J. Korean Math. Soc. **44** (2007), No. 4, pp. 779–786

BLOW-UP RATE ESTIMATES FOR A SYSTEM OF REACTION-DIFFUSION EQUATIONS WITH ABSORPTION

Zhaoyin Xiang, Qiong Chen, and Chunlai Mu

Abstract. In this note, we consider a system of two reaction-diffusion equations with absorption, under homogeneous Dirichlet boundary. Using scaling methods, we establish the blow-up rate estimates.

1. Introduction

In this note, we investigate the following system of reaction-diffusion equations with absorption:

$$
\begin{aligned}
&u_t - \Delta u = v^p - au^r, && x \in \Omega, \quad t > 0,\\
&v_t - \Delta v = u^q - bv^s, && x \in \Omega, \quad t > 0,\\
&u(x,t) = v(x,t) = 0, && x \in \partial\Omega, \quad t > 0,\\
&u(x,0) = u_0(x), \quad v(x,0) = v_0(x), && x \in \Omega,
\end{aligned}
\tag{1}
$$

where $p, q, r, s, a, b > 0$, $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary $\partial\Omega$, $u_0(x)$, $v_0(x)$ are continuous and nonnegative functions, and vanish on $\partial\Omega$.

Equations in (1) provide a simple model to describe, for instance, the cooperative interaction of two diffusing biological species (see [2]). We also refer to [1, 16, 17] for details on physical models involving more general reaction-diffusion systems.

In [2], Bedjaoni and Souplet proved that: (i) If $pq > \max(r,1)\max(s, 1)$, then there exist solutions of (1) which blow up in finite time; (ii) If $pq < \max(r,1)\max(s,1)$, then all solutions are global; (iii) in the critical case $pq = \max(r,1)\max(s,1)$, the issue may depend on the size of the coefficients a and b.

The main purpose of this paper is to establish the blow-up rate estimates for the blow-up solution (u,v) of (1). Since the system is completely coupled, u and v have simultaneous blow-up, which is necessarily for studying the blow-up profiles of blow-up solutions for the systems. Throughout this paper, we

Received September 28, 2005.

2000 *Mathematics Subject Classification.* 35K55, 35K60.

Key words and phrases. reaction-diffusion systems, absorption, blowup rate estimates.

This work is supported in part by NNSF of China (10201024, 10571126) and in part by Program for New Century Excellent Talents in University.

assume $pq > \max(r,1)\max(s,1)$ and denote

$$\alpha = \frac{p+1}{pq-1},\ \beta = \frac{q+1}{pq-1},\ M_u(t) = \sup_{\bar{\Omega}\times(0,t]} u(x,\tau),\ M_v(t) = \sup_{\bar{\Omega}\times(0,t]} v(x,\tau).$$

Our main results are the following.

Theorem 1.1. *Assume $r \le p\frac{q+1}{p+1}$ and $s \le q\frac{p+1}{q+1}$. Let (u,v) be a solution of (1), which blows up at finite time T. Then there exists a constant $c > 0$ such that*

$$\max_{\bar{\Omega}\times[0,t]} u(x,\tau) \ge c(T-t)^{-\alpha}, \quad \max_{\bar{\Omega}\times[0,t]} v(x,\tau) \ge c(T-t)^{-\beta}.$$

Theorem 1.2. *Assume $r < p\frac{q+1}{p+1}$ and $s < q\frac{p+1}{q+1}$, and $\max(\alpha,\beta) \ge \frac{N+1}{2}$. Let (u,v) be a solution of (1), which blows up at finite time T. Then there exists a constant $C > 0$ such that*

$$\max_{\bar{\Omega}\times[0,t]} u(x,\tau) \le C(T-t)^{-\alpha}, \quad \max_{\bar{\Omega}\times[0,t]} v(x,\tau) \le C(T-t)^{-\beta}.$$

