Low Pilot Ratio Channel Estimation for OFDM Systems Based on GCE-BEM

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Abstract

Doubly-selective channel estimator for orthogonal frequency division multiplexing(OFDM) systems is proposed in this paper. Based on the generalized complex exponential basis expansion model(GCE-BEM), we describe the time-variant channel with time-invariant coefficients over multiple OFDM blocks. The time variation of the channel destroys the orthogonality between subcarriers, and the resulting channel matrix in the frequency domain is no longer diagonal, but the main interference comes from the near subcarriers. Based on this, we propose a channel estimator with low pilot ratio. We first develop a least-square(LS) estimator under the assumption that only the maximum Doppler frequency and the channel order are known at the receiver, and then verify that the correlation matrix of inter-channel interference(ICI) is a scaled identity matrix based on which we derive an optimal pilot insertion scheme for the LS estimator in the sense of minimum mean square error. The proposed estimator has the advantages of low pilot ratio and robustness against inter-carrier interference.

Key words : Channel Estimation, OFDM, Doubly-Selective Channel, GCE-BEM.

Ⅰ. Introduction

The demand for high data rate transmissions calls for broadband communication systems. When the transmission bandwidth gets larger, the sampling period becomes smaller than the delay spread of the channel, especially in multipath scenarios, which gives rise to frequency-selective channels. Carrier frequency-offset due to the oscillators' mismatch, together with high relative mobility between the transmitter and receiver causes the transmission channel to change rapidly in time, which is referred to as the time-selectivity of the channel. The channel with both the frequency-selectivity and the time-selectivity is called doubly-selective in wireless communications. In OFDM systems, the bandwidth of each subcarrier is small enough that each subcarrier is considered to experience flat fading in the frequency-selective channel. The narrowband nature of subcarriers makes the signal robust against the frequency-selectivity. However, OFDM is relatively sensitive to the time-selectivity of the mobile channel. Time variations of the channel within an OFDM symbol duration result in inter-carrier interference(ICI)[1] and lead to an irreducible error floor in conventional receivers. In this paper, we deal with the doubly-selective channels that vary significantly over the duration of a long block of OFDM symbols.

Complex exponential basis expansion model(CE-BEM) is often used to represent the doubly-selective channel for its parsimony of coefficients to be estimated and its algebraic ease. However, the CE-BEM channel model can bring about high modeling error since the channel estimation period of CE-BEM is limited to the length of one OFDM symbol. An improved channel model so called generalized CE-BEM(GCE-BEM) can provide lower modeling error than CE-BEM. The frequency resolution of GCE-BEM can be also increased arbitrarily at the expense of higher computational complexity. The influence of the observation window length in GCE-BEM(oversampling factor) on the performance of channel estimation was studied in [2].

Based on CE-BEM, an optimal pilot insertion scheme for block transmission was proposed in [3] for time-varying channel estimation, in which the pilots are inserted in the time domain. For channel estimation in OFDM systems, another optimal pilot insertion scheme was also proposed in [4], in which the channel to be estimated is modeled as time invariant at lease in one OFDM symbol. Based on the more accurate GCE-BEM, a time varying channel estimation for OFDM was propose in [5] which is carried out within one OFDM block, and another time varying channel estimation method for OFDM was proposed in [6], the pilot insertion scheme of which capitalizes all subcarriers in certain OFDM symbol resulting in low data ratio especially for the estimation of the rapidly time-varying channel.

In this paper, based on GCE-BEM we propose a new

Manuscript received October 29, 2007 ; December 4, 2007. (ID No. 20071029-033J)
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channel estimation scheme with low pilot ratio for the OFDM systems, assuming that only the maximum Doppler frequency and the maximum delay spread are known at the receiver. In the proposed scheme, pilots are inserted in both time and frequency domain, which is different from the scheme in [3]. We verify that the correlation matrix of ICI is a scaled identity matrix under the assumptions normally used in the literature, based on which we propose an optimal pilot insertion scheme for the least square(LS) estimator in the sense of minimum mean square error. The proposed scheme has the advantage of comparatively low pilot ratio and thus high data efficiency.

