International Journal of Fuzzy Logic and Intelligent Systems, vol. 7, no. 4, December 2007 pp. 262-266

Fuzzy pairwise γ-irresoluteness

Young Bin Im

Department of Mathematics, Seonam University, Namwon 590-711, Korea

Abstract

We characterize a fuzzy pairwise γ-irresolute continuous mapping on a fuzzy bitopological space.

Key words : fuzzy pairwise γ-irresolute continuous mapping, (τ_i, τ_j)-γ-interior, (τ_i, τ_j)-γ-closure

1. Introduction

Azad [1], Singal and Prakash [9] introduced the concepts of a fuzzy semiopen set and a fuzzy preopen set on a fuzzy topological space respectively, and characterized a fuzzy semicontinuous mapping and a fuzzy precontinuous mapping on a fuzzy topological space. Weaker forms of a fuzzy pairwise continuity on a fuzzy bitopological space as a natural generalization of a fuzzy topological spaces have been considered by several mathematicians using a (τ_i, τ_j)-fuzzy semiopen set and a (τ_i, τ_j)-fuzzy preopen set. In particular, Sampath Kumar [7, 8] defined a $(\tau_i, \tau_j)-$fuzzy semiopen set and a $(\tau_i, \tau_j)-$fuzzy preopen set, and characterized a fuzzy pairwise semicontinuous mapping and a fuzzy pairwise precontinuous mapping on a fuzzy bitopological space.

Recently, Hanafy [2] defined a fuzzy γ-open set, and studied a fuzzy γ-continuous mapping on a fuzzy topological space. The author et al. [3, 5] defined a fuzzy γ-irresolute (fuzzy γ-irresolute open) mapping on a fuzzy topological space and investigated some of their properties. Also, he et al. [4, 6] defined a fuzzy pairwise γ-continuous mapping and a fuzzy pairwise pre-irresolute mapping on a fuzzy bitopological space and characterized.

In this paper, we characterize a fuzzy pairwise γ-irresolute continuous mapping on a fuzzy bitopological space.

2. Preliminaries

A system (X, τ_1, τ_2) consisting of a set X with two fuzzy topologies τ_1 and τ_2 on X is called a *fuzzy bitopological space* $[fbts]$. Throughout this paper, the indices i, j take values in $\{1, 2\}$ with $i \neq j$.

Manuscript received Mar. 25, 2007; revised Dec. 17, 2007.
CorrespondingAuthor:philpen@naver.com

Let μ be a fuzzy set in a $fbts$ (X, τ_1, τ_2). Then $\tau_i - fo$ set μ and $\tau_j - fc$ set μ mean τ_i-fuzzy open set μ and τ_j-fuzzy closed set μ respectively. Also, $\tau_i - \mathrm{Int}\,\mu$ and $\tau_j - \mathrm{Cl}\,\mu$ mean the interior and closure of μ for the fuzzy topologies τ_i and τ_j respectively.

A mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \tau_1^*, \tau_2^*)$ is *fuzzy pairwise continuous* $[fpc]$ if and only if the induced mapping $f : (X, \tau_k) \rightarrow (Y, \tau_k^*)$ is fuzzy continuous for $(k = 1, 2)$.

Definition 2.1. [7, 8] Let μ be a fuzzy set of a $fbts$ X. Then μ is called;

(1) $(\tau_i, \tau_j)-$fuzzy semiopen $[(\tau_i, \tau_j) - fso]$ in X if $\mu \leq \tau_j - \mathrm{Cl}(\tau_i - \mathrm{Int}\,\mu)$,

(2) $(\tau_i, \tau_j)-$fuzzy semiclosed $[(\tau_i, \tau_j) - fsc]$ in X if $\tau_j - \mathrm{Int}(\tau_i - \mathrm{Cl}\,\mu) \leq \mu$,

(3) $(\tau_i, \tau_j)-$fuzzy preopen $[(\tau_i, \tau_j) - fpo]$ in X if $\mu \leq \tau_i - \mathrm{Int}(\tau_j - \mathrm{Cl}\,\mu)$,

(4) $(\tau_i, \tau_j)-$fuzzy preclosed $[(\tau_i, \tau_j) - fpc]$ in X if $\tau_i - \mathrm{Cl}(\tau_j - \mathrm{Int}\,\mu) \leq \mu$.