We remark that $r = p\frac{q+1}{p+1}$ and $s = q\frac{p+1}{q+1}$ are critical exponents on the existence of *forward self-similar solutions* to system (1) (see [2, 18]). We also remark that for system (1) without absorption (i.e., $a = b = 0$), some authors have established the blow-up estimates. In the case of $p, q \ge 1, pq > 1$, Deng [4] and Wang [20] gave similar estimates as Theorem 1.1, 1.2 for the *time-increasing solutions* (generally, which can be ensured under the assumption that (u_0, v_0) is a pair of subsolutions of the system). On the other hand, using scaling methods introduced by [9] (developed by [3, 7, 10]), Fila and Souplet [7] also obtained similar results as Theorem 1.2. Our results are consistent with them if we take $r = s = 0$. The advantage of the scaling method is that we only need to assume $u_0(x), v_0(x)$ are nonnegative. In general, some growth restrictions on the reaction terms are necessary since the arbitrariness of initial data (see also [8, 11, 12]). Recently, Souplet and Tayachi in [19] also studied the following system

$$(2) \qquad u_t - \Delta u = v^p + u^r, \quad v_t - \Delta v = u^q + v^s, \qquad x \in \mathbb{R}^N, \quad t > 0,$$

with the same initial data as (1). They consider the condition of simultaneous blow-up and nonsimultaneous blow-up. Similar critical exponents $r = p\frac{q+1}{p+1}, s = q\frac{p+1}{q+1}$ occur when they established the blow-up estimates for the blow-up solutions of (2).

If $u_0(x) = v_0(x)$, $p = q$, $a = b$, $r = s$, in system (1), we have $u = v$. Then we get as a Corollary of these two theorems a similar result for single equation, whose blow-up rate estimates, as we know, has not been given in the literature previously.

Theorem 1.3. *Consider the problem*

$$
(3)\qquad
\begin{aligned}
&u_t - \Delta u = u^p - au^r, && x \in \Omega, \quad t > 0,\\
&u(x,t) = 0, && x \in \partial\Omega, \quad t > 0,\\
&u(x,0) = u_0(x), && x \in \Omega.
\end{aligned}
$$

Let $p > \max(r,1)$ and u be a solution of (3), which blows up in finite time T. Then there exist constants $C > c > 0$ such that

(i) $\max_{\bar{\Omega}\times[0,t]} u(x,\tau) \geq c(T-t)^{-\frac{1}{p-1}}$;

(ii) $\max_{\bar{\Omega}\times[0,t]} u(x,\tau) \leq C(T-t)^{-\frac{1}{p-1}}$ *provided that* $p \leq 1 + \frac{2}{N+1}$.

Remark 1.1. If $pq = \max(r,1)\max(s,1)$, $r, s > 1$ and a, b are sufficiently small, then there exist solutions of (1) which blow up in finite time (see [2]). Under this assumption, a carefully check of the proof of Lemma 2.1 and Theorem 1.1 shows that the lower estimates Theorem 1.1 (correspondingly, Theorem 1.3 (i)) are still valid.

The rest of this paper is organized as follows. In Section 2, we give the lower blow-up estimates. Then, we establish the upper estimates in Section 3. We will use some ideas of ([3, 6, 7, 15]) to prove our conclusions.

2. Lower blow-up estimates

We begin our arguments with a lemma, which gives the relationship between $M_u(t)$ and $M_v(t)$ near the blow-up time T. As the above mentioned, this lemma also implies that the simultaneous blowup occurs.

Lemma 2.1. *Assume $r \leq p\frac{q+1}{p+1}$ and $s \leq q\frac{p+1}{q+1}$. Let (u,v) be a solution of (1), which blows up at finite time T. Then there exists $\delta \in (0,1)$ such that*

$$
(4)\qquad \delta \leq M_u^{-\frac{1}{2\alpha}}(t) M_v^{\frac{1}{2\beta}}(t) \leq \delta^{-1}, \qquad t \in (\frac{T}{2}, T).
$$

Proof. Since the solution (u,v) blows up at finite time T, without loss of generality, we may assume M_u diverges as $t \to T$. It is sufficient to prove the first inequality in (4). On the contrary we assume it is not true, then there exists a sequence $\{t_n\}$ with $t_n \to T$ as $n \to +\infty$ such that

$$
(5)\qquad M_u^{-\frac{1}{2\alpha}}(t_n) M_v^{\frac{1}{2\beta}}(t_n) \to 0 \quad \text{as} \quad n \to \infty.
$$

