한국측량학회지, 제 25권 제 6-1호, pp. 537~543(2007년 12월 31일)

Times Series Analysis of GPS Receiver Clock Errors to Improve the Absolute Positioning Accuracy

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Abstract

Since the GPS absolute positioning with pseudorange measurements can significantly be affected by the observation error, the time series analysis of the GPS receiver clock errors was performed in this study. From the estimated receiver clock errors, the time series model is generated, and constrained back in the absolute positioning process. One of the CORS (Continuously Operating Reference Stations) network is used to analyze the behavior of the receiver clock. The dominant part of the model is the linear trend during 24 hours, and the seasonal component is also estimated. After constraining the modeled receiver clock errors, the estimated position error compared to the published coordinates is improved from ±11.4 m to ±9.5 m in 3D RMS.

Keywords : Time series, GPS, Clock error

1. Introduction

With the successful adaptation of GPS (and upcoming GNSS such as Galileo) for science and engineering, the need to improve the GPS positioning accuracy has greatly been increased. The analysis of GPS clock errors are predominantly focused on the satellite clock error estimation (Han et al., 2001; Oaks et al., 2005; Wright, 2007), which is very stable compared to the receiver clock errors. Generally speaking, the GPS clock errors can be eliminated by doubly-differencing the measurements involving two satellites and two receivers.

For the purpose of survey or navigation, however, the absolute positioning technique is commonly used with the one-way pseudorange measurements in the data processing. In this case, only the position and the receiver clock errors are usually estimated, while the ionospheric delay is ignored and the model values are used for the tropospheric delay. Therefore, depending on the measurement noise, the receiver clock errors can be abruptly increased and thus the position error gets worse accordingly. Since the receiver clock is relatively stable, thus it may be possible to model its behavior and apply to the absolute positioning.

In this study, the time series analysis of the GPS receiver clock errors is presented along with its practical application to real data. The basic theory of time series is described in the following section. The estimated receiver coordinates after constraining the receiver clock errors to their model values are compared with those of absolute positioning and the case of "true" receiver clock errors. The model of the time differenced receiver clock errors are checked for goodness of fit by plotting the sample autocovariance function (ACF), and the Q-Q plot for the normal distribution of the residuals.

2. Basic theory of time series

A time series is a set of observations x_t, each one being recorded at a specific time (Brockwell and Davis, 2002). The objective of the time series analysis is to find a model that can explain the dependency between different observations in time series data, and thus model the residuals as stationary process. Since the observations are usually made at discrete time epoch, especially in GPS measurements, the discrete-time time series is always

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assumed in this study.

It is usually known that a time series model is a specification of the probabilistic distribution of a sequence of random variables (RVs), $\{X_t : t \in T_0\}$ where T_0 is the set of all possible time points. The RVs can be realized as the observed series $\{x_t\}$. The distribution of the times series can be represented by the moments, therefore, the means and covariances of the RVs need to be specified in a time series model. The mean function of $\{X_t\}$ is defined by

$$\mu_X(t) = E(X_t), \tag{1}$$

where $E(\cdot)$ represents the expectation of the RVs. Accordingly, the variance and covariance function of $\{X_t\}$, respectively, can be represented by

$$\sigma_X^2(t) = \mathrm{var}(X_t) = E\left\{\left(X_t - \mu_X(t)\right)^2\right\}, \tag{2}$$

$$\gamma_X(s,t) = \mathrm{covar}(X_s, X_t) = E\left\{\left(X_s - \mu_X(t)\right)\left(X_t - \mu_X(t)\right)\right\}. \tag{3}$$

One of the important properties in time series analysis is the stationarity of the RVs. The RVs $\{X_t\}$ is considered as (weakly) stationary if

1) the mean μ_X is constant and does not depend on time, and

2) the covariance, $\mathrm{covar}(X_t, X_{t+h})$, is a finite constant and does not depend on t but h

where h is called the lag. If $\{X_t\}$ is stationary, the covariance becomes the autocovariance function (ACVF), that is,

$$\gamma_X(h) = \mathrm{covar}(X_t, X_{t+h}) = E\left\{\left(X_t - \mu_X\right)\left(X_{t+h} - \mu_X\right)\right\}, \tag{4}$$

and the corresponding autocorrelation function (ACF) can be defined as

$$\rho_X(h) = \frac{\gamma_X(h)}{\gamma_X(0)}, \tag{5}$$

2.1 The white noise process

Assuming $E\{X_t\} = \mu$ and $\mathrm{var}\{X_t\} = \sigma^2 < \infty$, then $\{X_t\}$ is a white noise or $WN(\mu, \sigma^2)$ process if $\gamma_X(h) = 0$ for $h \neq 0$. Therefore, the white noise process is stationary.

