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RELIABILITY-BASED DESIGN OPTIMIZATION OF AUTOMOTIVE SUSPENSION SYSTEMS

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ABSTRACT–Design variables for suspension systems cannot always be realized in the actual suspension systems due to tolerances in manufacturing and assembly processes. In order to deal with these tolerances, design variables associated with kinematic configuration and compliance characteristics of suspensions are treated as random variables. The reliability of a design target with respect to a design variable is defined as the probability that the design target is in the acceptable design range for all possible values of the design variable. To compute reliability, the limit state, which is the boundary between the acceptable and unacceptable design, is expressed mathematically by a limit state function with value greater than 0 for acceptable design, and less than 0 for unacceptable design. Through reliability analysis, the acceptable range of design variables that satisfy a reliability target is specified. Furthermore, through sensitivity analysis, a general procedure for optimization of the design target with respect to the design variables has been established.

KEY WORDS : Suspension, Static design factors, Tolerance, Reliability analysis, Optimization

1. INTRODUCTION

Suspension systems play an important role in providing good ride and handling performance of vehicles. Kinematic and compliance (K&C) characteristics of suspensions determine wheel alignments during vehicle maneuvers, which set the force and moment transmission capabilities of a tire at the contact patches. Since various effects of wheel alignments on vehicle dynamics are well explained by SDF's (Static Design Factors), suspension design targets are usually specified in terms of SDF's. Some SDF's such as camber, toe, caster, and kingpin inclination are purely kinematic quantities, while other SDF's are related to elasto-kinematic behavior of suspensions such as lateral force compliance toe, fore and aft deflections due to longitudinal force, etc. In order to deal with K&C characteristics, both kinematic design variables (such as suspension hard points), and compliance design variables (such as stiffness of springs and bushings), should be considered. Both kinematic and compliance design variables must be determined in order to achieve the design target.

Design variables cannot always be realized in actual suspension systems due to errors in manufacturing and assembly processes. In order to control these errors, a common practice in suspension design is to allocate a maximum allowable tolerance limit to each design variable. However this does not guarantee that the tolerance of the design target will be satisfied, since relationship between design variable tolerances and the design target are not straightforward. Strict tolerance is better for performance, but it may increase the manufacturing and/or assembly costs significantly. Therefore, it is important to find optimal tolerances for design variables that can simultaneously satisfy performance and cost.

In order to account for tolerances in suspension systems, design variables as well as SDF's can be regarded as distributed variables. A distinction should be made between acceptable and unacceptable design among many distributed values of design variables. For this purpose the concept of reliability is very effective. Reliability of a SDF with respect to a design variable can be defined as the probability that the SDF is in the acceptable design range for all possible values of the variable. To determine whether the SDF is within the acceptable region or not, the limit state, which is the boundary between the acceptable and unacceptable design region, is expressed mathematically by a limit state function with value greater than 0 for acceptable design, and less than 0 for unacceptable design (Nowak and Collins, 2000). Through reliability analysis, the acceptable range for design variables that satisfies the

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reliability target can be specified. For reliability based design, efficient and accurate calculation of reliability is the key factor.

Monte-Carlo simulation (Dubi, 1999) is the fundamental method for obtaining reliability. For each design variable with known distribution characteristics, random numbers are numerically generated, and for each generated number, analysis is carried out to determine whether the generated number satisfies the design target. Reliability can be obtained by dividing the number of random numbers that produced the acceptable results by the total number of generated random numbers. However, since the accuracy of the Monte-Carlo method depends on the sample size, it sometimes becomes impossible to apply if considerably long computation time is required for a single analysis.

In order to compute reliability more efficiently, various approximation methods, such as the MVFO (Mean Value First Order) method, and the FORM (First Order Reliability Method) method have been proposed. Cornell (1969) proposed the MVFO method where nonlinear limit state functions are linearized using Taylor series expansion. By applying the central limit theorem to the linearized limit state function, the reliability index is obtained. The MVFO method is an efficient method in terms of computing time, however, it becomes less accurate as the nonlinearity of the limit state function increases. Hasofer and Lind (1974), proposed FORM which is more accurate than the MVFO method. The reliability index is defined as the shortest distance from the origin of z-space to the failure surface, and optimization is performed in order to find this shortest distance. FORM has some advantages, first of all, accuracy is less affected by non-linearity of the limit state function, and second, it can perform reliability analysis effectively with variables that do not precisely follow the normal distribution. However, FORM takes a longer time to compute because of the optimization process. In efforts to improve efficiency and accuracy of reliability analysis, various methods have been proposed, such as the Rackwitz-Fiessler method (Rackwitz and Fiessler, 1976, 1978), Wang and Grandhi's method (1994, 1996) and Yu's method (Yu *et al.*, 1997), a combination of MVFO and FORM.

K&C behavior of suspensions can be effectively investigated by a multibody approach where all components that constitute a suspension system can be accounted for. Tolerances of suspension systems are caused by various factors such as deviations in link length, joint clearances, direction of joint motion axes, and stiffness of springs and bushings. Since traditional methods for multibody systems (Haug, 1989) do not account for these mechanical tolerances, a new definition for multibody systems that can handle mechanical tolerances should be developed. Chun (2005) proposed a stochastic tolerance model for spatial kinematic systems that can handle tolerances of link length, joint motion axis, and joint clearance. His model can be expanded to carry out elasto-kinematic analysis of spatial suspension systems with mechanical tolerances. Based on the tolerance model, a general procedure for reliability analysis of suspensions can be developed, and using the reliability analysis, an optimal design process for tolerance control of suspension systems is proposed.