Concerning notations, we use boldface letters to denote matrices or column vectors, and capital and lower case letters to stand for frequency and time domain symbols, respectively. $(\cdot)^H$, $(\cdot)^\dagger$, $E\{\cdot\}$, $\|\cdot\|$ and tr $\{\cdot\}$ denote conjugate transpose, pseudo-inverse, expected value, Euclidean norm and trace, respectively. diag$\{\mathbf{b}\}$ stands for a diagonal matrix with $\mathbf{b}$ on its main diagonal and $\boldsymbol{I}_b$ means $b\times b$ identity matrix.

Ⅱ. System Model

Considering an OFDM system with N subcarriers(see Fig. 1), the sequence to be transmitted $X(k)$ from the QPSK or QAM constellations is parsed into blocks of N symbols and then transformed into a time-domain sequence using an N-point inverse fast Fourier transform (IFFT). For simple implementation of the FFT, the number of subcarriers N is chosen to be a power of 2 in practical systems. To avoid inter-block interference(IBI), a cyclic prefix(CP) of length L which is equal to the channel order(channel is assumed to consist of L+1 discrete paths) is inserted at the head of each block. The time-domain signal $x(n)$ can be serially transmitted over the fading channel.

$$x(n)=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}X(k)e^{j2\pi nk/N}, n=-L,\ldots,N-1. \tag{1}$$

After CP is removed at the receiver, the received signal in time domain $y(n)$ can be expressed as

$$y(n)=\sum_{l=0}^{L}h(n,l)x(n-l)+v(n) \tag{2}$$

where $v(n)$ is additive white Gaussian noise(AWGN)

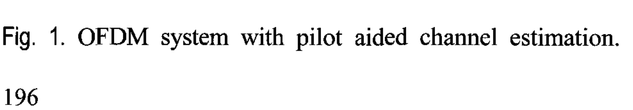

Fig. 1. OFDM system with pilot aided channel estimation.

with variance σ_n^2, and $h(n,l)$ is the baseband-equivalent doubly-selective channel impulse response of the lth path which includes the physical channel as well as the transmit and receive filters.

We denote the sampling period by T_s, the maximum delay spread and maximum Doppler frequency by $t_{\max}$ and $f_{\max}$, respectively, which are assumed to be known at the receiver beforehand, so DFT of $h(n, l)$, i.e. $H(k, l)=0$ for $k\Delta f>f_{\max}$ or $l\Delta t>t_{\max}$, where Δf and Δt are the Doppler frequency resolution and delay spread resolution, respectively. Now we adopt the parsimonious model GCE-BEM to approximate the channel's time variations in a discrete form. Then the channel can be expressed by BEM as [7]

$$h(n,l)=\sum_{q=-Q}^{Q}h_q(l)e^{j\omega_q n} \tag{3}$$

where $h_q(l)$ is the qth BEM coefficient of the lth channel path. The order of BEM is $2Q$+1, and $\omega_q=2\pi q/N_s$ where $N_s=G(N+L)$ is the total number of symbols in G OFDM blocks, which defines the size of the observation window. If G=1, GCE-BEM represented by (3) will be degraded to CE-BEM. Within the period of N_s samples, the BEM coefficients $h_q(l)$ are assumed to be constant. Q indicates the order of the BEM and should be selected such that$(2Q+1)\geq f_{\max}(2N_sT_s)$. GCE-BEM can be viewed as the over sampling BEM. With the increasing values of G, the resolution in Doppler frequency domain increases which in turn decreases the modeling mismatch. With larger values chosen for G and Q, the smaller reconstruction error for the channel can be obtained at the expense of system complexity.

The idea of BEM allows us to represent the time varying property of the channel by a set of known basis functions and thus converts the general time-varying channel to the block frequency-selective fading channel, and it allows for an accurate representation of the doubly-selective channels with a limited number of parameters for complex exponentials.

