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STOCHASTIC CHARACTERISTICS OF FATIGUE CRACK GROWTH RESISTANCE OF SM45C STEEL

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ABSTRACT–Reliability analysis based on fracture mechanics requires knowledge of the on statistical parameters m and C in the fatigue crack growth law $da/dN=C(\Delta K)^m$. The purpose of the present study is to investigate if it is possible to explain the change of parameter m by the fluctuation of C only. In this study, we apply the Paris-Erdogan law treating the parameter C as random and the parameter m as constant. Fluctuations in crack growth rate are assumed to be dependent only on C. The material resistance to fatigue crack growth ($Z=1/C$) is treated as a spatially random process, that varies along the crack path. The theoretical crack growth rates at various stress intensity factors are discussed. Additionally, the results of constant ΔK fatigue crack growth tests are reported for the structural steel, SM45C. The experimental data have been analyzed to determine the probability distribution of fatigue crack growth resistanc.

KEY WORDS : Fatigue, Crack growth retsistance, Spatial random process, Probability distribution, Reliability analysis

1. INTRODUCTION

In automotive engineering structural modifications are typically required to satisfy the design specifications such as fatige lifetime and strength (Song *et al.*, 2004).

A good example is the joining technology used to assemble automotive bodies, a key capability in light-weight automotive manufacturing (Kim *et al.*, 2006).

To establish rigorous methods of reliability analysis for machines and structures, the probability characteristics of the fatigue crack growth rate should be made clear In the viewpoint of fracture mechanics, this problem rests on the clarification of the probability characteristics of the coefficient C and index m of the Paris-Erdogan equation. Since the fatigue crack growth rate is influenced by various uncertain factors (e.g., external force, temperature, nonuniformity of material, and specimen geometry), stochastic fluctuation is inevitable.

Failure of material structures or mechanical systems is most likely to result from the effects of direct or indirect fatigue (Song *et al.*, 2004). Many experiments and analytical studies have been conducted on the influence of the factors listed above on fatigue crack growth. In these studies, any one of the following five models may be used:

(1) consider both C and m as fixed values;
(2) consider m as a fixed value, C as a probability variable;
(3) consider C as a fixed value, m as a probability variable;
(4) consider both m and C as probability variables with no correlation between the two;
(5) consider both m and C as probability variables with correlation between the two.

Models (1), (2), and (5) are the ones most frequently found in the literature. However, the question of which set of assumptions is most appropriate is still unanswered and requires more accurate experimental data. The focus of many current studies is the quantitative analysis of the uncertainty of material strength and the influence of this uncertainty on fatigue crack growth behavior.

In this study, we focus on the following Paris-Erdogan equation of fatigue crack growth rate and analyze crack growth rate with respect to the treatment of m and C.

$$\frac{da}{dN}=C(\Delta K)^m \quad (1)$$

It is often assumed that m is a parameter that depends only on the material, with the following justification: to assume the opposite, that m can vary in any one material, leads to the contradiction that the dimensionality of the material constant C also varies. Therefore, we assume the

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propagation index m to be constant along the crack growth path and treat the propagation coefficient C as the only fast varying variable - that is, the only stochastic variable along the crack path. We also assume that variations in C affect the fatigue crack growth rate. Here we study the correlation between variations in the propagation coefficient C and variations in the fatigue crack growth rate; we also investigate whether the propagation index m varies as well.

2. ANALYSIS OF THE STOCHASTIC CHARACTERISTICS OF FATIGUE CRACK GROWTH RESISTANCE

This study adopts the Paris-Erdogan Law shown in equation (1), in which m has a fixed value along the crack length and C is a spatial stochastic variable. The fatigue crack growth rate da/dN can therefore be considered to be a stochastic process that can be expressed by the equation below;

$$\frac{da}{dN}=C(x)(\Delta K)^m \tag{2}$$

The reciprocal dN/da, which is the resistance of the material to the fatigue crack growth, is more useful in characterizing material resistance against fatigue crack growth than the growth rate da/dN, as shown by Ortiz *et al.* This is because the propagation resistance at a point along the crack growth path represents the material's strength against crack growth at that point. Therefore, the selection of the variance of the resistance against crack propagation of material instead of the variance of the fatigue crack growth rate conforms with the goal of considering the spatial distribution of material characteristics. Equation (2) can thus be expressed as below;

$$\frac{da}{dN}=\frac{1}{Z(x)}(\Delta K)^m \tag{3}$$

Hereinafter, $Z(x)$ - that is, $1/C(x)$ - is referred to as the fatigue crack growth resistance coefficient (growth resistance coefficient) of the material against fatigue crack growth.

