An Orthogonal Phase-Superimposed Peak-to-Average Power Ratio Reduction Technique

Tae-Young Han[1] · Nam Kim[1] · Jung-Hun Choi[1] · Jae-Hwan Lee[2]

Abstract

This paper presents a method of superimposing the rotation phases over the pilot and data symbols in order to reduce the peak-to-average power ratio(PAPR) in orthogonal frequency division multiplexing(OFDM). The phases of the rotation vector are added to those of the pilot symbols and those of the data symbols by interlaying them between any two pilot symbols. The receiver restores the data symbols by utilizing the channel estimation of the pilot symbols. Therefore, the bandwidth efficiency is improved by not using the subcarriers that are assigned for the reduction of the PAPR. Also, the enormous increase of the bit error rate which would be caused by incorrectly receiving the side information, i.e. the phases of the rotation vector, is prevented. The simulation results of the bit error rate performance for the BPSK are given using the COST-207 channel model.

Key words : OFDM, PAPR, Channel Estimation, PTS.

Ⅰ. Introduction

The motivation of this paper stems from the question as to why the phase rotation vector in the selected mapping(SLM)[1] and the partial transmit sequences (PTS)[2] used for the reduction of the peak-to-average power ratio (PAPR) in orthogonal frequency division multiplexing (OFDM) should be transmitted to the receiver when the pilot symbols can be used to correct the frequency offset and to estimate the channel. As long as the change of the phase of the pilot symbols does not distract from their principal purposes, we can use them to reduce the PAPR[3].

Therefore, the partial transmit sequences technique can be used to reduce the PAPR without sending the phase rotation information superimposed on the pilot symbols and information data. In [4]~[7] and [11] this concept was developed. The organization of this paper is as follows. Section II gives an overview of the discrete OFDM signal model and the concept of the PAPR. Section III discusses the proposed superimposing method. Section IV shows the simulation method and its results. Section V concludes this paper.

Ⅱ. OFDM Signal Model and PAPR Reduction

The discrete-time OFDM system baseband model shown in Fig. 1 is used throughout this paper. In the transmitter, the time-domain data $\boldsymbol{X}_l$ of the lth OFDM symbol can be represented as $X_l = F^H S_l$, where $X_l = [x_{l,0}, x_{l,1}, \cdots, x_{l,N-1}]^T$ is the T/N-spaced discrete representation, T is an OFDM symbol period, F^H is a $N \times N$ FFT matrix whose component in the row p and column q is $F_{p,q} = e^{j2\pi pq/N}$, and $(\cdot)^H$ is Hermitian. The lth OFDM data block $S_l = [S_{l,0}, S_{l,1}, \cdots, S_{l,N-1}]^T$ of the distinct complex-valued frequency-domain data selected from the corresponding constellation is transformed into the time-domain data using the IFFT. For the rest of the paper, the OFDM symbol block index l will be dropped for notational simplicity when the context makes it clear. The first column of the $(N+G)\times(N+G)$ square Toeplitz matrix $\boldsymbol{H}_{ISI}$ is $[h_{0,\dots,}h_{G,}0,\cdots,0]$ and the first row of the $(N+G)\times(N+G)$ square Toeplitz matrix $\boldsymbol{H}_{IBI}$ is $[0,\cdots,h_{G,}h_{G-1},\cdots,h_1]^T$. The identity matrix $\boldsymbol{I}_G$ is a $G \times G$ matrix and $I_{CP} = [\ 0_G \times (N-G),\ I_G]^T$.The $T_{CP} = [\ I^T_{CP,}\ I^T_N]^T$ is a cyclic prefix insertion matrix. The $R_{CP} = [\ 0_{N\times G},\ I_N]^T$ is a cyclic removal matrix. So, $R_{CP} H_{IBI} = 0$ and the interblock interference(IBI) is

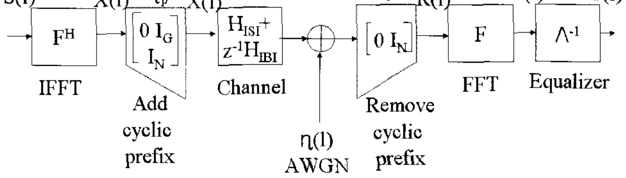

Fig. 1. Baseband discrete model of OFDM.

