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# Cone-beam CT에서 웨이브렛 역치값을 이용한 x-ray 영상에서의 노이즈 제거

## ( Noise Reduction of medical X-ray Image using Wavelet Threshold in Cone-beam CT )

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요 약

X-ray 영상 시스템에서는, 크게 2 종류의 noise 성분이 함유되어있다. 먼저 x-ray 방사선이 조사되어질 때, 검출기에서의 방사선의 상호작용으로부터 발생되어지는 것으로서 랜덤하게 발생되어지는 Poisson noise 성분이다. 다음으로 noise 성분은 readout electronics noise, pixel pattern noise 그리고 off-set noise 등으로부터 발생되어지는 Gaussian noise 성분이다. 그러나, x-ray 영상에서는 Gaussian noise가 아닌, Poisson noise로 모델링 되어진다. Gaussian noise에 의해서 발생되어지는 noise 성분은 위너필터 혹은 웨이브렛을 사용하여 쉽게 제거가 가능하지만, Poisson noise와 같은 랜덤 noise를 제거하기 위해서는 복잡한 분석기법이 필요하게 한다. 이 논문에서는 웨이브렛 영역에서 x-ray 영상의 Poisson noise를 제거하고자 하였으며, 적용된 분석 기법은 최적화된 웨이브렛 분석기법인 IBS(Improved BayesShrink)을 사용하였다. 적용된 IBS 기법은 cone-beam CT의 x-ray 영상에서의 기존의 방법에 비해 향상된 결과를 보여주었다.

Abstract

In x-ray imaging system, two kinds of noises are involved. First, the charge generated from the radiation interaction with the detector during exposure. Second, the signal is then added by readout electronics noise. But, x-ray images are not modeled by Gaussian noise but as the realization of a Poisson process. In this paper, we apply a new approach to remove Poisson noise from medical X-ray image in the wavelet domain, the applied methods shows more excellent results in cone-beam CT

**Keywords :** wavelet, x-ray noise, Poisson, denoising

## I. Introduction

Noise in X-ray images[1] is primarily categorized into quantum mottle, which is related to the number of incident X-ray, and artificial noise is due to the grid etc. The effect of quantum mottle is manifested as an increase in the graininess of an X-ray image. Moreover, in order to minimize graininess, the dose must be increased, which is undesirable in terms of patient exposure. Therefore, noise reduction is great significance in medical X-ray images. The noise of X-ray images obeys a Poisson process and, hence, is highly dependent on the underlying light intensity pattern being imaged[2].

Classically, denoising methods have been based on apply linear filters as the wiener filter to the image, however linear methods tend to blur the edge structure of the image. Several denoising methods

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based on nonlinear filters have been introduced to avoid this problem[3~4].

Recent work[5~6] has proposed a wavelet-based technique for estimating signal from a noisy image. This technique exploits the image signal using a 2-D discrete wavelet transform (DWT). Wavelet domain thresholding techniques are employed to suppress noise after decorrelating the data via the 2-D DWT.

But, in the case of Poisson noise, where the noise variance is proportional to image pixels, there is a disadvantage. It is only effective for small amplitude noise coefficients, and not effective for large-amplitude noise coefficients which exceed the thresholding value. This paper develops a new wavelet-domain filtering approach for medical X-ray image that addresses the drawbacks of conventional filtering techniques and the BayesShrink method. We use improved BayesShrink (IBS) to kill small amplitude noise coefficients, in order to reduce large amplitude noise coefficients, we use that we apply the new type of Directional Adaptive Median Filter (DAMF).

In this work, a comparative study of several denoising techniques for x-ray images is presented. The filters considered are: 1) a local Wiener filter, 2) a filter based on the denoising method of Donoho based on the minimax thresholding strategy, roughly speaking, based on a soft thresholding of the wavelet transformed coefficients of the image, and, 3) a filter based on the improved BayesShrink of the image. Denoised x-ray image is used data for reconstruction.