2. RELIABILITY ANALYSIS FOR ELASTO-KINEMATICS OF SUSPENSION SYSTEMS

For reliability analysis of elasto-kinematic behavior of suspension systems where considerable computation time is required, MVFO is a suitable method because of its computational efficiency. In applying MVFO, the limit state function should be linear, and random variables should follow a normal distribution. Since limit state functions associated with SDF's of suspension systems are quite linear in the tolerance limits (as will be seen later), and mechanical tolerances in general follow the normal distribution (Choi *et al.*, 1998; Park *et al.*, 1996) MVFO is a good choice for suspension systems.

A variablex_i is assumed to have mean value μ_{x_i} and tolerance T_{x_i}

$$x_i=\mu_{x_i}+T_{x_i} \tag{1}$$

In order to compute reliability, MVFO requires computation of the sensitivity of the limit state function with respect to the design variables. When a limit state function g is nonlinear, it can be linearized using a Taylor series expansion:

$$\begin{aligned} g(x_1,x_2,\ldots,x_n) &\approx g(\mu_{x_1},\mu_{x_2},\ldots,\mu_{x_n}) \\ &\quad +\sum_{i=1}^{n}(x_i-\mu_{x_1})\frac{\partial g}{\partial x_i}\bigg|_{(\mu_{x_1},\mu_{x_2},\ldots,\mu_{x_n})} \\ &=\mu_g+\sum_{i=1}^{n}T_{x_i}a_i \end{aligned} \tag{2}$$

where $g(\mu_{x_1},\mu_{x_2},\ldots,\mu_{x_n})\equiv\mu_g$ is the mean value of the limit state function, and $a_i\equiv(\partial g/\partial x_i)|_{\mu_{x_i}}$ is the gradient of g with respect to x_i at the mean value of μ_{x_i}. The reliability index β of limit state function g, is obtained by applying the central limit theorem to Equation (2) as

$$\beta=\frac{\mu_g}{\sigma_g}=\frac{g(\mu_{x_1},\mu_{x_2},\ldots,\mu_{x_n})}{\sqrt{\sum_{i=1}^{n}(a_i\sigma_{x_i})^2}} \tag{3}$$

where $\sigma_{x_i}^2$ is the variance of x_i. For reliability index β, the probability P of failure can be computed using the normal cumulative distribution function Ψ.

$$P(g < 0)=\Psi[-\beta] \quad (4)$$

Reliability can be obtained by deducting P from the total probability of 1.

$$\text{Reliability}=P(g > 0)=1-P(g < 0) \quad (5)$$

Since the limit state function g for suspension systems is a function of the generalized coordinate vector $\mathbf{q}=[q_1, q_2, \cdots, q_n]^T$, the derivative of g with respect to x_i, $\partial g(\mathbf{q})/\partial x_i$, can be calculated using the chain rule.

$$\frac{\partial g(\mathbf{q})}{\partial x_i}=\frac{\partial g(\mathbf{q})}{\partial \mathbf{q}}\frac{\partial \mathbf{q}}{\partial x_i} \quad (6)$$

In order to compute $\partial g(\mathbf{q})/\partial x_i$ in Equation (6), $\partial \mathbf{q}/\partial x_i$, which is the sensitivity of generalized coordinates with respect to x_i, should be known.

The elasto-kinematic behavior of a suspension system can be represented by m-kinematic constraint equations, which are functions of generalized coordinates $\mathbf{q}$, and k-design variables $\mathbf{x}=[x_1, x_2, \cdots, x_k]^T$ which represent tolerance of the system,

$$\mathbf{\Phi}[\mathbf{q}(\mathbf{x}), \mathbf{x}]=0 \quad (7)$$

$$\mathbf{\Phi}_{\mathbf{q}}^{T}\lambda-\mathbf{Q}=0 \quad (8)$$

where $\mathbf{\Phi}[\mathbf{q}(\mathbf{x}), \mathbf{x}]=0$ are the constraint equations for suspension systems, $\mathbf{\Phi}_{\mathbf{q}}$ is the Jacobian of $\mathbf{\Phi}$, λ are Lagrange multipliers, and $\mathbf{Q}$ are generalized forces due to external forces and forces associated with compliance elements of springs, dampers and bushings. Since Equations (7) and (8) are non-linear, they can be solved using the Newton-Raphson iteration method.

For computing the reliability index, the derivative of generalized coordinates $\mathbf{q}$ with respect to x_i should be known as demonstrated in Equation (6). For notational convenience, variable x_i is replaced by a new variable b without an index. The elasto-kinematic equations of Equation (7) and (8) are differentiated with respect to x_i.

$$\frac{d\mathbf{\Phi}}{db}=0 \quad (9)$$

$$\frac{d\mathbf{\Phi}_q^T}{db}\lambda+\mathbf{\Phi}_q^T\frac{d\lambda}{db}-\frac{d\mathbf{Q}}{db}=0 \quad (10)$$

Applying the chain rule, the first term of Equation (9) can be expressed in terms of partial derivatives

$$\frac{d\mathbf{\Phi}}{db}=\frac{\partial\mathbf{\Phi}}{\partial\mathbf{q}}\frac{d\mathbf{q}}{db}+\frac{\partial\mathbf{\Phi}}{\partial b}\frac{db}{db} =\mathbf{\Phi}_{\mathbf{q}}\mathbf{q}_b+\mathbf{\Phi}_b \quad (11)$$

where the subscript expresses the partial derivative. The terms in Equation (10) can be similarly calculated as

$$\frac{d\mathbf{\Phi}_{\mathbf{q}}^T}{db}\lambda=\frac{\partial(\mathbf{\Phi}_{\mathbf{q}}^T\lambda)}{\partial\mathbf{q}}\frac{d\mathbf{q}}{db}+\frac{\partial(\mathbf{\Phi}_{\mathbf{q}}^T\lambda)}{\partial b}\frac{db}{db} =(\mathbf{\Phi}_{\mathbf{q}}^T\lambda)_q\mathbf{q}_b+(\mathbf{\Phi}_{\mathbf{q}}^T\lambda)_b \quad (12)$$

$$\frac{d\lambda}{db}=\lambda_{\mathbf{q}}\mathbf{q}_b+\lambda_b \quad (13)$$

$$\frac{d\mathbf{Q}}{db}=\mathbf{Q}_{\mathbf{q}}\mathbf{q}_b+\mathbf{Q}_b \quad (14)$$

