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Controllability for the Semilinear Fuzzy Integrodifferential Equations with Nonlocal Conditions and Forcing Term with Memory

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Abstract

Balasubramaniam and Muralisankar (2004) proved the existence and uniqueness of fuzzy solutions for the semilinear fuzzy integrodifferential equations with nonlocal initial condition. Park et al. (2006) found the sufficient condition of this system. Recently, Kwun et al. (2006) proved the existence and uniqueness of solutions for the semilinear fuzzy integrodifferential equations with nonlocal initial conditions and forcing term with memory in E_N. In this paper, we study the controllability for this system by using the concept of fuzzy number whose values are normal, convex, upper semicontinuous and compactly supported interval in E_N.

Key words : Fuzzy number, semilinear, fuzzy integrodifferential equation, nonlocal.

1. Introduction

Many authors have studied several concepts of fuzzy systems. Kaleva [3] studied the existence and uniqueness of solution for the fuzzy differential equation on E^n where E^n is normal, convex, upper semicontinuous and compactly supported fuzzy sets in R^n. Seikkala [8] proved the existence and uniqueness of fuzzy solution for the following equation:

$$\dot{x}(t) = f(t, x(t)) \ , \ \ x(0) = x_0,$$

where f is a continuous mapping from $R^+ \times R$ into R and x_0 is a fuzzy number in E^1. Diamond and Kloeden [2] proved the fuzzy optimal control for the following system:

$$\dot{x}(t) = a(t)x(t) + u(t), \ \ x(0) = x_0,$$

where $x(\cdot), u(\cdot)$ are nonempty compact interval-valued functions on E^1. Kwun and Park [4] proved the existence of fuzzy optimal control for the nonlinear fuzzy differential system with nonlocal initial condition in E^1_N using by Kuhn-Tucker theorems. Balasubramaniam and Muralisankar [1] proved the existence and uniqueness of fuzzy solutions for the semilinear fuzzy integrodifferential equation with nonlocal initial condition. Park, Park and Kwun [7] find the sufficient condition of nonlocal controllability for the semilinear fuzzy integrodifferential equation with nonlocal initial condition. Recently, Kwun et al. [5] proved the existence and uniqueness of solutions for the following semilinear fuzzy integrodifferential equations with nonlocal initial conditions and forcing term with memory ($u(t) \equiv 0$):

$$\frac{dx(t)}{dt} = A\left[x(t) + \int_0^t G(t-s)x(s)ds\right] \qquad (1)$$

$$+f(t, x, \int_0^t k(t, s, x(s))ds) + u(t), \ t \in I = [0, T],$$

$$x(0) + g(x) = x_0 \in E_N, \qquad (2)$$

where $A : I \to E_N$ is a fuzzy coefficient, E_N is the set of all upper semicontinuous convex normal fuzzy numbers with bounded α-level intervals, $f : I \times E_N \times E_N \to E_N$ and $k : I \times I \times E_N \to E_N$ are nonlinear continuous functions, $G(t)$ is $n \times n$ continuous matrix such that $\frac{dG(t)x}{dt}$ is continuous for $x \in E_N$ and $t \in I$ with $\|G(t)\| \le k$, $k > 0$, $u : I \to E_N$ is control function and $g : E_N \to E_N$ is a nonlinear continuous function.

In this paper, we study the controllability for the above semilinear fuzzy integrodifferential equations with nonlocal condition and forcing term with memory (1)-(2) in E_N.

2. Preliminaries

A fuzzy subset of R^n is defined in terms of member-

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ship function which assigns to each point $x \in R^n$ a grade of membership in the fuzzy set. Such a membership function $m : R^n \to [0,1]$ is used synonymously to denote the corresponding fuzzy set. We shall restrict attention here to the normal fuzzy sets which satisfy

Assumption 1. m maps R^n onto $[0,1]$.
Assumption 2. $[m]^0$ is a bounded subset of R^n.
Assumption 3. m is upper semicontinuous.
Assumption 4. m is fuzzy convex.

We denote by E^n the space of all fuzzy subsets m of R^n which satisfy assumptions 1-4; that is, normal, fuzzy convex and upper semicontinuous fuzzy sets with bounded supports. In particular, we denote by E^1 the space of all fuzzy subsets m of R which satisfy assumptions 1-4 [2].