## II. Denoising Algorithm

### 1. Adaptive wiener filter

The classical denoising filter is the Wiener filter, defined as the linear filter that minimizes the mean squared error (MSE). The first denoising method used in this work consists in applying a Wiener filter to an image adaptively, tailoring itself to the local image variance. Where the variance is large, the Wiener filter performs little smoothing. Where the variance is small, the Wiener filter performs more smoothing. This approach often produces better results than linear filtering. The adaptive Wiener filter preserves edges and contours of the ridges in the fingerprint image. Filtering is performed on a pixelwise level based on statistics estimated from a local neighbourhood of each pixel. The optimal window size was found to be 33. The function estimates the local neighbourhood mean and standard deviation and performs filtering accordingly. Consecutively all holes would be smoothed. The adaptive filter is more selective than a comparable linear filter, preserving edges and other high frequency parts of an image.

### 2. Wavelet transform

The wavelet shrinkage methods achieve symptotically near optimal minimax mean-square error for a wide range of signals corrupted by an additive white Gaussian noise. The methods derive from the basic idea that the energy of a signal will often be concentrated in a few coefficients in wavelet domain while the energy of noise is spread among all coefficients in wavelet domain. Therefore, the nonlinear shrinkage function in wavelet domain will tend to keep a few larger coefficients representing the signal while the noise coefficients will tend to reduce to zero.

The wavelet shrinkage can be described as three-step procedure:

1. A noisy signal is transformed to the wavelet domain by DWT.
2. The coefficients representing details are shrunk.
3. The shrunk wavelet coefficients are returned to the time domain by the inverse DWT. The result is a wavelet shrinkage estimator of the denoised signal.

Donoho et al. showed that in estimating denoised signals simple shrinkage rules have asymptotic optimality properties for a rich class of function spaces. Examples are hard and soft thresholding rules, given by

$$\delta^{hard}(x,\lambda) = x \cdot 1(|x| > \lambda) \quad (1)$$

$$\delta^{soft}(x,\lambda) = (x - sign(x)\lambda) \cdot 1(|x| > \lambda) \quad (2)$$

where λ is a threshold parameter and $1(\cdot)$ is an indicator function.

For a fixed shrinkage policy, the parameters of shrinkage (eg. threshold λ) can be selected in many ways. The method of selecting threshold λ in soft thresholding rule (2) proposed by Donoho has a form:

$$\lambda = \frac{\hat{\delta}}{\sqrt{n}}\sqrt{2\log n} \quad (3)$$

where $\hat{\delta}$ is the estimator of standard deviation of details coefficients, and n is the number of details coefficients.

## 3. BayesShrink Method

BS[7] was proposed for image signals with detail subband coefficients having a Generalized Gaussian (GG) distribution. It has been experimentally shown and used in many applications that for a large class of images, the wavelet coefficients of the detail subbands (HH, HL, LH) obey a GG distribution for all decomposition levels. Each subband can be thought of as a random vector with elements that are independently and identically distributed GG random variables. The probability density function of a GG random variable is defined as

$$f_{\sigma_x\beta} = \sigma_x^{-1}\frac{\beta \cdot \alpha(\alpha_x,\beta)}{2\Gamma(1/\beta)} \quad (4)$$

$$\alpha(\alpha_x,\beta) = \left[\frac{\Gamma(3/\beta)}{\Gamma(1/\beta)}\right]^{-1/2} \quad (5)$$

where $\Gamma(t) = \int_0^\infty e^{-u}u^{t-1}du$ is the gamma function, $\sigma_x$ is the standard deviation and $\beta$ is the shape parameter. Assuming such a distribution for the wavelet coefficients, we empirically estimate $\beta$ and $\sigma_x$ for each subband and try to find the threshold $T_B$. We often use the threshold value of BS which is calculated by Eq. (6).

$$T_B = \frac{\sigma^2}{\sigma_X^2} \quad (6)$$

is very close to numerical value.

We use the Robust Median Estimator to estimate the value of noise standard deviation.

$$\sigma = \frac{Median[Y_{ij}]}{0.6745} \; [Y_{ij}] \in \text{subband } w^{(1,d)} \quad (7)$$

where $Y_{ij}$ is wavelet coefficient in the diagonal direction when the decomposition level is 1. The variance of the degraded model (noisy image) is

$$\sigma_Y^2 = \frac{1}{n^2}\sum_{i,j=1}^{n} X_{ij}^2 \quad (8)$$

where $n^2$ is the size of the subband under consideration and $X_{i,j}$ is the coefficient of subband under consideration.