Definition 2.2. [6] Let μ be a fuzzy set of a $fbts$ X. Then μ is called;

(1) a $(\tau_i, \tau_j)-$fuzzy $\gamma-$open $[(\tau_i, \tau_j) - f\gamma o]$ set of X if $\mu \leq \tau_j - \mathrm{Cl}(\tau_i - \mathrm{Int}\,\mu) \vee \tau_i - \mathrm{Int}(\tau_j - \mathrm{Cl}\,\mu)$,

(2) a $(\tau_i, \tau_j)-$fuzzy $\gamma-$closed $[(\tau_i, \tau_j) - f\gamma c]$ set of X if $\tau_i - \mathrm{Cl}(\tau_j - \mathrm{Int}\,\mu) \wedge \tau_j - \mathrm{Int}(\tau_i - \mathrm{Cl}\,\mu) \leq \mu$.

Remark that every $(\tau_i, \tau_j) - fso$ set is a $(\tau_i, \tau_j) - f\gamma o$ set and every $(\tau_i, \tau_j) - fpo$ set is a $(\tau_i, \tau_j) - f\gamma o$ set. The converses need not be true in general [6].

Proposition 2.3. [6] (1) The union of $(\tau_i, \tau_j) - f\gamma o$ sets is a $(\tau_i, \tau_j) - f\gamma o$ set.

(2) The intersection of $(\tau_i, \tau_j) - f\gamma c$ sets is a $(\tau_i, \tau_j) - f\gamma c$ set.

The intersection (union) of any two $(\tau_i, \tau_j) - f\gamma o$ ($(\tau_i, \tau_j) - f\gamma c$) sets need not be a $(\tau_i, \tau_j) - f\gamma o$ ($(\tau_i, \tau_j) - f\gamma c$) set [6].

Proposition 2.4. [6] Let μ be a fuzzy set of a $fbts$ X.

(1) If μ is a $(\tau_i, \tau_j) - f\gamma o$ and $\tau_j - fc$ set, then μ is a $(\tau_i, \tau_j) - fso$ set.

(2) If μ is a $(\tau_i, \tau_j) - f\gamma c$ and $\tau_j - fo$ set, then μ is a $(\tau_i, \tau_j) - fsc$ set.

Proposition 2.5. [6] Let (X, τ_1, τ_2) and (Y, η_1, η_2) be $fbts$'s such that X is product related to Y. Then the product $\mu \times \nu$ of a $(\tau_i, \tau_j) - f\gamma o$ set μ of X and a $(\eta_1, \eta_2) - f\gamma o$ set ν of Y is a $(\sigma_i, \sigma_j) - f\gamma o$ set in the fuzzy product bitopological space $(X \times Y, \sigma_1, \sigma_2)$, where σ_k is the fuzzy product topology generated by τ_k and η_k $(k = 1, 2)$.

Definition 2.6. [6] Let μ be a fuzzy set of a $fbts$ X.

(1) The $(\tau_i, \tau_j) - \gamma$–interior of μ [$(\tau_i, \tau_j) - \gamma$ Intμ] is defined by

$$(\tau_i, \tau_j) - \gamma \text{ Int}\mu = \sup\{\nu \mid \nu \leq \mu,\ \nu \text{ is a } (\tau_i, \tau_j) - f\gamma o \text{ set}\}.$$

(2) The $(\tau_i, \tau_j) - \gamma$–closure of μ [$(\tau_i, \tau_j) - \gamma$ Clμ)] is defined by

$$(\tau_i, \tau_j) - \gamma \text{ Cl}\mu = \inf\{\nu \mid \nu \geq \mu,\ \nu \text{ is a } (\tau_i, \tau_j) - f\gamma c \text{ set}\}.$$

Obviously, $(\tau_i, \tau_j) - \gamma$ Clμ is the smallest $(\tau_i, \tau_j) - f\gamma c$ set which contains μ, and $(\tau_i, \tau_j) - \gamma$ Intμ is the largest $(\tau_i, \tau_j) - f\gamma o$ set which is contained in μ. Also, $(\tau_i, \tau_j) - \gamma$ Cl$\mu = \mu$ for any $(\tau_i, \tau_j) - f\gamma c$ set μ and $(\tau_i, \tau_j) - \gamma$ Int$\mu = \mu$ for any $(\tau_i, \tau_j) - f\gamma o$ set μ.