A fuzzy number a in real line R is a fuzzy set characterized by a membership function m_a as $m_a : R \to [0,1]$. A fuzzy number a is expressed as $a = \int_{x\in R} m_a(x)/x$, with the understanding that $m_a(x) \in [0,1]$ represent the grade of membership of x in a and $\int$ denotes the union of $m_a(x)/x$'s [6].

Let E_N be the set of all upper semicontinuous convex normal fuzzy number with bounded α-level intervals. This means that if $a \in E_N$ then the α-level set

$$[a]^\alpha = \{x \in R : m_a(x) \geq \alpha,\ 0 < \alpha \leq 1\}$$

is a closed bounded interval which we denote by

$$[a]^\alpha = [a_l^\alpha, a_r^\alpha]$$

and there exists a $t_0 \in R$ such that $a(t_0) = 1$ [4].

The support Γ_a of a fuzzy number a is defined, as a special case of level set, by the following

$$\Gamma_a = \{x \in R : m_a(x) > 0\}.$$

Two fuzzy numbers a and b are called equal $a = b$, if $m_a(x) = m_b(x)$ for all $x \in R$. It follows that

$$a = b \Leftrightarrow [a]^\alpha = [b]^\alpha \text{ for all } \alpha \in (0,1].$$

A fuzzy number a may be decomposed into its level sets through the resolution identity

$$a = \int_0^1 \alpha[a]^\alpha,$$

where $\alpha[a]^\alpha$ is the product of a scalar α with the set $[a]^\alpha$ and $\int$ is the union of $[a]^\alpha$'s with α ranging from 0 to 1.

We denote the suprimum metric d_∞ on E^n and the suprimum metric H_1 on $C(I : E^n)$.

Definition 2.1. Let $a, b \in E^n$.

$$d_\infty(a,b) = \sup\{d_H([a]^\alpha, [b]^\alpha) : \alpha \in (0,1]\},$$

where d_H is the Hausdorff distance.

Definition 2.2. Let $x, y \in C(I : E^n)$

$$H_1(x,y) = \sup\{d_\infty(x(t), y(t)) : t \in I\}.$$

Let I be a real interval. A mapping $x : I \to E_N$ is called a fuzzy process. We denote

$$[x(t)]^\alpha = [x_l^\alpha(t), x_r^\alpha(t)],\ t \in I,\ 0 < \alpha \leq 1.$$

The derivative $x'(t)$ of a fuzzy process x is defined by

$$[x'(t)]^\alpha = [(x_l^\alpha)'(t), (x_r^\alpha)'(t)],\ \ 0 < \alpha \leq 1$$

provided that is equation defines a fuzzy $x'(t) \in E_N$.

The fuzzy integral

$$\int_a^b x(t)dt, \quad a, b \in I$$

is defined by

$$\left[\int_a^b x(t)dt\right]^\alpha = \left[\int_a^b x_l^\alpha(t)dt, \int_a^b x_r^\alpha(t)dt\right]$$

provided that the Lebesgue integrals on the right exist.

Definition 2.3. [1] The fuzzy process $x : I \to E_N$ is a solution of equations (1)-(2) without the inhomogeneous term if and only if

$$(\dot{x}_l^\alpha)(t) = \min\Big\{A_l^\alpha(t)\big[x_j^\alpha(t) + \int_0^t G(t-s)x_j^\alpha(s)ds\big],\ i,j = l,r\Big\},$$
$$(\dot{x}_r^\alpha)(t) = \max\Big\{A_r^\alpha(t)\big[x_j^\alpha(t) + \int_0^t G(t-s)x_j^\alpha(s)ds\big],\ i,j = l,r\Big\},$$

and

$$(x_l^\alpha)(0) = x_{0l}^\alpha - g_l^\alpha(x),\ \ (x_r^\alpha)(0) = x_{0r}^\alpha - g_r^\alpha(x).$$

Now we assume the following:

(H1) The nonlinear function $f : [0,T] \times E_N \times E_N \to E_N$ satisfies a global Lipschitz condition, there exists a finite constants $k_1, k_2 > 0$ such that

$$d_H\big([f(s,\xi_1(s),\eta_1(s))]^\alpha,\ [f(s,\xi_2(s),\eta_2(s))]^\alpha\big) \leq k_1 d_H([\xi_1(s)]^\alpha, [\xi_2(s)]^\alpha) + k_2\, d_H([\eta_1(s)]^\alpha, [\eta_2(s)]^\alpha)$$

for all $\xi_1(s), \xi_2(s), \eta_1(s), \eta_2(s) \in E_N$.