The parameter $Z(x)$ is a probability variable related to material strength and fatigue crack growth. If the growth resistance is larger, that is, if Z in equation (3) is larger, it is difficult for the fatigue crack to grow. The following distributions have been used to characterize Z in metals:

1) log-normal size distribution
2) Weibull distribution .

In this study, the probability distribution of Z is modeled as follows. The material is acknowledged to be nonuniform, and considering that the strength is dominated by the weakest region of the material, Z is assumed to follow the minimum extreme value distribution. As material strength in most cases follows the Weibull distribution, we assume that the probability distribution of Z also follows the Weibull distribution. Assuming that the distribution of Z is a 2-parameter Weibull distribution, its probability distribution function and density function are given as below;

$$F_z(z)=1-\exp\left(-\left(\frac{z}{\beta}\right)^{\alpha}\right) \tag{4}$$

$$f_z(z)=\frac{\alpha}{\beta}\left(\frac{z}{\beta}\right)^{\alpha-1}\exp\left(-\left(\frac{z}{\beta}\right)^{\alpha}\right) \tag{5}$$

where α is a shape parameter and β is a scale parameter. In this model, if a constant-•fK fatigue crack growth test is conducted, the mean value $E[da/dN]$ of the fatigue crack growth rate is expected to be proportional to the mean value of $1/Z$, as shown below;

$$\begin{aligned}\int_0^{\infty}\frac{1}{z}f_z(z)dz&=\int_0^{\infty}\frac{1}{z}\frac{\alpha}{\beta}\left(\frac{z}{\beta}\right)^{\alpha-1}\exp\left\{-\left(\frac{z}{\beta}\right)^{\alpha}\right\}dz\\&=\int_0^{\infty}\frac{1}{z}\frac{\alpha}{\beta}\left(\frac{z}{\beta}\right)^{\alpha-1}\exp(-x)\frac{\beta}{\alpha}\left(\frac{\beta}{z}\right)^{\alpha-1}dx\\&=\frac{1}{\beta}\int_0^{\infty}x^{-1/\alpha}\exp(-x)dx\\&=\frac{1}{\beta}\Gamma\left(1-\frac{1}{\alpha}\right)\end{aligned} \tag{6}$$

where, Γ is the gamma function.

3. EXPERIMENT

3.1. Specimen

The specimen material used in this study was general machine structure steel SM45C; the chemical composition and mechanical properties of this material are shown in Table 1 and Table 2. The specimen dimensions were in compliance with the CT specimen defined in ASTM E647-83, as shown in Figure 1, including dimensions of 100 mm width (W) and 12 mm thickness (B). The specimens were prepared so that the direction of roll was

Table 1. Chemical composition (wt.%).

Material	C	Si	Mn	P	S	Al
SM45C	0.47	0.20	0.74	0.01	0.18	0.01

Table 2. Mechanical properties.

Material	Tensile strength	Yield strength	Elongation	Hardness (H_B)
SM45C	610 MPa	342 MPa	23.0%	170

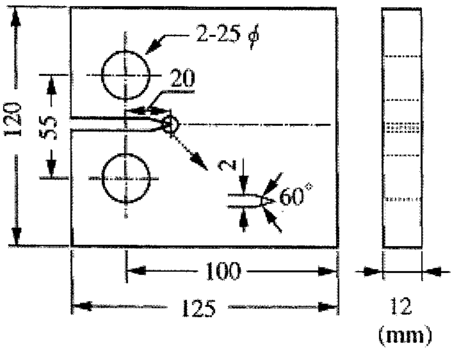

Figure 1. Shape and dimension of specimen.

vertical to the direction of crack propagation (L-T direction).

3.2. Experiment Method

The fatigue experiment system consisted of an ten-ton electric-hydraulic servo fatigue tester, a hydraulic motor for the servo tester, a computer for system control, a measuring device for the crack opening displacement to measure the crack length, and peripheral devices. The system was operated in automatic mode.

From a macroscopic point of view, fatigue crack growth tests that use constant-amplitude loads provide constant stresses; however, ΔK, which is the fracture condition at the crack edge, increases as the crack grows. This increase is undesirable for investigating the spatial variation of fatigue crack growth resistance. Therefore, constant-ΔK fatigue tests were conducted in this study.