Hence we have

$$\tau_i - \text{Int}\mu \leq (\tau_i, \tau_j) - \text{sInt}\mu \leq (\tau_i, \tau_j) - \gamma \text{ Int}\mu \leq \mu,$$

$$\mu \leq (\tau_i, \tau_j) - \gamma \text{ Cl}\mu \leq (\tau_i, \tau_j) - \text{sCl}\mu \leq \tau_i - \text{Cl}\mu$$

and

$$\tau_i - \text{Int}\mu \leq (\tau_i, \tau_j) - \text{pInt}\mu \leq (\tau_i, \tau_j) - \gamma \text{ Int}\mu \leq \mu,$$

$$\mu \leq (\tau_i, \tau_j) - \gamma \text{ Cl}\mu \leq (\tau_i, \tau_j) - \text{pCl}\mu \leq \tau_i - \text{Cl}\mu.$$

Definition 2.7. [6, 7, 8] Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ be a mapping. Then f is called;

(1) a fuzzy pairwise semicontinuous [$fpsc$] mapping if $f^{-1}(\nu)$ is a $(\tau_i, \tau_j) - fso$ set of X for each $\tau_i^* - fo$ set ν of Y,

(2) a fuzzy pairwise precontinuous [$fppc$] mapping if $f^{-1}(\nu)$ is a $(\tau_i, \tau_j) - fpo$ set of X for each $\tau_i^* - fo$ set ν of Y,

(3) a fuzzy pairwise γ–continuous [$fp\gamma c$] mapping if $f^{-1}(\nu)$ is a $(\tau_i, \tau_j) - f\gamma o$ set of X for each $\tau_i^* - fo$ set ν of Y.

From the above definitions it is clear that every $fpsc$ is a $fp\gamma c$ mapping and every $fppc$ is a $fp\gamma c$ mapping. But the converses are not true in general [6].

Theorem 2.8. [6] Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ be a mapping. Then the followings are equivalent:

(1) f is $fp\gamma c$.

(2) The inverse image of each $\tau_i^* - fc$ set of Y is a $(\tau_i, \tau_j) - f\gamma o$ set of X.

(3) $f((\tau_i, \tau_j) - \gamma \text{ Cl}\mu) \leq \tau_i^* - \text{Cl}(f(\mu))$ for each fuzzy set μ of X.

(4) $(\tau_i, \tau_j) - \gamma \text{ Cl}(f^{-1}(\nu)) \leq f^{-1}(\tau_i^* - \text{Cl}\nu)$ for each fuzzy set ν of Y.

(5) $f^{-1}(\tau_i^* - \text{Int}\nu) \leq (\tau_i, \tau_j) - \gamma \text{ Int}(f^{-1}(\nu))$ for each fuzzy set ν of Y.

Proposition 2.9. [6] Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ be a $fp\gamma c$ mapping. Then for each fuzzy set ν of Y,

$$f^{-1}(\tau_i^* - \text{Int}\nu) \leq \tau_j - \text{Cl}(\tau_i - \text{Int}(f^{-1}(\nu))) \vee \tau_i - \text{Int}(\tau_j - \text{Cl}(f^{-1}(\nu))).$$

Proposition 2.10. Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ be a $fp\gamma c$ mapping. Then for each fuzzy set ν of Y,

$$\tau_i - \text{Cl}(\tau_j - \text{Int}(f^{-1}(\nu))) \wedge \tau_j - \text{Int}(\tau_i - \text{Cl}(f^{-1}(\nu))) \leq f^{-1}(\tau_i^* - \text{Cl}\nu).$$

Proof. Let ν be a fuzzy set of Y. Then $\tau_i^* - \text{Cl}\nu$ is a $(\tau_i^*, \tau_j^*) - f\gamma c$ set of Y and so $f^{-1}(\tau_i^* - \text{Cl}\nu)$ is a $(\tau_i, \tau_j) - f\gamma c$ set of X. Hence

$$\begin{aligned}
&\tau_i - \text{Cl}(\tau_j - \text{Int}(f^{-1}(\nu))) \wedge \tau_j - \text{Int}(\tau_i - \text{Cl}(f^{-1}(\nu))) \\
&\leq \tau_i - \text{Cl}(\tau_j - \text{Int}(f^{-1}(\tau_i^* - \text{Cl}\nu))) \wedge \\
&\tau_j - \text{Int}(\tau_i - \text{Cl}(f^{-1}(\tau_i^* - \text{Cl}\nu))) \\
&\leq f^{-1}(\tau_i^* - \text{Cl}\nu).
\end{aligned}$$

