effective.

REFERENCES

[1] M. H. Hsueh and H. A Sehitoglu, "New eigenvalue placement method for linear dynamic systems," *Proc. of the 36th Conference on Decision and Control,* San Diego, California USA, pp. 4286-4287, December 1997.

[2] A. Eisinberg, G. Franze, and N. Salerno, "The pole placement problem by output-gain feedback: An alternative formulation," *Proc. of the 37th Conference on Decision and Control*, Tampa, Florida USA, pp. 1742-1744, December 1998.

[3] L. John and K. Nicos, "The pole placement map, its properties, and relationships to system invariants," *IEEE Trans. on Automatic Control,* vol. 38, no. 8, pp. 1266-1270, 1993.

[4] K. Hitoshi and I. Akira, "Pole assignment by state transition graph," *IEEE Trans. on Automatic Control*, vol. 37, no. 8, pp. 1196-1201, 1992.

[5] A. Dirk and L. W. Jacques, "Pole assignment for linear periodic systems by memoryless output feedback," *IEEE Trans. on Automatic Control*, vol. 40, no. 4, pp. 735-739, 1995.

[6] G. R. Duan, "Eigenstructure assignment and response analysis in descriptor linear systems with state feedback control," *Int. J. Control*, vol. 69, no. 5, pp. 663-694, 1998.

[7] G. R. Duan, "Eigenstructure assignment in descriptor linear systems via output feedback," *Int. J. Control*, vol. 72, no. 4, 1999, pp. 345-364.

[8] G. R. Duan and R. J. Patton, "Eigenstructure assignment in descriptor systems via state feedback-A new complete parametric approach," *Int. J. Systems Science*, vol. 29, no. 2, pp. 167-178, 1998.

[9] G. R. Duan, G. W. Irwin, and G. P. Liu, "Disturbance attenuation in linear systems via dynamical compensators-A parametric eigen-structure assignment approach," *IEE Proceeding, Control Theory and Applications*, vol. 146, no. 2, pp. 129-136, 2000.

[10] G. S. Wang and G. R. Duan, "State feedback eigenstructure assignment with minimum control effort," *Proc. of the 5th World Congress on Intelligent Control and Automation*, Hangzhou, China, vol. 1, pp. 35-38, June 2004.

[11] G. R. Duan and G. S. Wang, "P-D feedback eigenstructure assignment with minimum control effort in second-order dynamic systems," *Proc. of IEEE International Symposium of Computer Aid Control System Design*, Taibei, China, pp. 344-349, September 2004.

[12] G. S. Wang, B. Liang, and G. R. Duan, "Reconfiguring second-order dynamic systems via state feedback eigenstructure assignment," *International Journal of Control, Automation, and Systems*, vol. 3, no. 1, pp. 109-116, 2005.

[13] G. S. Wang and G. R. Duan, "Robust pole assignment via P-D feedback in a class of second-order dynamic systems," *Proc. of International Conference of Automation, Robots and Computer Vision*, Kunming, China, pp. 1152-1156, 2004.

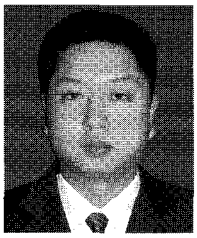
[14] L. J. Chiang and H. D. Thorp, "Partial eigenstructure assignment and its application to large-scale systems," *IEEE Trans. on Automatic Control*, vol. 36, no. 3, pp. 340-347, 1991.

[15] W. W. Lin and J. N. Wang, "Partial pole assignment for the vibrating system with aerodynamic effect," *Numerical Linear Algebra with Applications*, vol. 11, pp. 41-59, 2004.

[16] S. Kenji, "Partial pole placement by LQ regulators: An inverse problem approach," *IEEE Trans. on Automatic Control,* vol. 43, no. 5, pp.706-708, 1998.

[17] S. Atsushi, H. Ichijyo, and S. Kenji, "Partial pole placement with specified stability margins," *Proc. of the 38th Conference on Decision and Control,* Phoenix, Arizona USA, pp. 1389-390, December 1999.

[18] G. R. Duan and G. S. Wang, "Partial eigenstructure assignment for descriptor linear systems: A complete parametric approach," *Proc. of the Conference on Decision and Control*, Hawaii, US, pp. 3402-3407, December 2003.



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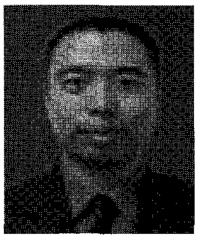

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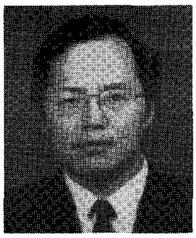

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