

# Observer-based Fault Tolerant Control for Constrained Switched Systems

Hao Yang, Bin Jiang\*, and Vincent Cocquempot

**Abstract:** An observer-based fault tolerant control (FTC) method is proposed for constrained switched systems (CSS) with input constraints. A family of Lyapunov-based bounded controllers are designed to ensure that, whenever actuator faults occur at the dwell time period of each continuous mode, the mode is always within its corresponding stability region. A set of switching laws are designed to guarantee the asymptotic stability of the overall CSS. The fixed stability regions on which the FTC method is based are also relaxed by the proposed variable stability regions. An example of CPU processing illustrates the effectiveness of proposed method.

**Keywords:** Constrained switched system, fault tolerant control, observer.

## 1. INTRODUCTION

Many tools have been developed for stability analysis of switched systems [1]. Control action of the majority of practical switched system is often subject to hard actuator constraints, the general synthesis of control for CSS is based on concepts of stability region and multiple Lyapunov functions (MLFs) e.g., [2]. Most of related methods only consider the CSS in the fault-free case and with full state measurements.

Fault may lead to an unacceptable anomaly in the system performance. Fault detection and diagnosis (FDD) and fault tolerant control (FTC) procedures are designed to guarantee that the system goal is still achieved in spite of the faults [3-5]. This paper focuses on designing a FTC strategy for CSS with actuator faults and without full state measurements. The novelty of this work are twofold: 1) to design a family of Lyapunov-based bounded controllers for

ensuring that, whenever actuator faults occur at the dwell time period of each mode, the mode is always in its corresponding stability region; 2) to design a set of switching laws based on MLFs, to guarantee the asymptotical stability of the overall CSS.

## 2. PRELIMINARIES

Consider the following switched system with input constraints:

$$\begin{aligned} \dot{x}(t) &= f_{\sigma(t)}(x(t)) + G_{\sigma(t)}(x(t))u_{\sigma(t)}(t), \\ \|u_{\sigma(t)}\| &\leq u_{\sigma}^{\max}, \quad \sigma(t) : [t_0, \infty) \rightarrow M = \{1, 2, \dots, N\}, \end{aligned} \quad (1)$$

where  $x \in \mathfrak{R}^n$  is the state vector,  $u_{\sigma}$  denotes the input vector taking values in the nonempty compact subset  $U_{\sigma} := \{u_{\sigma} \in \mathfrak{R}^m : \|u_{\sigma}\| \leq u_{\sigma}^{\max}\}$ .  $u_{\sigma}^{\max} > 0$  is the magnitude of the input constraints,  $f_{\sigma}(x)$  and  $G_{\sigma}(x)$  are sufficiently smooth.  $\sigma : [t_0, \infty) \rightarrow M$  is a switching signal, which is assumed to be a piecewise constant function that is continuous from the right.

The switching laws are defined as: mode  $k$  switches to mode  $(k+1)$ , if the dwell period of mode  $k$  is  $\Delta t_{kj}$ , where  $k \in M$ ,  $j = 1, 2, \dots$

$\Delta t_{kj}$  belongs to a series of dwell time periods for mode  $k$  when it is activated for the  $j$ th time. Also assume that the switching sequence is fixed and the initial mode is random.  $x$  is continuous everywhere.

Consider system (1) with a fixed  $\sigma(t) = k$  for some  $k \in M$ , for which a control Lyapunov function [2]  $V_k$  exists, using the results in [2], the following continuous bounded control law can be constructed

$$u_k(x) = -K_k(L_{f_k}^* V_k(x), x)(L_{G_k} V_k)^T(x) \triangleq b_k(x), \quad (2)$$

