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계단형 수송비용을 고려한 동적 로트 크기 결정

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Dynamic Lot-Sizing with Stepwise Transportation Costs

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본 논문에서는 운송비용과 재고유지비용의 합을 최소화하는 것을 목적으로 유한 계획기간 동안의 수요를 충족시키는 동적 랏사이징 문제를 다룬다. 운송비용을 고려하는 기존의 랏사이징 모형들과는 달리 운송 트럭의 대수에 따라 계단형으로 운송비용이 증가하는 경우를 다루고 있다. 이 문제를 선형정수모형으로 모델링하며 그리디 방식의 휴리스틱을 제안한다. 제안된 휴리스틱의 성능을 평가하기 위해 계산실험을 수행하며, 그 결과 매우 짧은 시간 안에 최적해에 가까운 해를 찾을 수 있음을 보여준다.

Keywords : Dynamic Lot-sizing, Stepwise Transportation Costs, Heuristic Algorithm

1. Introduction

Lot-sizing, which is one of important production planning and inventory control problems, has been a generic industrial optimization problem. In general, the problem can be classified into static and dynamic ones (Johnson and Montgomery, 1974). In the static version, such as economic order quantity (EOQ) and economic production quantity (EPQ), the demand is assumed to be constant over time, while in the dynamic version, the demand changes over a planning horizon. The dynamic lot-sizing problem is introduced by Wagner and Whitin (1958), and independently by Manne (1958). They suggest an exact algorithm using the well-known zero inventory property, and later, it is improved by Federgruen and Tzur (1991), Wagelmans *et al.* (1992), and Agrawal and Park (1993). Although transportation costs form a substantial part of the total logistics costs for satisfying demands, they are usually ignored in the bulk of lot-sizing research (van Norden and van de Velde, 2004). In general, lot-sizing and transportation decisions are closely interrelated, especially in the supply chain environment. This is because an integrated supply chain plan requires the coordination of production and logistics operations as well as other functional specialties within the firm (Lee *et al.*, 2003).

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There are a number of research articles on the lot-sizing models in which transportation costs are considered explicitly. As in the ordinary lot-sizing literature, the models with transportation costs can be classified into static and dynamic ones. In the static version, the previous research articles extend the EOQ or EPQ models (Hall, 1996). Swenseth and Godfrey (2002) and Zhao *et al.* (2004) are recent research results on the static version of lot-sizing with transportation costs. Also, in the dynamic version of the problem, the algorithms suggested in the previous research, which are differ in the structure of transportation costs, extend the basic dynamic lot-sizing algorithm of Wagner and Whitin (1958). For example, see van Norden and van de Velde (2004). A more general model is considered by Kim and Kim (2000) on the multi-period inventory and distribution planning problem with one warehouse and multiple retailers. They suggest a Lagrangean heuristic for the problem of minimizing the sum of inventory holding and distance-dependent and quantity-dependent linear transportation costs. Also, Bertazzi *et al.* (2000) suggest a model to make a trade-off between transportation costs and inventory costs by determining a cyclic scheme of cost in order to minimize transport frequencies for multiple products transported over a single link. For others, see Qu *et al.* (1999) and Vroblefski *et al.* (2000).

This paper is concerned with the single-item dynamic lot-sizing problem in which the transportation costs are computed by the number of trucks, i.e., stepwise transportation costs. The objective is to minimize the sum of transportation and inventory holding costs. Unlike the previous research articles that assume the linear transportation cost, we consider the case in which the linear transportation cost is computed by the number of trucks used. In general, companies try to fully utilize the trucks if possible to minimize the transportation costs, the associated inventory holding costs increase. These results in the basic trade-off between the transportation and inventory holding costs, and the focus of this paper is to suggest the algorithm that can give the solution that balances these cost factors.

In this paper, the problem is formulated as an integer linear programming model, which is an extension of the classical dynamic lot-sizing formulation. Since the zero inventory property does not hold, the problem considered in this paper is more difficult than the classical dynamic lot-sizing problem. Due to the problem complexity, we suggest a greedy heuristic algorithm in which the quantities that exceeds the integer multiples (including zero multiple) of the truck capacity are reassigned while considering the associated cost changes. To show the performance of the heuristic algorithm, computational tests were done on randomly generated test problems, and the results are reported.

The remainder of this paper is organized as follows. In the next section, the problem considered in this paper is described in more details and the corresponding integer linear programming model is presented. Section 3 presents the heuristic algorithm, and the results of computational tests are presented in Section 4. Finally, Section 5 concludes the paper with a summary and discussion of future research.