Proposition 2.11. Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ be a $fp\gamma c$ mapping. Then for each fuzzy set μ of X,

$$f(\tau_i - \text{Cl}(\tau_j - \text{Int}\mu) \wedge \tau_j - \text{Int}(\tau_i - \text{Cl}\mu)) \\ \leq \tau_i^* - \text{Cl}(f(\mu)).$$

Proof. Let μ be a fuzzy set of X. Then, by the above theorem,

$$\tau_i - \text{Cl}(\tau_j - \text{Int}\mu) \wedge \tau_j - \text{Int}(\tau_i - \text{Cl}\mu) \\ \leq \tau_i - \text{Cl}(\tau_j - \text{Int}(f^{-1}(f(\mu)))) \wedge \\ \tau_j - \text{Int}(\tau_i - \text{Cl}(f^{-1}(f(\mu)))) \\ \leq f^{-1}(\tau_i^* - \text{Cl}(f(\mu))).$$

and

$$f(\tau_i - \text{Cl}(\tau_j - \text{Int}\mu) \wedge \tau_j - \text{Int}(\tau_i - \text{Cl}\mu)) \\ \leq f(f^{-1}(\tau_i^* - \text{Cl}(f(\mu)))) \\ \leq \tau_i^* - \text{Cl}(f(\mu)).$$

Proposition 2.12. [6] Let (X_1, τ_1, τ_2), $(X_2, \tau_1^*, \tau_2^*)$, (Y_1, η_1, η_2) and $(Y_2, \eta_1^*, \eta_2^*)$ be $fbts$'s such that X_1 is product related to X_2. Then the product $f_1 \times f_2 : (X_1 \times X_2, \theta_1, \theta_2) \to (Y_1 \times Y_2, \sigma_1, \sigma_2)$, where θ_k (respectively σ_k) is the fuzzy product topology generated by τ_k and τ_k^* (respectively η_k and η_k^*) $(k = 1, 2)$, of $fp\gamma c$ mappings $f_1 : (X_1, \tau_1, \tau_2) \to (Y_1, \eta_1, \eta_2)$ and $f_2 : (X_2, \tau_1^*, \tau_2^*) \to (Y_2, \eta_1^*, \eta_2^*)$, is a $fp\gamma c$ mapping.

3. Fuzzy pairwise γ-irresolute continuous mappings

Definition 3.1. Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ be a mapping. Then f is called a fuzzy pairwise γ-irresolute [$fp\gamma$-irresolute] continuous mapping if $f^{-1}(\nu)$ is a $(\tau_i, \tau_j) - f\gamma o$ set of X for each $(\tau_i^*, \tau_j^*) - f\gamma o$ set ν of Y.

From the above definitions it is clear that every $fp\gamma$-irresolute continuous mapping is a $fp\gamma c$ mapping. But the converse is not true in general. A $fpsc$ mapping and a $fp\gamma$-irresolute continuous mapping do not have specific relations. Also, $fppc$ mapping and $fp\gamma$-irresolute continuous mapping are independent.

Example 3.2. Let μ_1, μ_2, μ_3 and μ_4 be fuzzy sets of $X = \{a, b, c\}$, defined as follows:

$$\mu_1(a) = 0.9, \mu_1(b) = 0.9, \mu_1 = 0.9, \\ \mu_2(a) = 0.2, \mu_2(b) = 0.2, \mu_2 = 0.2, \\ \mu_3(a) = 0.4, \mu_3(b) = 0.4, \mu_3 = 0.4, \\ \mu_4(a) = 0.7, \mu_4(b) = 0.7, \mu_4 = 0.7.$$

Consider fuzzy topologies

$$\tau_1 = \{0_X, \mu_4, 1_X\}, \tau_2 = \{0_X, \mu_3, 1_X\}, \\ \tau_1^* = \{0_X, \mu_1, 1_X\}, \tau_2^* = \{0_X, 1_X\}.$$

Then the identity mapping $i_X : (X, \tau_1, \tau_2) \to (X, \tau_1^*, \tau_2^*)$ is $fp\gamma c$. Also, i_X are $fpsc$ and $fppc$. But i_X is not $fp\gamma$-irresolute continuous.