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with

$$K_k(L_{f_k}^* V_k, x) = \frac{L_{f_k}^* V_k + \sqrt{(L_{f_k}^* V_k)^2 + (u_k^{\max})^2 \|(L_{G_k} V_k)^T\|^4}}{\|(L_{G_k} V_k)^T\|^2 [1 + \sqrt{1 + (u_k^{\max})^2 \|(L_{G_k} V_k)^T\|^2}]} \quad (3)$$

for  $(L_{G_k} V_k)^T \neq 0$ , and  $K_k(L_{f_k}^* V_k, x) = 0$  for  $(L_{G_k} V_k)^T = 0$ , where  $L_{f_k}^* V_k = L_{f_k} V_k + \rho_k V_k$  and  $\rho_k > 0$ . One can show that for all initial states of  $k$ th mode within the stability region described by the set

$$\Phi_k = \{x \in \mathfrak{R}^n : L_{f_k}^* V_k(x) < u_k^{\max} \|(L_{G_k} V_k)^T(x)\|\} \quad (4)$$

the controller (2) respects the constraints and ensures that the states of the  $k$ th mode remain within the region  $\Phi_k$  and converge to origin asymptotically. An estimate of  $\Phi_k$  is described by  $\Omega_k = \{x \in \mathfrak{R}^n : V_k(x) \leq c_k^{\max}\}$ , where  $\Omega_k$  is expected to be the largest invariant set of  $\Phi_k$ ,  $c_k^{\max}$  is the largest number for which  $\Omega_k \setminus \{0\} \subseteq \Phi_k$ .

### 3. FTC FOR CSS

#### 3.1. FTC with full state measurement

Consider the linear form of (1) with  $\sigma(t) = k$  for some  $k \in M$ , under actuator faulty conditions:

$$\dot{x}(t) = A_k x(t) + B_k u_k(t) + E_k f_k^a(t), \quad \|u_k\| \leq u_k^{\max}, \quad (5)$$

where  $x$  is measurable,  $(A_k, B_k)$  is a controllable pair. The actuator fault vector is modelled by  $f_k^a(t) \in \mathfrak{R}^q$ , assume that  $\|f_k^a\| \leq \bar{f}_k$ , where  $\bar{f}_k > 0$ .

Consider a Lyapunov function candidate  $V_k = x^T P_k x$ , where  $P_k$  is a positive definite symmetric matrix that satisfies the Riccati equation  $A_k^T P_k + P_k A_k - P_k B_k B_k^T P_k = -Q_k$  for some positive definite matrix  $Q_k$ . Define

$$\bar{\Omega}_k = \{x \in \mathfrak{R}^n : L_{f_k}^{**} V_k < u_k^{\max} \|(L_{G_k} V_k)^T\|\}, \quad (6)$$

where  $L_{f_k}^{**} V_k = L_{f_k} V_k + \rho_k V_k + \|L_{E_k} V_k\| \bar{f}_k$ , with  $L_{f_k} V_k = x^T (A_k^T P_k + P_k A_k)x$ ,  $(L_{G_k} V_k)^T = 2B_k^T P_k x$ ,  $\rho_k > 0$ .

Denote  $\bar{\Omega}_k = \{x \in \mathfrak{R}^n : V_k(x) \leq \bar{c}_k^{\max}\}$  as a set of  $V_k$ , completely contained in  $\bar{\Phi}_k$  for some  $\bar{c}_k^{\max} > 0$ .

**Lemma 1:** Consider system (5) with any initial condition  $x(t_{kj}) \in \bar{\Omega}_k$ , under the continuous bounded controller

$$u_k(x) = -K_k(L_{f_k}^{**} V_k(x), x)(L_{G_k} V_k)^T(x) \triangleq \bar{b}_k(x) \quad (7)$$

the states remain within the region  $\bar{\Omega}_k$ , and the origin of the  $k$ th mode is asymptotically stable, whenever the fault occurs in  $[t_{kj}, t_{kj} + \Delta t_{kj})$ , where  $j$  denotes the  $j$ th time that the  $k$ th mode is switched in,  $\Delta t_{kj}$  is the dwell period.

