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# Similarity Analysis Between Fuzzy Set and Crisp Set

**Park, Hyun Jeong* and Sang H. Lee****

***Department of Mathematics Education, Ewha Womans University**
****School of Mechatronics, Changwon National University**

**Abstract**

The similarity analysis for fuzzy set pair or crisp set pair are carried out. The similarity measure that is based on distance measure is derived and proved. The proposed similarity measure is considered with the help of analysis for uncertainty or certainty part of the membership functions. The usefulness of proposed similarity is verified through the computation of similarity between fuzzy set and crisp set or fuzzy set and fuzzy set. Our results are also compared with those of previous similarity measure which is based on fuzzy number.

**Key Words** : Fuzzy entropy, Similarity measure, Distance measure, Fuzzy number

## 1. Introduction

Computation of similarity between two or more informations is essential for the fields of decision making, pattern classification, or *etc*.. Quantity of difference between two or more information can be useful to discriminate or cluster for various informations. Until now the research of designing similarity measure has been made by numerous researchers[1-6]. For fuzzy set, there is an uncertainty knowledge in fuzzy set itself. Hence information of the data can be obtained from analyzing the fuzzy membership function. Thus most studies about fuzzy set are focussed on designing similarity measure based on membership function. With the previous results it is vague to obtain degree of similarity between fuzzy set and crisp set or crisp set and crisp set. In this paper we try to obtain similarity measure between fuzzy set and fuzzy set. After proving the similarity measure, we applied the similarity measure to calculate the degree of similarity. In our paper, we derive the similarity measure via well known-Hamming distance. We compare and analyze our similarity measure with previous measure. One similarity measure which was designed with fuzzy number is introduced. Two similarity measure have their own strong points, fuzzy number methods is simple and easy to compute similarity if membership function is trapezoidal or triangular. Whereas similarity with distance method needs more time and consideration, however that can be applied to the general membership function. At this point, it is interesting to study and compare two similarity measure for the fuzzy set and crisp set.

In the next section, fuzzy number, center of gravity, and the similarity measure are introduced. In Section 3, similarity measures with distance measure and fuzzy number are derived and proved. Also two similarity measures are compared and discussed in Section 4. In the example, we obtain similarity measure values that have proper meaning. Conclusions are followed in Section 5. Notations of Liu's are used in this paper [7].

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** Corresponding author
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## 2. Similarity measure based on fuzzy number and distance measure

In this section, we introduce some preliminary results for the degree of similarity. Introduction of the similarity measures are carried out, those similarity measure are based on fuzzy number and distance measure. Definition of fuzzy number, center of gravity, and axiomatic definitions of similarity measure are included.

### 2.1 Similarity measure via fuzzy number

A generalized fuzzy number $\widetilde{A}$ is defined as $\widetilde{A}=(a,b,c,d,\omega)$, where $0<\omega\leqq 1$ and $a,b,c,$ and $d$ are real numbers [1,2]. Trapezoidal membership function $\mu_{\widetilde{A}}$ of fuzzy number $\widetilde{A}$ satisfies the following conditions[4]:

1) $\mu_{\widetilde{A}}$ is a continuous mapping from real number $R$ to the closed interval $[0,1]$
2) $\mu_{\widetilde{A}}(x)=0$, where $-\infty<x\leqq a$
3) $\mu_{\widetilde{A}}(x)$ is strictly increasing on $[a,b]$
4) $\mu_{\widetilde{A}}(x)=\omega$, where $b\leqq x\leqq c$
5) $\mu_{\widetilde{A}}(x)$ is strictly decreasing on $[c,d]$
6) $\mu_{\widetilde{A}}(x)=0$, where $d\leqq x<\infty$.

If $b=c$ is satisfied, then it is natural to satisfy triangular type. Four fuzzy number operations are also found in literature

[4]. In addition to the fuzzy number, traditional center of gravity(COG) is also necessary. Thus the definition of COG is illustrated as follows

$$x_{\tilde{A}}^{*}=\frac{\int x\mu_{\tilde{A}}(x)dx}{\int \mu_{A}(x)dx}$$

$\mu_{\tilde{A}}$ is the membership function of the fuzzy number $\tilde{A}$. $\mu_{\tilde{A}}(x)$ indicates the membership value of the element $x$ in $\tilde{A}$, and generally, $\mu_{\tilde{A}}(x)\in[0,1]$. Chen and Chen presented a new method to calculate COG point of a generalized fuzzy number [4]. They derived the new COG calculation method based on the concept of the medium curve. These COG points play an important role in the calculation of similarity measure with fuzzy number.

