New Evaluation on the Selective Diversity Systems for the Detection of *M*-ary PSK & DPSK Signals over Rayleigh Fading Channels

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Abstract

When the *M*-ary signal experiences the Rayleigh fading, the diversity schemes can reduce the effect of fading since the probability that all the signals components will fade simultaneously is reduced considerably. The symbol error probabilities for various *M*-ary signals, such as MDPSK(*M*-ary DPSK) and MPSK(*M*-ary PSK), are mathematically derived for the Selection Combining 2(SC-2) and Selection Combining 3(SC-3) demodulation system which requires a less complex receiver than maximum ratio combining(MRC). The propagation model used in this paper is the frequency-nonselective slow Rayleigh fading channel corrupted by the additive white gaussian noise(AWGN). The numerical results presented in this paper are expected to provide information for the design of radio system using *M*-ary modulation method for above mentioned channel environment.

Key words : SC-2, SC-3, Rayleigh Fading, MDPSK, MPSK.

Ⅰ. Introduction

The statistics for various fading channel models and the resulting communication evaluation have been considerably studied as summarized in [1]. The statistical properties of mobile radio environments can be often specified by the following propagation effects: 1) long-term fading 2) short-term fading[2]. In long-term fading, the change of effective height for mobile communication antenna exists due to the nature of the terrain. Its statistics follow the log-normal distribution. But the Rician model can be obtained from the direct wave and its scattering components, and both waves carry information[3]. On the other hand, in short-term fading, the scattering mechanism only results in numerous reflected components[4]. The Rayleigh model is used to characterize this fading in small geographical areas and sometimes does not account for large scale effects like shadowing by building and hills. When the fading index in the Rician model, *K*, goes to 0, the error performances lead to those of Rayleigh fading model.

An alternative solution to the problem of obtaining acceptable performances on a fading channel is the diversity technique, which is widely used to combat the fading effects of time-variant channels. When *M*-ary signals experience the fading channels, diversity schemes can minimize the effects of these fadings since deep fades seldom occur simultaneously during the same time intervals on two or more paths.

In [5], the general formula for evaluating the error performance for either BPSK or DPSK signals with SC-2(SC-3), whereby the two(three) signals with the two(three) largest amplitudes are coherently combined among the *L* branches, under a Rayleigh fading channel. If one of $M>2$ possible values is available, it is referred as the *M*-ary signaling. In *M*-ary signaling schemes, we may send one of *M* possible signals during each signaling interval of duration *T*. For almost all applications, the number of possible signals $M=2^n$, where n is an integer. The symbol duration *T* is nT_b, where T_b is the bit duration.

M-ary signaling schemes are preferred over binary signaling schemes for transmitting a digital information over bandpass channels when the requirement is to conserve a bandwidth at the expense of increased power. In practice, we rarely find the communication channel that has the exact bandwidth required for transmitting the output of an information source by means of binary signaling schemes. Thus, when the bandwidth of the channel is less than the required value, we may use *M*-ary signaling schemes so as to utilize the channel efficiently.

In this paper, we can represent the average symbol error rate(SER) by these SC-2 and SC-3 systems, in

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receiving MDPSK and MPSK signals on Rayleigh fading channels. It has been reported that given average signal-to-noise ratio(SNR) per symbol, the exact performance of the MPSK signals is almost fitted to the approximation[6]. We evaluate the approximation of the error performance for coherent MPSK signals using SC-2 diversity systems over the slow and flat fading channels. On the other hand, the exact performance of SC-3 diversity system, simpler in some mathematical manipulations than the closed-form approximation, can be accomplished by averaging the exact SER of coherent MPSK under a nonfading channel. Next we compare the performance of SC-2 and SC-3 diversity reception for MDPSK and MPSK signals in slowly frequeney-nonselective Rayleigh fading channels with an AWGN. The analytical results of these performance evaluations presented in this paper are expected to provide designers with important informations in designing M-ary modulation systems under the Rayleigh fading channel.

The remainder of this paper is organized as follows. The next section presents a system model with selective combining diversity reception in a Rayleigh fading channel. The analytical results for the performance of the M-ary signals are explained in Section Ⅲ. Numerical results will be shown in Section Ⅳ. Finally, we summarize some known results in Section Ⅴ.