Substituting Equations (11)~(14) into Equations (9) and (10), the sensitivity equations of the elasto-kinematic equations with respect to design variable b can be written as

$$\begin{bmatrix}(\mathbf{\Phi}_{\mathbf{q}}^T\lambda-\mathbf{Q})_{\mathbf{q}} & \mathbf{\Phi}_{\mathbf{q}}^T\\ \mathbf{\Phi}_{\mathbf{q}} & 0\end{bmatrix}\begin{bmatrix}\mathbf{q}_b\\ \lambda_b\end{bmatrix}=-\begin{bmatrix}(\mathbf{\Phi}_{\mathbf{q}}^T\lambda)_b-\mathbf{Q}_b\\ \mathbf{\Phi}_{\mathbf{b}}\end{bmatrix} \quad (15)$$

To solve the equations, $\mathbf{\Phi}_b$, $(\mathbf{\Phi}_{\mathbf{q}}^T\lambda)_b$ and $\mathbf{Q}_b$ should be obtained. Constraint equations $\mathbf{\Phi}$ for various types of spatial joints such as revolute, spherical, and universal joints and the corresponding Jacobian $\mathbf{\Phi}_q$ are well defined along with the generalized forces associated with the compliance elements in reference (Haug, 1989). Once $\mathbf{\Phi}$, $\mathbf{\Phi}_q$, and $\mathbf{Q}$ and are computed, derivatives $\mathbf{\Phi}_b$,, and $\mathbf{\Phi}_b$ can be obtained by directly differentiating $\mathbf{Q}$, $(\mathbf{\Phi}_{\mathbf{q}}^T\lambda)_b$, and $\mathbf{Q}$ with respect to b, which represents the tolerance of the design variables (Tak *et al.*, 2000).

3. TOLERANCE MODELING

Suspension systems can be regarded as interconnections of bodies by kinematic joints and compliance elements such as spherical, revolute, cylindrical, and universal joints, as well as springs and bushings. Design variables can be classified into two groups, kinematic design variables and compliance design variables. Kinematic design variables define kinematic configurations of the suspension system such as body length, joint location, and joint motion axis direction, while compliance design variables specifies the magnitude of spring stiffness of springs and bushings. If design variables are treated as random variables with predetermined distribution characteristics in order to deal with tolerances, then design variables that account for tolerances in suspension system should be identified.

Since tolerances in compliance design variables do not affect the kinematic configuration of the suspension system, each design variable can be regarded as an independent variable with some tolerance about the nominal (mean) value in Equation (1). However, if kinematic tolerances are considered, the kinematic constraint equation should be expressed both in terms of generalized coordinates $\mathbf{q}$ and kinematic design variables $\mathbf{x}$ as $\mathbf{\Phi}[\mathbf{q}(\mathbf{x}), \mathbf{x}]=0$, since tolerances of kinematic design variables change the kinematic configuration of the suspension system.

A MacPherson suspension system, as shown in Figure 1, is considered as an example of many types of suspension systems. Since other types of suspensions such as

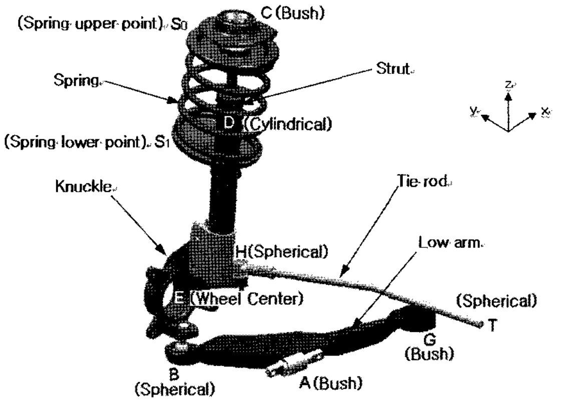


Figure 1. MacPherson suspension.

Table 1. Design variables of a MacPherson suspension.

Design Variable			x	y	z
Design points (mm)		A	1014	351	-5
		B	975.4	693.3	-26
		G	1322	351	3
		C	1000	551	605
		D	990.2	575.6	352.5
		T	1136	296	158
		H	1102	641	150.5
		E	980	720	100
		S0	998.2	567	558
		S1	992.7	605	418
Bush	C	Translational (N/mm)	3000	3000	300
		Torsional (Nmm/deg)	30000	30000	3000
	A	Translational (N/mm)	2000	2000	200
		Torsional (Nmm/deg)	20000	20000	2000
	G	Translational (N/mm)	1000	1000	100
		Torsional (Nmm/deg)	10000	10000	1000
Spring(N/mm)			21.67 (free length: 310 mm)		

double wishbone suspensions have common kinematic joints and compliance elements, tolerance analysis can be done in a manner similar to that of a MacPherson suspension. The suspension has a knuckle, a lower arm, and a strut, and they are interconnected by spherical and

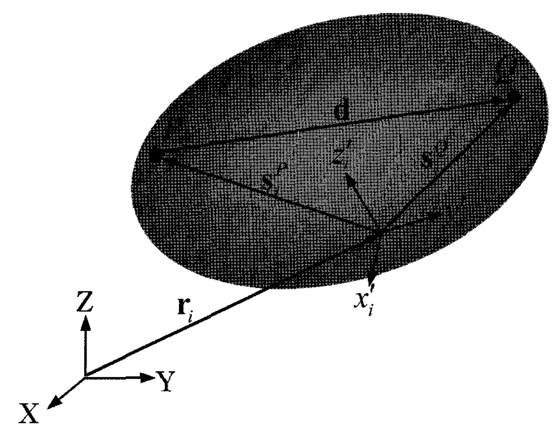


Figure 2. Tolerance model for body geometry.

cylindrical joints as well as by bushings and springs. A list of design variables and their nominal values is given in Table 1. The values of design variables in Table 1 are not always realized in the actual suspension products due to the errors in manufacturing and assembly process.