Example 3.3. Let μ_1, μ_2, μ_3 and μ_4 be fuzzy sets of $X = \{a, b, c\}$ in Example 3.2. Consider fuzzy topologies

$$\tau_1 = \{0_X, \mu_4^c, 1_X\}, \tau_2 = \{0_X, \mu_2, 1_X\}, \\ \tau_1^* = \{0_X, \mu_4, 1_X\}, \tau_2^* = \{0_X, \mu_1, 1_X\}.$$

Then the identity mapping $i_X : (X, \tau_1, \tau_2) \to (X, \tau_1^*, \tau_2^*)$ is $fp\gamma$-irresolute continuous. But i_X is not $fppc$.

Example 3.4. Let μ_1, μ_2, μ_3 and μ_4 be fuzzy sets of $X = \{a, b, c\}$ in Example 3.2. Consider fuzzy topologies

$$\tau_1 = \{0_X, \mu_4^c, 1_X\}, \tau_2 = \{0_X, \mu_2, 1_X\}, \\ \tau_1^* = \{0_X, \mu_1^c, 1_X\}, \tau_2^* = \{0_X, \mu_1, 1_X\}.$$

Then the identity mapping $i_X : (X, \tau_1, \tau_2) \to (X, \tau_1^*, \tau_2^*)$ is $fp\gamma$-irresolute continuous. But i_X is not $fpsc$.

Theorem 3.5. Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ be a mapping. Then the followings are equivalent:

(1) f is $fp\gamma$-irresolute continuous.

(2) The inverse image of each $(\tau_i^*, \tau_j^*) - f\gamma c$ set of Y is a $(\tau_i, \tau_j) - f\gamma c$ set of X.

(3) $f((\tau_i, \tau_j) - \gamma\text{Cl}\mu) \leq (\tau_i^*, \tau_j^*) - \gamma\text{Cl}(f(\mu))$ for each fuzzy set μ of X.

(4) $(\tau_i, \tau_j) - \gamma\text{Cl}(f^{-1}(\nu)) \leq f^{-1}((\tau_i^*, \tau_j^*) - \gamma\text{Cl}\nu)$ for each fuzzy set ν of Y.

Proof. (1) implies (2): Let ν be a $(\tau_i^*, \tau_j^*) - f\gamma c$ set of Y. Then ν^c is a $(\tau_i^*, \tau_j^*) - f\gamma o$ set. Since f is $fp\gamma$-irresolute continuous, $f^{-1}(\nu^c) = (f^{-1}(\nu))^c$ is a $(\tau_i, \tau_j) - f\gamma o$ set of X. Hence $f^{-1}(\nu)$ is a $(\tau_i, \tau_j) - f\gamma o$ set of X.

(2) implies (1): Let ν be a $(\tau_i^*, \tau_j^*) - f\gamma o$ set of Y. Then ν^c is a $(\tau_i^*, \tau_j^*) - f\gamma c$ set and $f^{-1}(\nu^c) = (f^{-1}(\nu))^c$

is a $(\tau_i,\tau_j)-f\gamma c$ set of X. Since $f^{-1}(\nu)$ is $(\tau_i,\tau_j)-f\gamma o$ set of X, f is $fp\gamma$-irresolute continuous.

(2) implies (3): Let μ be a fuzzy set of X. Since $f((\tau_i,\tau_j)-\gamma\,\mathrm{Int}\mu)$ is a $(\tau_i^*,\tau_j^*)-f\gamma c$ set of Y, $f^{-1}(f((\tau_i,\tau_j)-\gamma\,\mathrm{Int}\mu))$ is a $(\tau_i,\tau_j)-f\gamma c$ set of X. Hence

$$\begin{aligned}&(\tau_i,\tau_j)-\gamma\,\mathrm{Cl}\mu\\&\le(\tau_i,\tau_j)-\gamma\,\mathrm{Cl}(f^{-1}(f(\mu)))\\&\le(\tau_i,\tau_j)-\gamma\,\mathrm{Cl}(f^{-1}((\tau_i^*,\tau_j^*)-\gamma\,\mathrm{Cl}(f(\mu)))\\&=f^{-1}((\tau_i^*,\tau_j^*)-\gamma\,\mathrm{Cl}(f(\mu))).\end{aligned}$$

and

$$\begin{aligned}&f((\tau_i,\tau_j)-\gamma\,\mathrm{Cl}\mu)\\&\le f(f^{-1}((\tau_i^*,\tau_j^*)-\gamma\,\mathrm{Cl}(f(\mu))))\\&\le(\tau_i^*,\tau_j^*)-\gamma\,\mathrm{Cl}(f(\mu)).\end{aligned}$$