Test conditions were set for a stress repeat speed of 10 Hz and a stress ratio of (R) 0.2. The value of ΔK was maintained at 25, 30, 37.5 or 45 MPa$\sqrt{m}$. Crack lengths were calculated with the equation proposed by Ashok Saxena *et al.*, with the crack opening displacement automatically measured by load and clip gauges during the test. At every 0.5 mm multiple of crack length, the crack length a, stress endurance number N, stress intensity coefficient range ΔK, maximum and minimum loads, and maximum and minimum crack opening displacements were recorded automatically by computer. The stress intensity coefficient K was calculated with the equation below in accordance with ASTM E647-83, and the fatigue crack growth rate was obtained by the secant method.

$$K=P\left(2+\frac{a}{W}\right)/B\sqrt{W}\left(1-\frac{a}{W}\right)^{3/2}$$

$$\left\{C_0+C_1\frac{a}{W}+C_2\left(\frac{a}{W}\right)^2+C_3\left(\frac{a}{W}\right)^3+C_4\left(\frac{a}{W}\right)^4\right\} \quad (7)$$

where, C_0=0.886, C_1=4.64, C_2=−13.32, C_3=14.72, and C_4=−.5.6.

4. RESULT AND REVIEW OF THE EXPERIMENT

4.1. Examination of Fatigue Crack Growth Rate

Figure 2 shows the crack length a as a function of the stress endurance number N for different values of ΔK. As shown in the figure, although the relationship between a and N in each specimen is essentially linear, the slope varies even for identical values of ΔK. This variation can be attributed to differences in fatigue crack growth resistance between the specimens. In addition, the overlapping a-N curves indicate changes in the crack growth resistance during crack propagation, even in the same material. It is proposed that m changes from specimen to specimen and that C is characterized simultaneously by relatively smooth change and by white-noise-like change. As m is taken to be a constant in this study, these changes are attributed to C.

Figure 3 shows the degree of constant control of ΔK during the experiment, along with da/dN. It can be seen

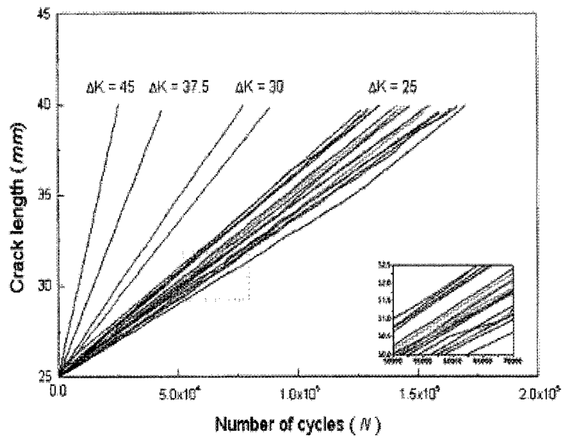

Figure 2. a~N curve.

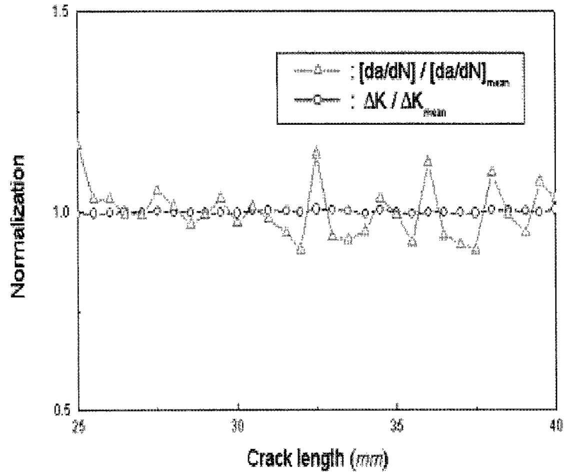

Figure 3. Variation of da/dN and ΔK.

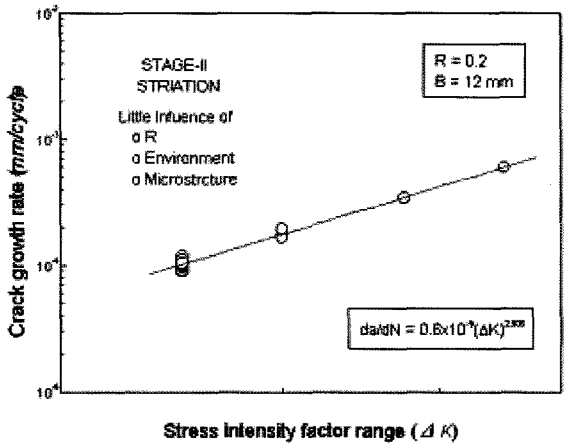

Figure 4. Relationship between *da/dN* and ΔK.

that changes in ΔK are negligible compared to the large changes in *da/dN* as crack length increased.