Let u's consider the tolerance of the body dimensions first. The body of the suspension system is generalized as body i as shown in Figure 2. The coordinate system $x_i'-y_i'-z_i'$ represents the body reference frame with respect to inertial frame X-Y-Z. Vector $\mathbf{r}_i$ is the position vector at the origin of the $x_i'-y_i'-z_i'$ frame, and points P and Q are the joint definition points through which body i is connected to other bodies by kinematic joints. Vectors $\mathbf{s}_i^P$ and $\mathbf{s}_i^Q$, respectively, define the position vectors of points P and Q, and vector $\mathbf{d}$ is the position vector from point P to Q, which can be expressed as

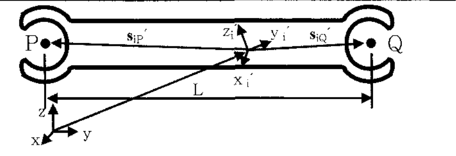
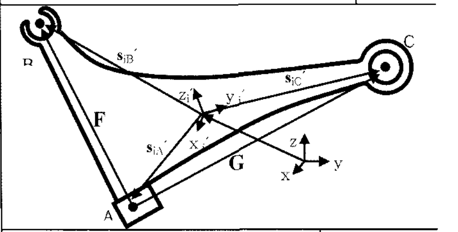
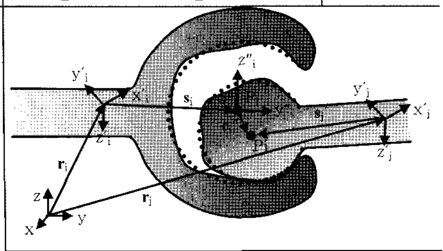
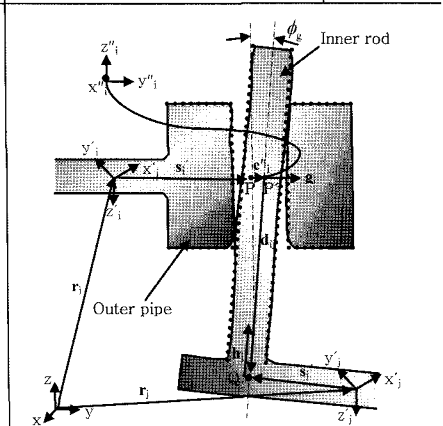
$$\mathbf{d}=\mathbf{s}_i^Q-\mathbf{s}_i^P=\mathbf{A}_i\mathbf{s}_i'^Q-\mathbf{A}_i\mathbf{s}_i'^P \tag{16}$$

where $\mathbf{A}_i$ is the transformation matrix from the $x_i'-y_i'-z_i'$ frame to the inertial frame, and $\mathbf{s}_i'^P$ and $\mathbf{s}_i'^Q$ are the local vector representations of $\mathbf{s}_i^P$ and $\mathbf{s}_i^Q$ in the $x_i'-y_i'-z_i'$ frame, respectively.

Both $\mathbf{s}_i'^P$ and $\mathbf{s}_i'^Q$ are time-constant vectors that define the joint definition points, which in fact determine the kinematic configuration of a body. Thus, variations in $\mathbf{s}_i'^P$ and $\mathbf{s}_i'^Q$ can account for the tolerances of the geometry of a body. In some cases, the relative position between P and Q determines the geometry of a body, and in other cases, only the distance between P and Q is meaningful, depending on the type of joints at these points. For example, if spherical joints are at both P and Q, then only the distance between them is kinematically significant; however, when a revolute joint is at P and a translational joint is at Q, then the relative position between P and Q affects the kinematic behavior of the system.

To deal with the relative position tolerance between P and Q, either $\mathbf{s}_i'^P$ or $\mathbf{s}_i'^Q$ is considered to be fixed, and

Table 2. Tolerance model for links, joints and force elements of a MacPherson suspension.

element	Figures: Constraint Equation/ generalized Force	Figures: Random variables
Tie rod		
	$L^2 = (\mathbf{s}'_{iQ} - \mathbf{s}'_{iP})^T (\mathbf{s}'_{iQ} - \mathbf{s}'_{iP})$	L
Lower arm		
	$\mathbf{F} = [F_x \ \ F_y \ \ F_z]^T = \mathbf{s}'_{iB} - \mathbf{s}'_{iA}$ $\mathbf{G} = [G_x \ \ G_y \ \ G_z]^T = \mathbf{s}'_{iC} - \mathbf{s}'_{iA}$	F_x, F_y, F_z G_x, G_y, G_z
Spherical joint		
	$\mathbf{\Phi}_s = \mathbf{r}_j + \mathbf{A}_j \mathbf{s}'_j - \mathbf{r}_i - \mathbf{A}_i \mathbf{s}'_i - \mathbf{A}_i \mathbf{D}'_i \mathbf{c}''_i = 0$	$c''_{ix}, c''_{iy}, c''_{iz}$
Cylindrical joint		
	$\mathbf{\Phi}_{cy} = \begin{bmatrix} \mathbf{f}_i^T \mathbf{d}_{ij} - \lvert \mathbf{d}_{ij} \rvert \sin(\phi_f) \\ \mathbf{g}_i^T \mathbf{d}_{ij} - \lvert \mathbf{d}_{ij} \rvert \sin(\phi_g) \\ \mathbf{f}_i^T \mathbf{h}_j + \sin(\phi_f) \\ \mathbf{g}_i^T \mathbf{h}_j + \sin(\phi_g) \end{bmatrix} = 0$ $\mathbf{d}_{ij} = \mathbf{r}_j + \mathbf{A}_j \mathbf{s}'_j - \mathbf{r}_i - \mathbf{A}_i \mathbf{s}'_i - \mathbf{A}_i \mathbf{D}'_i \mathbf{c}''_i$	c''_{ix}, c''_{iy} ϕ_f, ϕ_g

the other can be regarded as a random variable. Here let u's assume that $\mathbf{s}_i'^P$ is fixed and $\mathbf{s}_i'^Q$ is the random variable. Since the tolerance between P and Q can be in any direction, each element of $\mathbf{s}_i'^Q = [s_{ix}'^Q, s_{iy}'^Q, s_{iz}'^Q]^T$ can be an independent random variable with mean and tolerances given as

$$\mathbf{s}_i'^Q = \mu(\mathbf{s}_i'^Q) + \mathbf{T}(\mathbf{s}_i'^Q) \tag{17}$$