(H2) The nonlinear function $k : [0,T] \times [0,T] \times E_N \to E_N$ satisfies a global Lipschitz condition, there exists a finite constant $M > 0$ such that

$$d_H\big([k(t,s,\psi_1(s)]^\alpha, [k(t,s,\psi_2(s))]^\alpha\big) \leq M d_H([\psi_1(s)]^\alpha, [\psi_2(s)]^\alpha)$$

for all $\psi_1(s), \psi_2(s) \in E_N$.

(H3) The nonlinear function $g : E_N \to E_N$ satisfies following inequality

$$d_H\big([g(\xi_1)]^\alpha, [g(\xi_2)]^\alpha\big) \le L d_H([\xi_1(\cdot)]^\alpha, [\xi_2(\cdot)]^\alpha),$$

where constant $L > 0$.

(H4) $S(t)$ is a fuzzy number satisfying, for $y \in E_N$ and $S'(t)y \in C^1(I : E_N) \cap C(I : E_N)$, the equation

$$\begin{aligned}\frac{d}{dt}S(t)y &= A\left[S(t)y + \int_0^t G(t-s)S(s)yds\right]\\ &= S(t)Ay + \int_0^t S(t-s)AG(s)yds, \quad t \in I,\end{aligned}$$

such that

$$[S(t)]^\alpha = [S_l^\alpha(t), S_r^\alpha(t)],$$

and $S_i^\alpha(t)$ $(i = l, r)$ is continuous. That is, there exists a constant $c > 0$ such that $|S_i^\alpha(t)| \le c$ for all $t \in I$.

(H5) $c(L + k_1T + k_2MT^2) < 1$.

3. Nonlocal controllability

In this section, we consider the controllability for the equations (1)-(2).

The equations (1)-(2) is related to the following fuzzy integral equation:

$$\begin{aligned}x(t) = S(t)(x_0 - g(x)) + \int_0^t S(t-s)u(s)ds \quad (3)\\ + \int_0^t S(t-s)f(s, x(s), \int_0^s k(s,\tau,x(\tau))d\tau)ds,\end{aligned}$$

where $S(t)$ satisfy (H4).

Theorem 3.1. [5].Let $T > 0$, assume that the function f, k and g satisfy hypotheses (H1)-(H5). Then, for every $x_0 \in E_N$, equation (3)($u(t) \equiv 0$) has a unique fuzzy solution $x \in C([0,T] : E_N)$.

Definition 3.2. The equation (3) is nonlocal controllable if, there exists $u(t)$ such that the fuzzy solution $x(t)$ of (3) satisfies $x(T) = x^1 - g(x)$ (i.e., $[x(T)]^\alpha = [x^1 - g(x)]^\alpha$) where x^1 is target set.

We assume that the linear fuzzy control system with respect to nonlinear fuzzy control system (3) is nonlocal controllable. Then

$$\begin{aligned}x(T) &= S(T)(x_0 - g(x)) + \int_0^T S(T-s)u(s)ds\\ &= x^1 - g(x)\end{aligned}$$

and

$$\begin{aligned}&[x(T)]^\alpha\\ &= \left[S(T)(x_0 - g(x)) + \int_0^T S(T-s)u(s)ds\right]^\alpha\\ &= \Big[S_l^\alpha(T)(x_{0l}^\alpha - g_l^\alpha(x)) + \int_0^T S_l^\alpha(T-s)u_l^\alpha(s)ds,\\ &\quad .S_r^\alpha(T)(x_{0r}^\alpha - g_r^\alpha(x)) + \int_0^T S_r^\alpha(T-s)u_r^\alpha(s)ds\Big]\\ &= \left[(x^1 - g(x))_l^\alpha, (x^1 - g(x))_r^\alpha\right].\end{aligned}$$

