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Tank Model using Kalman Filter for Sediment Yield

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A tank model in conjunction with Kalman filter is developed for prediction of sediment yield from an upland watershed in Northwestern Mississippi. The state vector of the system model represents the parameters of the tank model. The initial values of the state vector were estimated by trial and error. The sediment yield of each tank is computed by multiplying the total sediment yield by the sediment yield coefficient. The sediment concentration of the first tank is computed from its storage and the sediment concentration distribution (SCD); the sediment concentration of the next lower tank is obtained by its storage and the sediment infiltration of the upper tank; and so on. The sediment yield computed by the tank model using Kalman filter was in good agreement with the observed sediment yield and was more accurate than the sediment yield computed by the tank model.

Key Words : Kalman filter, sediment yield, tank model, state vector, system model, sediment yield coefficient, sediment distribution, sediment concentration distribution, sediment infiltration

1. Introduction

Estimates of watershed sediment yield are required for design of dams and reservoirs, soil conservation practices, and debris basins; determination of pollutants; depletion of reservoirs, lakes and wetlands; determination of the effects of basin management; and cost evaluation. The SCD was assumed to be an exponential function for each event and its parameters were correlated with the effective rainfall characteristics. The sediment concentration of the IUSG was assumed to vary with the effective rainfall amount. A sediment routing function, using the travel time and sediment particle size, was used to determine the SCD. Runoff influencing on sediment yield is, in general, nonlinear and time-variant. The parameters of tank model vary in time and space, and when they are assumed constant, they are so only by assumption. The coefficients of the tank model for runoff and sediment yield are assumed to be the same. Thus the estimation of the parameters by Kalman filter has accomplished for runoff and the sediment yield is calculated by the parameters. The errors in a tank model may arise due to inadequacy of the model itself, parameter uncertainty, errors in the data used for parameter estimation, and inadequate understanding of the rainfall-runoff-sediment yield process due, in part, to randomness. The error in the prediction of sediment yield (runoff) due to the uncertainty caused by the physical process, the model, and the input data can be reduced if Kalman filter is incorporated in a tank model. Lee and Singh[1] analyzed sediment yield by coupling Kalman filter with the IUSG. And Lee and Singh[2] analyzed sediment yield by the tank model.

The objective of this study is to develop a tank model by using Kalman filter for prediction of sediment yield, test it on an upland watershed in northwestern Mississippi, and compare it with the tank model.

2. Theory

2.1. Tank model for sediment yield

The tank model (Sugawara[3]) considered in this study is represented by a cascade of conceptual tanks as shown in Fig.1. For determining the sediment yield by the tank model, the SCD of the first tank is produced by the incremental source runoff (or the effective rainfall) and sediment concentration of the next lower tank is computed from the sediment infiltration of the upper tank. The sediment concentration of the first tank is computed from its storage and the SCD; the sediment concentration

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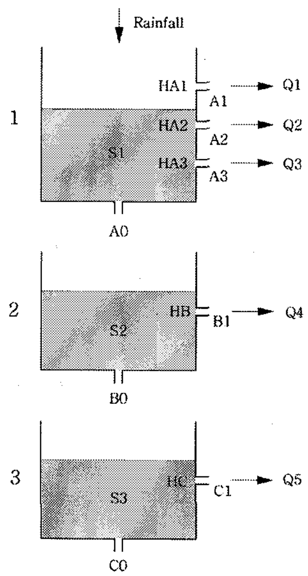

Fig. 1. Schematic representation of the tank model.

of the next lower tank is obtained by its storage and the sediment infiltration of the upper tank; and so on. The sediment yield through the side outlet is obtained by multiplying the total sediment yield, obtained by the product of runoff and the sediment concentration, by the sediment yield coefficient. The sediment infiltration through the bottom outlet is obtained by multiplying the total sediment infiltration, obtained by the product of infiltration and the sediment concentration, by the sediment infiltration coefficient.

The coefficients of the tank model for runoff and sediment yield are assumed to be the same. In Fig. 1, the runoff and sediment yield coefficients of the first tank are A1, A2, A3, those of the second tank are B1, and those of the third tank are C1 and the units of the runoff and sediment yield coefficients are 1/hr and dimensionless, respectively. Coefficients A0, B0, and C0 are, respectively, the infiltration and sediment infiltration coefficients of each tank and their units are 1/hr and dimensionless, respectively. The quantities, HA1, HA2, HA3, HB, and HC, are, respectively, the heights of side outlets (orifices) of each tank in mm.