Ⅱ. System Model

Since we are interested in performance analysis, we will present the signal description at the receiver frontend. At the input of the receiver, the signal under the Rayleigh fading channel can be written as, assuming L branch signals

$$r(t) = Re[R(t)e^{j\omega_c t}] \tag{1}$$

where

$$R(t) = \sum_{i=1}^{L-1} \alpha_i e^{j\theta_i} S_i(t) + N(t) \tag{2}$$

and where $\{\alpha_i\}$ are Rayleigh-distributed signal amplitudes for each branch, $\{\theta_i\}$ are the uniformly distributed phases, $\{S_i\}$ are the low-pass equivalent branch signal functions, and $N(t)$ is the low-pass, complex valued AWGN.

The interest for SC application has been recently increased for the high-capacity mobile radio system. With reference to SC, applied here are the order statistics to select μ branches from the branch with the largest amplitude, or to choose μ branches from the branch with the largest SNR at the L diversity branches, for a data recovery while assuming that the noise power is constant across all branches when the probability density function(PDF) for this combining is analyzed to evaluate the error rate performance of M-ary signals on Rayleigh fading channels[7],[8].

Then the statistics of an instantaneous SNR γ are as follows:

$$\gamma = \gamma_L + \gamma_{L-1} + \gamma_{L-2} + \cdots + \gamma_{L-\mu+1}, \quad L \ge \mu. \tag{3}$$

It is assumed that the statistical characteristics of diversity branches are independent each other over the Rayleigh fading channels.

Now, to derive the PDF for the instantaneous SNR, it is worthwhile to note that we introduce the following joint PDF of the statistics γ_L, $\gamma_{L-1}, \cdots$, and γ_1, where $0 \le \gamma_1 \le \gamma_2 \le \cdots \le \gamma_L$[7],[8]:

$$f(\gamma_L, \gamma_{L-1}, \cdots, \gamma_1) = f(\gamma_L) f(\gamma_{L-1}) \cdots f(\gamma_1) \tag{4}$$

Given each integration interval for the unnecessary random variables (rvs) $\gamma_{L-\mu}$, $\gamma_{L-\mu-1}, \cdots$, and γ_1, we can perform the integration of those statistics and yield the marginal joint PDF of γ_L, $\gamma_{L-1}, \cdots$, and $\gamma_{L-\mu+1}$.

Next, we can write, after the transform of rvs, the PDF of γ, $\gamma_{L-1}, \cdots, \gamma_{L-\mu+2}$, and $\gamma_{L-\mu+1}$ as

$$f(\gamma_L, \cdots, \gamma_{L-\mu+2}, \gamma_{L-\mu+1}) \frac{1}{|J|} = f(\gamma, \gamma_{L-1}, \cdots, \gamma_{L-\mu+2}, \gamma_{L-\mu+1}) \tag{5}$$

Jacobian of the transformation, $|J| = 1$ and $0 \le \gamma_{L-\mu+1} \le \gamma_{L-\mu+2} \le \cdots \le \gamma$.

It follows that upon performing the integrations with respect to $\gamma_{L-1}, \cdots, \gamma_{L-\mu+2}$, and $\gamma_{L-\mu+1}$, we can obtain the PDF of γ.

The performance improvement of SC is especially noticeable for noncoherent reception, where equal gain combining(EGC) is seen to provide the best performance only for low SER values. For coherent modulation with independent branch fading, although it is known that MRC is the optimal linear combining technique, it is seldom implementable in a multipath fading channel because the receiver complexity for MRC is directly proportional to the number of resolvable paths, L, available at the receiver.

If each branch has equal fading parameter and average SNR γ_0, we can represent the conditional PDF of the received instantaneous SNR in a SC-2 and SC-3 diversity system, one having

$$f_{SC2}(\gamma) = \int_0^{\gamma/2} f(\gamma_{L-1}, \gamma) d\gamma_{L-1} \equiv f_1(\gamma) + f_2(\gamma) \tag{6}$$

where

$$f_1(\gamma)=\frac{L(L-1)}{\gamma_0}e^{-\frac{\gamma}{\gamma_0}}\frac{\gamma}{2\gamma_0} \qquad (7)$$

and

$$f_2(\gamma)=\frac{L(L-1)}{\gamma_0}e^{-\frac{\gamma}{\gamma_0}}\sum_{k=1}^{L-2}\binom{L-2}{k}\frac{(-1)^k}{k}(1-e^{-\frac{k\gamma}{2\gamma_0}}), \qquad (8)$$

and the other having

$$f_{SC3}(\gamma)\equiv f_3(\gamma)+f_4(\gamma) \qquad (9)$$

where

$$f_3(\gamma)=\frac{L(L-1)(L-2)}{2\gamma_0}\exp\left(-\frac{\gamma}{\gamma_0}\right)\frac{\gamma^2}{6\gamma_0^2} \qquad (10)$$

