# ◎논문

# Mistuning Intensity Effect to Optimization of Mistuning Pattern for Bladed Disk

블레이드 디스크의 Mistuning 패턴 최적화에 미치는 Mistuning 강도의 영향

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## **ABSTRACT**

일반적으로 터보 기계에서 mistuning은 구조적, 기하학적인 면에서의 blade와 blade 사이의 미소한 특성의 차이를 의미하며, 이러한 미소한 차이가 강제 진동 시, 발생하는 진동 에너지가 몇 개의 blade 또는 하나의 blade에 집중되어 매우 큰 진동을 야기 시킬 수 있다는 사실이 여러 논문들에 언급되고 있다. 또한 mistuning에 대한 blade 시스템의 강제진동응답의 감도를 줄이기 위하여, intentional mistuning개념을 설계단계에 도입하려고 하는 연구도 진행되고 있다. 따라서 본 논문에서는 두 가지 형태의 blade만을 사용하여 bladed disk의 강제진동응답을 최소화 할 수있는 intentional mistuning의 pattern 최적화에 미치는 mistuning intensity 영향을 조사하였고 최적화 알고리즘으로 유전알고리즘을 사용하였다.

In turbomachinery rotor, there are small differences in the structural and/or geometrical properties of individual blades, which are referred to as blade mistuning. Mistuning effects of the forced response of bladed disks can be extremely large as often reported in many studies. In this paper, the pattern optimization of intentional mistuning for bladed disks considering with intentional mistuning intensity effect is the focus of the present investigation. More specifically, the class of intentionally mistuned disks considered here is limited, for cost reasons, to arrangements of two types of blades (A and B, say) and Genetic Algorithm is used to optimize the arrangement of these blades around the disk to reduce the forced response of blade with intentional mistuning intensity levels.

#### 1. Introduction

In a dynamic analysis of a turbomachinery rotor, one traditionally has assumed that the blades are iden-

tical. But, in practice there are small differences in the structural and/or geometrical properties of individual blades, which are referred to as blade mistuning. Much of the vast literature on this topic<sup>(1)~(5)</sup> has assumed these differences to be small and to arise either during the manufacturing process and/or as a consequence of in service wear. The motivation for considering such

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small variations is that their effects of the forced response of bladed disks can be extremely large as often reported in the above studies. Interestingly, the large sensitivity of the tuned system to these small variations has been linked<sup>(3)</sup> to its high level of symmetry.

In this light, it would appear beneficial to design bladed disk not to be tuned, namely to exhibit intentional mistuning, to reduce the sensitivity of the forced response to unintentional mistuning. Certainly, the consideration of intentional mistuning is not new<sup>(1),(2)</sup> however, in the context of forced response, some papers<sup>(6),(7)</sup> have only recently investigated the use of harmonic patterns of mistuning. Recently, the authors have investigated and identified the effect of intentional mistuning which can significantly reduce the magnification of the forced response due to unintentional random mistuning<sup>(8),(9)</sup>.

In this paper, the pattern optimization of intentional mistuning for bladed disks considering with intentional mistuning intensity effect is the focus of the present investigation using the two sets of blades A and B. Genetic Algorithm is used to obtain the pattern(s) that yields small/the smallest value of the largest amplitude of response to a given excitation in the absence of unintentional mistuning using simple model (one-degree-of-freedom per blade) of bladed disks.

#### 2. Optimization Approach

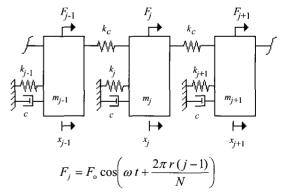
In view of the complexity and cost of intentional mistuning, one should not look simply at set patterns, for example the harmonic patterns<sup>(7)</sup> but rather one should optimize the pattern to reduce as much as possible the amplification of the forced response to a given excitation or set thereof. Accordingly, it was suggested by authors that the use of intentional mistuning is probably not a standard design tool but would be very valuable if: (a) it yields a large decrease in sensitivity to unintentional mistuning, and (b) it involves a minimum number of types of blades, ideally  $2^{(8),(9)}$ .