Let SY_1, SY_2, and SY_3 (t/d) represent the components of the sediment yield of first tank due to surface runoff, and SY_4 represent the sediment yield of the second tank due to intermediate runoff, and SY_5 represent the sediment yield of the third tank due to ground water runoff. Thus, the computed sediment yield is obtained by summing the sediment yield components of each tank. Let SYf_1, SYf_2, and SYf_3 (t/d) be, respectively, the sediment infiltration rates of lower tanks. If there were a fourth tank, then SYf_3 would infiltrate into the fourth tank. In the absence of the fourth tank, however, in this study, SYf_3 is regarded as f_3.

The amount of sediment yield or sediment infiltration through a side outlet is linearly proportional to the concentration in the tank and runoff through the side outlet and can be expressed as

$$\begin{aligned} SY_1 &= A1 \cdot CON1 \cdot Q_1 \\ SY_2 &= A2 \cdot CON1 \cdot Q_2 \\ SY_3 &= A3 \cdot CON1 \cdot Q_3 \\ SY_4 &= B1 \cdot CON2 \cdot Q_4 \\ SY_5 &= C1 \cdot CON3 \cdot Q_5 \\ SYf_1 &= A0 \cdot CON1 \cdot f_1 \\ SYf_2 &= B0 \cdot CON2 \cdot f_2 \\ SYf_3 &= C0 \cdot CON3 \cdot f_3 \end{aligned} \tag{1}$$

where CON1, CON2 and CON3 (mg/l), respectively, are the sediment concentrations of the first tank, the second tank, and the third tank. And f_1, f_2, and f_3 (mm/h) are, respectively, infiltration rates of tank 1, 2, and 3; and Q_1, Q_2, and Q_3 (mm/h) represent the components of surface runoff from first tank, and Q_4 (mm/hr) represents the intermediate runoff (or interflow) from the second tank, and Q_5 (mm/hr) represents the ground water runoff known as sub-base runoff from the third tank.

2.2. Sediment Concentration Distribution (SCD)

The SCD caused by rainfall was estimated by considering the sediment-routing equation:

$$Y = Y_0 \exp(-aTd^{0.5}) \tag{2}$$

in which Y is the sediment yield at a particular channel section (metric tons), Y_0 is the sediment yield at an upstream section (metric tons), a is the routing coefficient, T is the travel time between the two sections(hours), and d is the median sediment particle diameter(microns).

The SCD used in the tank model can be expressed as

$$C_{0i} = \frac{Yv_i}{\left[H \sum_{j=1}^{m} v_j^2 \right]} \tag{3}$$

To use (3) v_i, Y, H must be determined. In which v_i is the incremental source runoff (centimeters). Y was predicted with the modified universal soil loss equation, MUSLE (Williams[4]):

$$Y = 11.8(Vq_p)^{0.56} KCP(LS) \tag{4}$$

In which V is the volume of runoff(cubic meters), q_p is the peak flow rate(cubic meters per second), K is the soil factor, C is the crop management factor, P is the erosion control practice factor, and LS is the slope length and gradient factor.

$H = \int_0^\infty h(w)\exp(-awd^{0.5})dw$. To determine H requires knowledge of a and $h\ (w)$. The routing coefficient a can be determined from (2) by replacing T by the time to peak T_p and predicting Y and Y_0 with (4). Using (4), (2) becomes

$$11.8(Vq_p)^{0.56} KCP(LS) = 11.8(VQ_p)^{0.56} KCP(LS)\exp(-aT_p d^{0.5}) \tag{5}$$

in which q_p is peak flow rate, cubic meters per second, Q_P is the peak source runoff rate(cubic meters per second) and T_P is the watershed time to peak(hours). From (5), one gets

$$a = -\frac{\ln(q_p/Q_p)^{0.56}}{T_p d^{0.5}} \tag{6}$$

$h\ (w)$ is IUH ordinate(cubic meters per second).

Thus, the SCD produced by rainfall is used to compute the sediment yield in the tank model. The SCD is used in the 1st tank. The sediment concentration of the 1st tank is attributed to the SCD estimated by (3), rainfall and the storage in the first tank. The sediment concentration of the next lower tank is attributed to sediment infiltration from the upper tank and the storage of the next lower tank.