**Proof:** (sketch) From the time-derivative of  $V_k$  along the closed-loop trajectories (omitted due to space), we have that whenever  $L_{f_k}^{**} V_k(x) < u_k^{\max} \|(L_{G_k} V_k)^T(x)\|$ ,  $V_k < -\rho_k V_k$ . Since  $\bar{\Omega}_k$  is the largest invariant set, the inequality (6) holds for all  $x \neq 0$ , then, the origin of the system is asymptotically stable.  $\square$

**Theorem 1:** Consider the switched system (5) under a family of bounded controllers  $\{u_k(x) = \bar{b}_k(x) : k \in M\}$ , with the initial states  $x(t_0) \in \bar{\Omega}_k$ . If, at any time instant  $T$ , the following conditions hold:

$$x(T) \in \bar{\Omega}_{k+1}, \quad (8)$$

$$V_{k+1}(x(T)) < V_{k+1}(x(t_{(k+1)(j-1)})), \quad j > 0, \quad (9)$$

then, choosing  $\Delta t_{kj} \geq T - t_{kj}$  and setting  $\sigma(t) = k + 1$  for the  $j$ th time at  $t = t_{kj} + \Delta t_{kj}$ , guarantees that the origin of the overall CSS is asymptotically stable.

**Proof:** (sketch) From Lemma 1,  $\dot{V}_\sigma(t) < 0$ ,  $\forall \sigma = k$ . From (9), we have that for any admissible switching time  $t_{kj}$ ,  $V_{k+1}(x(t_{(k+1)j})) < V_{k+1}(x(t_{(k+1)(j-1)}))$ , MLFs Theorem [1] can be applied to conclude that the origin of the switched system is Lyapunov stable. Also note that for each switching time  $t_{kj}$ ,  $j = 1, 2, \dots$  such that  $\sigma(t_{kj}^+) = k$ , the sequence  $V_{\sigma(t_{kj})}$  is decreasing and positive, so there exists a class  $K$  function  $\alpha$  such that  $0 = \lim_{j \rightarrow \infty} [V_{k+1}(x(t_{(k+1)(j+1)})) - V_{k+1}(x(t_{(k+1)j}))] \leq \lim_{j \rightarrow \infty} [-\alpha(\|x(t_{(k+1)j})\|)] \leq 0$ . This means that  $x(t)$  converges to the origin, which together with Lyapunov stability, leads to the asymptotical stability of the origin.  $\square$

#### 3.2. FTC without full state measurement

Consider the system (5)

$$\begin{aligned} \dot{x}(t) &= A_k x(t) + B_k u_k(t) + E_k f_k^a(t), \quad \|u_k\| \leq u_k^{\max}, \\ y &= C_k x(t) = [C_k^1 \ 0]x(t), \end{aligned} \quad (10)$$

where  $y \in \mathfrak{R}^r$  is the output vector, with  $q < r$ , where  $q$  is the dimension of  $f_k^a$ .  $C_k^1$  is an  $r \times r$

nonsingular matrix, and  $(C_k, A_k)$  is an observable pair.

Denoting  $\hat{x}(t)$  as the state estimate and  $e(t) \triangleq x - \hat{x}$ . The following Lemma 2 can be obtained based on [2].

**Lemma 2:** Consider system (10) with  $x(t_{kj}) \in \bar{\Omega}_k$ , under the controller  $u_k = \bar{b}_k(\hat{x})$  in (7). There exists a positive real number  $e_{u,k}$  related to  $u_k$ , such that if  $\|e(t)\| \leq e_{u,k}$ ,  $\forall t \in [t_{kj}, t_{kj} + \Delta t_{kj})$ , then the states remain within the region  $\bar{\Omega}_k$ , and the origin of the  $k$ th mode is asymptotically stable.