The $\sigma_x$ is estimated by Eq. 9.

$$\sigma_X = \begin{cases} \sqrt{{\sigma_Y}^2 - \sigma_2}, & for\ {\sigma_Y}^2 > \sigma^2 \\ 0, & for\ {\sigma_Y}^2 \le \sigma^2 \end{cases} \quad (9)$$

In this case of $\sigma_Y^2 \le \sigma^2$, $\sigma_X$ is taken to be 0. That is, $T_B$ is 1 or, in practice, $T_B = \max(|X_{i,j}|)$, and all coefficients are set to 0.

## 4. Improved BayesShrink method

It is well known that the variance of a Poisson random variable is equal to mean. Therefore, the variability of the noise is proportional to the intensity and, hence, signal dependent. This signal dependence has dashed traditional noise removal attempts. We can knowthat noise level depends on the local pixel in the image. It is easily shown that the signal-to-noise ratio (SNR) for noise is linear in signal. Regarding the characteristics of the Poisson noise, the noise power will differ between wavelet coefficients according to the image pixel under the support of the associated wavelet basis function. This spatial variation of the noise must be accounted for in the wavelet domain filter design. The BS method does not adjust for these differences. In this section,

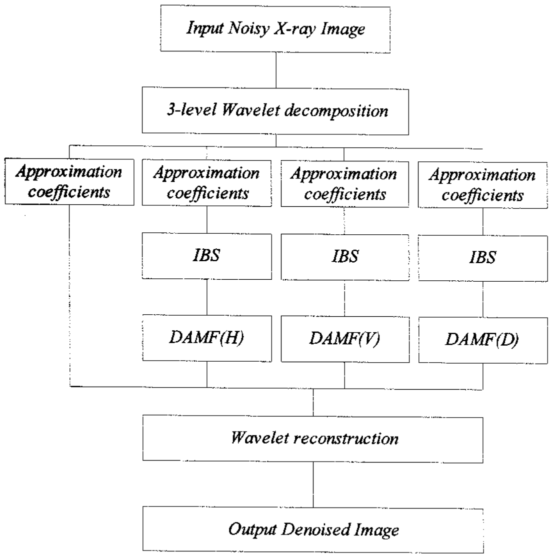



그림 1. 노이즈 제거 시스템의 처리 기법
Fig. 1. Procedure of the denoising system.

we provide a new filter framework in the wavelet domain based on the BS method. Fig. 1 is the compositive diagram of the denoising system.

Firstly, we analyze the coefficients of wavelet transform. We do experiments by NoisyX-ray image with Poisson noise and X-ray image. The values are obtained by the magnitudes of the wavelet coefficients. H, V and D represent horizontal directional coefficients, vertical directional coefficients, and diagonal directional coefficients, respectively. The suffix j of $H_j$, $V_j$, $D_j$ denotes the decompositional level. As the wavelet coefficients of X-ray image are smaller than common image, the obtained thresholds by BS method are not suitable for X-ray image, so that noise cannot be removed well. In order to obtain the optimum threshold for X-ray image, we improve count-method of threshold of BS method by a mount of simulation for X-ray images. The difference of IBS and BS is that the threshold is calculated by $T_B = \sigma^2/\sigma_X^2$ in BS, but the threshold is calculated by $T_B = \alpha\sigma^2/\sigma_X^2$ in IBS, we add a parameter $\alpha$ in the equation of BS. $\alpha$ changes with the size of the subband under consideration and decompositional level. We do experiments using different combination of denominator and numerator, we obtain the best PSNR by the following equation,

$$\alpha = \sqrt{\frac{\log(n^2)}{2 \times j}} \tag{10}$$

where $n^2$ is the size of the subband under consideration. We experiment using IBS for X-ray image.

## Ⅲ. Experimental

In this article, we will concentrate on meeting the x-ray imaging and perform a study of effectiveness for denoising algorithms. We demonstrate the effectiveness of the proposed method using 512 × 512 x-ray images. 360 projection data are collected at every angle from 0°to 360°by cone-beam CT. Denoised images are used data for image reconstruction. The reconstruction algorithm we used is the extended filtered back projection(FBP) reconstruction algorithm by Feldkamp[8]. The algorithms of the denoising methods have been implemented by using Matlab 7.0.