Next we introduce the degree of similarity which were contained in the previous literatures [1-4]. Which are all based on the fuzzy number. In the literatures [1-4], degree of similarities are derived through membership function fuzzy number and center of gravity. We introduce the conventional fuzzy measure that is based on the fuzzy number. Chen introduced the degree of similarity for trapezoidal or triangular fuzzy membership function of $\tilde{A}$ and $\tilde{B}$ as [1]

$$S(\tilde{A},\tilde{B})=1-\frac{\sum_{i=1}^{n}|a_i-b_i|}{4} \tag{1}$$

where $S(\tilde{A},\tilde{B})\in[0,1]$. If $\tilde{A}$ and $\tilde{B}$ are trapezoidal or triangular fuzzy numbers, then the $n$ can be 4 or 3, respectively. For trapezoidal membership function fuzzy number satisfy $\tilde{A}=(a_1,a_2,a_3,a_4,1)$ and $\tilde{B}=(b_1,b_2,b_3,b_4,1)$.

Hsieh et. al. also proposed similarity measure for the trapezoidal and triangular fuzzy membership function as follows [2]:

$$S(\tilde{A},\tilde{B})=\frac{1}{1+d(\tilde{A},\tilde{B})} \tag{2}$$

where $d(\tilde{A},\tilde{B})=|P(\tilde{A})-P(\tilde{B})|$, and if $\tilde{A}$ and $\tilde{B}$ are triangular fuzzy number, then the graded mean integration of $\tilde{A}$ and $\tilde{B}$ are defined as follows:

$$P(\tilde{A})=\frac{a_1+4a_2+a_3}{6} \text{ and } P(\tilde{B})=\frac{b_1+4b_2+b_3}{6},$$

if $\tilde{A}$ and $\tilde{B}$ are trapezoidal fuzzy number, then the graded mean integration of $\tilde{A}$ and $\tilde{B}$ are also defined as follows:

$$P(\tilde{A})=\frac{a_1+2a_2+2a_3+a_4}{6} \text{ and } P(\tilde{B})=\frac{b_1+2b_2+2b_3+b_4}{6}.$$

Lee derived the trapezoidal similarity measure using fuzzy number operation and norm definition. That is

$$S(\tilde{A},\tilde{B})=1-\frac{\|\tilde{A}-\tilde{B}\|_{l_p}}{\|U\|}\times 4^{-1/p} \tag{3}$$

where $\|\tilde{A}-\tilde{B}\|_{l_p}=(\sum_i(|a_i-b_i|))^{1/p}$, $\|U\|=max(U)-min(U)$, and $p$ is the natural number greater or equal 1, finally $U$ is the universe of discourse.

Chen and Chen propose similarity measure to overcome the drawbacks of existing similarity:

$$S(\tilde{A},\tilde{B})=[1-\frac{\sum_i|a_i-b_i|}{4}]\times(1-|x_{\tilde{A}}^{*}-x_{\tilde{B}}^{*}|)^{B(S_{\tilde{A}},S_{\tilde{B}})} \times\frac{min(y_{\tilde{A}}^{*},y_{\tilde{B}}^{*})}{max(y_{\tilde{A}}^{*},y_{\tilde{B}}^{*})} \tag{4}$$

where $(x_{\tilde{A}}^{*},y_{\tilde{A}}^{*})$ and $(x_{\tilde{B}}^{*},y_{\tilde{B}}^{*})$ are the COG of fuzzy number $\tilde{A}$ and $\tilde{B}$, $S_{\tilde{A}}$ and $S_{\tilde{B}}$ are expressed by $S_{\tilde{A}}=a_4-a_1$ and $S_{\tilde{B}}=b_4-b_1$ if they are trapezoidal. $B(S_{\tilde{A}},S_{\tilde{B}})$ is denoted by 1 if $S_{\tilde{A}}+S_{\tilde{B}}>0$, and 0 if $S_{\tilde{A}}+S_{\tilde{B}}=0$. In (4), $B(S_{\tilde{A}},S_{\tilde{B}})$ is used to determine whether we consider the COG distance or not. All these results (1)~(4) are obtained through fuzzy number, those are restricted with triangular or trapezoidal fuzzy membership function. Hence this restriction is the fatal to the general fuzzy membership function application. Now we design similarity measure for general fuzzy membership function with distance measure.