Distance l between P and Q is given as

$$l^2 = \mathbf{d}^T\mathbf{d} = (\mathbf{A}_i\mathbf{s}_i'^Q - \mathbf{A}_i\mathbf{s}_i'^P)^T(\mathbf{A}_i\mathbf{s}_i'^Q - \mathbf{A}_i\mathbf{s}_i'^P) \tag{18}$$

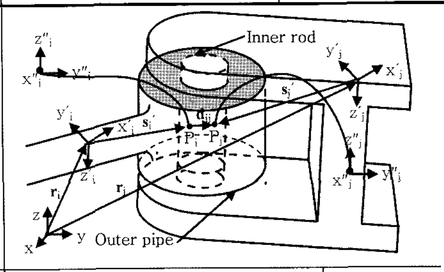
If it is assumed that $\mathbf{s}_i'^P$ is fixed and $\mathbf{s}_i'^Q$ is designated as a random variable to account for the link length variation, then the link length tolerance δl can be obtained by taking the variation of Equation (18) as

$$l \cdot \delta l = (\mathbf{A}_i\mathbf{s}_i'^Q - \mathbf{A}_i\mathbf{s}_i'^P)^T(\mathbf{A}_i\delta\mathbf{s}_i'^Q - \mathbf{A}_i\mathbf{s}_i'^P) \tag{19}$$

In the above equation, variation $\delta\mathbf{s}_i'^Q$ is the same as the tolerance $\mathbf{T}(\mathbf{s}_i'^Q)$ in Equation (17). Thus if the tolerance of $\delta\mathbf{s}_i'^Q$ is defined, then the tolerance in the link length can be calculated by Equation (19).

Tolerance models for suspension components such as links, kinematic joints, and bushings, are summarized in Table 2. The tie rod and lower control arm have tolerances associated with link length. In the case of a tie rod that has two spherical joints at each end, L, which is the length from the joint location point P to point Q can be defined as a design variable. In the case of the lower arm, vector **F** can be defined as a vector from a joint location point A to the other joint location point B, and vector **G** can be defined as a vector from a joint location

Table 2. Continued.

element	Figures: Constraint Equation/ generalized Force	Figures: Random variables
Bush		
	$\mathbf{Q}_i = \begin{bmatrix} \mathbf{F}_i \\ \mathbf{A}_i\mathbf{\Gamma}_i + \tilde{\mathbf{s}}'_i\mathbf{A}_i^T\mathbf{F}_i \end{bmatrix}$ $\mathbf{Q}_j = \begin{bmatrix} \mathbf{F}_j \\ \mathbf{A}_j^T\mathbf{\Gamma}_j + \tilde{\mathbf{s}}'_j\mathbf{A}_j^T\mathbf{F}_j \end{bmatrix}$	k_{xx}, k_{yy}, k_{zz} kt_{xx}, kt_{yy}, kt

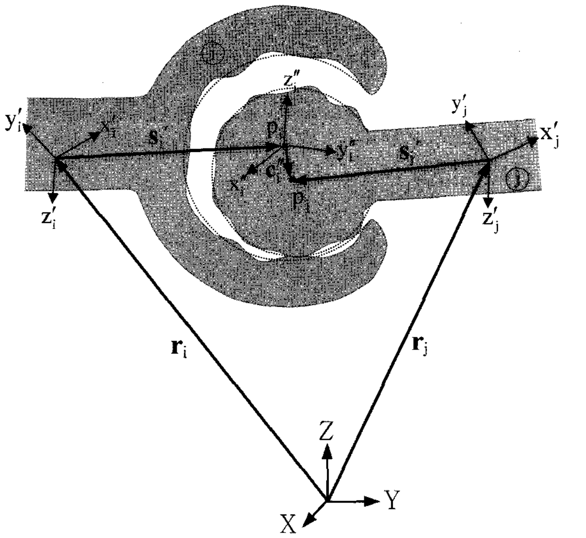


Figure 3. Tolerance model of a spherical joint.

point A to the other joint location point C. Each element of vectors **F** and **G**, (F_x, F_y, F_z, G_x, G_y and G_z) represent random variable.

A spherical joint as shown in Figure 3 is composed of the housing and ball, and a possible clearance between them. The joint clearance can be caused by a difference between the diameter of the housing and the ball, and also by a non-perfect circular shape of the housing and ball. Spherical joint random variables are components of the clearance vector between the ball and housing $\mathbf{c}_i''$ such as c_{ix}'', c_{iy}'' and c_{iz}''. The tolerance model and corresponding random design variables for other types of kinematic joints and bushing elements in MacPherson suspension systems are summarized in Table 2.