Although this is only an example of the test results, similar results were obtained from all the specimens.

Figure 4 shows the relationship between the fatigue crack growth rate *da/dN* and the stress intensity coefficient range ΔK. Though the experimental data at higher ΔK are sparse, the value of m and C are estimated to be 2.935 and 1.88×10^{-9} respectively by fitting the Paris-Erdogan Law. In the statistical analysis, m is set to a fixed value.

This experiment was performed in the second-stage, or stable-growth, area of fatigue crack growth. As noted in the figure, the wave front propagated via striation formation.

4.2. Probability Characteristics of Growth Resistance

Figure 5 shows an example of the fatigue crack growth

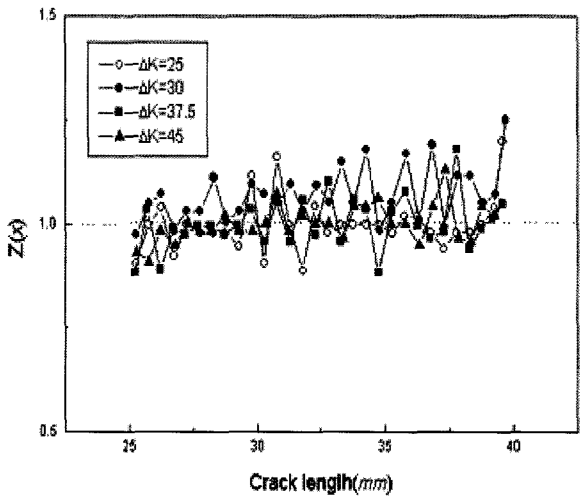

Figure 5. Scatter of the material resistance within a specimen.

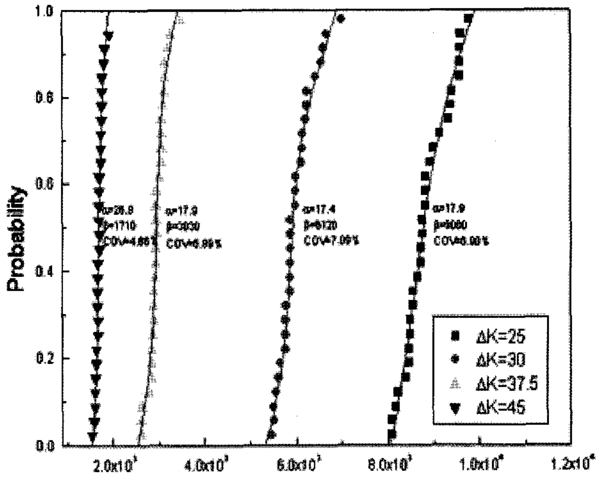

Figure 6. Weibull plots of material resistance obtained from experiments.

resistance coefficient $Z(x)$ as a function of the crack length a. In the figure, (a), (b), (c) and (d) correspond to ΔK values of 25, 30, 37.5 and 45 MPa$\sqrt{m}$ respectively; the value of Δa is 0.5 mm. It can be seen that variation in $Z(x)$ is reduced as ΔK is increased.

Figure 6 shows Weibull plots of experiment data for different values of ΔK. It can be seen that the probability distribution of the growth resistance coefficient depends upon ΔK. As ΔK increases, the fatigue crack growth resistance of the material decreases, as shown by a decrease in $Z(x)$.