(3) implies (4): Let ν be a fuzzy set of Y. Then

$$\begin{aligned}&f((\tau_i,\tau_j)-\gamma\,\mathrm{Cl}(f^{-1}(\nu)))\\&\le(\tau_i^*,\tau_j^*)-\gamma\,\mathrm{Cl}(f(f^{-1}(\nu)))\\&\le(\tau_i^*,\tau_j^*)-\gamma\,\mathrm{Cl}\nu.\end{aligned}$$

Thus

$$\begin{aligned}&(\tau_i^*,\tau_j^*)-\gamma\,\mathrm{Cl}(f^{-1}(\nu))\\&\le f^{-1}(f((\tau_i^*,\tau_j^*)-\gamma\,\mathrm{Cl}(f^{-1}(\nu))))\\&\le f^{-1}((\tau_i^*,\tau_j^*)-\gamma\,\mathrm{Cl}\nu).\end{aligned}$$

(4) implies (2): Let ν be a $(\tau_i^*,\tau_j^*)-f\gamma c$ set of Y. Then

$$\begin{aligned}(\tau_i,\tau_j)-\gamma\,\mathrm{Cl}(f^{-1}(\nu))&\le f^{-1}((\tau_i^*,\tau_j^*)-\gamma\,\mathrm{Cl}\nu)\\&=f^{-1}(\nu).\end{aligned}$$

Therefore, $f^{-1}(\nu)$ is a $(\tau_i,\tau_j)-f\gamma c$ set of X.

Theorem 3.6. A mapping $f:(X,\tau_1,\tau_2)\to(Y,\tau_1^*,\tau_2^*)$ is a $fp\gamma$-irresolute continuous if and only if for each fuzzy set ν of Y,

$$f^{-1}((\tau_i^*,\tau_j^*)-\gamma\,\mathrm{Int}\nu)\le(\tau_i,\tau_j)-\gamma\,\mathrm{Int}(f^{-1}(\nu)).$$

Proof. Let ν be a fuzzy set of Y. Then $(\tau_i^*,\tau_j^*)-\gamma\,\mathrm{Int}\nu\le\nu$. Since f is $fp\gamma$-irresolute continuous, $f^{-1}((\tau_i^*,\tau_j^*)-\gamma\,\mathrm{Int}\nu)$ is a $(\tau_i,\tau_j)-f\gamma o$ set of X. Hence

$$\begin{aligned}&f^{-1}((\tau_i^*,\tau_j^*)-\gamma\,\mathrm{Int}\nu)\\&=(\tau_i,\tau_j)-\gamma\,\mathrm{Int}(f^{-1}((\tau_i^*,\tau_j^*)-\gamma\,\mathrm{Int}\nu))\\&\le(\tau_i,\tau_j)-\gamma\,\mathrm{Int}(f^{-1}(\nu)).\end{aligned}$$

Conversely, let ν be a $(\tau_i^*,\tau_j^*)-f\gamma o$ set of Y. Then

$$\begin{aligned}f^{-1}(\nu)&=f^{-1}((\tau_i^*,\tau_j^*)-\gamma\,\mathrm{Int}\nu)\\&\le(\tau_i,\tau_j)-\gamma\,\mathrm{Int}(f^{-1}(\nu)).\end{aligned}$$

Therefore, $f^{-1}(\nu)$ is a $(\tau_i,\tau_j)-f\gamma o$ set of X and consequently f is $fp\gamma$-irresolute continuous.