4. RELIABILITY ANALYSIS

In order to carry out reliability analysis for the elasto-kinematics of a MacPherson suspension system, a limit state function should be defined. Figure 4 shows toe angle using the design variables of Table 1, when the wheel moves vertically from full rebound (−80 mm) to full bump (90 mm). Also, the upper and lower limit of acceptable toe angles is specified as ±0.2° of the nominal toe angle. In this case, the limit state functions of the toe angle can be defined as

$$g_{TOE\_up}=T_{TOE\_up}-TOE \tag{20}$$

$$g_{TOE\_low}=TOE-T_{TOE\_low} \tag{21}$$

where g_{TOE_up} and g_{TOE_low} represent upper and lower limit state function respectively, and TOE is toe angle, T_{TOE_up} is the upper limit state and T_{TOE_low} is lower limit state. If the toe angle is greater than the upper limit state or less than the lower limit state, toe angle is out of the limit state and the value becomes less than 0.

If limit states of the camber angle are defined as ±0.1° as shown in Figure 5, the limit state functions of the camber angle are defined as

$$g_{CAMBER\_up}=T_{CAMBER\_up}-CAMBER \tag{22}$$

$$g_{CAMBER\_low}=CAMBER-T_{CAMBER\_low} \tag{23}$$

Also, if the limit states of the caster angle are specified as ±0.1° as shown in Figure 6, the limit state functions of the

Table 3. Design variables and tolerances for a MacPherson suspension.

	Design variables		Nominal dimensions	Tolerances
	Joints, bushes & links	Directions		
1	Tie rod	Length(mm)	346.75	0.5
2	Low arm (Vector F)	x(mm)	−38.6	0.5
3		y(mm)	342.3	0.5
4		z(mm)	−21.0	0.5
5	Spherical(B) clearance	Radial(mm)	0	0.1
6	Spherical(H) clearance	Radial(mm)	0	0.1
7	Spherical(T) clearance	Radial(mm)	0	0.1
8	Cylindrical(D) clearance	Radial(mm)	0	0.5
9		Conical(°)	0	0.5
10	Bush (C) Stiffness	Radial(N/mm)	3000	300
11		Axial(N/mm)	300	30
12		Conical (Nmm/deg)	30000	3000
13		Torsional (Nmm/deg)	3000	300
14	Bush (A) Stiffness	Radial(N/mm)	2000	200
15		Axial(N/mm)	200	20
16		Conical (Nmm/deg)	20000	2000
17		Torsional (Nmm/deg)	2000	200
18	Bush (G) Stiffness	Radial(N/mm)	1000	100
19		Axial(N/mm)	100	10
20		Conical (Nmm/deg)	10000	1000
21		Torsional (Nmm/deg)	1000	100

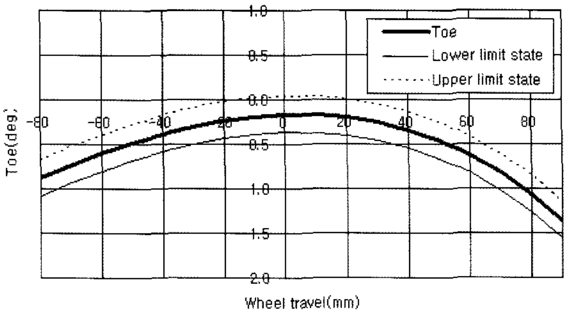


Figure 4. Limit state of toe.

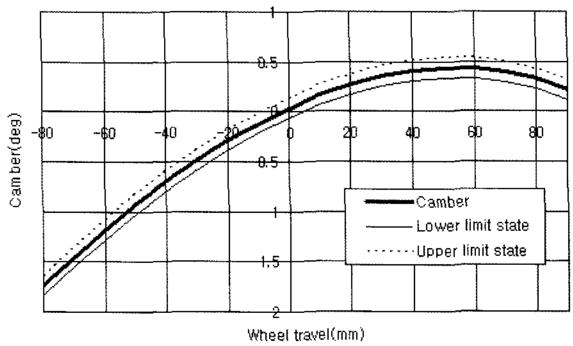


Figure 5. Limit state of camber

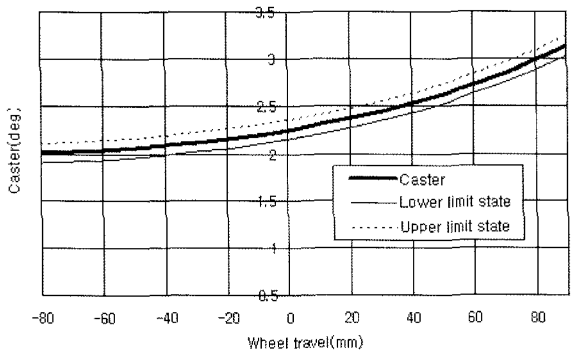


Figure 6. Limit state of caster.

caster angle can be given as

$$g_{CASTER\_up}=T_{CASTER\_up}-CASTER \quad (24)$$

$$g_{CASTER\_low}=CASTER-T_{CASTER\_low} \quad (25)$$

Design variables for link length, joint clearance and stiffness of bushing and their tolerances are presented in Table 3. In order to perform reliability analysis of the suspension system, the probability density function of random variables should be known. It is known that the tolerance of mechanical systems can be represented by the normal distribution in general (Choi *et al.*, 1998). If random variables of a suspension system do not follow the normal distribution, other distribution functions can be used with an appropriate reliability analysis method.

If the mean value of a random variable is μ and the variance is σ^2, the probability that the variable is within $\mu \pm 3\sigma$ becomes 99.7%. In this case, an interrelation between tolerance T and variance σ^2 can be represented as

$$\sigma^2=T^2/9 \quad (26)$$

When random variables from the MacPherson suspension system have mean values and tolerances as in Table 3, the variances of random variables are obtained using Equation (26). By substituting the variances of random variables and the sensitivity of the limit state function (with respect to random variables) into Equation (3), the reliability index can be computed. For example, the reliability index for the upper limit state function of the toe angle can be calculated as

$$\beta_{TOE\_up}=\frac{\mu_{g_{TOE\_up}}}{\sigma_{g_{TOE\_up}}}=\frac{g_{TOE\_up}(\mu_{x_1},\mu_{x_2},\ldots,\mu_{x_n})}{\sqrt{\sum_{i=1}^{n}(a_i\sigma_{x_i})^2}} \quad (27)$$

where $a_i=\lfloor \partial g_{TOE\_up}/\partial x_i \rfloor_{(\mu_{x_1},\mu_{x_2},\ldots,\mu_{x_n})}$.