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eliminated. And $\tilde{X} = T_{CP} X$. The first row of the matrix $H_{cyc} = R_{CP}(H_{ISI} + z^{-1} H_{IBI}) I_{CP}$ is $[h_0, \cdots, 0, h_G, h_{G-1}, \cdots, h_1]$. So, H_{cyc} is the circulant matrix if the guard interval G is greater than the length of the wireless multipath channel length and is then decomposed as $H_{cyc} = F^{-1} \Lambda F$. Therefore, the received signal $\boldsymbol{Y}$ after FFT is

$$Y = \Lambda S + F \eta. \tag{1}$$

Here, we have a diagonal matrix $\Lambda = diag[H(e^{j0}), H(e^{j2\pi/N}), \cdots, H(e^{j2\pi(N-1)/N})]$ where $H(e^{j2\pi k/N})$ is a channel transfer function at the subcarrier k. The intersymbol interference(ISI) is eliminated by the insertion of the cyclic prefix.

2-1 PAPR

Because of the statistical independence of the data at the subcarriers, the corresponding time-domain complex-valued samples $\boldsymbol{X}$ of the OFDM symbol are approximately modeled as a Gaussian distribution by applying the central limit theorem when assuming that $\boldsymbol{N}$ is sufficiently large. The PAPR of the transmitted OFDM signal $\boldsymbol{X}$ is defined as

$$PAPR \triangleq \frac{\max_{0 \le k \le N-1} |x_k|^2}{E[|x_k|^2]} . \tag{2}$$

Then, the complementary cumulative distribution function(CCDF) of the PAPR of an OFDM signal for a given PAPR level γ is the probability that the PAPR of a randomly generated OFDM data block exceeds the given threshold γ, which can be theoretically calculated as follows:

$$\Pr[PAPR > \gamma] = 1 - (1 - e^{-\gamma})^N. \tag{3}$$

2-2 PTS-OFDM

The OFDM symbol is divided into $\boldsymbol{M}$ subblocks $S_m, m = 0, \cdots, M-1$ which are called partitions. The original OFDM symbol is represented as the sum of these subblocks, $S = \sum_{m=0}^{M-1} S_m$. Here, the data symbol $S_{m,k}$ at the kth subcarrier of the mth subblock S_m is

$$S_{m,k} = \begin{cases} S_k & \text{if } mM \le k < (m+1)M \\ 0 & otherwise. \end{cases} \tag{4}$$

So, $X_m = F^H S_m$ which is called a partial transmit sequence is the time-domain signal of the subblock S_m. An optimal solution for PAPR is obtained by the following optimization criterion.

$$\arg\min_{\tilde{\phi}_0, \cdots, \tilde{\phi}_{M-1}} \sum_{m=0}^{M-1} X_m e^{j\phi_m} \tag{5}$$

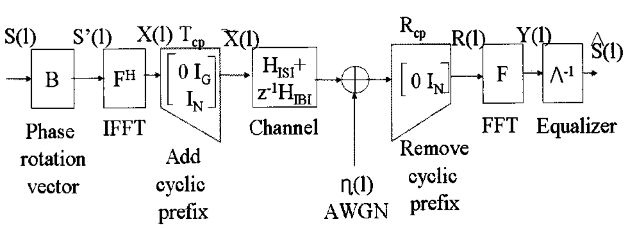


Fig. 2. Discrete-time DFT implementation model superimposing the phase rotation vector over pilot tone.

The low PAPR can be obtained by changing this phase rotation vector or factor $\phi = [\phi_0, \cdots, \phi_{M-1}]^T$. We can find the optimal values $\tilde{\phi}_0, \cdots, \tilde{\phi}_{M-1}$ by using the Nedler-Mead simplex method[8] but it is very complex. Instead, it is usually found the suboptimal solution obtained by limiting the mth rotation phase ϕ_m to $\{+1, -1\}$ and using one of the various noncomplex fast searching algorithms such as the successive iterative method[9], or gradient search algorithm[10] and so on. The use of a greater number of phase rotation vectors and greater M gives a lower PAPR. But the CCDF reaches saturation when the number of partitions M is greater than 8. The use of the Hadamard and Golay complementary sequence reduces the search time but the PAPR performance is lower than that obtained using all possible phase rotation vectors. It is necessary to send the side information bits $\log_2 M$ representing the index of the rotation phase vector. It is very important to protect these side information bits by adding the redundant channel coding bits so as to recover the correct data in the receiver.

Ⅲ. Superimposed Scheme

From the transmitter to the receiver shown in Fig. 2, the following expression can be given when we use the rotation phase $B = diag[b_0, b_1, \cdots, b_{N-1}]$, $b_k = e^{j\phi_m}$.

$$Y = \Lambda' S + F \eta \tag{6}$$

Here, we have a rotated diagonal matrix $\Lambda' = diag[H_0 b_0, H_1 b_1, \ldots, H_{N-1} b_{N-1}]$. If we let $H'_k = H_k b_k$, then $Y_k = H'_k S_k + W_k$. Therefore, without the information of the phase rotation vector B in the receiver, the information data can be correctly restored by using the channel estimation $\tilde{H}'_k$. So, even if there is no information on the phase rotation vector in the receiver, the data can be recovered by simply estimating the channel with the pilot symbols.