**Assumption 1:** Rank  $(C_k E_k) = q$ .

Define a transformation  $x = N_k^{-1}z$ , where  $N_k = \begin{bmatrix} C_k^1 & 0 \\ 0 & I \end{bmatrix}$ . The system (10) can be transformed into

$$\begin{aligned} \dot{z} &= \bar{A}_k z + \bar{B}_k u_k + \bar{E}_k f_k^a \\ &= \begin{bmatrix} \bar{A}_{k1} \\ \bar{A}_{k2} \\ \bar{A}_{k3} \end{bmatrix} z + \begin{bmatrix} \bar{B}_{k1} \\ \bar{B}_{k2} \\ \bar{B}_{k3} \end{bmatrix} u_k + \begin{bmatrix} \bar{E}_{k1} \\ \bar{E}_{k2} \\ \bar{E}_{k3} \end{bmatrix} f_k^a, \end{aligned} \quad (11)$$

$$y = \bar{C}_k z = \begin{bmatrix} I_{(r-q) \times (r-q)} & 0 & 0 \\ 0 & I_q & 0 \end{bmatrix} z, \quad (12)$$

where  $\bar{A}_k = N_k A_k N_k^{-1}$ ,  $\bar{B}_k = N_k B_k$ ,  $\bar{E}_k = N_k E_k$ .  $z$  can be represented as  $[z_1 z_2 z_3]^T = [y_1 y_2 z_3]^T$ , where  $z_3 \in \mathfrak{R}^{n-r}$ . We just need to estimate  $z_3$ . Define

$$S_k = \begin{bmatrix} I & -\bar{E}_{k1} \bar{E}_{k2}^{-1} & 0 \\ 0 & I & 0 \\ 0 & -\bar{E}_{k3} \bar{E}_{k2}^{-1} & I \end{bmatrix}. \quad (13)$$

Pre-multiplying (13) into (11), we have

$$\begin{aligned} \begin{bmatrix} \dot{y}_1 - \bar{E}_{k1} \bar{E}_{k2}^{-1} \dot{y}_2 \\ \dot{y}_2 \\ \dot{z}_3 - \bar{E}_{k3} \bar{E}_{k2}^{-1} \dot{y}_2 \end{bmatrix} &= \begin{bmatrix} \bar{A}_{k1} - \bar{E}_{k1} \bar{E}_{k2}^{-1} \bar{A}_{k2} \\ \bar{A}_{k2} \\ \bar{A}_{k3} - \bar{E}_{k3} \bar{E}_{k2}^{-1} \bar{A}_{k2} \end{bmatrix} z \\ &+ \begin{bmatrix} \bar{B}_{k1} - \bar{E}_{k1} \bar{E}_{k2}^{-1} \bar{B}_{k2} \\ \bar{B}_{k2} \\ \bar{B}_{k3} - \bar{E}_{k3} \bar{E}_{k2}^{-1} \bar{B}_{k2} \end{bmatrix} u_k + \begin{bmatrix} 0 \\ \bar{E}_{k2} \\ 0 \end{bmatrix} f_k^a. \end{aligned} \quad (14)$$

Define  $G_{ki} = \bar{A}_{ki} - \bar{E}_{ki} \bar{E}_{k2}^{-1} \bar{A}_{k2}$ ,  $H_{ki} = \bar{B}_{ki} - \bar{E}_{ki} \bar{E}_{k2}^{-1} \bar{B}_{k2}$  for  $i=1,3$ , and partitioning  $G_{ki}$  as  $[G_{ki1} G_{ki2} G_{ki3}]$ ,  $i=1,3$ , then the first and third block rows of system (14) can be written as  $\dot{z}_3 = G_{k33} z_3 + s$ ,  $v = G_{k13} z_3$ , where  $s = G_{k33} y_1 + G_{k32} y_2 + \bar{E}_{k3} \bar{E}_{k2}^{-1} \dot{y}_2 + H_{k3} u_k$ ,  $v =$

$\dot{y}_1 - \bar{E}_{k1} \bar{E}_{k2}^{-1} \dot{y}_2 - G_{k11} y_1 - G_{k12} y_2 - H_{k1} u_k$ . To estimate  $z_3$ , an observer can be designed as

$$\dot{\hat{z}}_3 = G_{k33} \hat{z}_3 + s + \zeta_k (v - G_{k13} \hat{z}_3) \quad (15)$$

assume  $(G_{k33}, G_{k13})$  is an observable pair, the observer gain  $\zeta_k$  can be chosen to make  $(G_{k33} - \zeta_k G_{k13})$  stable.