and

$$f_4(\gamma)=\frac{L(L-1)(L-2)}{2\gamma_0}\exp\left(-\frac{\gamma}{\gamma_0}\right)\sum_{k=1}^{L-3}\binom{L-3}{k}\cdot\frac{(-1)^k}{k^2}\left[\frac{k\gamma}{\gamma_0}-3\left(1-\exp\left(-\frac{k\gamma}{3\gamma_0}\right)\right)\right], \qquad (11)$$

on a Rayleigh fading channel[5],[9].

III. Performance Analysis

Once the statistics of the instantaneous SNR are determined as the function of the average SNR, the error performance in the Rayleigh fading channels can be evaluated by averaging the conditional probability of error over the PDF of the instantaneous SNR.

3-1 Error Probability for MDPSK

When MDPSK signals experience no fading, the expression for the conditional probability of error is given by [10]

$$P_{s,MDPSK}=\frac{\sin\frac{\pi}{M}}{2\pi}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{\exp\left[-\gamma\left(1-\cos\frac{\pi}{M}\cos\theta\right)\right]}{1-\cos\frac{\pi}{M}\cos\theta}d\theta. \qquad (12)$$

The average SER in receiving MDPSK signals with SC-2 and SC-3 diversity branches, respectively, is accomplished by averaging (12) over the PDF of an instantaneous SNR under the effect of Rayleigh fading channel, i.e.,

$$P_{e,MDPSK,SC2}=\int_0^\infty P_{s,MDPSK}\,f_{SC2}(\gamma)\,d\gamma \qquad (13)$$

and

$$P_{e,MDPSK,SC3}=\int_0^\infty P_{s,MDPSK}\,f_{SC3}(\gamma)\,d\gamma \qquad (14)$$

where $P_{e,MDPSK,SC2}$ is the average SER of SC-2 detection of MDPSK signals and $P_{e,MDPSK,SC3}$ is the average SER of SC-3 detection of MDPSK signals.

Consequently, we can find the symbol error probabilities under the Rayleigh fading model to be (See Appendix A).

$$P_{e,MDPSK,SC2}=\frac{L(L-1)}{2\gamma_0^2}\frac{\sin\frac{\pi}{M}}{2\pi}\cdot\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{1}{1-\cos\frac{\pi}{M}\cos\theta}\left(\frac{\gamma_0}{1+\gamma_0-\gamma_0\cos\frac{\pi}{M}\cos\theta}\right)^2 d\theta$$
$$+\frac{L(L-1)}{\gamma_0}\frac{\sin\frac{\pi}{M}}{2\pi}\sum_{k=1}^{L-2}\binom{L-2}{k}\frac{(-1)^k}{k}\cdot\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{1}{1-\cos\frac{\pi}{M}\cos\theta}\left[\frac{\gamma_0}{1+\gamma_0-\gamma_0\cos\frac{\pi}{M}\cos\theta}-\frac{2\gamma_0}{k+2+2\gamma_0(1-\cos\frac{\pi}{M}\cos\theta)}\right]d\theta \qquad (15)$$

and

$$P_{e,MDPSK,SC3}=\frac{L(L-1)(L-2)}{2\gamma_0}\frac{\sin\frac{\pi}{M}}{2\pi}\left\{\frac{1}{6\gamma_0^2}\cdot\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{1}{1-\cos\frac{\pi}{M}\cos\theta}\frac{2}{\left(\frac{1}{\gamma_0}+1-\cos\frac{\pi}{M}\cos\theta\right)^3}d\theta\right.$$
$$+\sum_{k=1}^{L-3}\binom{L-3}{k}\frac{(-1)^k}{k^2}\left[\frac{k}{\gamma_0}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{1}{1-\cos\frac{\pi}{M}\cos\theta}\cdot\left(\frac{\gamma_0}{1+\gamma_0-\gamma_0\cos\frac{\pi}{M}\cos\theta}\right)^2 d\theta-3\left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{1}{1-\cos\frac{\pi}{M}\cos\theta}\cdot\frac{\gamma_0}{1+\gamma_0-\gamma_0\cos\frac{\pi}{M}\cos\theta}d\theta\right.\right.$$
$$\left.\left.\left.-\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{1}{1-\cos\frac{\pi}{M}\cos\theta}\frac{3\gamma_0}{k+3+3\gamma_0\left(1-\cos\frac{\pi}{M}\cos\theta\right)}d\theta\right)\right]\right\} \qquad (16)$$

which can be written in the integral-form, not in the closed-form.