Substituting (3) into (2), the received signal in the ith (i=0, 1, ⋯, $G-1$) OFDM block can be shown as

$$y^{(i)}(n)=\sum_{l=0}^{L}\sum_{q=-Q}^{Q}h_q(l)e^{j2\pi qn/N_s}x^{(i)}(n-l)+v^{(i)}(n). \tag{4}$$

We collect N transmitted and received signals in the ith OFDM block as $\mathbf{x}^{(i)}=[x(i(N+L)), x(i(N+L)+1), \cdots, x(i(N+L)+N-1)]^T$ and $\mathbf{y}^{(i)}=[y(i(N+L)), y(i(N+L)+1), \cdots, y(i(N+L)+N-1)]^T$. Changing the summation order and rewriting (4) in the matrix form, we have

$$\mathbf{y}^{(i)}=\sum_{q=-Q}^{Q}\mathbf{\Gamma}_q^{(i)}\mathbf{H}_q\mathbf{x}^{(i)}+\mathbf{v}^{(i)} \tag{5}$$

where the diagonal N by N matrix $\Gamma_q^{(i)} = \mathrm{diag}\{e^{j2\pi q(i(N+L))/N_s}, \cdots, e^{j2\pi q(i(N+L)+N-1)/N_s}\}$, $\mathbf{v}^{(i)}$ is a noise vector and $\mathbf{H_q}$ is an N by N inter-block interference free circulant matrix which can be expressed as

$$\mathbf{H}_q = \begin{bmatrix} h_q(0) & 0 & h_q(L) & \cdots & h_q(1) \\ \vdots & \ddots & & \ddots & \vdots \\ \vdots & & & & h_q(L) \\ h_q(L) & \cdots & h_q(0) & & 0 \\ & \ddots & & \ddots & \\ 0 & & h_q(L) & \cdots & h_q(0) \end{bmatrix}.$$

After the discrete Fourier transform(DFT) operation, (5) becomes

$$\mathbf{Y}^{(i)} = \sum_{q=-Q}^{Q} \mathbf{F}^H \Gamma_q^{(i)} \mathbf{F}\, \mathrm{diag}\{\mathbf{X}^{(i)}\} \mathbf{F}_L \mathbf{h}_q + \mathbf{V}^{(i)} \tag{6}$$

where $\mathbf{Y}^{(i)}=\mathbf{F}\mathbf{y}^{(i)}$, $\mathbf{X}^{(i)}=\mathbf{F}\mathbf{x}^{(i)}$, $\mathbf{V}^{(i)}=\mathbf{F}\mathbf{v}^{(i)}$ and $\mathbf{h}_q=[h_q(0),\cdots, h_q(L)]^T$, $\mathbf{F}$ denotes $N\times N$ unitary discrete Fourier transform matrix defined as

$$\mathbf{F} = \frac{1}{\sqrt{N}} \begin{bmatrix} e^{-j2\pi 0 \cdot 0/N} & \cdots & e^{-j2\pi 0(N-1)/N} \\ \vdots & \cdots & \vdots \\ e^{-j2\pi(N-1)0/N} & \cdots & e^{-j2\pi(N-1)(N-1)/N} \end{bmatrix}$$

and the N by $(L+1)$ matrix $\mathbf{F}_L$ is $\sqrt{N}$ times the first $L+1$ columns of $\mathbf{F}$.

Defining N by N matrix

$$\Psi_q^{(i)} := \mathbf{F}^H \Gamma_q^{(i)} \mathbf{F} \tag{7}$$

the element of $\Psi_q^{(i)}$ can be shown as $[\Psi_q^{(i)}]_{m,n} = \varphi_q^{(i)}(m-n)$, where

$$\varphi_q^{(i)}(k) = \frac{1}{N}\sum_{n=0}^{N-1} e^{j2\pi q(i(N+L)+n)/N_s} e^{-j2\pi kn/N} \tag{8}$$

Note that $\varphi_q^{(i)}(0)$ appears on the main diagonal of $\Psi_q^{(i)}$, $\varphi_q^{(i)}(1)$ on the first sub-diagonal, $\varphi_q^{(i)}(-1)$ on the first super-diagonal, and so on. $\Psi_q^{(i)}$ is the ICI matrix induced by the time variations of the channel. If the channel is time invariant, then Q=0 and $\Psi_q^{(i)}$ will reduce to an identity matrix.