2.3. Kalman filter

Kalman filter is a state estimation algorithm of a state-space model and optimally represents the system state of a deterministic or a stochastic model which has uncertainties in observed data, initial and boundary conditions, and parameters. The Kalman filter algorithm is constituted by three components: system model, measurement model, and Kalman filtering.

2.3.1. System model

A system which has a discrete dynamic behavior can be described in terms of the state vector as (Todini[5], Wood[6])

$$X(k) = \Phi(k|k-1)\cdot X(k-1) + \Gamma(k|k-1)\cdot w(k-1) \tag{7}$$

where $X(k)$ = state vector (n×1), $\Phi(k|k-1)$ = state transition matrix (n×n) for time k at time(k-1), $\Gamma(k|k-1)$= system error transition matrix(n×n), $w(k-1)$= system error vector (n×1).

2.3.2. Measurement model

The state vector X(k) of the system is observed through a measurement system, that inherently contains error. Therefore the measurement vector Z(k) can be described as a linear combination of a state vector X(k) and a measurement error vector v(k):

$$Z(k) = H(k)\cdot X(k) + v(k) \tag{8}$$

where $Z(k)$ = measurement vector (m×1), $H(k)$= measurement transition matrix (m×n), $v(k)$= measurement error vector (m×1).

The error vectors w(k) of (7) and v(k) of (8) are assumed independent Gaussian processes.

2.3.3. Kalman filtering

By (7) of the system model the state prediction value $\overline{X}(k|k-1)$ at time k, given its value at time (k-1), is (Wu[7])

$$\overline{X}(k|k-1) = \Phi(k|k-1)\cdot \hat{X}(k-1|k-1) \tag{9}$$

Knowing the state prediction value $\overline{X}(k|k-1)$ and the measurement vector Z(k) of the measurement model, the state estimation value $\hat{X}(k|k)$ is obtained by filtering the measurement error with use of the Kalman gain K(k) as

$$\hat{X}(k|k) = \overline{X}(k|k-1) + K(k)\left[Z(k) - H(k)\cdot\overline{X}(k|k-1)\right] \tag{10}$$

where

$$K(k) = P(k|k-1)\cdot H(k)^T\left[H(k)\cdot P(k|k-1)\cdot H(k)^T + R(k)\right]^{-1} \tag{11}$$

$\left[Z(k) - H(k)\cdot\overline{X}(k|k-1)\right]$ = measurement error.

The covariance P(k|k) of the state estimation error is

$$P(k|k) = \left[I - K(k)\cdot H(k)\right]\cdot P(k|k-1) \tag{12}$$

2.4. Tank model using Kalman filter

The tank model is used in this study as the fundamental model for predicting runoff and sediment yield.

2.4.1. System model

The state vector of system model is used the parameters of tank model. Therefore the state vector X(k) : (13×1) is as follows:

$$X(k) = [A1, A2, A3, B1, C1, A0, B0, C0, HA1, HA2, HA3, HB, HC]^T \quad (13)$$

and the state transition matrix , Φ(k) : (13×13) and the system error transition matrix(n×n), $\Gamma(k|k-1)$: (13×13) are assumed as unit matrix I, then the system model is described by

$$X(k) = X(k-1) + w(k);\ \mathrm{w(k)} \sim \mathrm{N}\ (0,\ \mathrm{Q(k)}) \quad (14)$$

2.4.2. Measurement model

The observation variable applicable to the tank model is runoff, Q. Therefore the measurement model can be described as

$$Q(k) = Z(k) = H(k) \cdot X(k) + v(k);\ \mathrm{v(k)} \sim \mathrm{N}\ (0,\ \mathrm{R(k)}) \quad (15)$$

where H(k) : (1×13) is the observation transition matrix expressed by

$$H(k) = [h_1, h_2, h_3, h_4, h_5, 0,0,0,0,0,0,0,0] \quad (16)$$

where h_1, h_2, h_3, h_4, h_5 are, respectively, the head of water at the outlets of each tank.

Thus Runoff Q(k), can be represents as (15) in tank model using Kalman filter, can be estimated by (12).

3. Application and Analysis

3.1. Study basin

A small upland watershed, W-5, a part of the Pigeon Roost basin located near Oxford in Marshall County, Mississippi, was selected for testing the tank model. The watershed has an area of approximately 4.04 km^2, is 1288 m long and 128.8 m wide. The watershed consists of a rather flat flood plain with natural channels and rolling, severely dissected interfluvial areas. The channels have a few straight reaches, and most have banks that scour easily. The average channel width-depth ratio is approximately 2:1 at the gaging station. A detailed description of this watershed is given by Bowie and Bolton[8].