For the special case of Nakagami fading index m=1, we can agree that the result of (15) for L=2 agrees with that of [11, Eq. (5.2.13)]. We can also find that the result of (16) for L=3 corresponds to the result of [11, Eq. (5.2.13)] for Nakagami fading index m=1.

3-2 Error Probability for MPSK

The exact SER of coherent MPSK under a nonfading channel can be represented as [10]

$$P_{s,exact,MPSK}=\frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}-\frac{\pi}{M}}\exp\left[-\gamma\sin^2\left(\frac{\pi}{M}\right)\sec^2\theta\right]d\theta. \qquad (17)$$

On the other hand, the approximation of coherent MPSK signals on the probability of symbol error for larger M may be represented as follows [12]:

$$P_{s,MPSK} = erfc\left(\sqrt{\gamma}\sin\frac{\pi}{M}\right) \quad (18)$$

where $erfc(\cdot)$ is the error function defined as

$$erfc(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty} e^{-t^2}dt. \quad (19)$$

We can find the approximate performance of MPSK signals under the effect of SC-2 diversity in a Rayleigh fading channel to be (See Appendix B).

$$P_{e,MPSK,SC2} = \frac{3}{16}\frac{L(L-1)}{\gamma_0^2}\frac{1}{\sin^4\frac{\pi}{M}} \cdot {}_2F_1\left(2, \frac{5}{2}; 3; -\frac{1}{\gamma_0\sin^2\frac{\pi}{M}}\right) + \frac{1}{2}\frac{L(L-1)}{\gamma_0} \cdot \frac{1}{\sin^2\frac{\pi}{M}}\sum_{k=1}^{L-2}\binom{L-2}{k}\frac{(-1)^k}{k} \cdot \left[{}_2F_1\left(1, \frac{3}{2}; 2; -\frac{1}{\gamma_0\sin^2\frac{\pi}{M}}\right) - {}_2F_1\left(1, \frac{3}{2}; 2; -\frac{2+k}{2\gamma_0}\frac{1}{\sin^2\frac{\pi}{M}}\right)\right]. \quad (20)$$

For the special case of $K=0$, we can thus find that the result of (20) for $L=2$ corresponds to that of [13, Eq. (B.3)].

Next, identity [14, p. 310, Eq. (3.351)], we can represent the exact performance of MPSK signals with SC-3 diversity in a Rayleigh fading channel as follows:

$$P_{e,MPSK,SC3} = \frac{L(L-1)(L-2)}{2r_o}\frac{1}{\pi}\left\{\frac{1}{6r_0^2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}-\frac{\pi}{M}} \cdot \frac{2}{\left(\frac{1}{r_0}+\sin^2\left(\frac{\pi}{M}\right)\sec^2\theta\right)^3}d\theta + \sum_{k=1}^{L-3}\binom{L-3}{k}\frac{(-1)^k}{k^2} \left[\frac{k}{r_0}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}-\frac{\pi}{M}}\left(\frac{\gamma_0}{1+\gamma_0\sin^2\left(\frac{\pi}{M}\right)\sec^2\theta^2}\right)^2 d\theta - 3\left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}-\frac{\pi}{M}}\frac{\gamma_0}{1+\gamma_0\sin^2\left(\frac{\pi}{M}\right)\sec^2\theta}d\theta - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}-\frac{\pi}{M}}\frac{3\gamma_0}{3+k+3\gamma_0\sin^2\left(\frac{\pi}{M}\right)\sec^2\theta}d\theta\right)\right]\right\}. \quad (21)$$

For the special case of K=0, it is noted find that the result of (21) for L=3 becomes [13, Eq. (B.3)].

Ⅳ. Numerical Results

It is shown that in Figs. 1, 2 given average SNR of the received signal, for smaller diversity branches, the distribution of the instantaneous SNR for the received signal becomes more shifted to the left and more decayed.