Ⅲ. Channel Estimation

The main strongpoint of adopting BEM is that we can avoid estimating all the values of $h(n,l)$ in (2), but need to estimate $(L+1)(2Q+1)$ BEM coefficients within G OFDM blocks. Now we describe the pilot insertion scheme as shown in Fig. 2. We partition the G OFDM blocks as information symbols containing only information bearing data and pilot symbols containing both information bearing data and pilots. The pilot symbols are denoted by the index $i_p \in \{0, \cdots, G-1\}$, where $p=(0, \cdots, P-1)$ and P is the total number of the pilot symbols. In each pilot

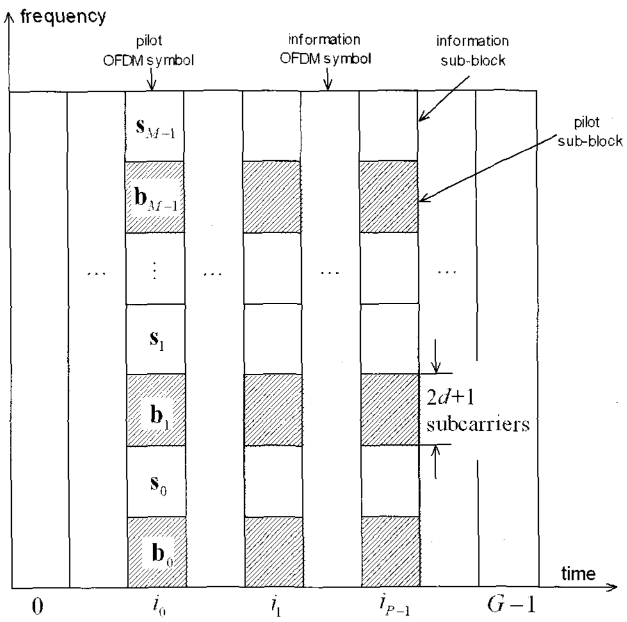

Fig. 2. Pilot insertion scheme for channel estimation.

symbol with N subcarriers, pilot sub-blocks are inserted in the frequency domain. The total number of the pilot sub-blocks in the i_pth pilot symbol is denoted by M_{ip}, so the i_pth pilot symbol can be given as $\mathbf{X}^{(i_p)} = [(\mathbf{b}_0^{(i_p)})^T, (\mathbf{s}_0^{(i_p)})^T, \cdots, (\mathbf{b}_{M_{ip}-1}^{(i_p)})^T, (\mathbf{s}_{M_{ip}-1}^{(i_p)})^T]^T$, where $\mathbf{b}_m^{(i_p)}$ and $\mathbf{s}_m^{(i_p)}$ are pilot sub-block and information sub-block, respectively. In order to eliminate ICI from the d neighboring subcarriers, the pilot sub-block $\mathbf{b}_m^{(i_p)}$ can be designed as a non-zero pilot $b_m^{(i_p)}$ surrounded by d zeros on both sides, i.e., $\mathbf{b}_m^{(i_p)} = [\mathbf{0}^T, b_m^{(i_p)}, \mathbf{0}^T]^T$, where 0 is a zero vector of length d. The value of d determines the robustness of the system against ICI. With higher d, the system performance gets better but with higher pilot ratio.

Now that we only take account of the pilot subcarriers in the i_pth received pilot symbol $\mathbf{Y}^{(i_p)}$, the received signals in (6) corresponding to the positions of pilot subcarriers can be expressed as

$$\tilde{\mathbf{Y}}^{(i_p)} = \sum_{q=-Q}^{Q} \Psi_q^p(i_p)\mathrm{diag}(\mathbf{p}^{(i_p)})\mathbf{F}_L^p(i_p)\mathbf{h}_q + \underbrace{\sum_{q=-Q}^{Q} \Psi_q^d(i_p)\mathrm{diag}(\mathbf{s}^{(i_p)})\mathbf{F}_L^d(i_p)\mathbf{h}_q}_{:=\mathbf{Ici}(i_p)} + \tilde{\mathbf{V}}^{(i_p)} \tag{9}$$