Fig 2 shows a photograph of the Cone-Beam CT prototype for dental application. The components of the system are the x-ray source, the rotation stage, and the flat panel detector. The x-ray source has a nominal focal spot size of 0.6mm, 40kV-120kV high voltage, and the maximum 12 mA target current. The imaging detector used in this investigation was a-Si:H flat panel detector, which wascoupled with CsI scintillator.

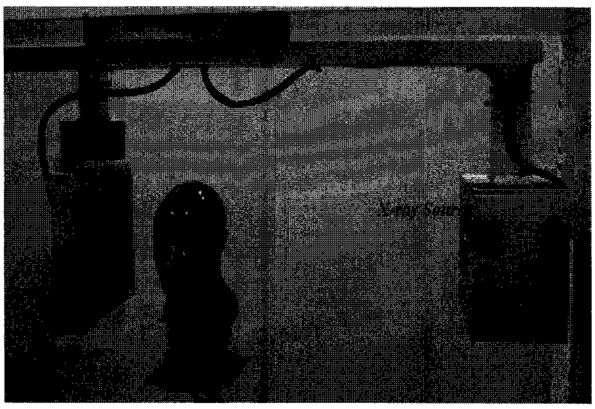


그림 2. 덴탈 시스템 적용을 위한 CBCT prototype.
Fig. 2. A photograph of the CBCT prototype for dental application.

We know that X-ray images have not original image to calculate the PSNR. Then, we regard the high dose image as original image. PSNR is used as the distortional measurement between the original image and denoised image. The wavelet transform employs Daubechies' least asymmetric compactly-supported wavelet with 4 vanishing moments with 3 scales of orthogonal decomposition.

To assess the performance of the applied method, it is compared with wiener filter and normal wavelet. To compare the results of different algorithms, PSNR is used as the distortional measurement between the original image and denoised image. It can be calculated by the following equation.

$$PSNR = 10 \times \log_{10}\left(\frac{Pixel_{\max}^{2}}{MSE}\right) \quad (11)$$

$$MSE = \frac{1}{H \times L}\left(\sum_{x=1}^{H}\sum_{y=1}^{L}(d(x,y)-o(x,y))\right)^{2} \quad (12)$$

## Ⅳ. Results

In order to perform the denoising experiments some images were selected from the Cone-beam CT data. Original images from the x-ray images were corrupted by adding a Poisson noise. We give the denoised images in Fig. 3, 4. (a)~(e) show original image, noisy image, denoised image by Wiener filter, denoised image by wavelet and denoised image by improved BayesShrink method, respectively.

It is well known that wiener filter is the optimal filter, which minimize the mean square error. Since it corresponds to linear filter, it may amplify the noise in the image. Therefore, we can see that a few stains are remained from Fig (c) when using wiener filter. (d), the noise is reduced and the edge is also prevented by using the wavelet. However, Fig (e) show that noise reduction and edge prevention in bright position can be improved by using the IBS method. Regarding Poisson noise characteristics, when the pixel value is high in the image, the noise amplitude is also larger. It is difficult to remove the large amplitude noise by using conventional wavelet transform. The applied method can remove small amplitude noise by IBS, acquiring the clearer image. From the figures and tables shown above, we know that the presented

표 1. 다른 노이즈제거 방법에 대한 PSNR 결과
Table 1. PSNR results with different denoising method.

| *Object* | *Noisy* | *Wiener* | *Wavelet* | *IBS* |
|---|---|---|---|---|
| *Head* | *21,5* | *27.4* | *29.0* | *31.2* |