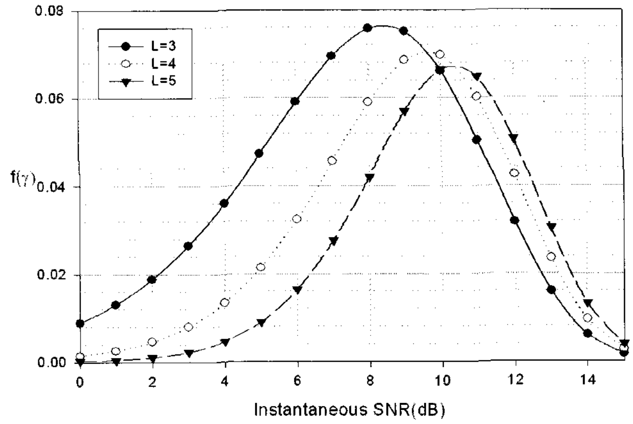

Fig. 1. The Rayleigh PDF of the instaneous SNR for L=3, 4, 5 and average SNR=6 dB in SC-2 diversity system.

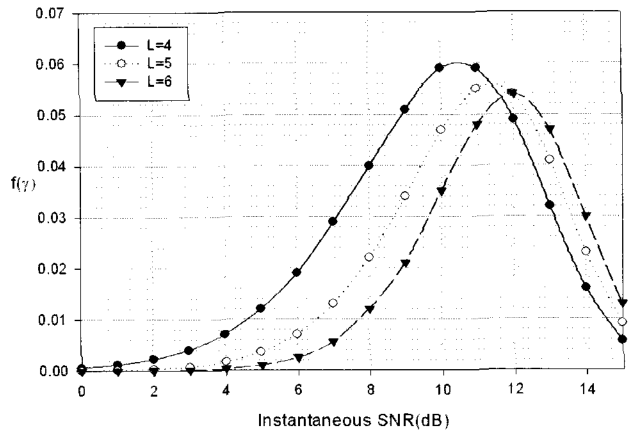

Fig. 2. The Rayleigh PDF of the instaneous SNR for L=4, 5, 6 and average SNR=6 dB in SC-3 diversity system.

Let us suppose the performance of coherent MPSK in SC-2 diversity system in the fading channels has an approximation while assuming that the coherent MPSK in SC-3 diversity system has an exact performance. With reference to the case of M=8 and L=4, 5, Figs. 3-4 shows the average SER of MDPSK and MPSK, respectively, versus average SNR using MRC, SC-2, and SC-3. When M-ary signals experience the Rayleigh fading channel, it is expected result that the performance of MRC with respect to SC is more improved with increasing the SNR per symbol for the single value of L. Given average SNR per symbol, the performance of L=5 in SC-2 diversity system almost becomes close to that of L=4 in MRC and SC-3 diversity system, in MDPSK signals. It is noted that by increasing average SNR, the departure of MRC and SC-3 in L=5 diversity system is more evident than that of MRC and SC-3 in L=4 diversity system.

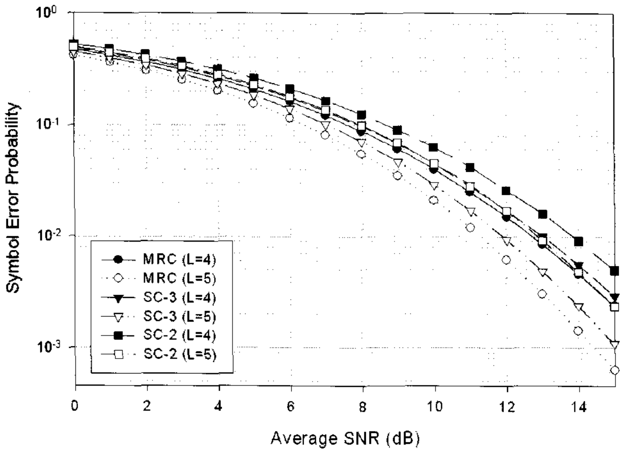

Fig. 3. Error performance comparisons of MDPSK signals adopting MRC, SC-2, and SC-3 diversity technique for M=8.

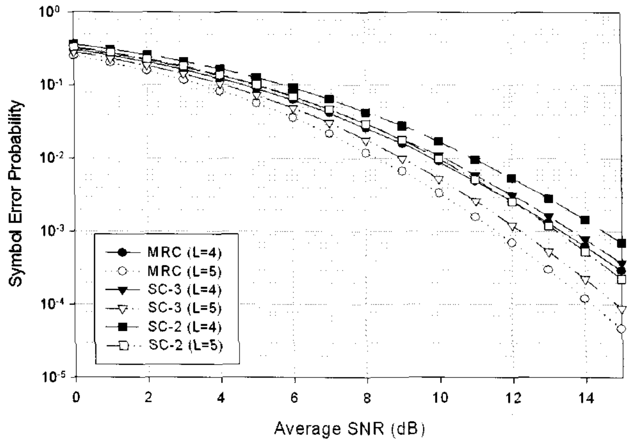

Fig. 4. Error performance comparisons of MPSK signals adopting MRC, SC-2, and SC-3 diversity technique for M=8.