where $\mathbf{p}^{(i_p)} = [b_0^{(i_p)}, \cdots, b_{M_{ip}-1}^{(i_p)}]^T$ is a length M_{ip} vector, the elements of which are nonzero pilot tones, $\mathbf{s}^{(i_p)}$ is a length D data vector(D=N$-(2d+1)M_{ip}$). $\tilde{\mathbf{Y}}^{(i_p)}$, $\mathbf{F}_L^p(i_p)$ and $\tilde{\mathbf{V}}^{(i_p)}$ are the M_{ip} rows of $\mathbf{Y}^{(i_p)}$, $\mathbf{F}_L$ and $\mathbf{V}^{(i_p)}$ respectively corresponding to the position of pilot subcarriers. $\mathbf{F}_L^d(i_p)$ is the D rows of $\mathbf{F}_L$ corresponding to the position of data. The $\Psi_q^p(i_p)$ is an $M_{ip}\times M_{ip}$ matrix, the rows and columns of which are related to the positions of the received and the transmitted non-zero pilots, respectively and $\Psi_q^d(i_p)$ is an $M_{ip}\times D$ matrix, the rows and columns of which are related to the positions of the received non-zero pilots and transmitted data, respectively. $\mathbf{Ici}(i_p)$ is the ICI vector induced by the data sub-blocks.

$\Psi_q^{(i)}$ is the ICI matrix, most energy of which is distributed over the diagonal and its neighbors, so we omit the interference from the subcarriers the distance of which is larger than d, and matrix $\Psi_q^{(i)}$ retains only diagonal entries and it d neighbors. Considering the pilot insertion scheme described above and the structure of $\Psi_q^{(i)}$, the effective entries for channel estimation in $\Psi_q^{(i)}$ turn out to be only the diagonal ones. From the definition of $\Psi_q^{(i)}$ in (7), we know that the diagonal entries in $\Psi_q^{(i)}$ can be expressed as

$$\begin{aligned}\left[\Psi_q^{(i)}\right]_{(m,m)} &= \frac{1}{N}\sum_{n=0}^{N-1} e^{j2\pi q(i(N+L)+n)/N_s} \\ &= e^{j2\pi qi(N+L)/N_s}\underbrace{\frac{1}{N}\sum_{n=0}^{N-1} e^{j2\pi qn/N_s}}_{\alpha_q} \\ &= \alpha_q e^{j2\pi qi/G}\end{aligned} \tag{10}$$

We note that the α_q is not a function of i, and the diagonal entry of $\Psi_q^{(i)}$ is not a function of m, so the diagonal entries have the same value. If we only take account of the pilot subcarriers in the pilot symbol, (9) can be rewritten as

$$\begin{aligned}\tilde{\mathbf{Y}}^{(i_p)} &= \sum_{q=-Q}^{Q} \underbrace{\alpha_q e^{j2\pi qi_p/G}\mathrm{diag}\{\mathbf{p}^{(i_p)}\}\mathbf{F}_L^P(i_p)}_{\mathbf{B}(q,i_p)}\mathbf{h}_q + \mathbf{Ici}(i_p) + \tilde{\mathbf{V}}^{(i_p)} \\ &= [\mathbf{B}(-Q,i_p),\cdots,\mathbf{B}(Q,i_p)]\mathbf{h}_b + \mathbf{Ici}(i_p) + \tilde{\mathbf{V}}^{(i_p)}\end{aligned} \tag{11}$$

where $\mathbf{h}_b = [\mathbf{h}_{-Q}^T,\cdots,\mathbf{h}_Q^T]^T$. Considering all the P pilot symbols, we can obtain

$$\tilde{\mathbf{Y}} = \underbrace{\begin{bmatrix}\mathbf{B}(-Q,i_0) & \cdots & \mathbf{B}(Q,i_0)\\ \vdots & \cdots & \vdots \\ \mathbf{B}(-Q,i_{P-1}) & \cdots & \mathbf{B}(Q,i_{P-1})\end{bmatrix}}_{\mathbf{D}}\mathbf{h}_b + \mathbf{Ici} + \tilde{\mathbf{V}} \tag{12}$$

where $\tilde{\mathbf{Y}} = [(\tilde{\mathbf{Y}}^{(i_0)})^T,\cdots,(\tilde{\mathbf{Y}}^{(i_{P-1})})^T]^T$, $\tilde{\mathbf{V}} = [(\tilde{\mathbf{V}}^{(i_0)})^T,\cdots,(\tilde{\mathbf{V}}^{(i_{P-1})})^T]^T$, and $\mathbf{Ici} = [(\mathbf{Ici}(i_0))^T,\cdots,(\mathbf{Ici}(i_{P-1}))^T]^T$. $\mathbf{D}$ is a $PM_{iP}\times(2Q+1)(L+1)$ matrix. If $PM_{iP}\geq(2Q+1)(L+1)$, then the LS estimation of $\mathbf{h}_b$ can be obtained as

$$\hat{\mathbf{h}}_b = \mathbf{D}^{\dagger}\tilde{\mathbf{Y}}. \tag{13}$$

The LS channel estimation does not need extra information about the channel correlation and noise variance beforehand, so LS channel estimation is more practical and we only address the LS channel estimation in this paper. Since the pilots and their positions are known at the receiver, the matrix $\mathbf{D}^{\dagger}$ can be calculated beforehand and this will not add any additional computation complexity for channel estimation.