### 2.2 Similarity measure with distance function

By the axiomatic definitions of Liu, distance measure and similarity measure can suggest the difference or closeness for different fuzzy membership functions [7]. By this definition, we design a similarity measure.

**Definition 2.1** A real function $d$: $F^2\to R^+$ is called a distance measure on $F$, if $d$ satisfies the following properties:

(D1) $d(A,B)=d(B,A),\forall A,B\in F(X)$

(D2) $d(A,A)=0,\ \forall A\in F(X)$

(D3) $d(D,D)=max_{A,B\in P}d(A,B),\ \forall D\in P(X)$

(D4) $\forall A,B,C\in F(X)$, if $A\subset B\subset C$, then $d(A,B)\leq d(A,C)$ and $d(B,C)\leq d(A,C)$.

Where $R^+=[0,\infty)$, $X$ is the universal set, $F(X)$ is the class of all fuzzy sets of $X$, $P(X)$ is the class of all crisp sets of $X$, and $D^c$ is the complement of $D$. There are a lot of distance measure satisfying Definition 2.1. Hamming distance is commonly used as distance measure between fuzzy sets $A$ and $B$,

$$d(A,B)=\frac{1}{n}\sum_{i=1}^{n}|\mu_A(x_i)-\mu_B(x_i)|$$

where $X=\{x_1,x_2,\cdots x_n\}$, $|k|$ is the absolute value of $k$. $\mu_A(x)$ is the membership function of $A\in F(X)$. Basically distance measure means the difference between two fuzzy membership functions. Next we will introduce the similarity measure, and it describes the degree of closeness between two fuzzy membership functions. It is also found in literature of Liu.

**Definition 2.2** [7] A real function $s$: $F^2\to R^+$ is called a similarity measure, if $s$ has the following properties:

(S1) $s(A,B)=s(B,A),\forall A,B\in F(X)$

(S2) $s(D, D^c) = 0 \ \ \forall D \in P(X)$
(S3) $s(C, C) = max_{A,B \in F} s(A, B), \ \ \forall C \in F(X)$
(S4) $\forall A, B, C \in F(X)$, if $A \subset B \subset C$, then $s(A, B) \geq s(A, C)$ and $s(B, C) \geq s(A, C)$.

Liu also pointed out that there is an one-to-one relation between all distance measures and all similarity measures, $d + s = 1$. Fuzzy normal similarity measure on $F$ is also obtained by the division of $\max_{C,D \in F} s(C, D)$. With Definition 2.1, we propose the following theorem as the similarity measure.

And the similarity measure construction with the distance measure is obtained in Theorem 2.1.

**Theorem 2.1** For any set $A, B \in F(X)$, if $d$ satisfies Hamming distance measure and $d(A, B) = d(A^C, B^C)$, then

$$s(A, B) = 1 - d((A \cap B^C), [0]) - d((A \cup B^C), [1]) \tag{5}$$

is the similarity measure between set $A$ and set $B$.

**proof.** Commutativity of (S1) is proved through

$$\begin{aligned} s(A, B) &= 1 - d((A \cap B^C), [0]) - d((A \cup B^C), [1]) \\ &= 1 - d((A \cap B^C)^C, [0]^C) - d((A \cup B^C)^C, [1]^C) \\ &= 1 - d((B \cup A^C), [1]) - d((B \cap A^C), [0]) \\ &= s(B, A). \end{aligned}$$

To show the property of (S2),

$$\begin{aligned} s(A, A^c) &= 1 - d((A \cap (A^c)^c), [0]) - d((A \cup (A^c)^c), [1]) \\ &= 1 - (d(A, [0]) + d(A, [1])) \\ &= 1 - 1 = 0 \end{aligned}$$

is clear. (S3) is clear from the relation

$$\begin{aligned} s(A, B) &= 1 - d((A \cap B^c), [0]) - d((A \cup B^c), [1]) \\ &\leqq 1 - d((D \cap D^c), [0]) - d((D \cup D^c), [1]) \\ &= s(D, D), \end{aligned}$$

where the inequality is proved by

$$d((A \cap B^c), [0]) \geqq d((D \cap D^c), [0])$$
$$\text{and } d((A \cup B^c), [1]) \geqq d((D \cup D^c), [1]).$$