For each t_n, there exists $(\hat{x}_n, \hat{t}_n) \in \Omega \times (0, t_n]$ such that $u(\hat{x}_n, \hat{t}_n) = M_u(t_n)$. Because $M_u(t_n) \to \infty$, it follows that $\hat{t}_n \to T$ as $n \to \infty$. Let $d_n := \operatorname{dist}(\hat{x}_n, \partial\Omega)$ and $\lambda_n := \lambda(t_n) := M_u^{-\frac{1}{2\alpha}}(t_n)$. We distinguish two cases depending on whether

$$
(i) \limsup_{n\to\infty} \frac{d_n}{\lambda_n} = \infty \quad \text{or} \quad (ii) \limsup_{n\to\infty} \frac{d_n}{\lambda_n} < \infty.
$$

Case (i) Choose a subsequence (denoted again by $\{t_n\}$) such that

$$
\lim_{n\to\infty} \frac{d_n}{\lambda_n} = \infty.
$$

We rescale the solution (u,v) about the corresponding point $(\hat{x}_n,\hat{t}_n)$ with the scaling factor λ_n as follows:

$$\phi^{\lambda_n}(y,\tau) := \lambda_n^{2\alpha}u(\lambda_n y+\hat{x}_n,\lambda_n^2\tau+\hat{t}_n),$$
$$\psi^{\lambda_n}(y,\tau) := \lambda_n^{2\beta}v(\lambda_n y+\hat{x}_n,\lambda_n^2\tau+\hat{t}_n),$$

where

$$(y,\tau)\in\bar{\Omega}_n\times(-\lambda_n^{-2}\hat{t}_n,\lambda_n^{-2}(T-\hat{t}_n)),\quad \Omega_n=\{y\in\mathbb{R}^N:\lambda_n y+\hat{x}_n\in\Omega\}.$$

Clearly, $(\phi^{\lambda_n},\psi^{\lambda_n})$ is a solution of the system

$$\phi_\tau-\Delta\phi=\psi^p-a\lambda_n^{2(\alpha+1-\alpha r)}\phi^r,\quad \psi_\tau-\Delta\psi=\phi^q-b\lambda_n^{2(\beta+1-\beta s)}\psi^s$$

in $\Omega_n\times(-\lambda_n^{-2}\hat{t}_n,\lambda_n^{-2}(T-\hat{t}_n))$. Then, in $\Omega_n\times(-\lambda_n^{-2}\hat{t}_n,0]$ it holds that

$$\text{(6)}\qquad \phi^{\lambda_n}(0,0)=1,\quad 0\le\phi^{\lambda_n}\le 1,\quad 0\le\psi^{\lambda_n}\le M_u^{-\frac{\beta}{\alpha}}(t_n)M_v(t_n).$$

Noticing that $r\le p\frac{q+1}{p+1}$ and $s\le q\frac{p+1}{q+1}$, we have $\alpha(r-1)\le 1$ and $\beta(s-1)\le 1$. Hence, $\lambda_n^{2(\alpha+1-\alpha r)}\to\kappa_1$, $\lambda_n^{2(\beta+1-\beta s)}\to\kappa_2$ as $\lambda_n\to 0$, where $\kappa_1,\kappa_2\in\{0,1\}$. It follows from the interior Schauder's estimates ([13]) that there exists a $\sigma\in(0,1)$ such that for any $K>0$,

$$\|\phi^{\lambda_n}\|_{C^{2+\sigma,1+\frac{\sigma}{2}}(\bar{\Omega}_n\cap\{|y|\le K\}\times[-K,0])}\le C_K,$$
$$\|\psi^{\lambda_n}\|_{C^{2+\sigma,1+\frac{\sigma}{2}}(\bar{\Omega}_n\cap\{|y|\le K\}\times[-K,0])}\le C_K,$$

where the constant C_K is independent of n. By compactness and diagonal arguments, we obtain a subsequence of $\{\phi^{\lambda_n},\psi^{\lambda_n}\}$ converging to a solution of

$$\text{(7)}\quad \phi_\tau-\Delta\phi=\psi^p-a\kappa_1\phi^r,\quad \psi_\tau-\Delta\psi=\phi^q-b\kappa_2\psi^s,\quad (y,\tau)\in\mathbb{R}^N\times(-\infty,0].$$

On the other hand, it follows from (5), (6) that $\phi(0,0)=1$ and $\phi\le 1$, $0\le\psi\equiv 0$ in $R^N\times(-\infty,0]$, which contradicts to the second equation in (7).