2.2 The moving average process

Let $\{Z_t\}$ be a $WN(0, \sigma^2)$ process, and θ_j be some real-valued constants. Then for each integer t, let

$$\begin{aligned} X_t &= Z_t + \theta_1 Z_{t-1} + \cdots + \theta_q Z_{t-q} \\ &= \sum_{j=0}^{q} \theta_j Z_{t-j} \end{aligned} \tag{6}$$

where $\theta_0 = 1$. The sequence of RVs $\{X_t\}$ is called the moving average process of order q. This $MA(q)$ process is a stationary time series model with q-correlated.

2.3 The autoregressive process

Let $\{Z_t\}$ be a $WN(0, \sigma^2)$ process, and $\{\phi_1, \cdots, \phi_p\}$ be a set of constants for some integer $p > 0$ ($\phi_p \neq 0$). Then the autoregressive process of order p, $AR(p)$, is defined as the solution to the equation

$$X_t = \sum_{j=1}^{p} \phi_j X_{t-j} + Z_t. \tag{7}$$

2.4 Sample mean and autocovariance function

Let $\{X_t\}$ be a stationary process with mean μ_X and autocovariance function $\gamma(\cdot)$. Then the unbiased sample mean can be represented as

$$\bar{X} = \frac{1}{N}\sum_{t=1}^{N} X_t, \tag{8}$$

and the sample autocovariance function (ACVF) (biased but standard) for $|h| < n$ is given by

$$\hat{\gamma}_X(h) = \frac{1}{n}\sum_{t=1}^{n-|h|}(X_t - \bar{X})(X_{t+|h|} - \bar{X}), \tag{9}$$

and the corresponding sample autocorrelation function (ACF) is defined by

$$\hat{\rho}(h) = \frac{\hat{\gamma}_X(h)}{\hat{\gamma}_X(0)}. \quad (10)$$

If the process is independent and identically distributed (IID) noise with finite variance, then the sample ACF will follow a normal distribution with zero mean and variance of 1/n, that is,

$$\hat{\rho}(h) \sim N\left(0, \frac{1}{n}\right) \quad (11)$$

Therefore, we can check the model by plotting $\hat{\rho}(h)$ and draw two horizontal lines at $\pm 1.96\sqrt{n}$ and see if about 95% of the $\hat{\rho}(h)$ be within the lines if we have IID noise.

3. Test data

For the time series analysis of the GPS receiver clock errors, COLB station, one of the CORS (Continuously Operating Reference Stations) located in Columbus, Ohio (USA), is used. The receiver type installed in COLB is Trimble NETR5 with the elevation cutoff setting of 5 degrees. GPS data was collected on 18 September 2007 (day of year 261) for 24 hours with 30-second interval. The published coordinates of COLB can be obtained from the website of National Geodetic Survey (NGS) at http://www.ngs.noaa.gov/CORS/. Table 1 shows the published coordinates of the receiver at L1 phase center (antenna offset from the reference point corrected) at 1997.0 along with its velocity in ITRF00 frame. The receiver coordinates are further corrected for its velocity to get the current position in the middle of observation (noon 18 September 2007). The absolute point positioning routine is implemented in MATLAB and all the times series analysis is performed in R. For further information on R, see the website of the R project for Statistics Computing (http://www.r-project.org/).

The pseudorange measurement from the GPS satellite is used to calculate the GPS receiver clock errors. The GPS satellite orbits can be calculated using the broadcast navigation message (Seeber, 2003; Remondi, 2004). The tropospheric delay model used in this study is the modified Hopfield model and the ionospheric delay is usually ignored in absolute positioning. Therefore the pseudorange observation model can be expressed as (Hofmann-Wellenhof et al., 2004)

Table 1. The position and velocity of COLB station in ITRF00 frame at epoch 1997.0.

	Position [m]	Velocity [m/yr]
X	592756.525	-0.0168
Y	-4859703.434	-0.0017
Z	4074680.999	0.0009

$$P_i^k = \rho_i^k + T_i^k + c(dt_i - dt^k) + e, \quad (12)$$

where

P : the pseudorange measurement in distance units;
ρ : the geometric range between the transmitter and the receiver at the time of signal emission and reception, respectively;
T : the tropospheric refraction;
c : the speed of light in vacuum;
dt : the clock error;
e : the measurement noise;

and the superscript represents the satellite, and the subscript the receiver. The satellite clock error can also be obtained from the navigation message. If the precise ephemeris data is used, the GPS satellite orbits as well as the satellite clock error can be interpolated at the time of transmission.