3-1 Proposed Pilot Symbol Pattern

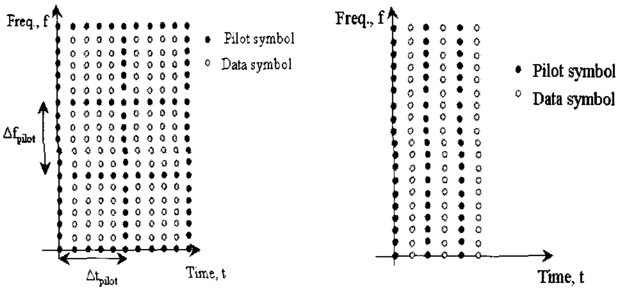
For the purpose of the proposed phase-superimposed scheme we suggest the following comb type pilot symbol pattern. A mathematical representation for how it is realized in the simulation is as follows:

$$S_{k,l}=\begin{cases} p_{k,l} & \text{if } 0\le k<N, l=rT_{blk}\\ q'_{k,l}e^{j\phi_m} & \text{if } k=mM, l\ne rT_{blk}\\ d_{k,l}e^{j\phi_m} & \text{if } mM<k<(m+1)M,\\ & \quad l\ne rT_{blk}.\end{cases} \qquad (7)$$

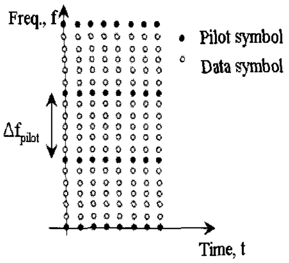
Here, the integers T_{freq} and T_{blk} are the frequency-domain pilot spacing and the time-domain OFDM symbol spacing, respectively, and r is also integer.

$$T_{blk}\le\frac{1}{\Delta f\tau_{max}},\quad T_{freq}\le\frac{1}{f_{Dmax}T/N}, \qquad (8)$$

where $\Delta f=1/T$, τ_{max} and f_{Dmax} are the subcarrier frequency spacing, the maximum delay of multipath and the maximum Doppler frequency. The integer M can be greater than the integer T_{freq}. The subcarrier index m and the OFDM symbol block index l are integers. The pilot symbol $p_{k,l}$ of the lth OFDM symbol and kth subcarrier must be known both to the transmitter and the receiver. The pilot symbol $q'_{k,l}$ of the lth OFDM symbol and kth subcarrier known both to the transmitter and the receiver is only retained for restoring the phase rotation vector. The information data symbols $d_{k,l}$ of the lth OFDM symbol and kth subcarrier is rotated by the phase $e^{j\phi_m}$ to reduce the PAPR using the PTS technique. Assuming that the channel frequency response $H_{k,l}(0\le k<N, l\ne jT_{blk})$ is the same as the $H_{k,l}(0\le k<N, l=jT_{blk})$ of the block type pilot OFDM symbol, we can detect the data symbol $d_{k,l}=Y_{k,l}/(\hat{H}_{k,l}e^{-j\hat{\phi}_m})$. The estimate $\hat{H}_{k,l}$ of the channel frequency response at the subcarrier k and the estimate $\hat{\phi}_m$ of the transmitted rotation phase at mth subcarrier is used in the receiver.



(a) Proposed pattern (b) Block type



(c) Comb type

Fig. 3. Pilot symbol pattern used for the phase superimposed scheme.

3-2 Block-Type Pilot Symbol Pattern

A mathematical representation for the block type pilot symbol pattern shown in Fig. 3(b) is as follows:

$$S_{k,l}=\begin{cases} p_{k,l}e^{j\phi_m} & \text{if } mM\le k<(m+1)M,\\ & \quad l=2r\\ d_{k,l}e^{j\phi_m} & \text{if } mM\le k<(m+1)M,\\ & \quad l=2r+1\end{cases} \qquad (9)$$

This is an ideal case for the proposed superimposed scheme. We assign the pilot symbols to one OFDM symbol and the data symbols to the other, and we assume that the fading channel is invariant during the two OFDM symbols. Since there is no need to restore the phase rotation vector in the receiver, any phase rotation vector can be allowed. However this would be inefficient in terms of the bandwidth.