2. Problem Description

The problem considered in this paper can be defined as the problem of determining the lot sizes while satisfying the given dynamic demand over a finite planning horizon for the objective of minimizing the sum of transportation and inventory holding costs. The planning horizon is divided into T periods, and demands over the planning horizon are given and deterministic. At the beginning of each time period, the item can be ordered in the form of a lot. It is assumed that backlogging is not allowed, and hence the demand should be satisfied on time. As stated earlier, the stepwise transportation costs are considered in this paper, i.e., the transportation costs are computed by the number of trucks used. Therefore, the cost of using one truck occurs although the amount of shipment is less than the truck capacity, i.e., less than truck load (LTL). An organization may own a fleet of trucks to serve its demands, and hence the quantity to be delivered to any of its customers is limited by the number and size of the available trucks. In this paper, however, we consider the situation in which the trucks are rented and hence the required trucks are computed based on the lot sizes. It is assumed that homogeneous trucks are used, i.e., truck capacities are the same. Also, the inventory holding cost is the cost required for storing items to satisfy future demand and assumed to be computed based on the inventory level at the end-of-period.

To describe the problem considered here mathematically, an integer linear programming model is presented below. The formulation includes the purchase costs for demanded items since time-variant purchase costs are considered in the general form of the model. In fact, this paper focuses on a special case with time-invariant costs and hence the purchase costs can be eliminated without loss of generality. The formulation of the special case considered in this paper will be presented later. Before describing the formulation, the notations used are summarized below.

- *Parameters*
  - D_t demand at period t
  - h_t inventory holding cost of one unit of the item in period t
  - s_t transportation cost in period t
  - c_t purchase cost of one unit of the item in period t
  - Q capacity of truck

- *Decision variables*
  - X_t lot size in period t
  - I_t inventory level at the end of period t (I_0 represents the initial inventory)
  - Z_t number of trucks needed in period t (general integer)

Now, the integer program is given below.

[P1] Minimize $\sum_{t=1}^{T}(s_t Z_t + h_t I_t + c_t X_t)$

subject to

$I_t = I_{t-1} + X_t - D_t$	for all t	(1)
$X_t \le Q Z_t$	for all t	(2)
$Z_t \ge 0$ and integer	for all t	(3)
$I_t \ge 0$ and integer	for all t	(4)
$X_t \ge 0$ and integer	for all t	(5)

In the formulation, the objective function denotes minimizing the sum of transportation, inventory holding, and purchase costs. Note that these costs are time-variant in that the cost values may differ in different planning periods. Constraint (1) defines the inventory level of the item at the end of each period, called the inventory flow conservation constraint. This implies that at the end of each period, we have inventory what we had the period before, increased by the amount of lot size, and decreased by the demand in that period. Constraint (2) represents the relation between the lot size and the number of trucks needed. That is, if no truck is used in a period, i.e., Z_t = 0, the corresponding lot size becomes 0, i.e., X_t = 0. Otherwise, the lot size should be less than or equal to the sum of capacities of the number of trucks used. Constraints (3), (4), and (5) represent conditions on the decision variables. In particular, constraint (3) represents the number of trucks required in each period, which results in the stepwise transportation cost, and constraint (4) ensures that backlogging is not allowed.

As stated earlier, this paper focuses on the case of time-invariant costs, i.e., the cost values are the same over the planning horizon. These results in the following integer programming model in which index t is removed in the transportation and inventory holding costs, and the purchase costs are eliminated. Note that in the case of time-invariant costs, the purchase costs are not needed since the total purchase quantity always remains the same, irrespective of the different lot sizes over the planning horizon.

[P2] Minimize $\sum_{t=1}^{T}(s Z_t + h I_t)$

subject to (1)~(5)

The above formulation is very similar to that of the single-item dynamic lot-sizing problem, i.e., the Wagner-Whitin model. The differences are : (1) the truck capacity Q is used instead of a big number; and (2) the number of trucks Z_t is used instead of the binary setup variable. However, these slight changes make the problem much more difficult since the well-known zero inventory property does not hold. In other words, the solution space to be searched in the problem considered in this paper is much greater than that of the Wagner-Whitin model.

The integer programming model [P2] can be solved using commercial optimization packages such as LINGO or CPLEX. However, as the problem size increases, the packages cannot guarantee the optimal solutions within a reasonable amount of computation time. In fact, there may be much variation in the computation times, depending on cost values, truck capacity, etc. In this paper, therefore, we suggest a fast heuristic that can give near optimal solutions within very short computation time.