Ⅳ. Optimal Training Design

In the following, we make following two assumptions, the rationale of which is explained in [3].

a1) Data symbols are of zero mean and unit variance, and are uncorrelated with each other, i.e. $E\{\mathbf{s}^{(d)}(\mathbf{s}^{(d)})^H\}=\mathbf{I}_D$, where $\mathbf{I}_D$ is a $D\times D$ identity matrix.

a2) The BEM coefficients $h_q(l)$ are complex Gaussian random variables with zero mean and variance $\sigma_{q,l}^2$, and are uncorrelated with each other for different q and l.

From (12) and (13), the mean square error(MSE) of the LS channel estimation is given by

$$\begin{aligned}\mathrm{MSE} &= E\{\|\hat{\mathbf{h}}_b - \mathbf{h}_b\|^2\}/[(L+1)(2Q+1)] \\ &= tr\{\mathbf{D}^{\dagger}E\{\tilde{\mathbf{V}}\tilde{\mathbf{V}}^H + \mathbf{Ici}(\mathbf{Ici})^H\}(\mathbf{D}^{\dagger})^H\}/[(L+1)(2Q+1)]\end{aligned} \tag{14}$$

Based on the above assumptions, we can see that the correlation matrix of ICI is a scaled identity matrix[8], i.e. $E\{\mathbf{Ici}(\mathbf{Ici})^H\} = \sigma_I^2\mathbf{I}_{MP}$. Since there is no channel state information(CSI) except for $f_{\max}$ and $\tau_{\max}$ supposed to be known at the transmitter and receiver, the above assumption will not affect the optimality of the training scheme described below. We can get

$$\mathrm{MSE} = \mathrm{tr}\{(\mathbf{D}^H\mathbf{D})^{-1}\}\sigma^2/[(L+1)(2Q+1)] \tag{15}$$

where $\sigma^2 = \sigma_n^2 + \sigma_I^2$. The minimum MSE can be achieved if the optimal condition $\mathbf{D}^H\mathbf{D}=C\mathbf{I}_{(2Q+1)(L+1)}$ is satisfied, where C is a constant, and then minimum MSE is given as σ^2/C[3].

Now we derive the optimal scheme in the sense of minimum MSE. If we partition the matrix $\mathbf{D}^H\mathbf{D}$ into $(2Q+1)$ rows and $(2Q+1)$ columns with the resultant$(L+1)$ by $(L+1)$ sub-matrices as entries, the sub-matrix of $\mathbf{D}^H\mathbf{D}$ in the mth row and the nth column $(m,n\in\{1,\cdots,2Q+1\})$ can be expressed from (11) and (12) as

$$[\mathbf{D}^H\mathbf{D}]_{(m,n)} = \sum_{p=0}^{P-1}\underbrace{\alpha_{(m,n)}e^{j2\pi(n-m)i_p/G}}_{\gamma(i_p,n-m)}\underbrace{(\mathbf{F}_L^P(i_p))^H(\mathrm{diag}\{\mathbf{p}^{(i_p)}\})^H\mathrm{diag}\{\mathbf{p}^{(i_p)}\}\mathbf{F}_L^P(i_p)}_{\mathbf{R}(i_p)} \tag{16}$$

where $\alpha_{(m,n)}=(\alpha_m)^H\alpha_n$. In order to satisfy the optimal condition as discussed in [3], the diagonal sub-matrix of $\mathbf{D}^H\mathbf{D}$, $[\mathbf{D}^H\mathbf{D}]_{(m,m)}$, should be an identity matrix while others should be zero matrices.