Figure 7 shows the correlation between the mean fatigue crack growth rate $E(da/dN)_{exp}$ obtained from experiment and the fatigue crack growth rate $E(da/dN)_{th}$ obtained by theory. It can be seen that the theoretical values are in good agreement with the experimental

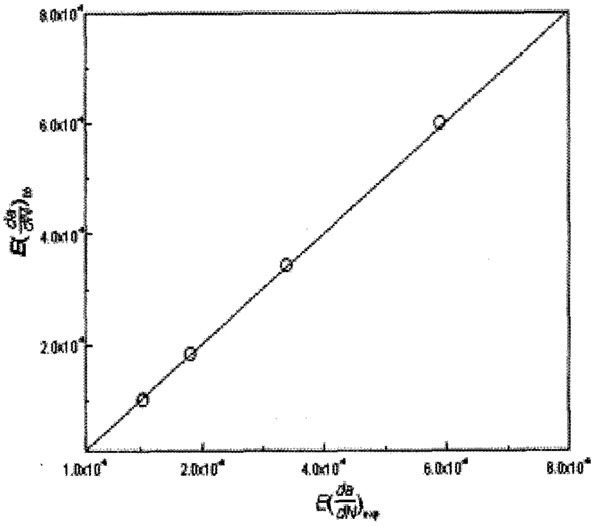

Figure 7. Relation between $E(da/dN)_{exp}$ and $E(da/dN)_{th}$.

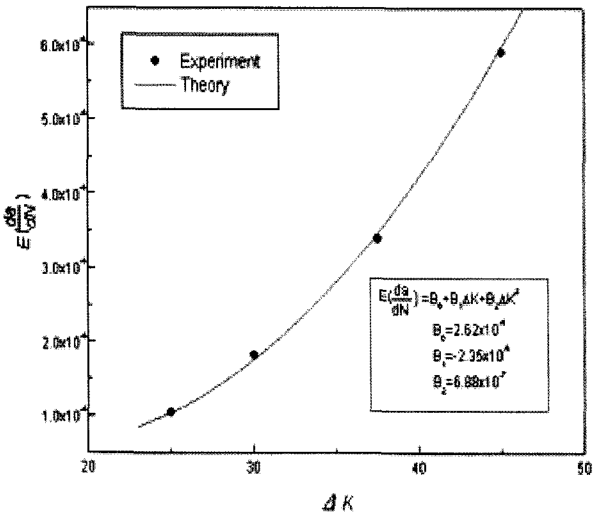

Figure 8. Growth rate vs. ΔK linear scale.

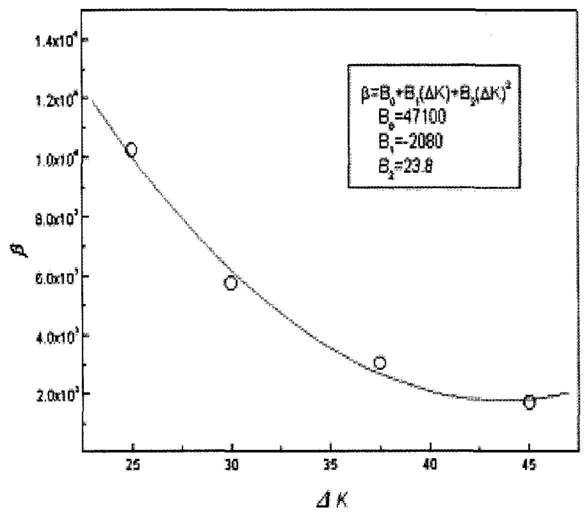

Figure 10. β versus ΔK.

values. However, the theoretical value is larger in the area of ΔK. Figure 8 shows $E(da/dN)$ as a function of ΔK on a linear scale. The markers in Figure 8 represent the experimental values and the curve represents the theoretical value of $E(da/dN)$. This may indicate an increase of m in the Paris Law. The theoretical equation can be approximated with the quadratic function below with good accuracy (correlation coefficient of 0.998)

$$E\left(\frac{da}{dN}\right)=B_0+B_1\Delta K+B_2\Delta K^2 \tag{8}$$

where the coefficients are B_0=2.62x10^{-4}, B_1=2.35x10^{-5}, and B_2=6.88x10^{-7}. As shown in the figure, the fatigue crack growth rate can be estimated with the equation (6).

Figure 9 and Figure 10 show the relation of α and β, which are the probability distribution parameters of Z in accordance with ΔK. It can be seen that α increases with the increase of ΔK while β decreases.

Figure 11 illustrates theoretical and experimental values of $E(da/dN)$ as a function of ΔK to examine the influence of Z on m in the Paris-Erdogan Law. Though the experimental and theoretical values seem to be the same in Figure 11 as in Figure 8, the theoretical value exceeds the experimental value at large ΔK. As a result, the value of m appears to increase with Z.