Figures 1 and 2 show the results of the absolute positioning of COLB station on September 18, 2007, the position and the receiver clock errors, respectively. The receiver position (L1 phase center actually) is calculated at each observation epoch and compared with the published coordinates as mentioned above. As can be seen in the figures, the receiver clock errors are highly correlated with the calculated coordinate error of the receiver, especially the UP component (not shown here). This is mainly due to the neglected ionospheric delay, tropospheric model error, and the random error of the pseudorange measurement (C1 in this study). Therefore, the idea is that once we can somehow model the behavior of the receiver clock errors and/or eliminate the outliers, then the absolute positioning error can be further improved.

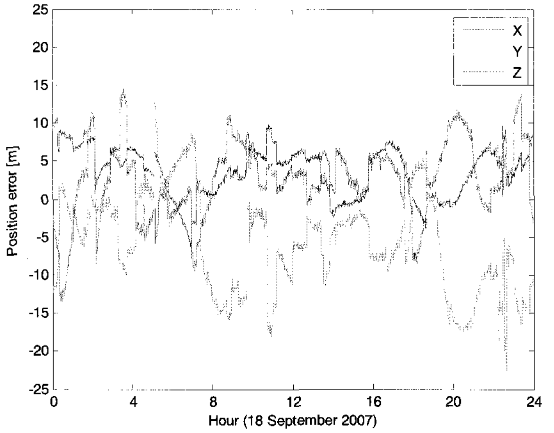


Fig. 1. The GPS receiver coordinates from absolute positioning.

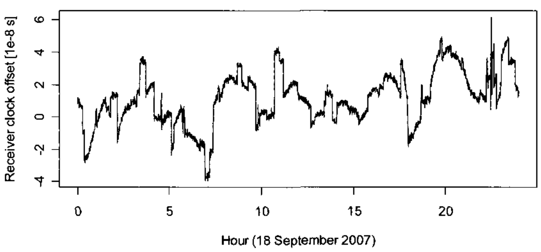


Fig. 2. The GPS receiver clock errors.

4. Time Series Analysis of the Receiver Clock Errors

A standard model for times series can be represented by

$$X_t = m_t + s_t + Y_t, \qquad (13)$$

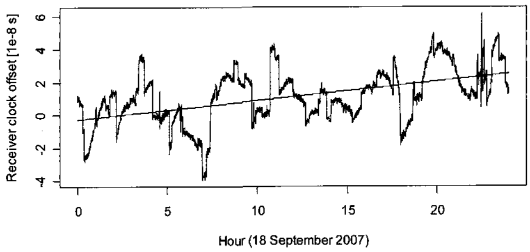
where

m_t: the trend component;

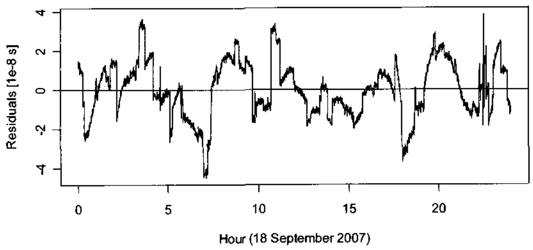
S_t : a seasonal component with period d, $\sum_{t=1}^{d} s_t = 0$;

Y_t : a random noise component with $E\{Y_t\} = 0$.

As can be seen in Figure 2, however, it is difficult to specify the time series of the receiver clock errors except the (upward) linear trend. This is because the GPS receiver clock is relatively stable for short term, although some shows a behavior of drift. Therefore, it is possible to model the receiver clock errors with the trend and a seasonal component, neglecting the random noise components from the absolute positioning.



(a) The receiver clock errors and its linear trend.



(b) The residuals after removing the linear trend.

Fig. 3. Estimation of the trend.

The first step of the time series analysis is to plot the data in order to check any abrupt changes in behavior and/or outliers (Figure 2), and then estimate the trend.

The linear model is used in this study to fit the time series data. According to Figure 3, there still have large randomness in the residuals of the receiver clock errors. Therefore, it is helpful to apply a filter to the residuals for better understanding of the behavior. Based on the assumption of the stable receiver clock, two hours of moving average (MA) filter is applied to obtain the smoothed residuals (Figure 4). As can be seen in the Figure 4, the filtered residuals fairly pick up the trend of each sub-interval through the time series with the zero mean residuals. After the smoothing process, the seasonality can be seen more clearly, although it does not look like a perfect seasonality.