Defined the fuzzy mapping $G : \widetilde{P}(R) \to E_N$ by

$$G^\alpha(v) = \begin{cases} \int_0^T S^\alpha(T-s)v(s)ds, & v \subset \overline{\Gamma_u}, \\ 0, & \text{otherwise.}\end{cases} \quad (4)$$

Then there exists G_i^α $(i = l, r)$ such that

$$G_l^\alpha(v_l) = \int_0^T S_l^\alpha(T-s)v_l(s)ds\ ,\ v_l(s) \in [u_l^\alpha(s), u^1(s)]\ ,$$

$$G_r^\alpha(v_r) = \int_0^T S_r^\alpha(T-s)v_r(s)ds\ ,\ v_r(s) \in [u^1(s), u_r^\alpha(s)]\ .$$

We assume that G_l^α, G_r^α are bijective mappings.

Hence α-level of $u(s)$ are

$$\begin{aligned}&[u(s)]^\alpha = [u_l^\alpha(s), u_r^\alpha(s)]\\ &= \big[(\widetilde{G}_l^\alpha)^{-1}\big((x^1)_l^\alpha - g_l^\alpha(x) - S_l^\alpha(T)(x_{0l}^\alpha - g_l^\alpha(x))\big),\\ &\quad (\widetilde{G}_r^\alpha)^{-1}\big((x^1)_r^\alpha - g_r^\alpha(x) - S_r^\alpha(T)(x_{0r}^\alpha - g_r^\alpha(x))\big)\big].\end{aligned}$$

Thus we can be introduced $u(s)$ of nonlinear system

$$\begin{aligned}&[u(s)]^\alpha = [u_l^\alpha(s), u_r^\alpha(s)]\\ &= \Bigg[(\widetilde{G}_l^\alpha)^{-1}\bigg((x^1)_l^\alpha - g_l^\alpha(x) - S_l^\alpha(T)\\ &\quad \times\big(x_{0l}^\alpha - g_l^\alpha(x)\big) - \int_0^T S_l^\alpha(T-s)\\ &\quad \times f_l^\alpha(s, x(s), \int_0^s k(s,\tau,x(\tau))d\tau)ds\bigg),\\ &\quad (\widetilde{G}_r^\alpha)^{-1}\bigg((x^1)_r^\alpha - g_r^\alpha(x) - S_r^\alpha(T)\\ &\quad \times\big(x_{0r}^\alpha - g_r^\alpha(x)\big) - \int_0^T S_r^\alpha(T-s)\\ &\quad \times f_r^\alpha(s, x(s), \int_0^s k(s,\tau,x(\tau))d\tau)ds\bigg)\Bigg].\end{aligned}$$

Then substituting this expression into the equation (3) yields α-level of $x(T)$.