Substituting β_{Toe_up} of Equation (27) into Equation (5), the reliability that the toe angle is in the upper limit states is obtained as

$$\begin{aligned}\text{Reliabilty}_{TOE\_up}&=P(g_{TOE\_up}>0)\\&=1-P_f(g_{TOE\_up}<0)\\&=1-\Psi(\beta_{TOE\_up})\end{aligned} \quad (28)$$

Figure 7 shows the reliability that the toe angle is in the limit states as Equation (28), which is the result of reliability analysis when wheel moves up and down.

$$\begin{aligned}&1-P_f(g_{TOE\_up}<0 \text{ and } g_{TOE\_low}<0)\\&=P(T_{TOE\_up}>TOE>T_{TOE\_low})\end{aligned} \quad (29)$$

The lowest reliability is about 0.963 when wheel center position is located at −80 mm. Figure 8 shows the reliability when the camber angle is in the limit states as in Equation (30), and the lowest reliability is about 0.989 when the wheel center position is located at 90 mm.

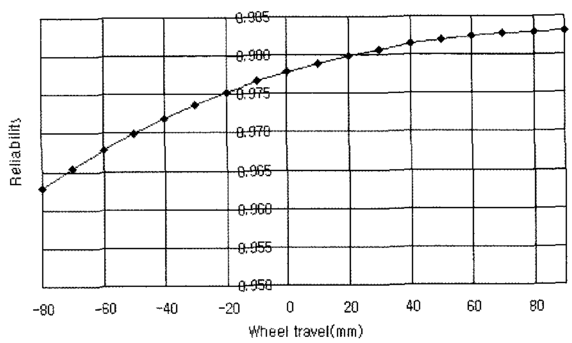


Figure 7. Reliability of toe.

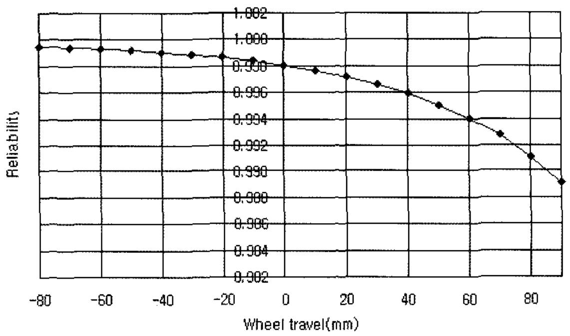


Figure 8. Reliability of camber.

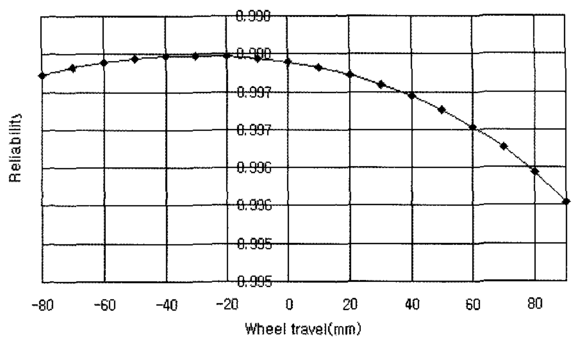


Figure 9. Reliability of caster.

$$1-P_f(g_{CAM\_up} < 0 \text{ and } g_{CAM\_low} < 0)$$
$$=P(T_{CAM\_up} > CAMBER > T_{CAM\_low}) \quad (30)$$

Figure 9 shows the reliability that the caster angle is in the limit state as in Equation (31), and the lowest reliability is about 0.996 when the wheel center position is located at 90 mm.

$$1-P_f(g_{CAS\_up} < 0 \text{ and } g_{CAS\_low} < 0)$$
$$=P(T_{CAS\_up} > CASTER > T_{CAS\_low}) \quad (31)$$

5. OPTIMIZATION

The optimization problem can be defined as in Equation (32) in order to perform reliability-based design optimization.

$$\underset{T}{Minimize}\ I(\mathbf{x})$$
$$Subject\ to\ x_{i\,lower} \le x_i \le x_{i\,upper}\quad i=1,2, ..., 21 \quad (32)$$
$$Cost(\mathbf{x}) < 690$$

Design variable x_i corresponds to each tolerance in Table 3. In order that the lowest reliability over the whole range of wheel travel becomes 0.997 for toe angle, camber angle, and caster angle, performance index I(**x**) is defined as the norm of unit vectors which are composed of differences between the lowest reliability and 0.997 for each SDF. where the target value 0.997 means the probability that each SDF is within $\mu_{SDF} \pm 3\sigma_{SDF}$. Since it

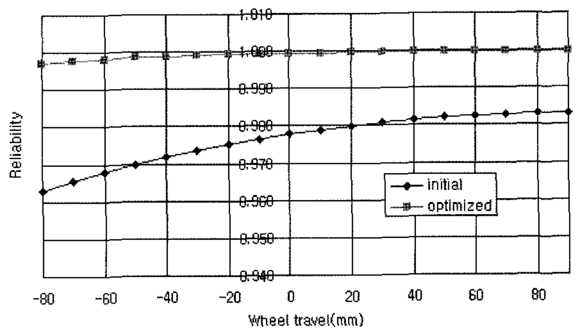


Figure 10. Reliability of toe after optimization.