The second step is to fit the detrended residuals to the harmonic model which is composed of sine and cosine terms. From Figure 5, it can be seen that the fitted harmonic model has much smaller amplitude compared to the smoothed residuals. The estimated harmonic model is given by

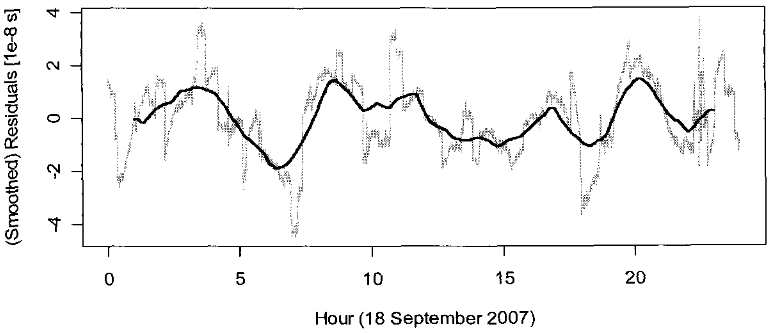


Fig. 4. (Smoothed) Residuals after moving average (MA) filter.

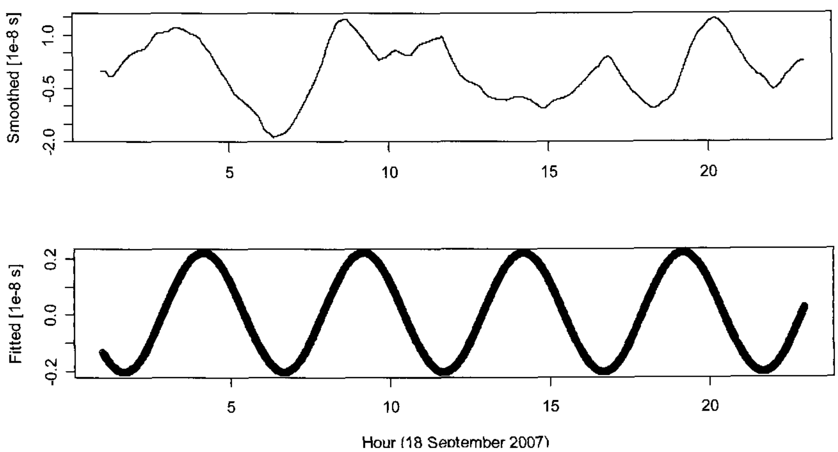


Fig. 5. The smoothed receiver clock errors and its harmonic model fit.

$$\hat{s}_t = 0.110533\cos(2\pi t) - 0.181834\sin(2\pi t) + 0.007114 \quad [10^{-8}s]. \tag{14}$$

Figure 6 shows the modeled receiver clock errors, combining the linear trend and the seasonality.

Table 2 shows the results for three cases of absolute positioning. The case 1 represents the standard absolute positioning which estimates the receiver clock errors along with the coordinates of the receiver. There are a few meters of bias in each component and the 3D RMS reaches about ±11.4 meters which are commonly expected from the absolute positioning. The case 2 indicates the results that the receiver clock errors are fixed to their "true" values. The "true" values here mean that they are computed from the absolute positioning by constraining the receiver position to its published (close to "true") coordinates. It should not be real "true" because the pseudorange measurements have random errors and the ionospheric and tropospheric delay terms are not estimated as mentioned earlier. Therefore, even in this case, the estimated receiver position has an error of about ±6.3 meters in 3D RMS. The third case that is emphasized in this study shows an estimation of the receiver position by constraining the receiver clock errors to the model as shown in Figure 6. Since the main component of the model is the linear trend of the estimated receiver clock errors, the biases of the Case 3 are very close to the Case 1, while the variations are significantly decreased resulting in ±9.5 meters in 3D RMS.