3.2. SCD

The SCD was determined from (3) and the IUH was determined by the Nash model for each event. The parameters for the sediment yield estimated by MUSLE in (4) for watershed W-5 are as follows: The soils factor, K, is 0.26, the crop management factor, C, is 0.07, the erosion control practice factor, P, is 0.47 and the slope length and gradient factor, LS, is 0.34. The routing coefficient *a*, for estimating H was estimated for each event by (3) and is given in Table 1. The initial concentration for one unit of runoff, C_0, the sediment yield, Y, estimated by MUSLE and H in (3) for each event are given in Table 1. The SCD estimated by (3), used to estimate the sediment yield by tank model and the tank model using the Kalman filter.

3.3. Tank model using Kalman filter

In (13) there is a total of 13 parameters to be calibrated by trial and error. In order not to bias the physical constraints the numerical values of the tank model parameters shown in Table 2 were used as the mean values to define the initial parameter values for the state vector of the tank model using Kalman filter, X(k) = [0.085 0.085 0.085 0.043 0.009 0.063 0.043 0.009 8 4 1 1 1 $]^T$.

The initial storage values S1, S2, and S3 of each tank for h1~h5 in the matrix H(k) were assumed zero. The initial diagonal elements values P(0|0) for the covariance

Table 1. Characteristic values for the determination of the SCD

Storm	*a*	H	C_0 (mg/l)	Y (t/h)
No.1	0.256	1596.27	195595.1	91.30
No.2	1.735	577.88	257880.3	110.08
No.3	0.592	888.48	245075.4	62.83
N0.4	0.837	697.05	210107.8	70.75

Table 2. Parameters of the tank model

Storm	A1	A2	A3	B1	C1	A0	B0	C0	HA1	HA2	HA3	HB	HC
No.1(72.12.9)	.09	.09	.09	.05	.01	.09	.05	.01	8	4	1	1	1
No.2(73.3.14)	.07	.07	.07	.02	.005	.07	.02	.005	8	4	1	1	1
No.3(75.1.10)	.10	.10	.10	.05	.01	.01	.05	.01	8	4	1	1	1
No.4(75.3.12)	.08	.08	.08	.05	.01	.08	.05	.01	8	4	1	1	1
Mean	.085	.085	.085	.043	.009	.063	.043	.009	8	4	1	1	1

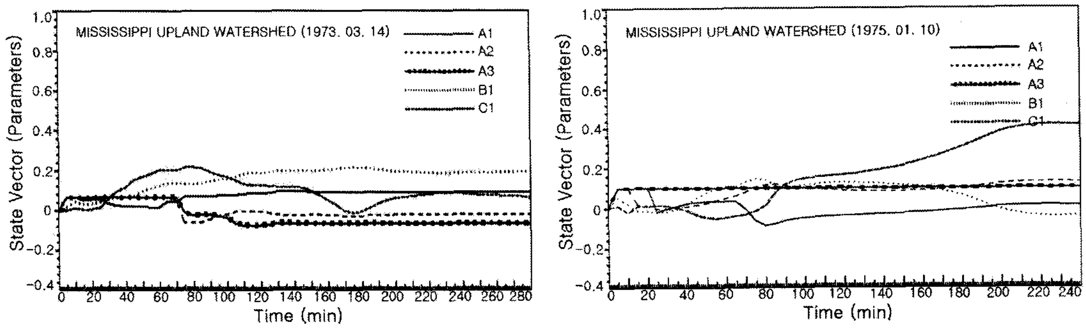

Fig. 2. Variation of state vector (the parameters of tank model).

matrix P(k) of the state estimation error were assumed P(0|0) = [0.06 0.06 0.01 0.06 0.06 10 10 10 10 10 10 10 10]. Other initial values were assumed as follows: the measurement error covariance matrix R(k) of the measurement error vector V(k), R(0) = 0; the system error covariance matrix Q(k) of the system error vector W(k), Q(0) = 0; the state transition matrix $\Phi(0)$ and the system error transition matrix $\Gamma(0)$ were assumed as unit matrix I.