$$
\begin{aligned}
&[x(T)]^\alpha \\
&= \Big[S_l^\alpha(T)(x_{0l}^\alpha - g_l^\alpha(x)) + \int_0^T S_l^\alpha(T-s) \\
&\quad \times f_l^\alpha(s,x(s), \int_0^s k(s,\tau,x(\tau))d\tau)ds \\
&\quad + \int_0^T S_l^\alpha(T-s)(\widetilde{G}_l^\alpha)^{-1}\big((x^1)_l^\alpha - g_l^\alpha(x) \\
&\quad - S_l^\alpha(T)(x_{0l}^\alpha - g_l^\alpha(x)) - \int_0^T S_l^\alpha(T-s) \\
&\quad \times f_l^\alpha(s,x(s), \int_0^s k(s,\tau,x(\tau))d\tau)ds\big)ds, \\
&\quad S_r^\alpha(T)(x_{0r}^\alpha - g_r^\alpha(x)) + \int_0^T S_r^\alpha(T-s) \\
&\quad \times (f_r^\alpha(s,x(s), \int_0^s k(s,\tau,x(\tau))d\tau)ds \\
&\quad + \int_0^T S_r^\alpha(T-s)(\widetilde{G}_r^\alpha)^{-1}\big((x^1)_r^\alpha - g_r^\alpha(x) \\
&\quad - S_r^\alpha(T)(x_{0r}^\alpha - g_r^\alpha(x)) - \int_0^T S_r^\alpha(T-s) \\
&\quad \times f_r^\alpha(s,x(s), \int_0^s k(s,\tau,x(\tau))d\tau)ds\big)ds\Big] \\
&= \Big[S_l^\alpha(T)(x_{0l}^\alpha - g_l^\alpha(x)) + \int_0^T S_l^\alpha(T-s) \\
&\quad \times f_l^\alpha(s,x(s), \int_0^s k(s,\tau,x(\tau))d\tau)ds \\
&\quad + G_l^\alpha \cdot (\widetilde{G}_l^\alpha)^{-1}\big((x^1)_l^\alpha - g_l^\alpha(x) \\
&\quad - S_l^\alpha(T)(x_{0l}^\alpha - g_l^\alpha(x)) - \int_0^T S_l^\alpha(T-s) \\
&\quad \times f_l^\alpha(s,x(s), \int_0^s k(s,\tau,x(\tau))d\tau)ds\big), \\
&\quad S_r^\alpha(T)(x_{0r}^\alpha - g_r^\alpha(x)) + \int_0^T S_r^\alpha(T-s) \\
&\quad \times f_r^\alpha(s,x(s), \int_0^s k(s,\tau,x(\tau))d\tau)ds \\
&\quad + G_r^\alpha \cdot (\widetilde{G}_r^\alpha)^{-1}\big((x^1)_r^\alpha - g_r^\alpha(x) \\
&\quad - S_r^\alpha(T)(x_{0r}^\alpha - g_r^\alpha(x)) - \int_0^T S_r^\alpha(T-s) \\
&\quad \times f_r^\alpha(s,x(s), \int_0^s k(s,\tau,x(\tau))d\tau)ds\big)\Big] \\
&= [(x^1)_l^\alpha - g_l^\alpha(x), (x^1)_r^\alpha - g_r^\alpha(x)] \\
&= [x^1 - g(x)]^\alpha.
\end{aligned}
$$

We now set

$$
\begin{aligned}
\Phi x(t) &= S(t)(x_0 - g(x)) + \int_0^t S(t-s) \\
&\quad \times f(s,x(s), \int_0^s k(s,\tau,x(\tau))d\tau)ds \\
&\quad + \int_0^t S(t-s)\widetilde{G}^{-1}\big(x^1 - g(x) \\
&\quad - S(T)(x_0 - g(x)) - \int_0^T S(T-s) \\
&\quad \times f(s,x(s), \int_0^s k(s,\tau,x(\tau))d\tau)ds\big)ds
\end{aligned}
$$

where the fuzzy mappings $\widetilde{G}^{-1}$ satisfied above statements.

Notice that $\Phi x(T) = x^1 - g(x)$, which means that the control $u(t)$ steers the equation (3) from the origin to $x^1 - g(x)$ in time T provided we can obtain a fixed point of the nonlinear operator Φ.

Assume that the following hypotheses:

(H6) Linear system of equation (3) ($f = 0$) is nonlocal controllable.

(H7) $c\{L(1+(1+c)T) + (k_1 + k_2MT)T(1+cT)\} < 1$.

Theorem 3.3. Suppose that hypotheses (H1)-(H7) are satisfied. Then the equation (3) is nonlocal controllable.

Proof. We can easily check that Φ is continuous function from $C([0,T]:E_N)$ to itself. For $x, y \in C([0,T]:E_N)$,

$$
\begin{aligned}
&d_H\big([\Phi x(t)]^\alpha, [\Phi y(t)]^\alpha\big) \\
&= d_H\Big(\big[S(t)(x_0 - g(x)) + \int_0^t S(t-s) \\
&\quad \times f(s,x(s), \int_0^s k(s,\tau,x(\tau))d\tau)ds \\
&\quad + \int_0^t S(t-s)\widetilde{G}^{-1}\big(x^1 - g(x) \\
&\quad\quad - S(T)(x_0 - g(x)) - \int_0^T S(T-s) \\
&\quad \times f(s,x(s), \int_0^s k(s,\tau,x(\tau))d\tau)ds\big)ds\big]^\alpha, \\
&\quad \big[S(t)(x_0 - g_l^\alpha(y)) + \int_0^t S(t-s) \\
&\quad \times f(s,y(s), \int_0^s k(s,\tau,y(\tau))d\tau)ds \\
&\quad + \int_0^t S(t-s)\widetilde{G}^{-1}\big(x^1 - g(y) \\
&\quad\quad - S(T)(x_0 - g(y)) - \int_0^T S(T-s) \\
&\quad \times f(s,y(s), \int_0^s k(s,\tau,y(\tau))d\tau)ds\big)ds\big]^\alpha\Big)
\end{aligned}
$$