Case (ii) *Choose a subsequence (denoted again by $\{t_n\}$) such that*

$$\lim_{n\to\infty}\frac{d_n}{\lambda_n}=c\ge 0.$$

Let $\tilde{x}_n\in\partial\Omega$ be such that $d_n=|\hat{x}_n-\tilde{x}_n|$ and let R_n be an orthonormal transformation in $\mathbb{R}^N$ that maps $-e_1:=(-1,0,\dots,0)$ onto the outer normal vector to $\partial\Omega$ at $\tilde{x}_n$. Now, we define the new scaling

$$\phi^{\lambda_n}(y,\tau) := \lambda_n^{2\alpha}u(\lambda_n R_n y+\hat{x}_n,\lambda_n^2\tau+\hat{t}_n),$$
$$\psi^{\lambda_n}(y,\tau) := \lambda_n^{2\beta}v(\lambda_n R_n y+\hat{x}_n,\lambda_n^2\tau+\hat{t}_n)$$

for

$$(y,\tau)\in\bar{\Omega}_n\times(-\lambda_n^{-2}\hat{t}_n,\lambda_n^{-2}(T-\hat{t}_n)),\quad \Omega_n=\{y\in\mathbb{R}^N:\lambda_n R_n y+\hat{x}_n\in\Omega\}.$$

Then $(\phi^{\lambda_n}, \psi^{\lambda_n})$ is a solution of the system

$$\begin{aligned}
&\phi_\tau - \Delta\phi = \psi^p - a\lambda_n^{2(\alpha+1-\alpha r)}\phi^r, && \Omega_n \times (-\lambda_n^{-2}\hat{t}_n, \lambda_n^{-2}(T-\hat{t}_n)),\\
&\psi_\tau - \Delta\psi = \phi^q - b\lambda_n^{2(\beta+1-\beta s)}\psi^s, && \Omega_n \times (-\lambda_n^{-2}\hat{t}_n, \lambda_n^{-2}(T-\hat{t}_n)),\\
&\phi(y,\tau) = \psi(y,\tau) = 0, && \partial\Omega_n \times (-\lambda_n^{-2}\hat{t}_n, \lambda_n^{-2}(T-\hat{t}_n)).
\end{aligned}$$

If we restrict τ to $\tau \in (-\lambda_n^{-2}\hat{t}_n, 0]$, then clearly

$$\phi^{\lambda_n}(0,0) = 1, \quad 0 \le \phi^{\lambda_n} \le 1, \quad 0 \le \psi^{\lambda_n} \le M_u^{-\frac{\beta}{\alpha}}(t_n)M_v(t_n), \quad \text{in } \Omega_n \times (-\lambda_n^{-2}\hat{t}_n, 0]. \tag{8}$$

Since Ω_n approaches the halfspace $H_c := \{y_1 > -c\}$ as $\lambda_n \to 0$ and $\partial\Omega$ is smooth, Schauder's estimates for $(\phi^{\lambda_n}, \psi^{\lambda_n})$ yield a subsequence converging to a solution (ϕ, ψ) of

$$\begin{aligned}
&\phi_\tau - \Delta\phi = \psi^p - a\kappa_1\phi^r, && H_c \times (-\infty, 0],\\
&\psi_\tau - \Delta\psi = \phi^q - b\kappa_2\psi^s, && H_c \times (-\infty, 0],\\
&\phi = \psi = 0, && \{y_1 = -c\} \times (-\infty, 0],
\end{aligned}$$

where κ_1, κ_2 is same as Case (i). Furthermore, using (5), (8), we obtain $\phi(0,0) = 1$, $\phi \le 1$ and $\psi \equiv 0$ in $\mathbb{R}^N \times (-\infty, 0]$, which lead to a contradiction.