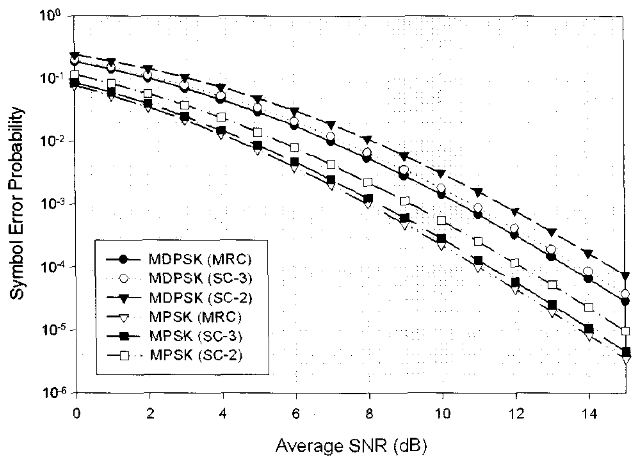

Fig. 5. Error performance comparisons of MDPSK and MPSK signals with MRC, SC-2, and SC-3 diversity receiver structures in fading channels. These parameters for this figure are M=4, L=4.

Fig. 5 shows MPSK and MDPSK performance comparison of the Rayleigh fading channel for M=4 and L=4 in MRC, SC-2, and SC-3 diversity system. We can find that the performance of M-ary signals with MRC diversity system is slightly better than that of M-ary signals with SC-3 diversity system.

V. Conclusion

The performance for MPSK and MDPSK signals in a Rayleigh fading channel has been evaluated. In coherent MPSK, the approximation to a symbol error probability in SC-2 diversity system and the exact SER of SC-3 diversity have been presented under a Rayleigh fading channel. The integral-form performances for MDPSK systems employing the multichannel SC-2 and SC-3 diversity in the presence of Rayleigh-distributed slow and nonselective fadings have been analyzed. These results indicate that the performance of MRC is almost fitted closely to that of SC-3, even when the higher average SNR is employed.

The results of the present works are sufficiently general in offering a convenient method to evaluate the performance of several current M-ary modulation systems that operate on channels with a wide variety of fading conditions in wireless personal communications.

Appendix A: The Integral-form Derivation of (15) and (16)

In this Appendix, given that v is real number, using the identity [14, p. 310, Eq. (3.351)]

$$\int_0^{\infty} x^n e^{-vx} dx = n!\, v^{-n-1}, \qquad Re\ v>0 \tag{A1}$$

we find the symbol error probabilities under the Rayleigh fading model to be

$$P_{e,MDPSK,SC2} = \frac{L(L-1)}{2\gamma_0^2}\,\frac{\sin\frac{\pi}{M}}{2\pi}\cdot \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{1}{1-\cos\frac{\pi}{M}\cos\theta}\left(\frac{\gamma_0}{1+\gamma_0-\gamma_0\cos\frac{\pi}{M}\cos\theta}\right)^2 d\theta + \frac{L(L-1)}{\gamma_0}\,\frac{\sin\frac{\pi}{M}}{2\pi}\sum_{k=1}^{L-2}\binom{L-2}{k}\frac{(-1)^k}{k}\cdot \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{1}{1-\cos\frac{\pi}{M}\cos\theta}\left[\frac{\gamma_0}{1+\gamma_0-\gamma_0\cos\frac{\pi}{M}\cos\theta} - \frac{2\gamma_0}{k+2+2\gamma_0\left(1-\cos\frac{\pi}{M}\cos\theta\right)}\right]d\theta \tag{A2}$$

and

$$P_{e,MDPSK,SC3} = \frac{L(L-1)(L-2)}{2\gamma_0}\,\frac{\sin\frac{\pi}{M}}{2\pi}\left\{\frac{1}{6\gamma_0^2}\cdot\right.$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{1-\cos\frac{\pi}{M}\cos\theta} \frac{2}{\left(\frac{1}{\gamma_0}+1-\cos\frac{\pi}{M}\cos\theta\right)^3} d\theta$$

$$+\sum_{k=1}^{L-3}\binom{L-3}{k}\frac{(-1)^k}{k^2}\left[\frac{k}{\gamma_0}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{1-\cos\frac{\pi}{M}\cos\theta}\cdot\right.$$