In the case of $m=n$, $[\mathbf{D}^H\mathbf{D}]_{(m,m)} = \sum_{p=0}^{P-1}\alpha_{(m,m)}(\mathbf{F}_L^P(i_p))^H\mathrm{diag}\{\mathbf{P}^{(i_p)}\}\mathbf{F}_L^P(i_p)$, where $\mathbf{P}^{(i_p)} = [\wp_0^{(i_p)},\cdots,\wp_{M_{i_p}-1}^{(i_p)}]^T$ and $\wp_m^{(i_p)}$ is the power of the *m*th pilot tone in the i_pth pilot symbol. To make the sub-matrix $[\mathbf{D}^H\mathbf{D}]_{(m,m)}$ be an identity matrix, the following conditions must be held.

1) $\varPhi_j^{(i_p)} = \varPhi_k^{(i_p)}$, $\forall j,k \in \{0,\cdots,M_{i_p}\}$ and $\forall p \in \{0,\cdots,P-1\}$.

2) $f_j^{i_p} = f_0^{i_p} + j\Delta f$, $\forall j \in \{0,\cdots,M_{i_p}\}$ and $\forall p \in \{0,\cdots,P-1\}$, where $f_j^{i_p}$ is the corresponding position index of the *j*th pilot tone in the ipth pilot symbol and Δf is the neighboring pilot tones interval.

In the case of $m \neq n$, assuming that the conditions 1) and 2) are held, $[\mathbf{D}^H\mathbf{D}]_{(m,n)}$ will be a zero matrix if $\mathbf{R}(i_p)$'s in (16) are the same for different i_p, and the vector $[\gamma(i_0,n-m),\cdots,\gamma(i_{P-1},n-m))]^T$ is a column of a DFT matrix. Then, we arrive at the following conditions.

3) $M_{i_p} = M_{i_k}$, $\forall p,k \in \{0,\cdots,P-1\}$.

4) $i_p = i_0 + p\Delta p$, $\forall p \in \{0,\cdots,P-1\}$, where Δp is the neighboring pilot symbols interval.

The condition 1) means that the pilot tones must be equipowered, 2) means that the pilot tones must be equispaced, 3) means that the number of pilot sub-blocks in any pilot symbol must be the same, so we omit the subscript of M_{ip}, that is $M=M_{ip}$, and 4) means that the pilot symbols must be equispaced in the *G* OFDM blocks.

The proposed optimal pilot insertion scheme has the similar form to the ones shown in [3], [4], but they are derived from different situations. In [4], the channel is assumed to be time invariant during at least one OFDM symbol, and all pilots are assigned to an OFDM symbol which can result in high pilot rate. In [3], the pilots are inserted in the time domain, so the scheme is not suitable for the OFDM systems. Meanwhile, the proposed scheme is derived for being used in the OFDM systems in the doubly-selective channels.

V. Simulation Results

Consider an OFDM system with N=64 subcarriers, channel length L+1=8, and QPSK modulation. The paths of the doubly-selective channel are assumed to be independent and identically distributed(i.i.d). The carrier frequency f_c is 2 GHz and the OFDM symbol duration NT_s is 0.32 ms. The maximum Doppler frequency $f_{\max}$ are set as 100 Hz, 200 Hz and 300 Hz corresponding to the vehicle speed of v=54, 108, and 162 km/h, respectively. The *G* in (2) is 64.

In the simulations, all the channel coefficients $h_q(l)$ are generated as independent complex Gaussian random variables. The power delay profile is selected as $\phi_\tau(l) = \exp(-0.1l)$, $\forall q$, and the Doppler power spectrum is selected as $\phi_D(f) = \left(\pi f_{\max}\sqrt{1-(f/f_{\max})^2}\right)^{-1}$, $|f| < f_{\max}$; otherwise, $\phi_D(f) = 0$, $\forall l$. We use the same variance of $h_q(l)$ as in [3], that is $\sigma_{q,l}^2 = \beta\phi_\tau(l)\phi_D(q/(N_sT_s))$, where the normalizing factor $\beta = (\sum_{l,q}\phi_\tau(l)\phi_D(q/(N_sT_s)))^{-1}$. The length of zero guard *d* is set as 0 or 2. We use system parameter values of Q=2, M=8, P=8 for $f_{\max}$=100 Hz, Q=5, M=8, P=16 for $f_{\max}$=200 Hz and Q=7, M=8, P=16 for $f_{\max}$=300 Hz, which satisfy the condition $(2Q+1) \geq f_{\max}(2N_sT_s)$. The pilot ratio $PM(2d+1)/GN$ in the proposed scheme is rather low, while in [6] the pilots are inserted into all the N subcarriers of the pilot symbol and the pilot ratio is $1/(1+k)$, corresponding to $M(2d+1)=N$ in the proposed scheme, where k is the number of data OFDM symbols between two pilot OFDM symbols.