3-3 Comb-Type Pilot Symbol Pattern

A mathematical representation for the comb type pilot symbol pattern in the simulation is as follows:

$$S_{k,l}=\begin{cases} p_{k,l}e^{j\phi_m} & \text{if } k=mM, l=rT_{blk}\\ d_{k,l}e^{j\phi_m} & \text{if } mM<k<(m+1)M,\\ & \quad l\ne rT_{blk}.\end{cases} \qquad (10)$$

There is no method to estimate the phase rotation ϕ_m which superimposed over the pilot symbols except by estimating the channel estimation. In this type of pilot pattern shown in Fig. 3(c), a good channel estimation method improves the BER performance.

Ⅳ. Simulation and Results

The data is randomly generated and encoded by the nonsystematic, nonrecursive convolutional encoder with constraint length 3, code rate 1/2 and the generator polynomial $(5, 7)_8$. The random interleaver is used and the signal is mapped by the BPSK and grey coded. The number of subcarriers for OFDM modulation is $N=1024$, and the sampling frequency is 10 MHz. The fading channel model is the time-variant channel model of COST-207 wide sense stationary uncorrelated sca-

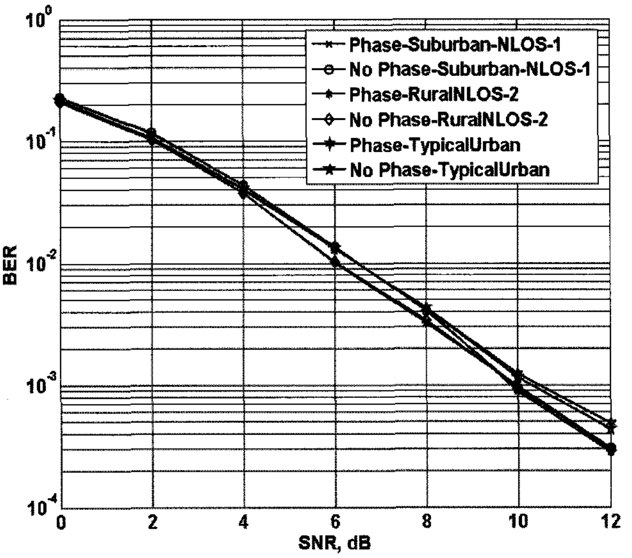


Fig. 4. Bit error rate performance for the block type pilot OFDM symbols.

ttering(WSSUS). The phase rotation is randomly chosen from the generated Walsh-Hadamard sequences of the rows and added to the phase of the OFDM pilot and data symbols. When we use the equi-spaced pilot symbols, the partial transmit sequences technique can be implemented. The Shapiro-Rudin, Golay complementary and the orthogonal variable spreading factor(OVSF) sequences can also be used.

Fig. 4 shows the bit error rate performance obtained from the simulation of BPSK modulation. The bit error rate (BER) performance shows no difference between the phased and nonphased cases(in the figure, we use the terms "Phase-" and "No Phase-" to mean that use is made or not made of the phase-superimposed technique, respectively), namely, the phase rotation can be included in the phase of the channel frequency transfer function. The simulations are performed in the various wireless channel environments (in the figure, "Suburban-NLOS", "RuralNLOS" and "TypicalUrban" mean the no line of sight of suburban, rural and typical urban environments, respectively). The original data can be restored by the channel estimation. The pilot symbol pattern is as shown in Fig. 3(b).

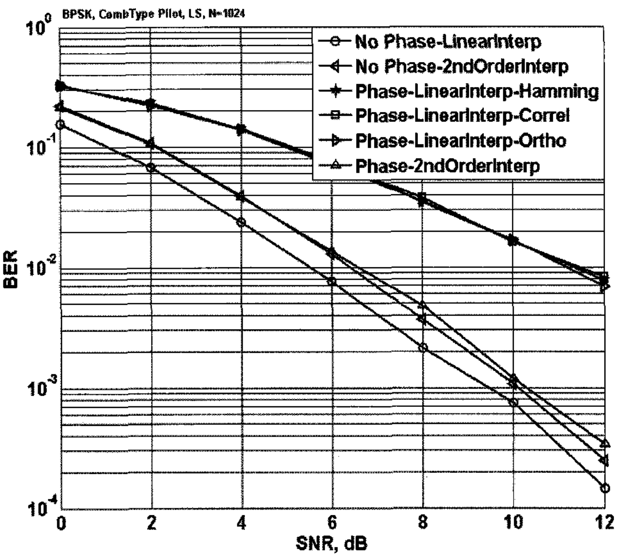


Fig. 5. Bit error rate performance for the comb type pilot OFDM symbols.