Finally, $\forall A, B, C \in F(X)$ and $A \subset B \subset C$ imply

$$\begin{aligned} s(A, B) &= 1 - d((A \cap B^c), [0]) - d((A \cup B^c), [1]) \\ &= 1 - d([0], [0]) - d((A \cup B^c), [1]) \\ &\geqq 1 - d(A \cap C^c, [0]) - d(A \cup C^c, [1]) \\ &= 1 - d([0], [0]) - d(A \cup C^c, [1]) \\ &= s(A, C). \end{aligned}$$

Also

$$\begin{aligned} s(B, C) &= 1 - d((B \cap C^c), [0]) - d((B \cup C^c), [1]) \\ &= 1 - d([0], [0]) - d((B \cup C^c), [1]) \\ &\geqq 1 - d(A \cap C^c, [0]) - d(A \cup C^c, [1]) \\ &= 1 - d([0], [0]) - d(A \cup C^c, [1]) \\ &= s(A, C) \end{aligned}$$

is satisfied with $d((A \cup B^c), [1]) \geqq d((A \cup C^c), [1])$ and $d((B \cup C^c), [1]) \geqq d((A \cup C^c), [1])$.

We have proposed the similarity measure that are induced from distance measure. This similarity is useful for the non interacting fuzzy membership function pair. Another similarity is also obtained, and it can be found in our previous literature [6].

**Theorem 2.2** For any set $A, B \in F(X)$ if $d$ satisfies Hamming distance measure, then

$$s(A, B) = 2 - d((A \cap B), [1]) - d((A \cup B), [0]) \tag{6}$$

To be a similarity measure, Theorem 2.2 does not need condition $d(A, B) = d(A^C, B^C)$. Because commutativity is clear from the theorem itself. Also this similarity (6) is useful for the interacting membership function pair. In the next section, we derive similarity measure that is generated by distance measure. Furthermore entropy is derived through similarity measure by the properties of Liu.

### 2.3 Similarity measure using distance function

It is obvious that Hamming distance measure satisfy the properties of Definition 2.2. Next Hamming distance is represented as

$$d(A, B) = d((A \cap B), [1]) - (1 - d((A \cup B), [0])).$$

Where $A \cap B = min(\mu_A(x_i), \mu_B(x_i))$, $A \cup B = max(\mu_A(x_i), \mu_B(x_i))$ are satisfied. With the Proposition 3.4 of Liu[7], we can generate the similarity measure or distance measure through distance measure or similarity measure.

**Proposition 2.1** [7] There exists an one-to-one correlation between all distance measures and all similarity measures, and a distance measure $d$ and its corresponding similarity measure $s$ satisfy $s + d = 1$. With the property of $s = 1 - d$, we can construct the similarity measure generated by distance measure $d$, that is $s < d >$.

$$\begin{aligned} d(A, B) &= d((A \cap B), [1]) + d((A \cup B), [0]) - 1 \\ &= 1 - s(A, B) \end{aligned} \tag{7}$$

Therefore we propose the similarity measure with above expression.

$$s < d > = 2 - d((A \cap B), [1]) - d((A \cup B), [0]) \tag{8}$$

This similarity measure can be found in our previous results [6]. The usefulness of this similarity measure also can be found same literature. At this point, we verified the one-to-one relation of distance measure and similarity measure. We also noticed the fact that the similarity measure is generated through distance measure. In the next subsection, we verify that the entropy of fuzz set is derived through similarity (7).

### 2.4 Entropy derivation with similarity measure

Liu also suggested propositions about entropy and similarity measure [7]. He also insisted that the entropy can be generated by similarity measure and distance measure, those are denoted by $e<s>$ and $e<d>$. In reference 7, Proposition 3.5 and 3.6 are summarized as follows.

**Proposition 2.2** [7] If $s$ is a similarity measure on $F$, define $e(A)=s(A,A^c),\ \forall A\in F$.

Then $e$ is an entropy on $F$. Similarly, it is obvious that the following proposition is satisfied.

**Proposition 2.3** [7] If $d$ is a distance measure on $F$, define $e(A)=1-d(A,A^c),\ \forall A\in F$.

Then $e$ is an entropy on $F$.

Now we check whether our similarity (6) satisfy Proposition 2.2.
**Proof** can be obtained by checking from (E1) to (E4) of Definition 2.1. Hence, entropy defined by similarity is $s(A,A^c)=2-d((A\cap A^c),[1])-d((A\cup A^c),[0])$.