Therefore, in this paper, the disk will first be assumed to support only two different types of blades (blades A and B, say) and their arrangement that yields the smallest amplification of the forced response will be sought with different intentional mistuning intensity. Many sets of blades A and B were selected to change the intentional mistuning intensity having natural frequencies lower and higher, respectively, than the tuned ones (type C) in this paper.

Mistuning also produces a very nonlinear effect on the forced response. That is, by switching the order of the blades around the disk, dramatic differences can be obtained in the variability of the blade-to-blade amplitudes of vibration as exemplified in particular by the harmonic mistuning analysis of Mignolet et al<sup>(4)</sup>. It might thus be suspected that there exists a series of local optima in the complex, high dimensional space over which the optimization must take place. In this light, the present optimization effort has relied on the use of Genetic Algorithm (GA)<sup>(10)</sup>.

### 3. Simple Genetic Algorithm

GA are particularly well suited for the present effort because the design variables only admit discrete values (i.e. a specific blade is only of type A or B), see References<sup>(10)</sup> for further details. The simple genetic algorithm (SGA) used here relies on a population of



 $m_t = 0.0114 \text{ kg}, k_t = 430,300 \text{ N/m},$  c = 1.3814 N·s/m, N = 4 blades, $F_0 = 1N, r = 3,$ 

 $K_j$ : normal distribution of mean  $k_t$  and standard deviation s

Fig. 1 Single-degree-of-freedom per blade disk model

npop bladed disks each of which is a random arrangement of N genes (the type A or B of the different blades). Accordingly, each bladed disk can be characterized by a sequence of N A and B letters, for example AABBBBAA…, which evolves from one generation to the next according to the rules of selection, crossover, and mutation until all the chromosomes yield essentially similar values of the fitness or objective function (the maximum amplitude of blade response).

The fitness proportionate selection, the single point crossover technique and an exponentially decreasing mutation function was used in the present investigation. Also, the one elite reservation strategy was used in this paper according to which the best disk is retained unchanged from one generation to the next.

#### 4. Simple Bladed Disk Model

To identify the effect of intentional mistuning intensity to optimum pattern of intentional mistuning, the SDOF per blade model shown in Fig. 1 was first considered with the coupling stiffness ( $k_c$  = 45430 N/m) and damping coefficient (c = 1.3814  $N \cdot s/m$ ) with only 4 blades was considered in this paper. Specifically, each of the N blades is represented as a single mass (m) which is connected to the ground (i.e. the disk) and the aerodynamic and structural coupling between blades are modeled by springs( $k_c$ ) and dashpots(c). In

the sequel, it is assumed that the coefficient vanishes in this paper. The values of mass  $(m = 0.0114 \, kg)$ , and stiffness  $(= 430,300 \, N/m)$  have already been used in a previous investigations to model a high-pressure turbine stage is used in this paper.

The computations proceeded as follows. The bladed disk model and engine order of the excitation were first selected. The SGA described above was then used to obtain the intentionally mistuned disk formed of blades A and B such that the maximum of its response over the entire frequency range was the smallest possible.

First, the intentional mistuning intensity to optimum pattern of intentional mistuning is identified. Many sets of blades A and B were selected to change natural frequencies lower and higher, respectively, than the tuned ones (type C). In analysis, the stiffness of A and B type blade is selected to have lower and higher than the tuned ones. For example, in table 1, A(0.9) and B(1.1) means that A and B have natural frequencies 5 % lower and 5 % higher than C. The optimum pattern of intentional mistuning is searched with 45,430 N/m (an average to strong blade–to–blade coupling level) by Genetic Algorithm.