As the runoff coefficients in the parameter set of the tank model are the most sensitive to runoff, runoff is estimated by varying runoff coefficients in time. The variation of model parameters is illustrated in Fig. 2.

A comparison of observed sediment yield, the sediment yield computed by the tank model, and the sediment yield computed by the tank model using Kalman filter is illustrated in Fig. 3. These error indices for the sediment yield by the tank model and the tank model using Kalman filter are given in Table 3. As shown in the table, the model efficiencies for each event are shown between 0.80 and 0.90 for tank model and between 0.87 and 0.93 for the tank model using Kalman filter. The sediment yield of the tank model using Kalman filter is in closer agreement with the observed sediment yield than that of the tank model alone.

The sediment yield computed by the tank model and the tank model with Kalman filter was quantitatively compared with the observed sediment yield, based on: (1) model efficiency, *ME*; (2) mean square error, *MSE*; (3) *Bias*; (4) volume error, *VER*; (5) peak runoff error, *PER*; and (6) peak time error, *TER*.

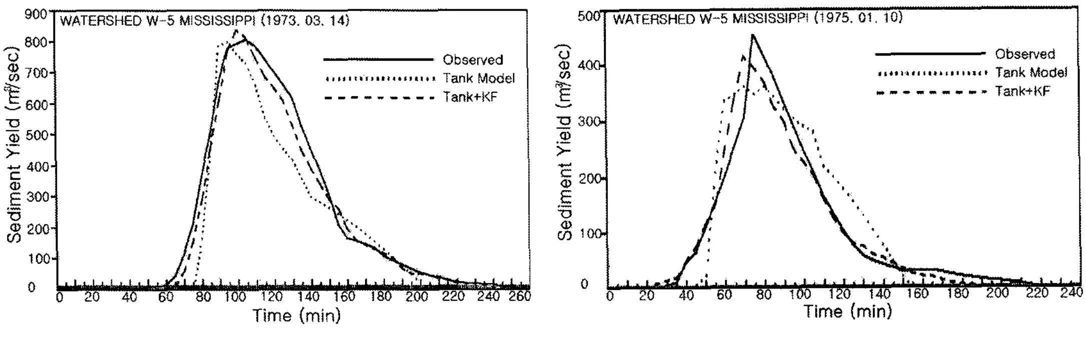

Fig. 3. Comparison of observed and computed sediment yield graphs.

Table 3. Error indices for sediment yield by tank model and tank model using Kalman filter

Storm	ME		MSE		Bias		VER(%)		PER(%)		TER(min)	
	Tank	Tankkf	Tank	Tankkf	Tank	Tankkf	Tank	Tankkf	Tank	Tankkf	Tank	Tankkf
No.1	0.80	0.87	30.69	21.83	10.16	5.21	15.36	-0.68	5.47	2.64	10	5
No.2	0.90	0.93	83.47	35.21	39.53	2.57	20.06	3.26	1.03	-1.12	10	5
No.3	0.85	0.92	50.22	26.87	20.87	3.26	20.22	2.18	20.55	9.24	5	0
No.4	0.83	0.89	56.18	23.56	-4.47	-1.03	-5.15	1.57	-2.28	1.68	5	5

$$ME = 1 - \frac{\sum (Q_{obs} - Q_{est})^2}{\sum \left((Q_{obs} - \bar{Q}_{obs} \right)^2} \quad (17)$$

$$MSE = \left[\frac{\sum (Q_{obs} - Q_{est})^2}{n} \right]^{0.5} \quad (18)$$

$$Bias = \frac{\sum (Q_{obs} - Q_{est})}{n} \quad (19)$$

$$VER = \frac{\sum Q_{obs} - \sum Q_{est}}{\sum Q_{obs}} \times 100 \quad (20)$$

$$QER = \left[\frac{Q_{pobs} - Q_{pest}}{Q_{pobs}} \right] \times 100 \quad (21)$$

$$TER = T_{pobs} - T_{pest} \quad (22)$$

4. Conclusions

The following conclusions can be drawn from this study. (1) The tank model and the tank model using Kalman filter satisfactorily simulated sediment yield. (2) The sediment yield computed by the tank model using Kalman filter was in good agreement with the observed sediment yield and was more accurate than the sediment yield computed by the tank model. (3) The values of the coefficients used for computing runoff and sediment yield by the tank model are the same. (4) The values of the coefficients used for computing runoff and sediment yield by the tank model using Kalman filter are the same.

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