$$
\begin{aligned}
&\leq d_H\Big([S(t)g(x)]^\alpha, [S(t)g(y)]^\alpha\Big)\\
&\quad + d_H\Big([\int_0^t S(t-s)\\
&\qquad \times f(s,x(s), \int_0^s k(s,\tau,x(\tau))d\tau)ds]^\alpha,\\
&\quad [\int_0^t S(t-s)\\
&\qquad \times f(s,y(s), \int_0^s k(s,\tau,y(\tau))d\tau)ds]^\alpha\Big)\\
&\quad + d_H\Big([\int_0^t S(t-s)\widetilde{G}^{-1}(-g(x)+S(T)g(x)\\
&\qquad - \int_0^T S(T-s)\\
&\qquad \times f(s,x(s), \int_0^s k(s,\tau,x(\tau))d\tau)ds)\, ds]^\alpha,\\
&\qquad [\int_0^t S(t-s)\widetilde{G}^{-1}(-g(y)+S(T)g(y)\\
&\qquad - \int_0^T S(T-s)\\
&\qquad \times f(s,y(s), \int_0^s k(s,\tau,y(\tau))d\tau)ds)\, ds]^\alpha\Big)\\
&\leq cLd_H([x(\cdot)]^\alpha, [y(\cdot)]^\alpha)\\
&\quad + c(\int_0^t \Big(k_1 d_H([x(s)]^\alpha, [y(s)]^\alpha)\\
&\qquad + k_2 M \int_0^s d_H([x(\tau)]^\alpha, [y(\tau)]^\alpha) d\tau\Big) ds\\
&\quad + c\int_0^t \Big\{(1+c)Ld_H([x(\cdot)]^\alpha, [y(\cdot)]^\alpha)\\
&\qquad + c\int_0^T \Big(k_1 d_H([x(s)]^\alpha, [y(s)]^\alpha)\\
&\qquad + k_2 M \int_0^s d_H([x(\tau)]^\alpha, [y(\tau)]^\alpha) d\tau\Big) ds\Big\} ds
\end{aligned}
$$

Therefore

$$
\begin{aligned}
&d_\infty(\Phi x(t), \Phi y(t)) = \sup_{\alpha\in(0,1]} d_H([\Phi x(t)]^\alpha, [\Phi y(t)]^\alpha)\\
&\leq cLd_\infty(x(\cdot), y(\cdot))\\
&\quad + c(\int_0^t \Big(k_1 d_\infty(x(s), y(s))\\
&\qquad + k_2 M \int_0^s d_\infty(x(\tau), y(\tau)) d\tau\Big) ds\\
&\quad + c\int_0^t \Big\{(1+c)Ld_\infty(x(\cdot), y(\cdot))\\
&\qquad + c\int_0^T \Big(k_1 d_\infty(x(s), y(s))\\
&\qquad + k_2 M \int_0^s d_\infty(x(\tau), y(\tau)) d\tau\Big) ds\Big\} ds
\end{aligned}
$$

Hence

$$
\begin{aligned}
&H_1(\Phi x, \Phi y) = \sup_{t\in[0,T]} d_\infty(\Phi x(t), \Phi y(t))\\
&\leq c\Big\{L\Big(1+(1+c)T\Big)\\
&\qquad + \Big(k_1 + k_2 MT\Big)T(\Big(1+cT\Big)\Big\} H_1(x,y).
\end{aligned}
$$

By hypotheses (H7), Φ is a contraction mapping. By the Banach fixed point theorem, (3) has a unique fixed point $x \in C([0,T]: E_N)$. □

4. Example

Consider the semilinear one dimensional heat equation on a connected domain (0,1) for a material with memory, boundary condition $x(t,0) = x(t,1) = 0$ and with initial condition $x(0,z) = x_0(z)$, $\sum_{k=1}^p c_k x(t_k, z) = g(x)$, where $x_0(z) \in E_N$. Let $x(t,z)$ be the internal energy and $f(t, x(t,z), \int_0^t k(t,s,x(t,z))ds) = \tilde{2}tx(t,z)^2 + \int_0^t (t-s)x(s)ds$ be the external heat with memory.