Combining Case (i) and Case (ii), we have completed the proof of lemma. □

Proof of Theorem 1.1. Let $G(x,t;y,\tau)$ be the Green's function of the heat equation in the bounded domain $\Omega \times (0,T]$ under the homogeneous Dirichlet boundary condition. Then for each $x \in \bar{\Omega}$ and $0 < z < t < T$, we have the representation formula:

$$\begin{aligned}
u(x,t) &= \int_\Omega G(x,t;y,z)u(y,z)dy + \int_z^t \int_\Omega (v^p - au^r)G(x,t;y,z)dyd\tau,\\
v(x,t) &= \int_\Omega G(x,t;y,z)v(y,z)dy + \int_z^t \int_\Omega (u^q - bv^s)G(x,t;y,z)dyd\tau.
\end{aligned}$$

Notice $G(x,t;y,\tau) \ge 0$ and $\int_\Omega G(x,t;y,z)dy \le 1$. Hence, Lemma 2.1 implies

$$\begin{aligned}
M_u(t) &\le M_u(z) + \int_z^t M_v^p(\tau)d\tau\\
&\le M_u(z) + (T-z)M_v^p(t) \le M_u(z) + C(T-z)M_u^{\frac{p\beta}{\alpha}}(t),\\
M_v(t) &\le M_v(z) + \int_z^t M_u^q(\tau)d\tau\\
&\le M_v(z) + (T-z)M_u^q(t) \le M_v(z) + C(T-z)M_v^{\frac{p\beta}{\alpha}}(t).
\end{aligned}$$

By our assumption, $M_u(t) \to \infty$ as $t \to T$. For any z, one can choose $t \in (z,T)$ such that $M_u(t) = 2M_u(z)$. Thus we have

$$2M_u(z) \le M_u(z) + 2^{\frac{p\beta}{\alpha}}C(T-z)M_u^{\frac{p\beta}{\alpha}}(z),$$

which implies that for some positive constant c

$$M_u(t) \geq c(T-t)^{-\alpha}, \quad 0 < t < T.$$

Similar arguments or using Lemma 2.1, we easily get the lower estimates of $M_v(t)$. □

3. Upper blow-up estimates

In this section, we use Lemma 2.1 to establish the upper blow-up rate estimates.

Proof of Theorem 1.2. Notice $M_u(t) \to \infty$ as $t \to T$. For any given $t_0 \in (0, T)$, we can define t_0^+ by

$$t_0^+ := \max\{t \in (t_0, T) : M_u(t) = 2M_u(t_0)\}.$$

Take $\lambda_0 = \lambda(t_0) := M_u^{-\frac{1}{2\alpha}}(t_0)$. We claim that

$$\lambda^{-2}(t_0)(t_0^+ - t_0) \leq \tilde{C}, \quad t_0 \in (\frac{T}{2}, T), \tag{9}$$

where the constant $\tilde{C} \in (0, +\infty)$ is independent of t_0. Suppose that this estimate were false. Then there would exist a sequence $t_n \to T$ such that $\lambda^{-2}(t_n)(t_n^+ - t_n) \to \infty$. For each t_n, we scale about the corresponding point $(\hat{x}_n, \hat{t}_n)$ and define the λ_n, d_n as Lemma 2.1. We divide the rest proof into two cases depending on whether:

$$(i) \limsup_{n\to\infty} \frac{d_n}{\lambda_n} = \infty \quad \text{or} \quad (ii) \limsup_{n\to\infty} \frac{d_n}{\lambda_n} < \infty.$$

Case (i) We use the same scaling as Case (i) in Lemma 2.1, and see $(\phi^{\lambda_n}, \psi^{\lambda_n})$ is a solution of the following system

$$\begin{aligned} \phi_\tau - \Delta\phi &= \psi^p - a\lambda_n^{2(\alpha+1-\alpha r)}\phi^r, \quad \Omega_n \times (-\lambda_n^{-2}\hat{t}_n, \lambda_n^{-2}(t_n^+ - \hat{t}_n)) \\ \psi_\tau - \Delta\psi &= \phi^q - b\lambda_n^{2(\beta+1-\beta s)}\psi^s, \quad \Omega_n \times (-\lambda_n^{-2}\hat{t}_n, \lambda_n^{-2}(t_n^+ - \hat{t}_n)). \end{aligned}$$