$$\left(\frac{\gamma_0}{1+\gamma_0-\gamma_0\cos\frac{\pi}{M}\cos\theta}\right)^2 d\theta$$

$$-3\left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{1-\cos\frac{\pi}{M}\cos\theta}\frac{\gamma_0}{1+\gamma_0-\gamma_0\cos\frac{\pi}{M}\cos\theta} d\theta\right.$$

$$\left.\left.\left.-\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{1-\cos\frac{\pi}{M}\cos\theta}\frac{3\gamma_0}{k+3+3\gamma_0\left(1-\cos\frac{\pi}{M}\cos\theta\right)} d\theta\right)\right]\right\} \quad \text{(A3)}$$

which can be written in the integral-form, not in the closed-form.

Appendix B: The Closed-form Derivation of (20)

In this Appendix, given that μ, ν, and β are real numbers, using the identity [14, p. 649, Eq. (6.286.1)]

$$\int_0^\infty erfc(\beta x)e^{\mu^2x^2}x^{\nu-1}dx$$

$$=\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi}\nu\beta^\nu}\ {}_2F_1\left(\frac{\nu}{2},\frac{\nu+1}{2};\ \frac{\nu}{2}+1;\ \frac{\mu^2}{\beta^2}\right),\ \cdot$$

$$Re\ \beta^2 > Re\ \mu^2, Re\ \nu > 0, \quad \text{(B1)}$$

where

$${}_2F_1(a,b;\ c;\ z)=\frac{\Gamma(c)}{\Gamma(a)\Gamma(b)}\sum_{n=0}^{\infty}\frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)}\frac{x^n}{n!}, \quad \text{(B2)}$$

we can find the approximate performance of MPSK signals under the effect of SC-2 diversity in a Rayleigh fading channel to be

$$P_{e,MPSK,SC2}=\int_0^\infty P_{s,MPSK}\ f(\gamma)\ d\gamma=\frac{3}{16}\frac{L(L-1)}{\gamma_0^2}$$

$$\frac{1}{\sin^4\frac{\pi}{M}}\ {}_2F_1\left(2,\frac{5}{2};\ 3;\ -\frac{1}{\gamma_0\sin^2\frac{\pi}{M}}\right)$$

$$+\frac{1}{2}\frac{L(L-1)}{\gamma_0}\frac{1}{\sin^2\frac{\pi}{M}}\sum_{k=1}^{L-2}\binom{L-2}{k}\frac{(-1)^k}{k}\cdot$$

$$\left[{}_2F_1\left(1,\frac{3}{2};\ 2;\ -\frac{1}{\gamma_0\sin^2\frac{\pi}{M}}\right)\right.$$

$$\left.-\ {}_2F_1\left(1,\frac{3}{2};\ 2;\ -\frac{2+k}{2\gamma_0}\frac{1}{\sin^2\frac{\pi}{M}}\right)\right]. \quad \text{(B3)}$$

Appendix C: The Finite-series Representation of ${}_2F_1(\cdot)$ in (20)

Given that n is real number more than 1/2, we assume that $G(z)$ is given by [15]

$$G(z)=z^n\ {}_2F_1(n,n+\frac{1}{2};\ n+1;\ -z)$$

$$=\sum_{i=0}^{\infty}\frac{n}{n+i}\frac{(2n-1+2i)!!}{2^i(2n-1)!!}\frac{(-1)^i}{i!}z^{n+i},\ Re\ n>\frac{1}{2} \quad \text{(C1)}$$

where

$$(2n-1)!!=(2n-1)(2n-3)\cdots 3\cdot 1. \quad \text{(C2)}$$

We note that since n is more than $1/2$, $G(0)$ is 0. Then, (C1) can be represented as follows:

$$G(z)=n\int_0^z x^{n-1}\left[\sum_{i=0}^{\infty}\frac{(2n-1+2i)!!}{2^i(2n-1)!!}\frac{(-x)^i}{i!}\right]dx+G(0). \quad \text{(C3)}$$

The infinite-series formula of (C3) becomes

$$\sum_{i=0}^{\infty}\frac{(2n-1+2i)!!}{2^i(2n-1)!!}\frac{(-x)^i}{i!}=(x+1)^{-n-\frac{1}{2}} \quad \text{(C4)}$$