For (E1), $\forall D\in P(X)$,

$$\begin{aligned} s(D,D^c) &= 2-d((D\cap D^c),[1])-d((D\cup D^c),[0]) \\ &= 2-d([0],[1])-d([1],[0])=0 \end{aligned}$$

(E2) represents that crisp set $1/2$ has the maximum entropy value. Therefore, the entropy $e([1/2])$ satisfies

$$\begin{aligned} s([1/2],[1/2]^c) &= 2-d(([1/2]\cap[1/2]^c),[1]) \\ &\quad -d(([1/2]\cup[1/2]^c),[0]) \\ &= 2-d([1/2],[1])-d([1/2],[0]) \\ &= 2-1/2-1/2=1 \end{aligned}$$

In the above equation, $[1/2]^c=[1/2]$ is satisfied.

(E3) shows that the entropy of the sharpened version of fuzzy set $A$, $e(A^*)$, is less than or equal to $e(A)$.

$$\begin{aligned} s(A^*,A^{*c}) &= 2-d((A^*\cap A^{*c}),[1])-d((A^*\cup A^{*c}),[0]) \\ &\leq 2-d((A\cap A^c),[1])-d((A\cup A^c),[0]) \\ &= s(A,A^c) \end{aligned}$$

Finally, (E4) is proved directly

$$\begin{aligned} s(A,A^c) &= 2-d((A\cap A^c),[1])-d((A\cup A^c),[0]) \\ &= 2-d((A^c\cap A),[1])-d((A^c\cup A),[0]) \\ &= s(A^c,A) \end{aligned}$$

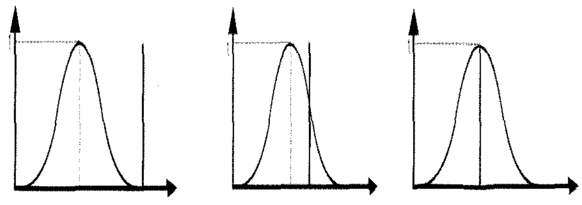
From the above proof, our similarity measure

$$s(A,A^c)=2-d((A\cap A^c),[1])-d((A\cup A^c),[0])$$

satisfies fuzzy entropy. In the following chapter we present the similarity measure for fuzzy set and crisp set by changing the fuzzy set as crisp set.

## 3. Computational Analysis of Similarity Measures

With the proposed similarity measure we have computed the degree of similarity between fuzzy membership function in our previous paper [6]. However it is often required to compute similarity between fuzzy set and deterministic data. For that case, how can we compute the degree of similarity between fuzzy set and crisp set ? In Fig. 1 there are three membership function pairs. All three pairs have the same degree of similarity ? Naturally it must have the different degree of similarity.



(a) (b) (c)

Fig. 1. Similarity between fuzzy set and crisp set

Now we replace fuzzy set $B$ (5) and (6) to the crisp set $A_{near}$ of $A$,

where $\mu_{A_{near}}(x)=\begin{cases}1 & \mu_A(x)\geq \frac{1}{2}\\ 0 & \mu_A(x)<\frac{1}{2}\end{cases}$.

$A_{near}$ is represented as Fig. 2, rectangular shape of membership function of $A$ reveal crisp set. If the width of rectangle become narrow, it becomes the singleton as in Fig. 1. Furthermore two times of graded area of Fig. 2 represent fuzzy entropy of fuzzy set $A$ [9]. Two membership functions are identical, then entropy can be zero, this means that degree of similarity becomes maximum value.

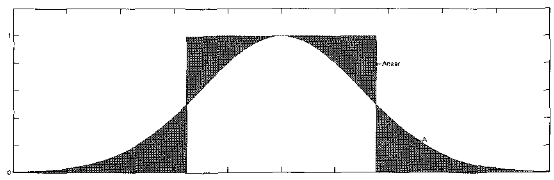


Fig. 2. Fuzzy set and crisp set membership function

Now it is possible to represent the degree of similarity between fuzzy set and crisp set. In next theorem we replace fuzzy set $B$ into crisp set $A_{near}$, this means degree of similarity between fuzzy set $A$ and crisp set $A_{near}$.

**Theorem 3.1** For fuzzy set $A\in F(X)$ and Hamming distance measure $d$,

$$s(A,A_{near})=2-2d((A\cap A_{near}^C),[0])-2d((A\cup A_{near}^C),[1]) \quad (9)$$

is the similarity measure of fuzzy set $A$ and crisp set $A_{near}$.

**Proofs** are found in reference [6].

Similarly we can extend Theorem 3.2 for the similarity measure between fuzzy set and crisp set.