3. Solution Algorithm

This section presents a fast greedy-type heuristic algorithm to find near optimal solutions for problem [P2]. As stated earlier, the basic idea of the algorithm is that the quantities that exceed the integer multiples of the truck capacity are reassigned while considering the associated cost changes. In general, it would be better to utilize the truck capacity as much as possible when the transportation cost is relatively higher than the holding cost while it would be better not to order more than needed in the opposite situation. Thus, we should take into accounts the trade-off between the two cost factors in determining the order sizes.

In the heuristic algorithm, the initial solution is obtained with $X_t = D_t$ for all t. This implies that transportation occurs in every period to satisfy the demands over the planning horizon and hence the maximum number of trucks is needed. This incurs the largest transportation cost and the smallest inventory holding cost. Therefore, we can improve the initial solution by adjusting the lot sizes while considering the trade-off between the two cost factors. Here, the trade-off is that the transportation cost in a period is decreased by s by moving the items that exceeds the corresponding integer multiples of the truck capacity to one or more earlier periods, while this move results in increase in the inventory holding cost. Therefore, it can be seen that there is an improvement if the amount of decrease in transportation cost is greater than the amount of increase in inventory holding cost. Note that this can be done because we consider the case of time-invariant costs.

The number of trucks needed in period t can be represented as

$$Z_t = \lceil X_t/Q \rceil ,$$

where $\lceil \bullet \rceil$ is the smallest integer that is greater than or equal to $\bullet$. Let R_t denote the amount of remaining items in period t after $Q(Z_t - 1)$ items are loaded to $(Z_t - 1)$ trucks and A_t denote the available extra truck capacity after loading X_t items in period t. Then, R_t and A_t are defined as below.

$$R_t = X_t - Q(Z_t - 1)$$
$$A_t = QZ_t - X_t$$

Then, based on the above two definitions, it can be seen that there is an improvement if R_t can be moved to one or more periods earlier than t so that the corresponding increase in inventory holding cost is less than the unit transportation cost s. In addition, we can obtain a condition under which the remaining items in a period t cannot be reassigned to its earlier periods. This can be expressed as follows.

$$\sum_{i=1}^{t-1} A_i < R_t \text{ for } t \text{ such that } R_t > 0 \qquad (6)$$

If the above condition holds, the total available truck capacity before period t is less than R_t, and hence the items cannot be reassigned to earlier periods.

Now, it is needed to specify the amount of improvement unless condition (6) holds. A mathematical description of the amount of improvement (cost saving) CS_t by moving R_t items from period t to its earlier periods can be represented as

$$CS_t = s - h\min[A_{t-1}, R_t] \text{ for } t = 2$$
$$CS_t = s - h\min[A_{t-1}, R_t] - h\sum_{i=2}^{t-1} i\min[A_{t-i}, \max(R_t - \sum_{j=1}^{i-1} A_{t-j}, 0)] \text{ for } 3 \le t \le T$$

In the algorithm suggested in this paper, the improvements are done in a greedy manner in such a way that the total cost is decreased in the steepest decent direction.

More formally, the algorithm can be summarized below.

Procedure 1

Step 1 : (Obtain an initial solution)

Let $X_t = D_t$ and $Z_t = \lceil X_t/Q \rceil$ for all t.

Step 2 : (Specify the period with the maximum improvement)

1) Calculate CS_t for all $2 \le t \le T$.

2) Find the period t^* that gives the maximum improvement, i.e.,

$$t^* = \underset{2 \le t \le T}{\operatorname{argmax}} CS_t .$$

Step 3 : (Update the solution)

If $CS_{t^*} \le 0$, stop and save the solution.

Otherwise, let

$$X_{t^*} = X_{t^*} - R_{t^*},$$
$$X_{t^*-1} = X_{t^*-1} + \min(A_{t^*-1}, R_{t^*}) \text{ and}$$
$$X_t = X_t + \min[A_t, \max(R_{t^*} - \sum_{j=1}^{t^*-t-1} A_{t^*-j}, 0)]$$

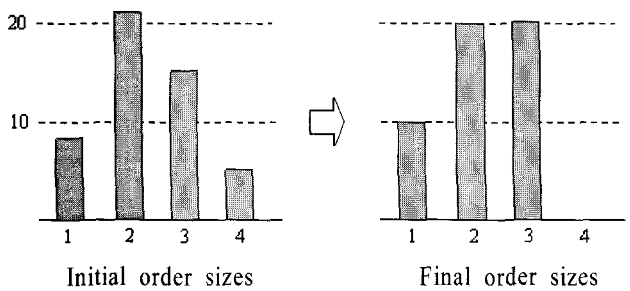
for $1 \le t \le t^* - 2$, and go to step 2.