Theorem 3.7. Let $f:(X,\tau_1,\tau_2)\to(Y,\tau_1^*,\tau_2^*)$ be a bijection. Then f is $fp\gamma$-irresolute continuous if and only if for each fuzzy set μ of X,

$$(\tau_i^*,\tau_j^*)-\gamma\,\mathrm{Int}(f(\mu))\le f((\tau_i,\tau_j)-\gamma\,\mathrm{Int}\mu)).$$

Proof. Let μ be a fuzzy set of X. Then by the above theorem,

$$f^{-1}((\tau_i^*,\tau_j^*)-\gamma\,\mathrm{Int}(f(\mu)))\le(\tau_i,\tau_j)-\gamma\,\mathrm{Int}(f^{-1}(f(\mu))).$$

Since f is a bijection,

$$\begin{aligned}&(\tau_i^*,\tau_j^*)-\gamma\,\mathrm{Int}(f(\mu))\\&=f(f^{-1}((\tau_i^*,\tau_j^*)-\gamma\,\mathrm{Int}(f(\mu))))\\&\le f((\tau_i,\tau_j)-\gamma\,\mathrm{Int}(f^{-1}(f(\mu))))\\&=f((\tau_i,\tau_j)-\gamma\,\mathrm{Int}\mu).\end{aligned}$$

Conversely, let ν be a $(\tau_i^*,\tau_j^*)-f\gamma o$ set of Y. Then

$$(\tau_i^*,\tau_j^*)-\gamma\,\mathrm{Int}(f(f^{-1}(\nu)))\le f((\tau_i,\tau_j)-\gamma\,\mathrm{Int}(f^{-1}(\nu))).$$

Since f is a bijection,

$$(\tau_i^*,\tau_j^*)-\gamma\,\mathrm{Int}\nu\le f((\tau_i,\tau_j)-\gamma\,\mathrm{Int}(f^{-1}(\nu))).$$

This implies that

$$\begin{aligned}&f^{-1}((\tau_i^*,\tau_j^*)-\gamma\,\mathrm{Int}\nu)\\&\le f^{-1}(f((\tau_i,\tau_j)-\gamma\,\mathrm{Int}(f^{-1}(\nu))))\\&=(\tau_i,\tau_j)-\gamma\,\mathrm{Int}(f^{-1}(\nu)).\end{aligned}$$

Therefore, by the above theorem, f is $fp\gamma$-irresolute continuous.

Theorem 3.8. Let $f:(X,\tau_1,\tau_2)\to(Y,\tau_1^*,\tau_2^*)$ be $fp\gamma$-irresolute continuous. Then for each fuzzy set ν of Y,

$$\begin{aligned}&f^{-1}((\tau_i^*,\tau_j^*)-\gamma\,\mathrm{Int}\nu)\le\\&\tau_j-\mathrm{Cl}(\tau_i-\mathrm{Int}(f^{-1}(\nu)))\vee\tau_i-\mathrm{Int}(\tau_j-\mathrm{Cl}(f^{-1}(\nu))).\end{aligned}$$

Proof. Let ν be a fuzzy set of Y. Then $(\tau_i^*, \tau_j^*)-\gamma$ Intν is a $(\tau_i^*, \tau_j^*) - f\gamma o$ set of Y and so $f^{-1}((\tau_i^*, \tau_j^*) - \gamma \text{ Int}\nu)$ is a $(\tau_i, \tau_j) - f\gamma o$ set of X. Hence

$$\begin{aligned}&f^{-1}((\tau_i^*, \tau_j^*) - \gamma \text{ Int}\nu) \leq \\ &\tau_j - \text{Cl}(\tau_i - \text{Int}(f^{-1}((\tau_i^*, \tau_j^*) - \gamma \text{ Int}\nu)))\vee \\ &\tau_i - \text{Int}(\tau_j - \text{Cl}(f^{-1}((\tau_i^*, \tau_j^*) - \gamma \text{ Int}\nu))) \leq \\ &\tau_j - \text{Cl}(\tau_i - \text{Int}(f^{-1}(\nu))) \vee \tau_i - \text{Int}(\tau_j - \text{Cl}(f^{-1}(\nu))).\end{aligned}$$

Theorem 3.9. Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ be $fp\gamma$-irresolute continuous. Then for each fuzzy set ν of Y,

$$\begin{aligned}&\tau_i - \text{Cl}(\tau_j - \text{Int}(f^{-1}(\nu))) \wedge \tau_j - \text{Int}(\tau_i - \text{Cl}(f^{-1}(\nu))) \\ &\leq f^{-1}((\tau_i^*, \tau_j^*) - \gamma \text{ Cl}\nu).\end{aligned}$$