(a) Original

(b) Noisy

(c) Wiener filter

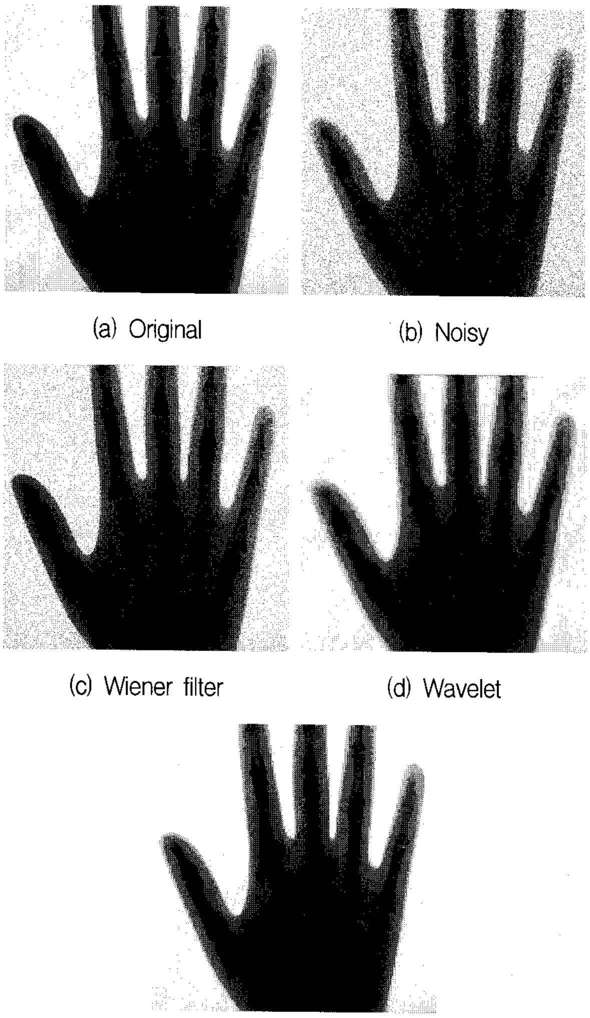
(d) Wavelet

(e) IBS

그림 3. 노이즈 영상과 노이즈가 제거 영상(머리)
Fig. 3. Noisy image and denoised images(Head).

표 2. 다른 노이즈제거 방법에 대한 PSNR 결과
Table 2. PSNR results with different denoising method.

| *Object* | *Noisy* | *Wiener* | *Wavelet* | *IBS* |
|---|---|---|---|---|
| *Hand* | *21.3* | *27.4* | *28.4* | *32.8* |

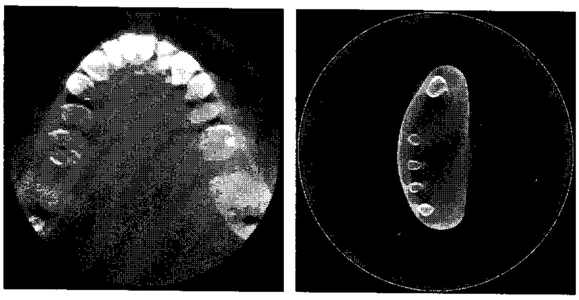


(a) Original (b) Noisy

(c) Wiener filter (d) Wavelet

(e) IBS

그림 4. 노이즈 영상과 노이즈가 제거 영상(손)
Fig. 4. Noisy image and denoised images(Hand).



(a) Head(dental) (b) Hand

그림 5. 노이즈제거 된 x-ray 영상을 이용한 3D 재구성
Fig. 5. Reconstruction of denoised x-ray image.

algorithm achieves the highest quality.

The denoised cone-beam CT image can be reconstructed from these denoised images. The denoised sectional images are shown in Fig. 5. The quality of CT images will be better with the increased quality of the X-ray images.

## V. Conclusions

We have studied the problem of denoising for medical X-ray image in this paper. We have seen that exiting methods need some assumptions and possess limitations. In order to remove noise better, we have proposed a new method in the wavelet domain. Firstly, we process wavelet coefficients using the IBS method. Secondly, we execute the process for edge detection with the Wiener filter. This method has shown better results and the PSNR values in visual images. Our future research will focus on finding more accommodating convergent values for wavelet coefficients. We also want to generalize it to all wavelets and employ it for other kinds of noise.

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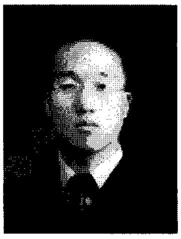
## 저 자 소 개



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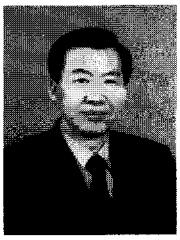

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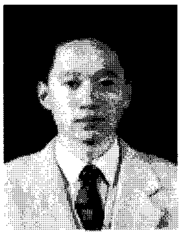

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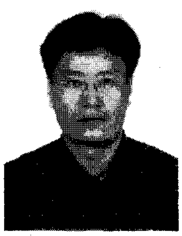

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