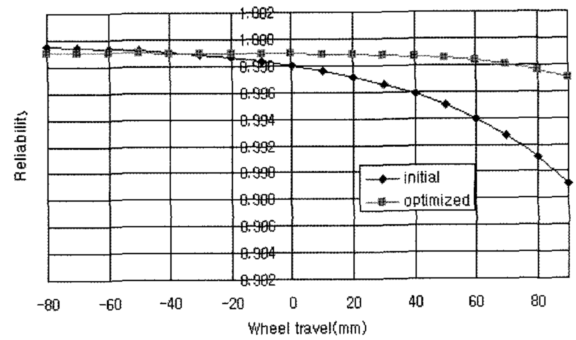


Figure 11. Reliability of camber after optimization.

is assumed that the cost of production is inversely proportional to the tolerance, the cost Cost(x) is defined as the summation of the reciprocal of the tolerance. The upper boundary of design variable $x_{i\ upper}$ and lower boundary $x_{i\ lower}$ are set up and Cost(**x**) is restricted to 690 or less, where 690 is defined as a constraint value in order that 10% or more of the initial cost is decreased.

$$I(\mathbf{x})=\left(\left(\frac{Min[P_z(T_{toe\_up} > toe > T_{toe\_low})]-0.977}{0.977}\right)^2 + \left(\frac{Min[P_z(T_{cam\_up} > camber > T_{cam\_low})]-0.977}{0.977}\right)^2 \left(\frac{Min[P_z(T_{cas\_up} > casber > T_{cas\_low})]-0.977}{0.977}\right)^2\right)^{1/2} \quad (33)$$

$$Cost(\mathbf{x})=\sum_{i=1}^{21} \frac{1}{x_i} \quad (34)$$

Figure 10 through Figure 13 show reliabilities that toe, camber, and caster angles are located in the limit state before and after optimization. It can be shown that each reliability of toe, camber, and caster angles is larger than 0.997. Figure 13 shows the history of the performance index I(**x**), which demonstrates the performance index reduction as iteration proceeds. The cost of production and the lowest reliability within the entire range of wheel travel for toe, camber and caster angle before and after optimization are arranged in Table 4. The cost of production becomes 685.75, which is an 11.5% decrease

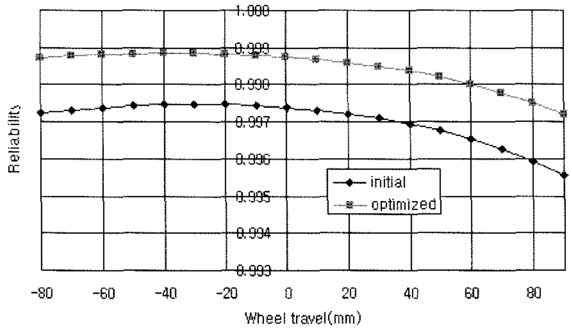


Figure 12. Reliability of caster after optimization.

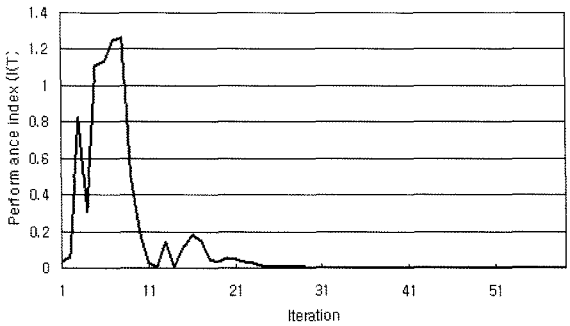


Figure 13. History of performance index for a MacPherson suspension.

Table 4. Cost and reliability of initial and optimal designs for a MacPherson suspension.

Performance function	Initial design	Optimal design	Reduction (%)
Cost	775.34	685.78	11.5
Minimum reliability for toe	0.963	0.997	–
Minimum reliability for camber	0.989	0.997	–
Minimum reliability for caster	0.995	0.997	–

Table 5. Initial and optimal tolerances of a MacPherson suspension.

	Design variables		Tolerances	
	Joints, bushes & links	Directions	Initial	Optimal
1	Tie rod	Length(mm)	0.5	0.209
2	Lower arm (Vector F)	x(mm)	0.5	0.653
3		y(mm)	0.5	0.148
4		z(mm)	0.5	0.768
5	Spherical (B)	Radial(mm)	0.1	0.158
6	Spherical (H)	Radial(mm)	0.1	0.131
7	Universal (T)	Radial(mm)	0.1	0.142
8	Cylindrical(D)	Radial(mm)	0.5	0.464
9		Conical(°)	0.5	0.146
10	Bush (C)	Radial(N/mm)	300	326.3
11		Axial(N/mm)	30	33.08
12		Conical (Nmm/deg)	3000	3102.4
13		Torsional (Nmm/deg)	300	318.4
14	Bush (A)	Radial(N/mm)	200	200.7
15		Axial(N/mm)	20	26.47
16		Conical (Nmm/deg)	2000	2045.1
17		Torsional (Nmm/deg)	200	212.3
18	Bush (G)	Radial(N/mm)	100	102.8
19		Axial(N/mm)	10	27.6
20		Conical (Nmm/deg)	1000	1106.2
21		Torsional (Nmm/deg)	100	102.8

compared to the initial cost. Tolerances before and after optimization are arranged in Table 5. The shaded tolerances, which are sensitive to toe, camber and caster angle, are decreased and the other less sensitive tolerances are increased to minimize cost and improve reliability at the same time.

6. CONCLUSION

In this study, the tolerances that exist in the links, joints, and bushings of suspension systems are modeled. Additionally, the reliability-based design optimization for elasto-kinematic behavior of suspensions is presented based on the tolerance model. MVFO is selected out of various available methods as the reliability method for suspension systems because of numerical efficiency. The sensitivity needed in the reliability analysis is evaluated by kimemato-static analysis and sensitivity analysis using the constraint equations and generalized forces. In order to minimize the cost of production and improve the reliability of the suspension system at the same time, reliability-based design optimization is presented. The optimization is applied to a MacPherson suspension system in order to validate the usefulness of the proposed method. The reliabilities of toe, camber, and caster angle are increased and the cost is simultaneously decreased.

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