From the above discussion, we let  $\hat{x} = N_k^{-1} \hat{z} = N_k^{-1} [y_1 y_2 \hat{z}_3]^T$ . Denote  $\tilde{z}_3 = z_3 - \hat{z}_3$ , then

$$\|e(t)\| \leq \mu(\lambda_k^*) \|\tilde{z}_3(t_{kj})\| \exp(-\lambda_k^* (t - t_{kj})), \quad (16)$$

where  $\lambda_k^*$  is such that all eigenvalues of  $(G_{k33} - \zeta_k G_{k13})$  satisfy  $\lambda_k \leq -\lambda_k^*$ ,  $\mu(\lambda_k^*) > 0$  is polynomial in  $\lambda_k^*$ .

From the second block row in (14), the fault estimate  $\hat{f}_k^a$  can be obtained as

$$\hat{f}_k^a = \bar{E}_{k2}^{-1} (\dot{y}_2 - \bar{A}_{k21} y_1 - \bar{A}_{k22} y_2 - \bar{A}_{k23} \hat{z}_3 - \bar{B}_{k2} u_k). \quad (17)$$

The above method can provide accurate state and fault estimates. For CSS, the observer (15) is switched according to the current mode at each switching time. The initial states of the current observer are chosen as the final states of the previous observer.  $\hat{f}_k^a$  are always obtained from (17) for each mode.

**Theorem 2:** Consider the switched system (10) under a family of bounded controllers  $\{u_k = \bar{b}_k(\hat{x}) : k \in M\}$ , with the initial states  $x(t_0) \in \bar{\Omega}_k$  and  $\hat{x}(t_0)$  such that  $\mu(\lambda_k^*) \|e(t_0^+)\| \leq e_{u,k}$ . If, at any time instant  $T$

$$\|e(T)\| \leq \bar{e}_{u,k+1}, \quad (18)$$

$$\hat{x}(T) \in \Psi_{k+1}, \quad (19)$$

$$V_{k+1}(\hat{x}(T)) + 2M_{k+1} < V_{k+1}(\hat{x}(t_{(k+1)(j-1)})), \quad (20)$$

where  $\bar{e}_{u,k+1}$  is such that  $\mu(\lambda_{k+1}^*) \|\bar{e}_{k+1}\| (T^+) \leq e_{u,k+1}$ ,  $\Psi_{k+1}$  is such that  $\hat{x} \in \Psi_{k+1} \rightarrow x \in \bar{\Omega}_{k+1}$  for  $\|e\| \leq e_{u,k+1}$ ,  $M_{k+1}$  is such that  $\|e\| \leq e_{u,k+1} \rightarrow |V_{k+1}(x) - V_{k+1}(\hat{x})| \leq M_{k+1}$ , then, choosing  $\Delta t_{kj} \geq T - t_{kj}$  and setting  $\sigma(t) = k + 1$  at  $t = t_{kj} + \Delta t_{kj}$ , guarantees that the origin of the overall CSS is asymptotically stable.

**Proof:** (sketch) Based on Lemmas 1, 2, if (18) and (19) hold at any time instant  $T$ , we set  $\sigma(t) = k + 1$  at  $t = t_{kj} + \Delta t_{kj}$ , then the origin of the  $(k + 1)$ th

mode is asymptotically stable. Due to the continuity of  $V_{k+1}(\cdot)$ , there exists a positive real number  $M_{k+1}$ , such that if  $\|e\| \leq e_{u,k+1}$ , then  $|V_{k+1}(x) - V_{k+1}(\hat{x})| \leq M_{k+1}$ , which, together with (20), leads to  $V_{k+1}(x(T)) < V_{k+1}(x(t_{(k+1)(j-1)}))$ . Similar to Theorem 1, we can conclude the result.  $\square$