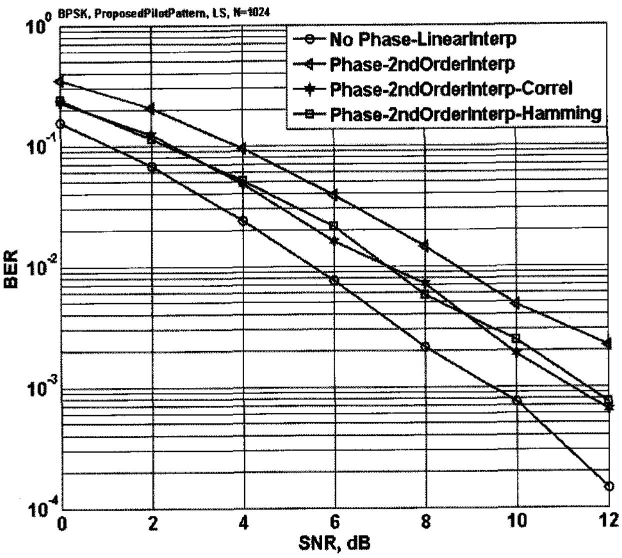


Fig. 6. Bit error rate performance for the proposed pilot OFDM symbols.

In the case of the comb type pilot symbol of Fig. 3(c) the second order polynomial interpolation ("2nd Order-Interp", in the figure) greatly improves the BER performance of the phase-superimposed scheme, as shown in Fig. 5. However, it is not better than the linear interpolation ("LinearInterp", in the figure) in the case of where the phase-superimposed scheme is nto used. The orthogonal property and Hamming distance of the Hadamard phase sequence ("Ortho" and "Correl", in the figure) do not improve the BER performance.

The proposed pilot pattern of Fig. 3(a) which is specific to the proposed phase-superimposed PAPR technique can enhance the BER performance by utilizing the orthogonal property of the Hadamard sequences, as shown in Fig. 6. The gain obtained from the orthogonality of the phase rotation vectors is 1.5 dB. The Hamming distance and correlation criteria have no distinction. The Hamming distance criterion is more preferable because of its convenience.

The CCDF of Fig. 3 (a), (b), and (c) is depicted in Fig. 7. The Block type pilot pattern has a poor PAPR.

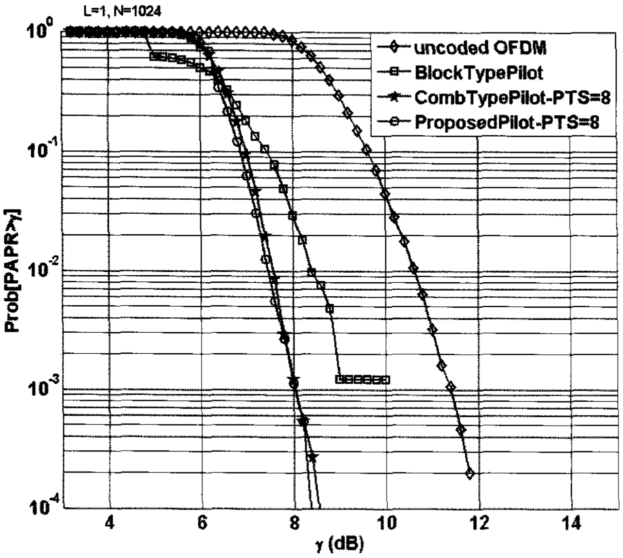


Fig. 7. CCDF of the PAPR of Fig. 3(a), (b), and (C).

V. Conclusion

In this paper, we theoretically verified whether the phase rotation vector can be included in the phase in a transfer function H_k of the wireless channel, and the simulation results confirm this hypothesis. Therefore, the phase change of pilot symbols, utilized for the frequency offset and channel estimation, does not affect its purpose. Thus, the channel estimation is sufficient to restore the data symbols in the receiver if we have accurate channel estimation for the pilot symbols. A 1 dB degradation of the BER occurred in the proposed pilot pattern because of the channel estimation error and the rotation phase estimation error.

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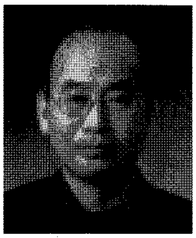

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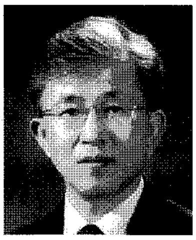

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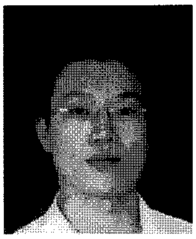

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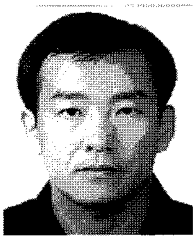

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