Let $A = \tilde{2}\frac{\partial^2}{\partial z^2}$ and $G(t-s) = e^{-(t-s)}$, then the balance equation becomes

$$
\begin{aligned}
\frac{dx(t)}{dt} &= \tilde{2}[x(t) - \int_0^t e^{-(t-s)}x(s)ds] \qquad (5)\\
&\quad + \tilde{2}tx(t)^2 + \int_0^t (t-s)x(s)ds + u(t),\ t\in I,\\
x(0) &= x_0 - \sum_{k=1}^p c_k x(t_k, z). \qquad (6)
\end{aligned}
$$

Since α-level set of fuzzy number $\tilde{2}$ is $[2]^\alpha = [\alpha+1, 3-\alpha]$ for all $\alpha \in [0,1]$, α-level set of $f(t,x(t), \int_0^t k(t,s,x(s))ds)$ is

$$
\begin{aligned}
&[f(t,x(t), \int_0^t k(t,s,x(s))ds)]^\alpha\\
&= [t(\alpha+1)(x_l^\alpha(t))^2 + \int_0^t (t-s)x_l^\alpha(t),\\
&\quad t(3-\alpha)(x_r^\alpha(t))^2 + \int_0^t (t-s)x_r^\alpha(t)].
\end{aligned}
$$

Further, we have

$$
\begin{aligned}
& d_H([f(t,x(t),\int_0^t k(t,s,x(s))ds)]^\alpha, \\
& \quad [f(t,y(t),\int_0^t k(t,s,y(s))ds)]^\alpha) \\
& = d_H\Big([t(\alpha+1)(x_l^\alpha(t))^2 + \int_0^t (t-s)x_l^\alpha(t), \\
& \quad t(3-\alpha)(x_r^\alpha(t))^2 + \int_0^t (t-s)x_r^\alpha(t)], \\
& \quad [t(\alpha+1)(y_l^\alpha(t))^2 + \int_0^t (t-s)y_l^\alpha(t), \\
& \quad t(3-\alpha)(y_r^\alpha(t))^2 + \int_0^t (t-s)y_r^\alpha(t)]\Big) \\
& = t\max\{(\alpha+1)|(x_l^\alpha(t))^2 - (y_l^\alpha(t))^2|, \\
& \quad (3-\alpha)|(x_r^\alpha(t))^2 - (y_r^\alpha(t))^2|\} \\
& \quad + \int_0^t (t-s)d_H([x_l^\alpha(s), x_r^\alpha(s)], [y_l^\alpha(s), y_r^\alpha(s)]) \\
& \leq 3T|x_r^\alpha(t) + y_r^\alpha(t)| \\
& \quad \times \max\{|x_l^\alpha(t) - y_l^\alpha(t)|, |x_r^\alpha(t) - y_r^\alpha(t)|\} \\
& \quad + \frac{T^2}{2}\max\{|x_l^\alpha(t) - y_l^\alpha(t)|, |x_r^\alpha(t) - y_r^\alpha(t)|\} \\
& = k_1 d_H([x(t)]^\alpha, [y(t)]^\alpha) + k_2 d_H([x(t)]^\alpha, [y(t)]^\alpha),
\end{aligned}
$$

where k_1 and k_2 are satisfies the inequality in hypotheses (H1)-(H2), and also we have

$$
\begin{aligned}
& d_H([g(x)]^\alpha, [g(y)]^\alpha) \\
& = d_H(\sum_{k=1}^p C_k[x(t_k)]^\alpha, \sum_{k=1}^p C_k[y(t_k)]^\alpha) \\
& \leq |\sum_{k=1}^p C_k| \max_k d_H([x(t_k)]^\alpha, [y(t_k)]^\alpha) \\
& = L d_H([x(t_k)]^\alpha, [y(t_k)]^\alpha),
\end{aligned}
$$

where L satisfies the inequality in hypothesis (H3).