### 3.3. Relaxation of the stability region

The region  $\bar{\Phi}_k$  in (6) is based on a fixed norm bound of faults, which could be relaxed. Define the fault detection threshold as  $\|\hat{f}_k^a\| = \|\bar{E}_{k2}^{-1} \bar{A}_{k23} e(t)\|$ , and define  $\hat{f}_k^a > 0$  such that  $\|\hat{f}_k^a\| \leq \hat{f}_k^a$ . A variable stability region is designed as

$$\bar{\Phi}_{k,\hat{f}} = \{x \in \mathbb{R}^n : L_{f_k}^\diamond V_k(x) < u_k^{\max} \|(L_{G_k} V_k)^T(x)\|\}, \quad (21)$$

where  $L_{f_k}^\diamond V_k = L_{f_k} V_k + \rho_k V_k + \|L_{E_k} V_k\| \hat{f}_k^a + \|L_{E_k} V_k \bar{E}_{k2}^{-1} \bar{A}_{k23}\| e_{u,k}$ , and  $e_{u,k}$  is related to  $u_k = b_k(\hat{x})$ . The region (21) is variable according to different  $\hat{f}_k^a$ . This region is less conservative than (6).

Let's similarly define  $\bar{\Omega}_{k,\hat{f}} = \{x \in \mathbb{R}^n : V_k(x) \leq \bar{c}_{k,\hat{f}}^{\max}\}$ .  $\bar{\Omega}_{k,\hat{f}}$  is a set of  $V_k$ , completely contained in  $\bar{\Phi}_{k,\hat{f}}$  for some  $\bar{c}_{k,\hat{f}}^{\max} > 0$ , and define  $\Psi_{k,\hat{f}}$  such that if  $\|e\| \leq e_{u,k}$ , then  $\hat{x} \in \Psi_{k,\hat{f}} \rightarrow x \in \bar{\Omega}_{k,\hat{f}}$ .

**Lemma 3:** Consider system (10) under control law  $u_k = b_k(\hat{x})$ ,  $x(t_{kj}) \in \Omega_k$ , and  $\hat{x}(t_{kj})$  is such that  $\mu(\lambda_k^*) \|e(t_{kj}^+)\| \leq e_{u,k}$ . Assume the faults occur at  $t = t_{kj}^f$ . If  $\hat{x}(t_{kj}^f) \in \Psi_{k,\hat{f}}$ , then there exists  $e_{u,k}^{\hat{f}} > 0$ , such that for all  $\|e\| \leq e_{u,k}^{\hat{f}} \quad \forall t \in [t_{kj}^f, t_{kj} + \Delta t_{kj})$ , the bounded controller

$$u_k^\diamond(\hat{x}) = \begin{cases} b_k(\hat{x}), & t \in [t_{kj}, t_{kj}^f) \\ \bar{b}_k(\hat{x}), & t \in [t_{kj}^f, t_{kj} + \Delta t_{kj}), \end{cases} \quad (22)$$

where  $\bar{b}_k(\hat{x}) \triangleq -K_k(L_{f_k}^\diamond V_k(\hat{x}), \hat{x})(L_{G_k} V_k)^T(\hat{x})$ , makes the origin of the  $k$ th mode asymptotically stable.

**Proof:** (sketch) Lemma 2 ensures the system is stable for  $t \in [t_{kj}, t_{kj}^f)$ . At  $t = t_{kj}^f$ , the faults occur, from the time-derivative of  $V_k$  along the closed-loop trajectories, we obtain that for any  $x(t_{kj}^f) \in \bar{\Omega}_{k,\hat{f}}$ ,  $x$

is input-to-state stable w.r.t.  $e$  for  $t \in [t_{kj}^f, t_{kj} + \Delta t_{kj})$ , the result follows.  $\square$