Hence, (C3) can be expressed as

$$G(z)=\int_0^z nx^{n-1}(x+1)^{-\frac{2n+1}{2}}dx. \quad \text{(C5)}$$

Consequently, (C5) may be presented as follows [14, p. 73, Eq. (2.221)]:

$$G(z)=-n\sum_{i=1}^{n-1}\frac{(-1)^i}{i+\frac{1}{2}}\binom{n-1}{i}\left[(z+1)^{-i-\frac{1}{2}}-1\right]$$

$$=-\sum_{i=0}^{n-1}\frac{(-1)^i}{i+\frac{1}{2}}\frac{\Gamma(n+1)}{\Gamma(i+1)\Gamma(n-i)}\left[(z+1)^{-i-\frac{1}{2}}-1\right]. \quad \text{(C6)}$$

Finally, ${}_2F_1(\cdot)$ in (C.1) can be written as

$${}_2F_1(n,n+\frac{1}{2};\ n+1;\ -z)=-z^{-n}\sum_{i=0}^{n-1}\frac{(-1)^i}{i+\frac{1}{2}}\cdot$$

$$\frac{\Gamma(n+1)}{\Gamma(i+1)\Gamma(n-i)}\left[(z+1)^{-i-\frac{1}{2}}-1\right]. \quad \text{(C7)}$$

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References

[1] J.-C. Lin, W.-C. Kao, Y. T. Su, and T. H. Lee, "Outage and coverage considerations for micro-celluar mobile radio systems in a shadowed-rician/shadowed-nakagami environments", *IEEE Trans. Veh. Technol.*, vol. 48, no. 1, pp. 66-75, Jan. 1999.

[2] C. S. Chang, P. J. McLane, "Bit- error-probability for noncoherent orthogonal signals in fading with optimum combining for correlated branch diversity", *IEEE Trans. Inform. Theory*, vol. 43, no. 1, pp. 263-274, Jan. 1997.

[3] J. G. Proakis, *Digital Communications*, New York: McGraw-Hill Inc., 1995.

[4] L. M. Song, P. Y. Kam, "Performance of adaptive reception of MPSK on the nonselective rayleigh fading channel", in *Proc. ICC 98*, pp. 176-180,

Nov. 1998.

[5] T. Eng, N. Kong, and L. B. Milstein, "Comparison of diversity combining techniques for rayleigh-fading channels", *IEEE Trans. Commun.*, vol. 44, no. 9, pp. 1117-1129, Sep. 1996.

[6] Chang Hwan Kim, "New evaluation on maximum ratio diversity for the reception of *M*-ary PSK and DPSK signals on rician fading channels", *Telecommunications Review*, vol. 12, no. 2, pp. 261-268, Mar.-Apr. 2002.

[7] A. Papoulis, *Probability, Random Variables, and Stochastic Processes*, New York: Graw-Hill Inc., 1981.

[8] A. M. Mood, F. A. Graybill, and D. C. Boes, *Introduction to the Theory of Statistics*, McGraw-Hill, 1982.

[9] S. Y. Choi, Comparison of diversity combining techniques for MPSK signals in Nakagami fading channels, M. Sc. thesis, Dep. Elec. Com. Eng., Hanyang Univ., Korea, Dec. 1998.

[10] J. Sun, I. S. Reed, "Performance of MDPSK, MPSK, and noncoherent MFSK in wireless Rician fading channels", *IEEE Trans. Commun.*, vol. 47, no. 6, pp. 813-816, Jun. 1999.

[11] C. H. Kim, "Performance analysis on reception of *M*-ary FSK and diversity *M*-ary DPSK signals in wireless fading channels", Doc. Sc. thesis, Dep. Elec. Com. Eng., Hanyang Univ., Jun. 2001.

[12] S. Stein, J. J. Jones, *Modern Communication Principles*, New York: McGraw-Hill Inc., 1995.

[13] Chang Hwan Kim, "New evaluation on maximum ratio diversity for the reception of *M*-ary PSK and DPSK signals on rician fading channels, *Telecommunications Review*, vol. 12, no. 2, pp. 261-268, Mar.-Apr. 2002.

[14] I. S. Gradshteyn, I. M. Ryzhik, *Table of Integral, Series and Products*, New York: Academic Press, Inc., 1980.

[15] K. K. Cho, "Analysis of MRC diversity for *M*-ary QAM signals in nakagami fading channels", M. Sc. thesis, Dep. Elec. Com. Eng., Hanyang Univ., Dec. 1996.

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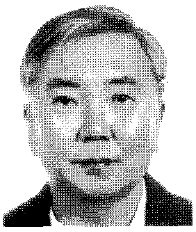

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