**Theorem 3.2** For any set $A \in F(X)$ if $d$ satisfies Hamming distance measure, then

$$s(A, A_{near}) = 2 - d((A \cap A_{near}), [1]) - d((A \cup A_{near}), [0]) \quad (10)$$

is the similarity measure of fuzzy set $A$ and crisp set $A_{near}$.

**Proofs** are also found in [6]. In the following Chapter 4 we compute the degree of similarity between fuzzy set and crisp set. The results are compared with previous example.

## 4. Computation of Similarity Measures

In this section, we compare the our similarity measure (9) and (10) with Chen and Chen's similarity measure (4) [4]. As mentioned before, similarity measures based on fuzzy number have to depend on the membership function shapes. They consider only trapezoidal or triangular type membership function. Whereas our similarity measure do not consider about shapes. In [4], Chen and Chen illustrated twelve membership function pairs and computed the degree of similarity. Among them one pair is shown for the computation of degree of similarity between fuzzy set and crisp set as Fig. 3.

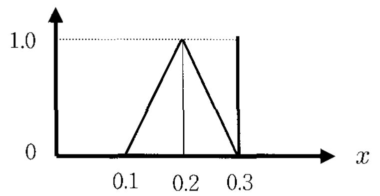



Fig. 3. Fuzzy set and crisp set pair [4]

In the above figure, fuzzy number of fuzzy set is denoted by $\widetilde{A} = (0.1, 0.2, 0.3, 1.0)$ and singleton fuzzy number is also denoted as $\widetilde{B} = (0.3, 0.3, 0.3, 1.0)$. Computation results are illustrated in Table 1. 4 previous results are all obtained through fuzzy number.

Table 1. Comparison with the result of Chen and Chen

| Previous computations | similarity value |
|---|---|
| Lee[3] | 0.5 |
| Hsieh and Chen[2] | 0.909 |
| Chen [1] | 0.9 |
| Chen and Chen[4] | 0.54 |

One of computation in Table 1, Chen and Chen computed the degree of similarity as follows

$$S(\widetilde{A}, \widetilde{B}) = [1 - \frac{0.2 + 0.1 + 0}{3}](1 - 0.1)^1 \times \frac{min(1/3, 0.5)}{max(1/3, 0.5)} = 0.54$$

This result is also obtained through fuzzy number, so computation was easy to obtain. However, the result is strictly limited for the trapezoidal or triangular membership functions. That means fuzzy membership function of fuzzy set $A$ has to keep triangular or trapezoidal shape. Whereas with proposed similarity measure fuzzy membership function shape is not needed to keep triangular or trapezoidal shape. Proposed similarity measures (9) and (10) are useful for general shapes of fuzzy membership functions.

We compute the similarity measure between fuzzy set and crisp set in Fig. 3. Computation circumstances are illustrated as follows.

Our computation conditions are:

Universe of discourse : 0.1 ~ 1.0
Data points : 100
Sample distance : 0.01

For fuzzy set $A$, domain can be from 0.1 to 0.3 among universe of discourse, whereas crisp set $B$ can only be 0.3. We apply similarity measure (9). We have obtained the degree of similarity of Fig. 3 as follows

The degree of similarity : 0.476

Our proposed similarity measures are also possible to compute the degree of similarity for the general membership function pairs.

## 5. Conclusions

We have introduced the fuzzy number and the similarity measure that is derived from fuzzy number. We also proposed a similarity measure based on the distance measure. The usefulness of proposed similarity measure is proved. By the comparison with degree of similarity based on the fuzzy number, we can see that proposed similarity measure can be applied to the general types of fuzzy membership functions.

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**Park, Hyun-Jeong**
She received the B.S. in Mathematics from Kyunghee University, in 1992, M.S. and Ph.D. degrees in Mathematics Education from Ewha Womans University, in 2001 and 2007, respectively. She served as a Researcher of Mathematics from Mar. 2002 to Dec. 2002 in The Korean Institute of Curriculum and Evaluation. Currently, she is an Instructor of Teaching Psychology of Mathematical Reasoning, Research and guiding curriculum planning in Ewha Womans University and Kyunghee University. Her research interests include fuzzy theory, In-depth examination on thinking processes in students when solving mathematical problems, Evaluation of thinking process using qualitative methods during the process of solving problem or understanding concepts.

E-mail : hyunjp@ewhain.net

**Sang-Hyuk Lee**
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E-mail : leehyuk@changwon.ac.kr