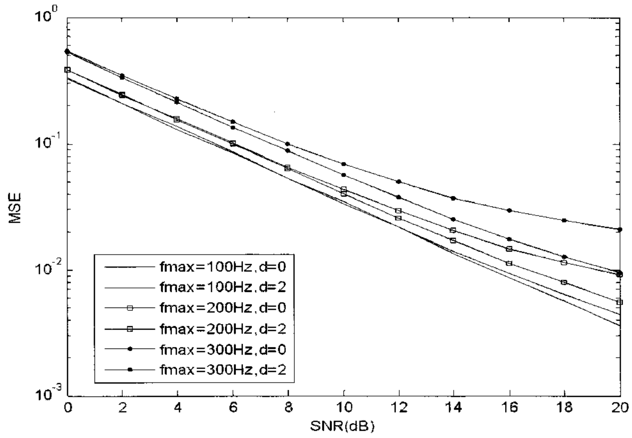

Fig. 3. Channel estimation MSE as a function of SNR.

Fig. 3 shows the MSE performance as a function of signal to noise ratio(SNR) for the different Doppler frequency and different length of zero guard *d*. we can see that increasing the length of zero guard *d* can raise the robustness against ICI especially at high SNR. It is because at high SNR, the main interference for the channel estimation comes from ICI but not from noise. The shortcoming of the larger zero guard is that it lowers the data ratio. Fig. 4 shows the performance comparison of

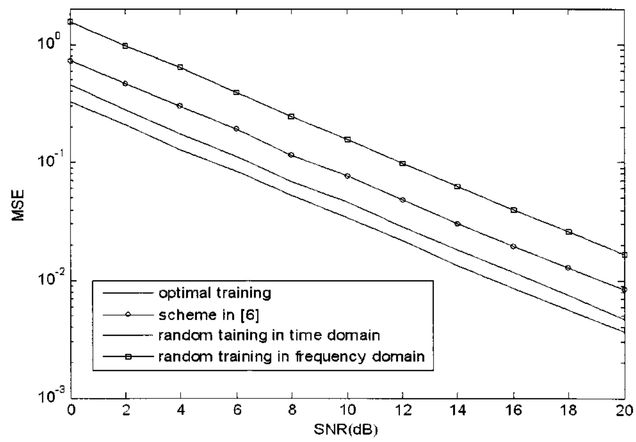

Fig. 4. Channel estimation MSE as a function of SNR for different schemes.

the proposed scheme and the channel estimation method in [6] and the non-optimal training estimations in which the pilot symbols are distributed randomly in the G OFDM symbols and distributed randomly in the frequency domain. The maximum Doppler frequency $f_{\max}$ is set as 100 Hz, and the pilot ratio in the proposed scheme is 7.8 %. As shown in Fig. 4, the proposed scheme performs better than the random training method and the scheme advocated in [6].

Ⅵ. Conclusions

We present a new method of estimating the doubly-selective channel for OFDM systems. The time variation of the channel is modeled by GCE-BEM over several OFDM blocks, by which the time-varying channel estimation is converted to estimating a limited number of the BEM coefficients. The time variation of the channel in OFDM systems can introduce ICI, most power of which comes from the neighboring subcarriers. The correlation of ICI can be verified to be a scaled identity matrix. Based on this property, we propose an optimal pilot insertion scheme for the LS estimator in the sense of minimum mean square error. Compared to the existing estimators, the proposed estimator requires low pilot ratio and thus entails higher data efficiency. The simulation results validate our conclusion.

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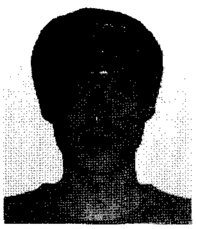
Lidong Wang



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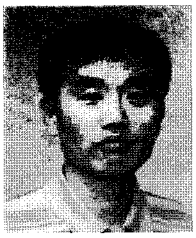

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