**Theorem 3:** Consider switched system (10) under a family of bounded controllers  $\{u_k^\diamond(\hat{x}), k \in M\}$ , the initial states  $x(t_0) \in \Omega_k$ , and  $\hat{x}(t_0)$  are such that  $\mu(\lambda_k^*) \|e(t_0^+)\| \leq e_{u,k}$ . If  $\hat{x}(t_{kj}^f) \in \Psi_{k,\hat{f}}$  and  $\|e(t_{kj}^f)\| \leq e_{u,k}^{\hat{f}} \quad \forall k \in M$ , and if at any time instant  $T \geq t_{kj}^f$

$$\|e(T)\| \leq \bar{e}_{u,k+1}, \quad (23)$$

$$\hat{x}(T) \in \Psi_{k+1}, \quad (24)$$

$$V_{k+1}(\hat{x}(T)) + 2M_{k+1} < V_{k+1}(\hat{x}(t_{(k+1)(j-1)})), \quad (25)$$

then, choosing  $\Delta t_{kj} \geq T - t_{kj}$  and setting  $\sigma(t) = k + 1$  at  $t = t_{kj} + \Delta t_{kj}$ , guarantees that the origin of the overall CSS is asymptotically stable.  $\square$

## 4. CPU PROCESSING CONTROL SYSTEM

A simplified CPU processing control system illustrates our approach. The system have two modes:

**Mode 1:** The amount of CPU tasks is large while CPU temperature is not too high.

**Mode 2:** The amount of CPU tasks is not large and more energy is used for decreasing the temperature.

Three states are respectively the amount of CPU tasks  $\pi$ , the temperature  $\rho$ , and angular velocity of a cooling fan  $\omega$ .  $c \in \mathbb{R}$  and  $v \in \mathbb{R}$  are the clock frequency and the voltage input of a cooling fan. The system model is omitted due to the page limit. In Mode 1,  $|c| \leq 5$ ,  $|v| \leq 10$ ,  $|f_1^a| \leq 2.5$ . In Mode 2,  $|c| \leq 2$ ,  $|v| \leq 5$ ,  $|f_2^a| \leq 1$ . We only illustrate the method in Section 3.2. Choosing  $x(0) = x(t_{11}) = [8 \ 9.5 \ 9]^T$ ,

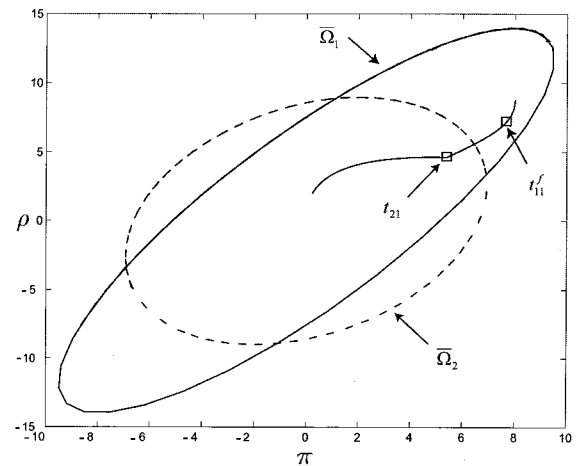


Fig. 1. State trajectories.

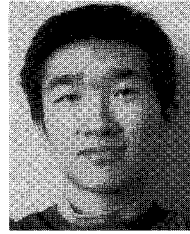
which is in  $\bar{\Omega}_1$ . Only  $f_1^a$  is considered:  $f_1^a = 0$ , for  $0s \leq t < t_{11}^f$ , and  $f_1^a = 2 + 0.2\sin(5t)$ , for  $t_{11}^f \leq t \leq 0.7s$ , with  $E_1 = [2 \ -0.2 \ 0]^T$ . The parameters are omitted. We switch the system to Mode 2 at  $t = t_{21} = 0.7s$ , Fig. 1 shows that the origin of CSS is asymptotical stable.

## 5. CONCLUSIONS

In this work, the FTC problem for CSS with input constraints is investigated. The proposed method will be also extended to more general nonlinear CSS with application to real systems in the future.

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