Clearly, $\phi^{\lambda_n}(0,0) \geq \frac{1}{2}$, $0 \leq \phi^{\lambda_n}(y,s) \leq 2$ in $\Omega_n \times (-\lambda_n^{-2}\hat{t}_n, \lambda_n^{-2}(t_n^+ - \hat{t}_n))$. By Lemma 2.1, we see

$$\begin{aligned} 0 \leq \psi^{\lambda_n}(y,\tau) \leq \lambda_n^{2\beta} M_v(t_n^+) \leq \lambda_n^{2\beta}\delta^{-2\beta} M_u^{\frac{\alpha}{\beta}}(t_n^+) = 2^{\frac{\beta}{\alpha}}\delta^{-2\beta}, \\ \text{for any} \quad (y,\tau) \in \Omega_n \times (-\lambda_n^{-2}\hat{t}_n, \lambda_n^{-2}(t_n^+ - \hat{t}_n)). \end{aligned}$$

Then, uniform Schauder's estimates for $(\phi^{\lambda_n}, \psi^{\lambda_n})$ yield a subsequence converging to a solution (ϕ, ψ) of

$$\phi_\tau - \Delta\phi = \psi^p, \quad \psi_\tau - \Delta\psi = \phi^q, \quad \mathbb{R}^N \times \mathbb{R}. \tag{10}$$

Meanwhile, we have

$$0 \leq \phi \leq 2, \quad 0 \leq \psi \leq 2^{\frac{\beta}{\alpha}}\delta^{-2\beta}, \quad \text{in} \quad \mathbb{R}^N \times \mathbb{R},$$

and $\phi(0,0) \geq \frac{1}{2}$. This is a contradiction, as it was shown in [5] that all positive solutions of (10) under our assumptions blow up in finite time.

Case (ii) First, we introduce the same scaling as Case (ii) in Lemma 2.1. Similarly as Case (i), we can show there exists nontrivial nonnegative solution (ϕ, ψ) of

$$\begin{aligned} &\phi_\tau - \Delta\phi = \psi^p, \quad \psi_\tau - \Delta\psi = \phi^q, \quad H_c \times \mathbb{R}, \\ &\phi(y,\tau) = \psi(y,\tau) = 0, \qquad \{y_1 = -c\} \times \mathbb{R}, \end{aligned}$$

such that $0 \le \phi \le 2$, $0 \le \psi \le 2^{\frac{\beta}{\alpha}}\delta^{-2\beta}$ in $H_c \times \mathbb{R}$ and $\phi(0,0) \ge \frac{1}{2}$, which lead to a contradiction to global nonexistence result from [14] since $\max(\alpha,\beta) \ge \frac{N+1}{2}$.

Now, we have completed the proof of (9). Then using the ideas of [10, 3], we can prove Theorem 1.2. Here, we give a sketch of proof for completeness. For given $t_0 \in (\frac{T}{2}, T)$, we define $t_1 = t_0^+ \in (t_0, T)$ such that $M_u(t_1) = 2M_u(t_0)$. It follows from (9) that $t_1 - t_0 \le \tilde{C} M_u^{-\frac{1}{\alpha}}(t_0)$. If we define $t_2 = t_1^+ \in (t_1, T)$, we easily obtain $t_2 - t_1 \le \tilde{C} M_u^{-\frac{1}{\alpha}}(t_1) = \tilde{C} M_u^{-\frac{1}{\alpha}}(t_0) 2^{-\frac{1}{\alpha}}$. Continuing this process we get a sequence $t_n \to T$ as $n \to \infty$ satisfying

$$t_{n+1} - t_n \le \tilde{C} M_u^{-\frac{1}{\alpha}}(t_0) 2^{-\frac{n}{\alpha}}, \quad n = 0, 1, 2, \dots.$$

Adding these inequalities, we have $T - t_0 \le \tilde{C} M_u^{-\frac{1}{\alpha}}(t_0)(1 - 2^{-\frac{n}{\alpha}})^{-1}$. Noticing $t_0 \in (\frac{T}{2}, T)$ is arbitrary, we have established the upper estimates

$$M_u(t) \le C(T-t)^{-\alpha}, \quad t \in (\frac{T}{2}, T).$$

The estimates of $M_v(t)$ follows from Lemma 2.1 and the above inequality. □

References

[1] J. Bebernes and D. Eberly, *Mathematical Problems from Combustion Theory*, Applied mathematical Sciences, 83, Springer-Verlag, New York, 1989.