Table 1 shows the comparison of optimization result by Genetic Algorithm for each intentional mistuning intensity level with the tune system (all C:1.0682e-4 m and all B) in  $3^{rd}$  engine order case.

Table 1 The comparison of optimization result	by Genetic Algori	hm for each intentiona	mistuning blade set with the
tune and other mistuning pattern			

Mistuning	blade type			
A	В	Optimum distribution		
0.7	1.3	BBBB = 9.5817e-5 m, (ABBA = 1.0026e-4 m)		
0.8	1.2	ABBA = 9.7014e-5 m, (BBBB = 9.9172e-5 m), 9.18 %		
0.9	1.1	ABBA = $9.9242e-5 m$ , (BBBB = $1.028e-4 m$ ), $7.09 \%$		
0.91	1.09	ABBA = $1.0186e-4 m$ , (BBBB = $1.0326e-4 m$ )		
0.9125	1.0875	ABBA = $1.0261e^{-4} m$ , (BBBB = $1.0305e^{-4} m$ )		
0.92	1.08	BBBB = 1.0344e-4 m, ( ABBA = 1.0536e-4 m)		
0.94	1.06	BBBB = 1.0425e-4 m, (ABBA = 1.163e-4 m)		
0.96	1.04	BBBB = 1.0529e-4 m, ( ABBA = 1.2457e-4 m)		
0.98	1.02	BBBB = 1.0619e-4 m, (ABBA = 1.1438e-4 m)		

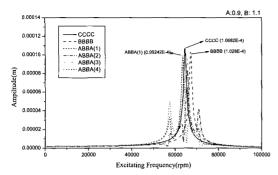


Fig. 2 Comparison with forced response of optimum, tuned and other A/B pattern (A:0.9, B:1.1)

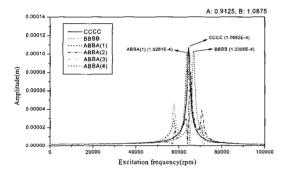


Fig. 3 Comparison with forced response of optimum, tuned and other A/B pattern(A:0.9125, B:1.0875)

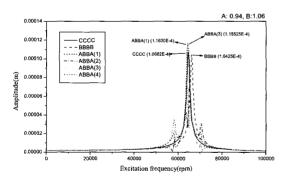


Fig. 4 Comparison with forced response of optimum, tunedand other A/B pattern (A:0.94, B:1.06)

In Table 1, the optimum pattern that yields small/ the smallest value of the largest amplitude of response to a given excitation by GA was changed according to the intentional mistuning intensity (mistuning blade type) level. The genetic optimization algorithm yielded the configuration ABBA and BBBB the highest responding

blade in each mistuning blade type. Figs.  $2\sim4$  show the forced response of optimum, tuned and other A/B patterns in three mistuning type. As known from Figs.  $2\sim4$ , A/B patterns have several resonance peak because the blade symmetry is destroyed by the mistuning, while the tuned pattern have one resonance peak.

It is also identified that ABBA are better then tuned and other patterns in case of A (0.9), B (1.1) and A (0.9125), B (1.0875). But in case of A (0.94) and B (1.06), BBBB is better than tuned and ABBA. The reason about the decrease in forced response of the mistuning pattern than that of tune pattern have been investigated in many paper  $^{(4)\sim(9)}$  but still does not prove clearly.

According to Table 1, in A (0.9), B (1.1) and A (0.8), B (1.2), the max amplitude of blade is reduced to 9.18% and 7.09% than tuned one respectively. So, in this paper, more detail analysis around the A (0.9) and B (1.1) is needed to identify the effect of intentional mistuning intensity to optimum pattern of intentional mistuning. Table 2 shows the comparison of optimization result for intentional mistuning intensity levels around A (0.9) and B (1.1) with the tune system (all C and all B) in  $3^{rd}$  engine order case.