Let initial value x_0 is $\widetilde{0}$. Target set is $x^1 = \widetilde{2}$. The α-level set of fuzzy numbers $\widetilde{0}$ is $[\widetilde{0}]^\alpha = [\alpha-1, 1-\alpha]$. We introduce the α-level set of $u(s)$ of equations (5)-(6).

$$
\begin{aligned}
& [u(s)]^\alpha = [u_l^\alpha(s), u_r^\alpha(s)] \\
& = \Bigg[\widetilde{G}_l^{-1}\Bigg((\alpha+1) - \sum_{k=1}^p c_k x_l^\alpha(t_k) - S_l^\alpha(T) \\
& \times((\alpha-1) - \sum_{k=1}^p c_k x_l^\alpha(t_k)) - \int_0^T S_l^\alpha(T-s) \\
& \times\Big(s(\alpha+1)(x_l^\alpha(s))^2 + \int_0^s (s-\tau)x_l^\alpha(\tau)d\tau\Big)ds\Bigg), \\
& \widetilde{G}_r^{-1}\Bigg((3-\alpha) - \sum_{k=1}^p c_k x_r^\alpha(t_k) - S_r^\alpha(T) \\
& \times((1-\alpha) - \sum_{k=1}^p c_k x_r^\alpha(t_k)) - \int_0^T S_r^\alpha(T-s) \\
& \times\Big(s(3-\alpha)(x_r^\alpha(s))^2 + \int_0^s (s-\tau)x_r^\alpha(\tau)d\tau\Big)ds\Bigg)\Bigg].
\end{aligned}
$$

Then substituting this expression into the integral system with respect to (5)-(6) yields α-level set of $x(T)$.

$$
\begin{aligned}
& [x(T)]^\alpha = \Bigg[S_l^\alpha(T)((\alpha-1) - \sum_{k=1}^p c_k x_l^\alpha(t_k)) \\
& + \int_0^T S_l^\alpha(T-s)\Big(s(\alpha+1)(x_l^\alpha(s))^2 \\
& + \int_0^s (s-\tau)x_l^\alpha(\tau)d\tau\Big)ds + \int_0^T S_l^\alpha(T-s) \\
& \times(\widetilde{G}_l^\alpha)^{-1}\Bigg((\alpha+1) - \sum_{k=1}^p c_k x_l^\alpha(t_k) - S_l^\alpha(T) \\
& \times((\alpha-1) - \sum_{k=1}^p c_k x_l^\alpha(t_k)) - \int_0^T S_l^\alpha(T-s) \times \\
& \Big(s(\alpha+1)(x_l^\alpha(s))^2 + \int_0^s (s-\tau)x_l^\alpha(\tau)d\tau\Big)ds\Bigg)ds, \\
& S_r^\alpha(T)((1-\alpha) - \sum_{k=1}^p c_k x_r^\alpha(t_k)) \\
& + \int_0^T S_l^\alpha(T-s)\Big(s(3-\alpha)(x_r^\alpha(s))^2 \\
& + \int_0^s (s-\tau)x_r^\alpha(\tau)d\tau\Big)ds + \int_0^T S_l^\alpha(T-s) \\
& \times(\widetilde{G}_r^\alpha)^{-1}\Bigg((3-\alpha) - \sum_{k=1}^p c_k x_r^\alpha(t_k) - S_r^\alpha(T) \\
& \times((1-\alpha) - \sum_{k=1}^p c_k x_r^\alpha(t_k)) - \int_0^T S_r^\alpha(T-s) \times \\
& \Big(s(3-\alpha)(x_r^\alpha(s))^2 + \int_0^s (s-\tau)x_r^\alpha(\tau)d\tau\Big)ds\Bigg)ds\Bigg] \\
& = \Bigg[(\alpha+1) - \sum_{k=1}^p c_k x_l^\alpha(t_k), \\
& \qquad (3-\alpha) - \sum_{k=1}^p c_k x_r^\alpha(t_k)\Bigg] \\
& = \Bigg[\widetilde{2} - \sum_{k=1}^p c_k x(t_k)\Bigg]^\alpha.
\end{aligned}
$$

Then all the conditions stated in Theorem 3.3 are satisfied, so the system (5)-(6) is nonlocal controllable on $[0,T]$.

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