[2] N. Bedjaoui and Ph. Souplet, *Critical blowup exponents for a system of reaction-diffusion equations with absorption*, Z. Angew. Math. Phys. **53** (2002), no. 2, 197–210.

[3] M. Chlebik and M. Fila, *From critical exponents to blow-up rates for parabolic problems*, Rend. Mat. Appl. (7) **19** (1999), no. 4, 449–470.

[4] K. Deng, *Blow-up rates for parabolic systems*, Z. Angew. Math. Phys. **47** (1996), no. 1, 132–143.

[5] M. Escobedo and M. A. Herrero, *Boundedness and blow up for a semilinear reaction-diffusion system*, J. Differentail Equations **89** (1991), no. 1, 176–202.

[6] M. Fila and P. Quittner, *The blow-up rate for a semilinear parabolic systems*, J. Math. Anal. Appl. **238** (1999), no. 2, 468–476.

[7] M. Fila and Ph. Souplet, *The blow-up rate for semilinear parabolic problems on general domains*, NoDEA Nonlinear Differentail Equations Appl. **8** (2001), no. 4, 473–480.

[8] S.-C. Fu and J.-S. Guo, *Blow-up for a semilinear reaction-diffusion system coupled in both equations ans boundary conditions*, J. Math. Anal. Appl. **296** (2002), no. 1, 458–475.

[9] B. Gidas and J. Spruck, *A priori bounds for positive solutions of nonlinear elliptic equations*, Commun. Paritial Differentail Equations **6** (1981), no. 8, 883–901.

[10] B. Hu, *Remarks on the blowup estimate for solutions of the heat equation with a nonlinear boundary condition*, Differential Integral Equations **9** (1996), no. 5, 891–901.

[11] B. Hu and H. M. Yin, *The profile near blowup time for solution of the heat equation with a nonlinear boundary condition*, Trans. Amer. Math. Soc. **346** (1994), no. 1, 117–135.

[12] K. I. Kim and Z. G. Lin, *Blowup estimates for a parabolic system in a three-species cooperating model*, J. Math. Anal. Appl. **293** (2004), no. 2, 663–676.
[13] O. A. Ladyženskaja, V. A. Solonnikov, and N. N. Ural'ceva, *Linear and quasilinear equations of parabolic type*, Amer. Math. Soc. Providence, 1967.
[14] H. A. Levine, *A Fujita type global existence – global nonexistence theorem for a weakly coupled system of reaction-diffusion equations*, Z. Angew. Math. Phys. **42** (1991), no. 3, 408–430.
[15] Z. G. Lin, *Blowup estimates for a mutualistic model in ecology*, Electron. J. Qual. Theory Differ. Equ. (2002), no. 8, 1–14.
[16] C. V. Pao, *Nonlinear Parabolic and Elliptic Equations*, Plenum Press, New York, 1992.
[17] F. Rothe, *Global Solutions of Reaction-diffusion Systems*, Lecture Notes in Mathematics, 1072, Springer-Verlag, Berlin, 1984.
[18] S. Snoussi and S. Tayachi, *Global existence, asymptotic behavior and self-similar solutions for a class of semilinear parabolic systems*, Nonlinear Anal. **48** (2002), no. 1, Ser. A : Theory Methods, 13–35.
[19] P. Souplet and S. Tayachi, *Optimal condition for non-simultaneous blow-up in a reaction-diffusion system*, J. Math. Soc. Japan **56** (2004), no. 2, 571–584.
[20] M. X. Wang, *Blow-up rate estimates for semilinear parabolic systems*, J. Differentail Equations **170** (2001), no. 2, 317–324.

Zhaoyin Xiang
School of Applied Mathematics
University of Electronic Science and Technology of China
Chengdu 610054, P. R. China
E-mail address: zhaoyin-xiang@sohu.com

Qiong Chen
Department of Mathematics
Sichuan University
Chengdu 610064, P. R. China
E-mail address: qiong-chen@sohu.come

Chunlai Mu
Department of Mathematics
Sichuan University
Chengdu 610064, P. R. China