In Table 2, as the gab between A and B is larger, all A become worse; max amplitude is increased while all B and ABBA become good; max amplitude is decreased. Also, if the gab between A and B is equal, the larger A or the larger B is better than others. That means that the max amplitude is more reduced than tuned one.

## 5. Summary

The investigation of this paper focused on the pattern optimization of intentional mistuning for bladed disks considering with intentional mistuning intensity effect is the focus of the present investigation using the two sets of blades A and B. Genetic Algorithm is used to obtain the pattern(s) that yields small/the smallest value of the largest amplitude of response to a given excitation in the absence of unintentional mistuning using simple model (one-degree-of-freedom per blade) of bladed disks.

A	В	B - A	All A	All B	ABBA	Compare	Compare to all C
0.9	1.1	0.2	1.1155e-4 m	1.028e-4 m	9.9242e-5 m	to all B	
0.9 + 0.01(0.91)	1.1 + 0.01(1.11)	0.2	1.1124e-4 m	1.0238e-4 m	9.8977e-5 m	3.46 %	7.09 %
0.9 - 0.01(0.89)	1.1 - 0.01(1.09)	0.2	1.1201e-4 m	1.0326e-4 m	9.9536e-5 m	3.32 %	7.34 %
0.9 + 0.01(0.91)	1.1 - 0.01(1.09)	0.18	1.1124e-4 m	1.0326e-4 m	1.0186e-4 m	3.6 %	6.82 %
0.9 - 0.01(0.89)	1.1 + 0.01(1.11)	0.22	1.1201e-4 m	1.0238e-4 m	9.7272e-5 m	1.36 %	4.64 %
0.9 + 0.02(0.92)	1.1 + 0.02(1.12)	0.2	1.1078e-4 m	1.0209e-4 m	9.8468e-5 m	4.99 %	8.94 %
0.9 + 0.02(0.92)	1.1 - 0.02(1.08)	0.16	1.1078e-4 m	1.0344e-4 m	1.0536e-4 m	3.55 %	7.82 %
0.9 - 0.02(0.88)	1.1 + 0.02(1.12)	0.24	1.1273e-4 m	1.0209e-4 m	9.5777e-5 m	-1.86 %	1.37 %
0.9 - 0.03(0.87)	1.1 + 0.03(1.13)	0.26	1.1332e-4 m	1.0153e-4 m	9.4605e-5 m	6.18 %	10.34 %
0.9 + 0.008(0.908)	1.1 + 0.008(1.108)	0.2	1.1112e-4 m	1.0252e-4 m	9.8741e-5 m	6.82 %	11.44 %
0.9 - 0.012(0.888)	1.1 - 0.012(1.088)	0.2	1.1239e-4 m	1.032e-4 m	9.9855e-5 m	3.69 %	7.56 %
0.9 + 0.01(0.91)	1.1 - 0.008(1.092)	0.182	1.1124e-4 m	1.0307e-4 m	1.0147e-4 m	3.24 %	6.52 %
0.9 - 0.008(0.892)	1.1 + 0.01(1.11)	0.218	1.1215e-4 m	1.0238e-4 m	9.7473e-5 m	1.55 %	5.0 %

Table 2 The comparison of optimization result by Genetic Algorithm for each intentional mistuning blade set with the tune and other mistuning pattern

Through the optimization, it is found that the optimum pattern that yields small/the smallest value of the largest amplitude of response to a given excitation by GA was changed according to the intentional mistuning intensity (mistuning blade type) level. The genetic optimization algorithm vielded the configuration ABBA and BBBB the highest responding blade in each mistuning blade type in Table 1. Also it is identified that the gab between A and B is larger, all A become worse (namely, max amplitude is increased) while all B and ABBA become better (namely, max amplitude is decreased). If the gab between A and B is equal, the larger A or the larger B is better than others, namely, the max amplitude of blade is more reduced than tuned one in Table 2. Therefore, the effect of intentional mistuning intensity should be considered to optimize the intentional mistuning pattern which can reduce the forced response in blade.

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