To illustrate the algorithm, consider a simple example with $T = 4$. The inventory holding (h) and transportation (s) costs are 1 and 10, respectively. The truck capacity Q is 10, and the demands are 8, 21, 16, and 5 for periods 1, 2, 3, and 4, respectively. <Table 1> shows the values of CS_t and X_t at each iteration of the algorithm. In this example, the algorithm gives an optimal solution with total cost of 58. (This was proved by solving the corresponding integer program with LINGO 8.0.) Note that as the iteration continues, the total transportation cost decreases while the total inventory holding cost increases. Also, <Figure 1> shows the lot sizes in the initial and final solutions, respectively, for the example problem.

<Table 1> Results of the example problem

Iteration	Amounts of improvement			
	CS_1	CS_2	CS_3	CS_4
0 (Initial)	-	9	4	4
1	-	-	-	3
2	-	-	-	-
Iteration	Lot sizes			
	X_1	X_2	X_3	X_4
0	8	21	16	5
1	9	20	16	5
2	10	20	20	0
Iteration	Costs			
	THC[1]	TTC[2]	TC[3]	
0	0	70	70	
1	1	60	61	
2	8	50	58	

Note) [1] Total inventory holding cost.
[2] Total transportation cost.
[3] Total cost.



<Figure 1> Initial and final lot sizes for the example problem

4. Computational Experiment

To show the performance of the heuristic algorithm suggested in this paper, this section reports the test results on various test problems. Two performance measures were used in the test. They are percentage deviations from optimal solution values and CPU seconds. Here, optimal solution values were obtained by solving the corresponding integer programs using LINGO 8.0, commercial integer programming software based on the branch and bound algorithm.

For the test, 10 problems with different data were generated randomly for each of six levels of the number of periods (5, 10, 20, 30, 40, 50), and hence 60 problems were generated in total. Demands were generated from $DU(10, 100)$, where $DU(a, b)$ denotes the discrete uniform distribution with range $[a, b]$. The transportation and inventory holding costs were generated from $10DU(15, 20)$ and $DU(1, 5)$, respectively. Also, the capacity of the truck was generated as the total demand divided by $2T$, i.e., $D_t/2T$. In this test, the algorithm and the program to generate integer programming formulations were coded in C and the test was done on a personal computer with a Pentium processor operating at 600 MHz clock speed.

Results of the test are summarized in <Table 2>, which shows the percentage deviations from optimal solutions and CPU seconds. It can be seen from the table that the heuristic algorithm suggested in this paper gives near optimal solutions for most of the test problems, i.e., 3.21% in average. Also, as expected, LINGO 8.0 required much longer computation times than the heuristic. In fact, the CPU seconds of the heuristic algorithm were less than 0.01 second.

<Table 2> Test results for the suggested heuristic

Num of periods	Gap[1]			CPU seconds	
	min	average	max	LINGO	Heuristic
5	0.00	3.99	9.05	0.30	0.00*
10	0.48	3.57	6.51	1.00	0.00
20	1.11	3.25	4.92	11.30	0.00
30	0.48	3.07	5.51	24.00	0.00
40	1.72	2.66	3.81	129.90	0.00
50	1.64	2.74	5.35	843.70	0.00

Note : [1] percentage deviations from optimal solutions.
* CPU second was less than 0.01 second.

5. Concluding Remarks

This paper considered the single-item dynamic lot-sizing problem with stepwise transportation costs, which is the problem of determining the lot sizes while satisfying the dynamic demand over the planning horizon for the objective of minimizing the sum of transportation and inventory holding costs. An integer linear programming formulation, which is an extension of the classical dynamic lot-sizing formulation, was suggested to model the problem mathematically, and the simple greedy algorithm was developed. The basic idea is to reassign the quantities that exceeds the integer multiples of truck capacity while considering the associated cost changes in a greedy manner. Computational tests were done on randomly generated test problems, and the results showed that the heuristic can give near optimal solutions within very short computation time.

This research can be extended in several ways. First, it is needed to consider the problem with time-variant costs. In this case, the basic idea of the algorithm suggested in this paper can also be used together with a more complicated method to compute cost changes. Second, the case of multiple items is an important consideration. If there are no interactions among the multiple items, we can treat them individually. However, there are various constraints that make the problem difficult. In this case, the single-item model suggested in this paper can be used as to solve the subproblems. Finally, like other lot-sizing problems, it is needed to extend the problem in a stochastic version, e.g., stochastic costs, stochastic demand, etc.

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