Proof. Let ν be a fuzzy set of Y. Then $(\tau_i^*, \tau_j^*)-\gamma$ Clν is a $(\tau_i^*, \tau_j^*) - f\gamma c$ set of Y and so $f^{-1}((\tau_i^*, \tau_j^*) - \gamma \text{ Cl}\nu)$ is a $(\tau_i, \tau_j) - f\gamma c$ set of X. Hence

$$\begin{aligned}&\tau_i - \text{Cl}(\tau_j - \text{Int}(f^{-1}(\nu))) \wedge \tau_j - \text{Int}(\tau_i - \text{Cl}(f^{-1}(\nu))) \\ &\leq \tau_i - \text{Cl}(\tau_j - \text{Int}(f^{-1}((\tau_i^*, \tau_j^*) - \gamma \text{ Cl}\nu)))\wedge \\ &\tau_j - \text{Int}(\tau_i - \text{Cl}(f^{-1}((\tau_i^*, \tau_j^*) - \gamma \text{ Cl}\nu))) \\ &\leq f^{-1}((\tau_i^*, \tau_j^*) - \gamma \text{ Cl}\nu).\end{aligned}$$

Theorem 3.10. Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ be $fp\gamma$-irresolute continuous. Then for each fuzzy set μ of X,

$$\begin{aligned}&f(\tau_i - \text{Cl}(\tau_j - \text{Int}\mu) \wedge \tau_j - \text{Int}(\tau_i - \text{Cl}\mu)) \\ &\leq (\tau_i^*, \tau_j^*) - \gamma \text{ Cl}(f(\mu)).\end{aligned}$$

Proof. Let μ be a fuzzy set of X. Then, by the above theorem,

$$\begin{aligned}&\tau_i - \text{Cl}(\tau_j - \text{Int}\mu) \wedge \tau_j - \text{Int}(\tau_i - \text{Cl}\mu) \\ &\leq \tau_i - \text{Cl}(\tau_j - \text{Int}(f^{-1}(f(\mu))))\wedge \\ &\tau_j - \text{Int}(\tau_i - \text{Cl}(f^{-1}(f(\mu)))) \\ &\leq f^{-1}((\tau_i^*, \tau_j^*) - \gamma \text{ Cl}(f(\mu))).\end{aligned}$$

and

$$\begin{aligned}&f(\tau_i - \text{Cl}(\tau_j - \text{Int}\mu) \wedge \tau_j - \text{Int}(\tau_i - \text{Cl}\mu)) \\ &\leq f(f^{-1}((\tau_i^*, \tau_j^*) - \gamma \text{ Cl}(f(\mu)))) \\ &\leq (\tau_i^*, \tau_j^*) - \gamma \text{ Cl}(f(\mu)).\end{aligned}$$

Theorem 3.11. Let $f : (X, \tau_1, \tau_2) \to (Y, \tau_1^*, \tau_2^*)$ be $fp\gamma$-irresolute continuous. Then for each $(\tau_i^*, \tau_j^*) - f\gamma o$ set ν of Y,

$$\begin{aligned}f^{-1}(\nu) \leq (\tau_i, \tau_j) - \gamma \text{ Int}(f^{-1}(\tau_j^* - \text{Cl}(\tau_i^* - \text{Int}\nu)\vee \\ \tau_i^* - \text{Int}(\tau_j^* - \text{Cl}\nu))).\end{aligned}$$

Proof. Let ν be a $(\tau_i^*, \tau_j^*) - f\gamma o$ set of Y. Then

$$\begin{aligned}&f^{-1}(\nu) \\ &\leq f^{-1}(\tau_j^* - \text{Cl}(\tau_i^* - \text{Int}\nu) \vee \tau_i^* - \text{Int}(\tau_j^* - \text{Cl}\nu)).\end{aligned}$$

Since $f^{-1}(\nu)$ is a $(\tau_i, \tau_j) - f\gamma o$ set of X,

$$\begin{aligned}f^{-1}(\nu) &= (\tau_i, \tau_j) - \gamma \text{ Int}(f^{-1}(\nu)) \\ &\leq (\tau_i, \tau_j) - \gamma \text{ Int}(f^{-1}(\tau_j^* - \text{Cl}(\tau_i^* - \text{Int}\nu)\vee \\ &\tau_i^* - \text{Int}(\tau_j^* - \text{Cl}\nu))).\end{aligned}$$

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Young Bin Im
Professor of Seonam University
Research Area: Fuzzy mathematics, Fuzzy topology, General topology
E-mail : philpen@naver.com