In order to investigate the influence of Z on m, assume that, in equation (6), β depends upon the area r which is controlled by ΔK. In such a case, let the probability distribution at reference area r0 be described by equation (4), and assume that it is in the minimum value distribution. If the distribution of minimum value dominates the (nxr0) area, then the probability distribution is as follows:

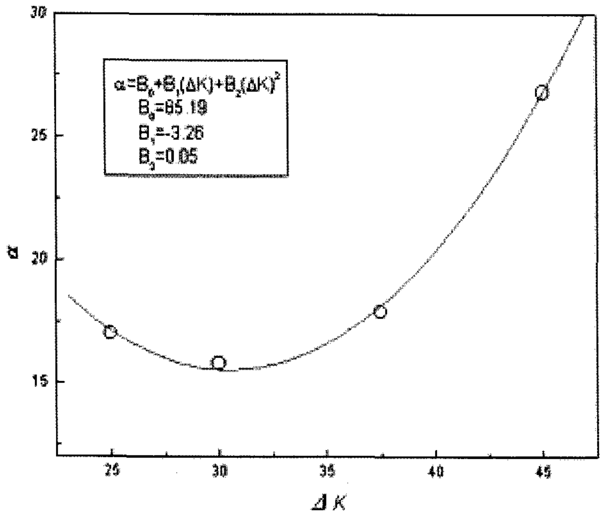

Figure 9. Growth rate vs. ΔK linear scale.

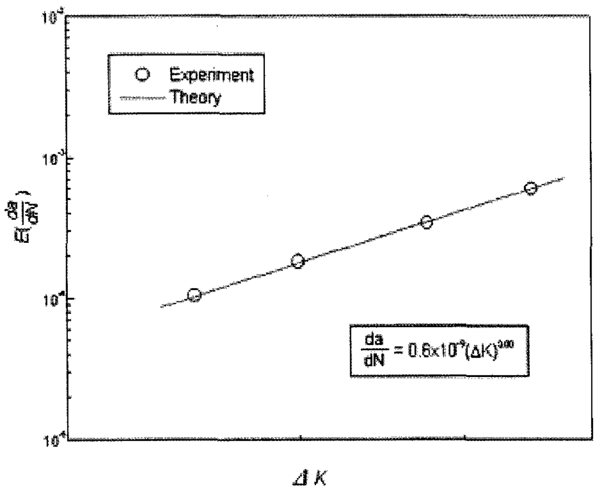

Figure 11. Crack growth rate vs. ΔK in Paris' law.

$$P(\min(z_1, z_2, \ldots, z_n \le x) = 1 - \exp\left\{-n\left(\frac{z}{\beta}\right)^c\right\} \quad (9)$$

The theoretical mean value of *da/dN* also changes in accordance with the following equation:

$$E\left(\frac{da}{dN}\right) \propto \frac{n^{1/c}}{\beta}\Gamma\left(1 - \frac{1}{a}\right) \quad (10)$$

Here, if we consider the case in which n is proportional to the p power of ΔK, then the mean value of *da/dN* is

$$E\left(\frac{da}{dN}\right) = \frac{n^{1/c}}{\beta}\Gamma\left(1 - \frac{1}{a}\right)(\Delta K)^m$$

$$= \frac{(f\Delta K)^{p/c}}{\beta}\Gamma\left(1 - \frac{1}{a}\right)(\Delta K)^m$$

$$= \frac{f^{p/c}}{\beta}\Gamma\left(1 - \frac{1}{a}\right)(\Delta K)^{m+p/c} \quad (11)$$

Consequently, it is in proportion to the $(m+p/\alpha)$ power of ΔK.

The above considerations show that m may vary with Z, the growth resistance coefficient, even though m was previously considered to be constant. However, more experiment data are required in the region of the highest stress intensity coefficients.

5. CONCLUSION

This study has applied the Paris-Erdogan Law of fatigue crack growth rate for the purpose of analyzing the parameters m, the propagation or growth index, and C, the propagation coefficient. The growth index m was considered to be fixed along the crack propagation path, and the parameter C was considered to be a stochastic variable that can vary considerably within a specimen. The reciprocal Z of the propagation coefficient C was defined as the fatigue crack growth resistance coefficient and used to analyze experimental data under the assumption that changes in Z influence the fatigue crack growth rate.

It was found that Z follows a 2-parameter Weibull distribution and that the fatigue crack growth rate could be estimated with the change of Z only. In addition, although the propagation index m in the Paris-Erdogan Law was assumed to be constant in this study, m may show apparent changes when changes in the specimen affect the value of Z.

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