It should be mentioned here that the time difference of the receiver clock errors can be modeled as the autoregressive process that has a stationary and unique solution. This is because the dominant component of the receiver clock errors is the linear trend, thus there re-

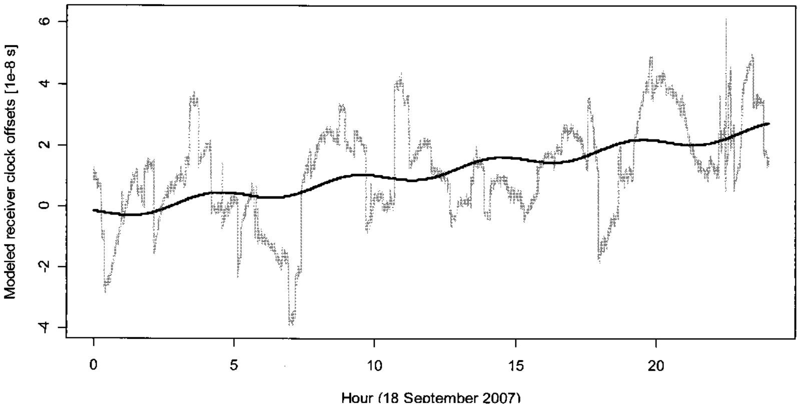


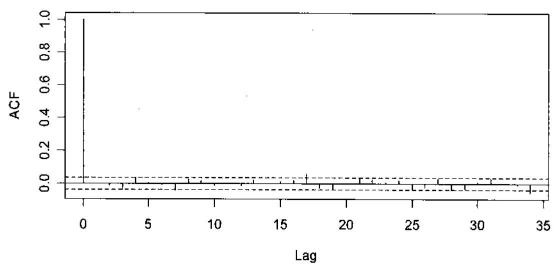
Fig. 6. The modeled receiver clock errors.

Table 2. Comparison of the absolute positioning results.

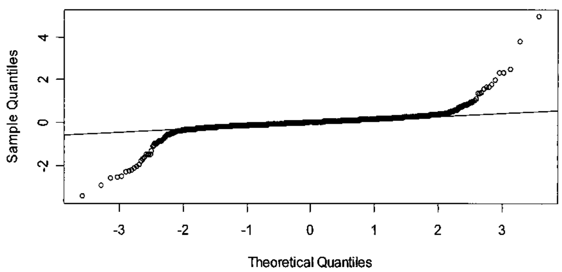
		X [m]	Y [m]	Z [m]	3D [m]
Case 1*	Mean	3.629	-5.821	2.482	
	Std.	±3.310	±6.437	±4.956	
	RMS	±4.912	±8.678	±5.542	±11.409
Case 2**	Mean	3.430	-2.092	-0.563	
	Std.	±3.095	±2.550	±2.673	
	RMS	±4.620	±3.299	±2.732	±6.300
Case 3***	Mean	3.788	-5.708	2.448	
	Std.	±2.825	±4.121	±3.405	
	RMS	±4.726	±7.040	±4.193	±9.460

* Case 1: The receiver clock errors are estimated.
** Case 2: The receiver clock errors are fixed to their "true" values.
*** Case 3: The receiver clock errors are fixed to their "model" values.

mains a white noise process after detrending. Figure 7 shows the sample autocorrelation function (ACF) of the differenced time series of the receiver clock errors after eliminating the linear trend. The sample ACF of the residuals of AR(2) model shows that the correlation of which the lag is greater 0 is almost zero, although there are lags that are slightly outside the confidence intervals. The Q-Q (theoretical quantiles versus sample quantiles) plot can be used to evaluate a family of distributions for the observation samples. In this case the normal Q-Q plot is very close to a straight line except some outliners due mostly to the observation errors.



(a) The sample ACF of the residuals of AR(2) model.



(b) The Q-Q plot.

Fig. 7. The residuals of AR(2) model.

5. Discussion and summary

For an improved GPS absolute positioning, the time series analysis of the receiver clock errors is performed in this study. Since the receiver clock is relatively stable (of course it is much less accurate than the satellite clock), the major component of the time series is modeled as a linear trend, and some minor fluctuations are estimated as harmonic series. This modeling is confirmed by checking the residuals of the time differenced data, which are modeled as AR(2) process. The residuals are centered around zero and a Gaussian assumption for the residual seems to be reasonable from the straight line in the normal Q-Q plot, except some outliers coming from the measurement noise and the mismodeling in the absolute positioning. With the modeling of the receiver clock error, the absolute positioning accuracy of the receiver position is improved about ±2 m in 3D RMS.

This study suggests that if the receiver clock errors can be modeled using the one-way phase measurements with more precise modeling of the ionospheric/tropospheric delay it would be improved significantly. Also the prediction or forecasting of the receiver clock errors based on the time series analysis can be used for real time application. The prediction of the time series and its prediction errors computed using the times series up to current time epoch should be investigated further in future study.

6. References

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(접수일 2007. 11. 16, 심사일 2007. 11. 27, 심사완료일 2007. 12. 26)